# Introduction to the Mars Subsurface Ice Model (MSIM) Program Collection

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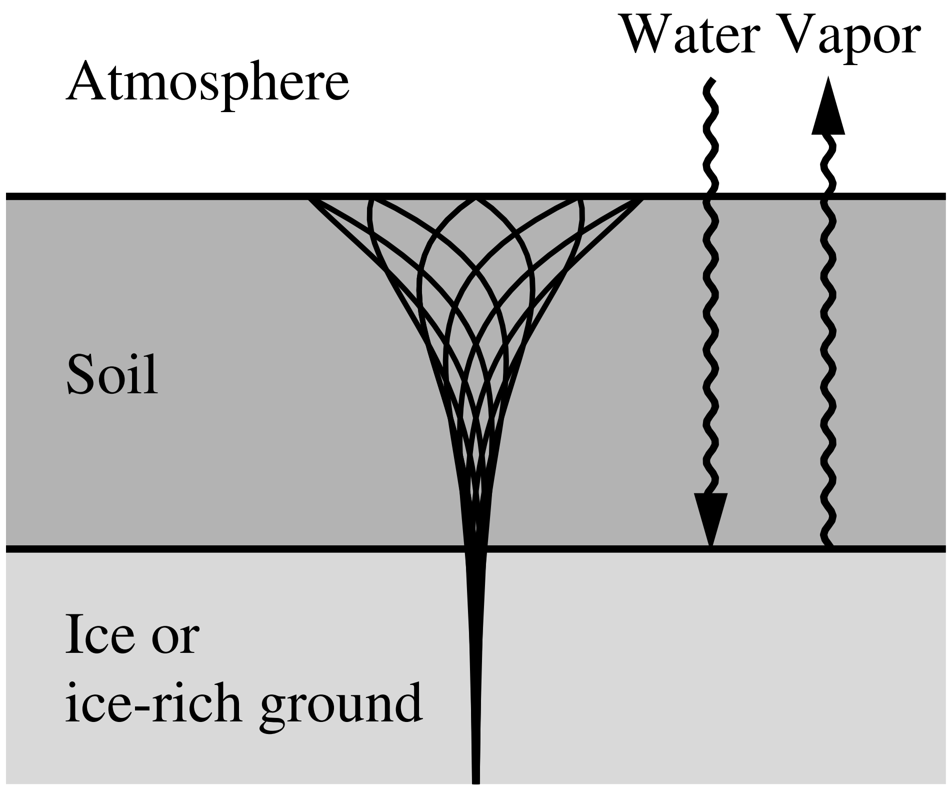
https://github.com/nschorgh/MSIM/

# Ground Ice in Diffusive Contact with Atmospheric Water Vapor




Vapor diffusion in the presence of temperature variations

# Mars Subsurface Ice Model (MSIM)

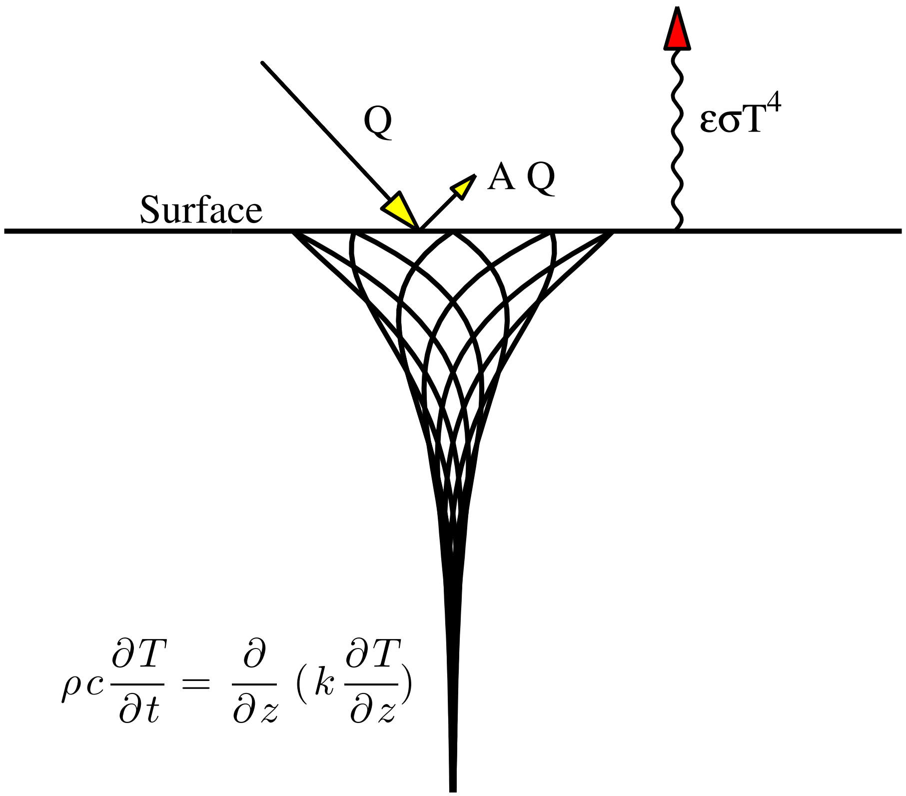
https://github.com/nschorgh/MSIM/

| Component | | Typical time step |
|---|---|---|
| Thermal model | semi-implicit + nonlinear b.c. | 15 mins |
| Microphysical model | diffusion, adsorption, sublimation | seconds |
| Equilibrium ice table | vapor equilibrium | – |
| Non-equilibrium ice | net vapor flux & ice content | 100 yr |

https://github.com/nschorgh/Planetary-Code-Collection/

| | | |
|---|---|---|
| 3D surface energy balance | slopes & shadows | Megapixels |

# Subsurface Heat Storage (1D)




$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right)$$

Heat equation with non-linear boundary condition

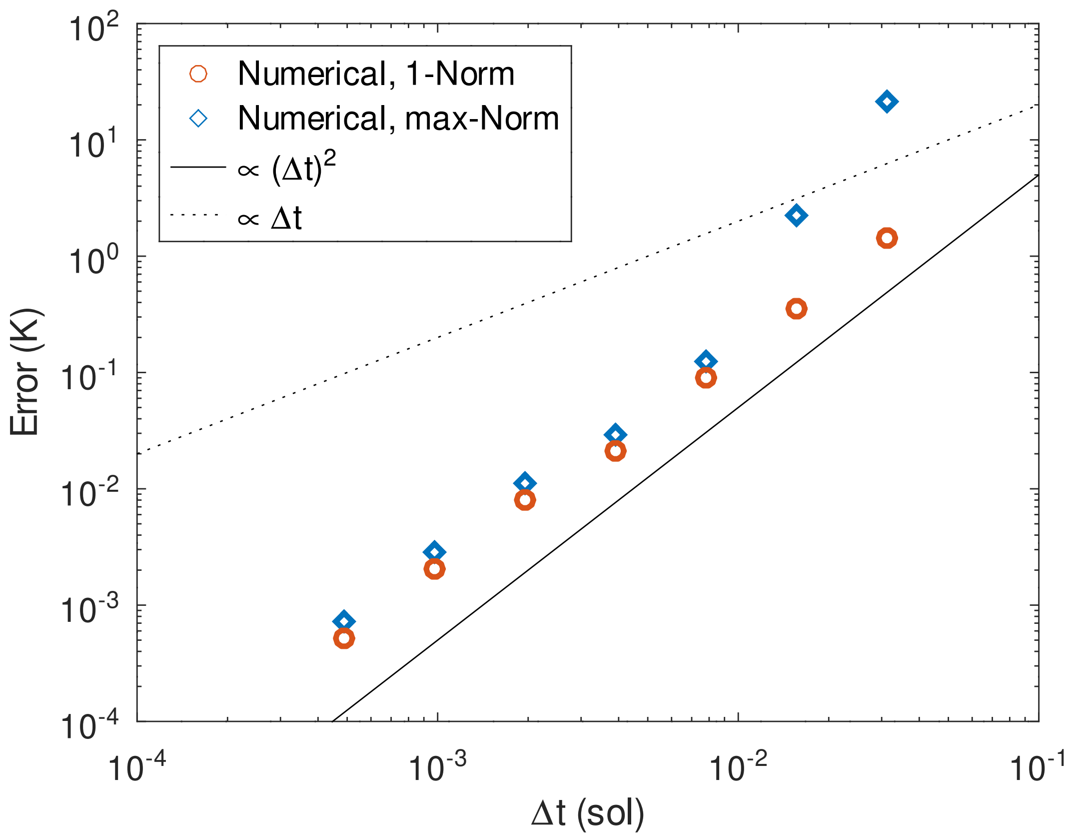
1) Flux-conservative discretization (heat conservation) on irregular grid

2) Time-marching scheme that is *not* subject to $\Delta t < \Delta z^2/(2\kappa)$.
Semi-implicit solver,
$\Delta t \lesssim$ (Solar Day)/100

# Numerical Solver for Heat Equation

Semi-implicit finite-difference method (Crank-Nicolson)

2nd order — unconditionally stable (for linear b.c.)




Convergence with time step $\Delta t$ for the Crank-Nicolson method with nonlinear boundary condition.

(Many common Mars thermal models use explicit time steps, which lack both of these advantages.)

# Surface Energy Balance

Surface energy balance:

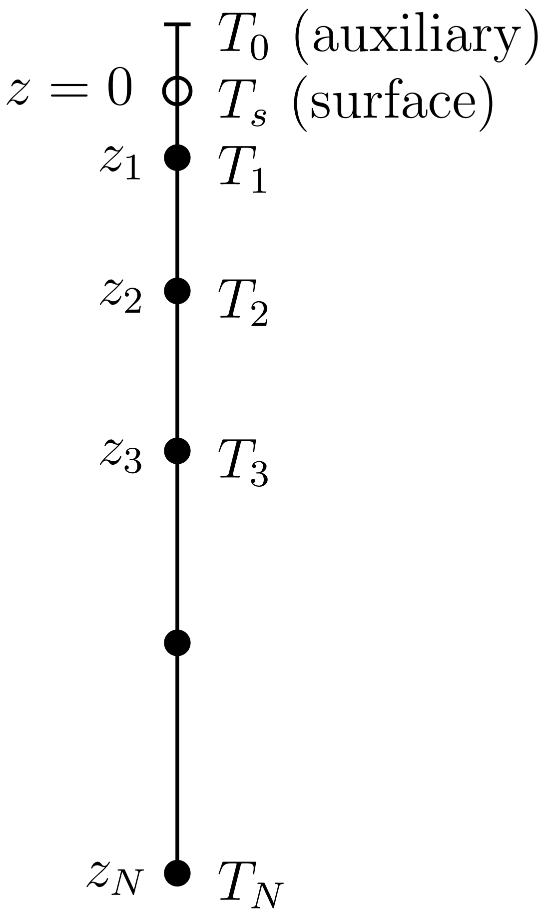
$$k\frac{\partial T}{\partial z} = \epsilon\sigma T^4 - Q(t)$$

nonlinear boundary condition, because dependence on $T$ itself (superposition fails)

- In MSIM, Stefan-Boltzmann term is linearized, so it can be incorporated into implicit method *(Schorghofer & Khatiwala 2004)*

- Nonlineary implies fully-implicit and semi-implicit schemes are no longer unconditionally stable *(Williams & Curry 1977)*
  For Mars and flat ground, stability is not an issue;
  For airless bodies and slopes, this can be a common issue at sunrise

# Grid Spacing




- Flux is defined in between grid points, so surface boundary condition should be imposed there too.
  $\Rightarrow (z_1 - 0) = \frac{1}{2}(z_2 - z_1)$
- Otherwise grid-spacing is arbitrary

# Thermal Model Performance

Full surface energy balance:

$$\underbrace{(1-A)Q_0 \sin\theta}_{\text{direct insolation}} + Q_{\text{sky}} + \underbrace{k\frac{\partial T}{\partial z}}_{\substack{\text{subsurf.}\\ \text{heat}}} = \underbrace{\epsilon\sigma T^4}_{\substack{\text{Stefan-}\\ \text{Boltzmann}}} - \underbrace{L\frac{dm_{CO_2}}{dt}}_{\substack{\text{latent heat of}\\ \text{seasonal CO}_2}}$$

Benchmark `(3.6 GHz Intel Xeon E5-1650, gfortran 11.4.0)`:
10 Mars years (6686 sols), 100 steps/sol (15 mins), 80 grid points: **only 1.1 seconds** (with 2nd order accuracy)

For example, 1 million thermal model runs over 10 Mars years on a 10-core (20 threads) CPU take 15 hours, hence feasible on a simple workstation.

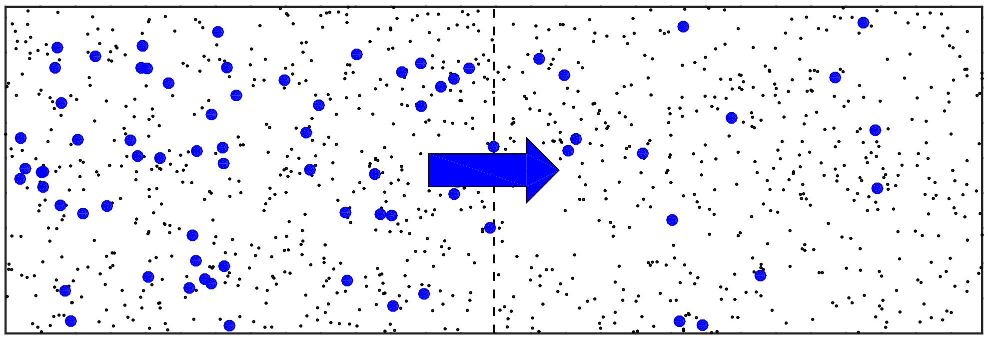
# Vapor Transport in Non-Isothermal Sublimation Environments

Phases of $H_2O$ incorporated: *vapor, adsorbed $H_2O$, and ice*

Vapor flux in non-isothermal environment *(Landau & Lifshitz )*:

$$\vec{J} = -D\rho_{\text{atm}}\,\vec{\nabla}\left(\frac{\rho_v}{\rho_{\text{atm}}}\right) + (\text{exotic terms}) + (\text{advection}) \approx -D\,\vec{\nabla}\rho_v$$

$\vec{J}$...vapor flux; $D$...diffusion coefficient; $\rho_v$...$H_2O$ vapor density



Vapor diffuses along gradient of **absolute humidity** (kg/m$^3$), not necessarily along gradients of relative humidity.
E.g., diffusion from warm unsaturated to cold saturated region

# Microphysical Model Equations

Diffusion of water vapor in porous medium with phase transitions (adsorption and sublimation) on 1-dimensional irregular grid

*Governing Equations*

**Conservation of mass:** $\frac{\partial}{\partial t}(\bar{\rho}_v + \bar{\rho}_f + \bar{\rho}_a) + \frac{\partial \bar{J}}{\partial z} = 0$

**Vapor transport:** $J = -D\frac{\partial \rho_v}{\partial z}$

**Adsorption:** $\bar{\rho}_a = \bar{\rho}_a(p, T)$ reversible and not kinetically-limited

**Sublimation:** $p \leq p_{sv}(T)$ excess vapor converted to ice

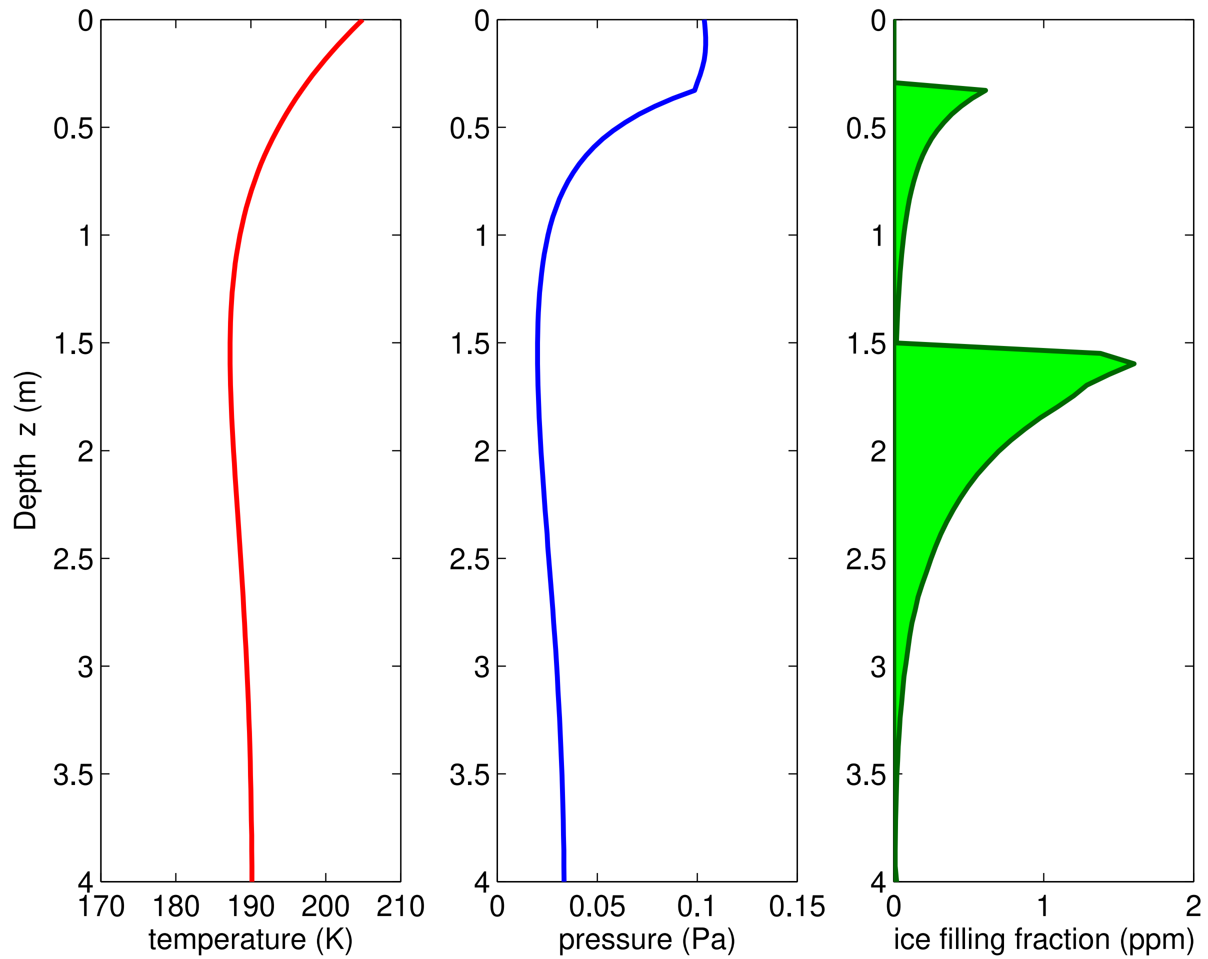
Indices: $v$ ... vapor, $f$ ... ice, $a$ ... adsorbed water

$\bar{\rho}$ ... mass per total volume, $\bar{J}$ ... vapor flux per total area

# Microphysical Model Numerics

- Flux-conservative discretization on irregular 1d grid (there are several options)
- Threshold for phase transition implies problem is non-linear $\Rightarrow$ forward-time discretization (explicit time step)

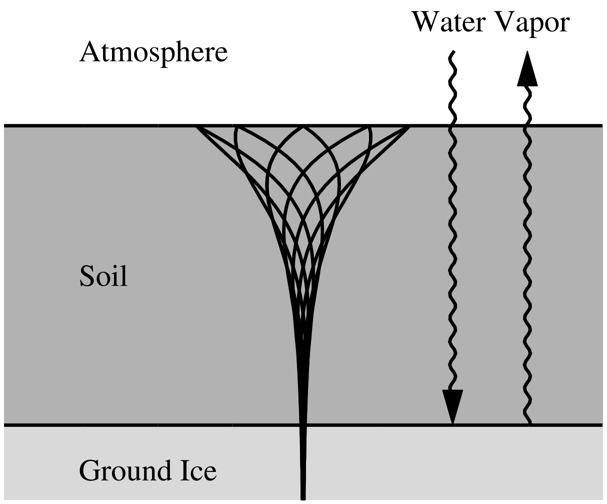
# Accumulation of Subsurface Ice




Snapshot of temperature cycles that pump atmospheric humidity into the ground; movie is on GitHub

# Time-Averaging of Transport Equations

Microphysics is complicated, but time averages can help us.




$$J = -D\frac{\partial \rho_v}{\partial z}$$

$J$ ... vapor flux; $D$ ... diffusion coefficient

$\rho_v$ ... $H_2O$ vapor density; $z$ ... vertical coordinate

Time average over period $P$ : $$\langle X \rangle = \frac{1}{P}\int_0^P X\,dt$$

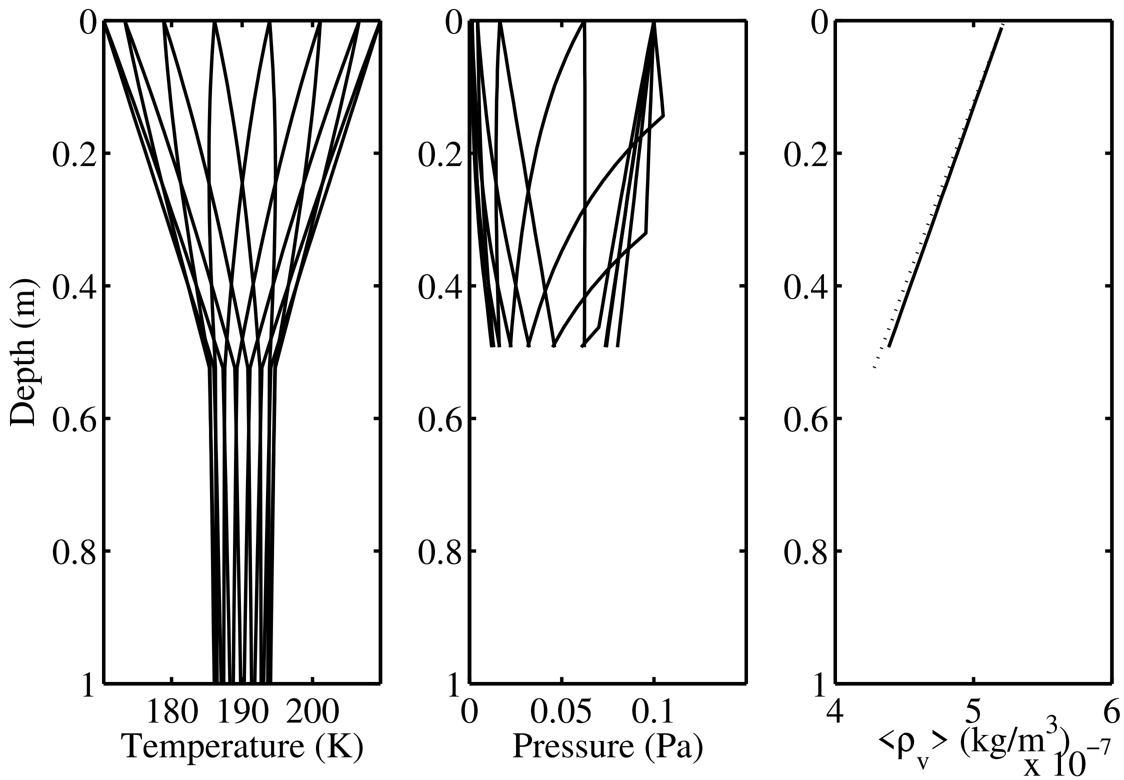
$$\langle J \rangle = -\left\langle D\frac{\partial \rho_v}{\partial z}\right\rangle \approx -\langle D \rangle \left\langle \frac{\partial \rho_v}{\partial z}\right\rangle = -\langle D \rangle \frac{\partial \langle \rho_v \rangle}{\partial z}$$

The **time-averaged flux** is given by the gradient of the **time-averaged vapor density**. $\Rightarrow$ Net vapor flux can be calculated without microphysical processes. $\Rightarrow$ "Fast" computational methods for subsurface ice evolution.

# Boundary-Value Formulation

If averaging period is one solar year, time-average of $\rho_v$ is determined by the boundary values at the surface and the ice table.

(mass conservation) $$\frac{\partial(\rho_v + \rho_{ads} + \rho_{ice})}{\partial t} + \frac{\partial J}{\partial z} = 0 \quad \Rightarrow \quad \left\langle \frac{\partial J}{\partial z} \right\rangle = 0$$

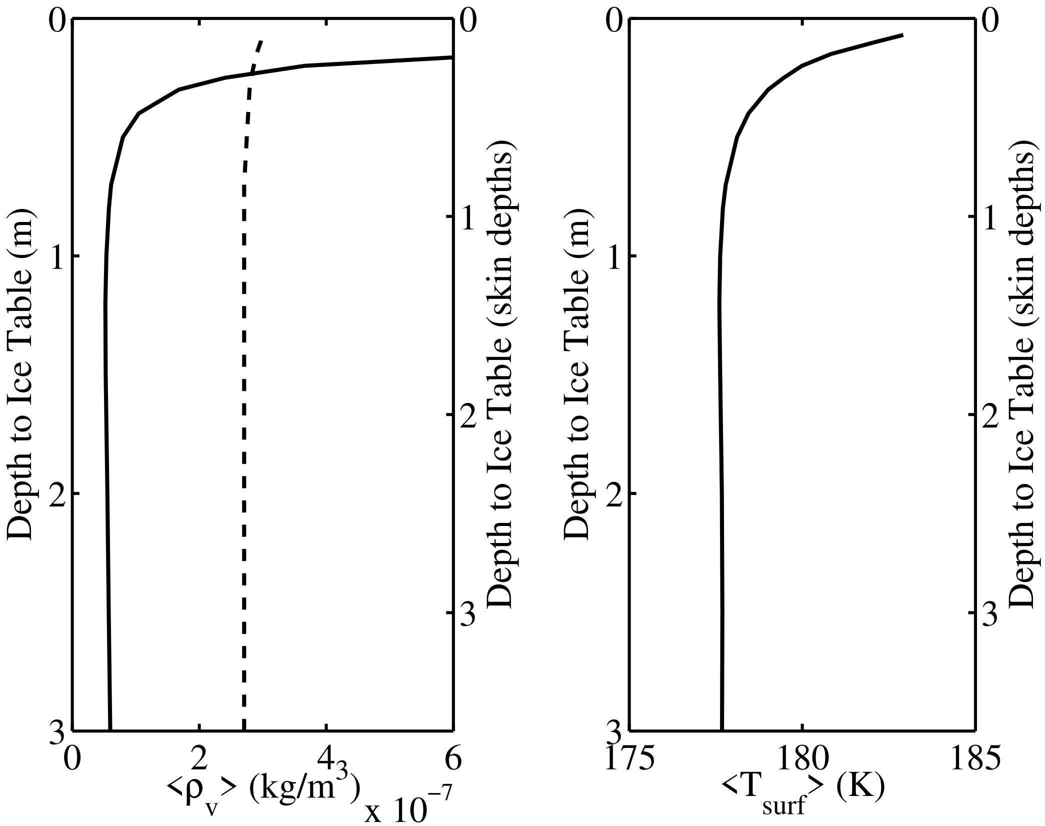



$\langle J \rangle$ is constant with depth; $\langle \rho_v \rangle$ is linear with depth; solid lines are microphysics simulation; dotted line connects boundary values

# Equilibrium Ice Table $\langle J \rangle = 0$

Depth of equilibrium ice table:

$$\boxed{\langle \rho_v(\text{surface}) \rangle = \langle \rho_v(\text{ice table}) \rangle}$$

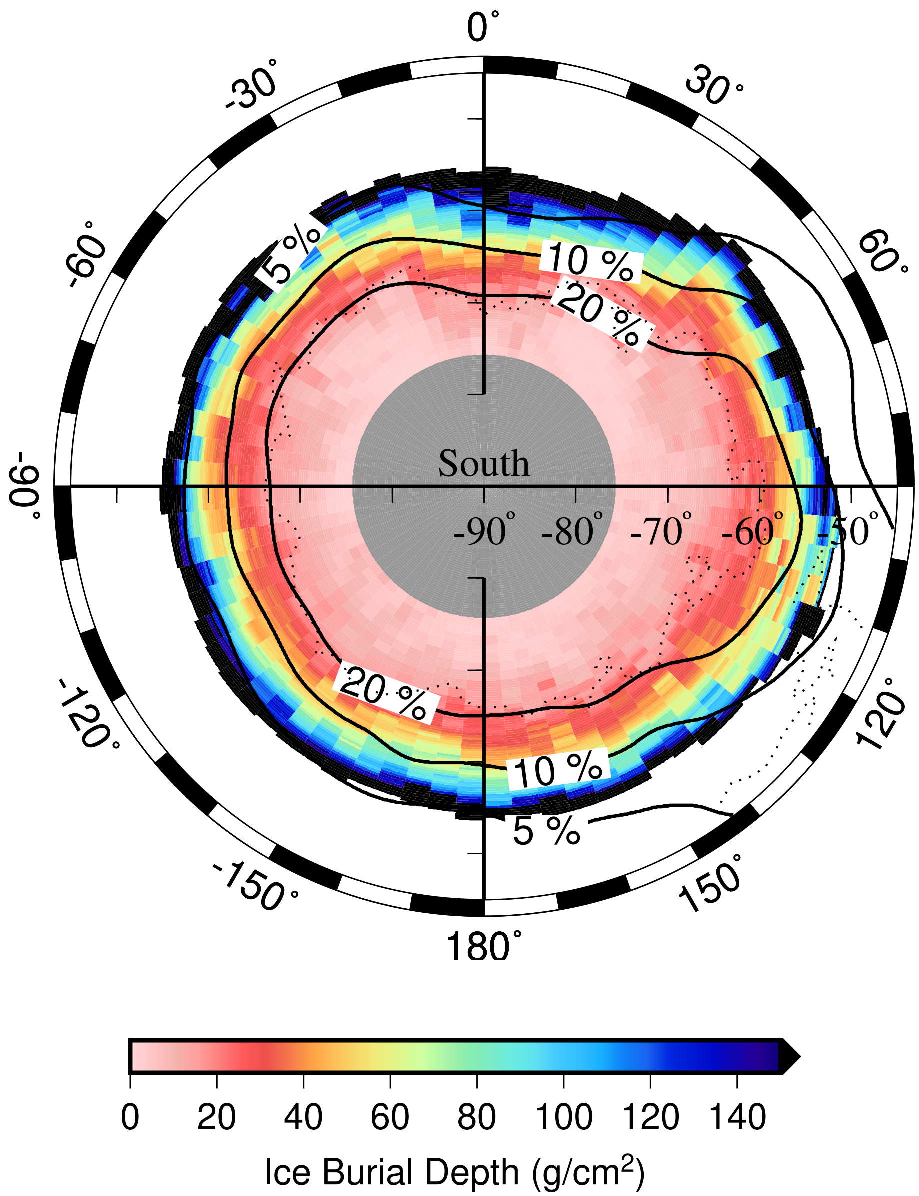



For each depth to ice table, thermal model was equilibrated and $\langle \rho_v \rangle$ was calculated for the last Mars year.

Depth to equilibrium ice table: **Nonlinear root-finding**

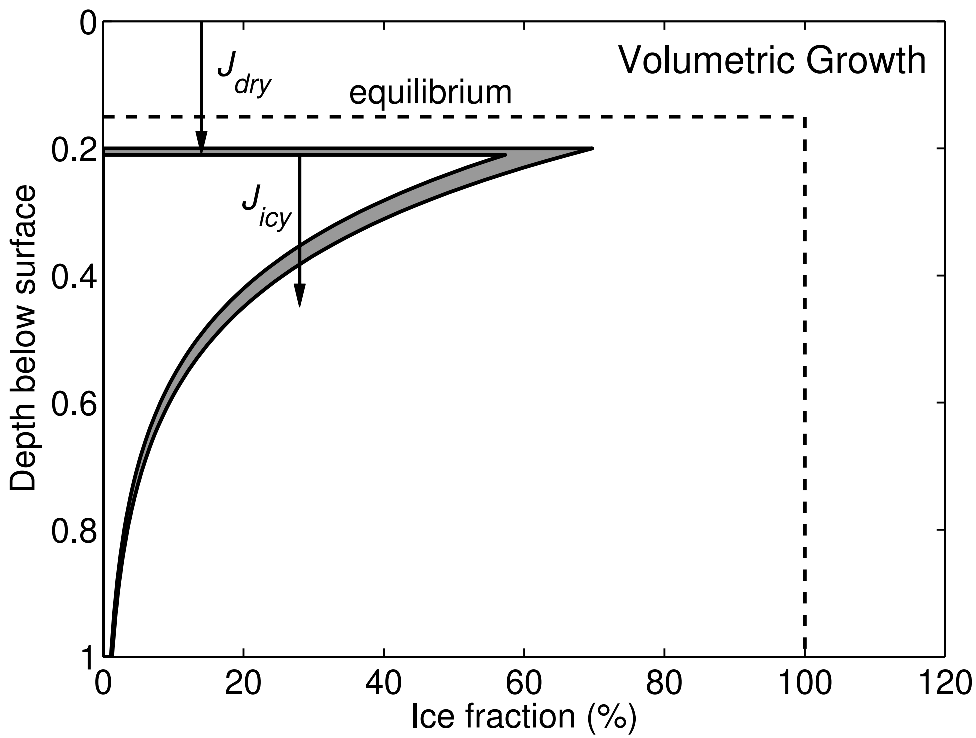
I usually use bisection method, and turn off geothermal heat to avoid missing a root.

# Example Result




Depth to equilibrium ice table (colors) in the south polar region of Mars compared to results from nuclear spectroscopy (solid contours). *Schorghofer & Aharonson (2005)*

# Beyond Equilibrium: Recharge of Subsurface Ice




- Subsurface ice can be sequestered from atmospheric vapor *(Mellon & Jakosky 1993)*
- Interface is below equilibrium depth
- Interface is sharp
- Interface is moving (i.e., between grid points)

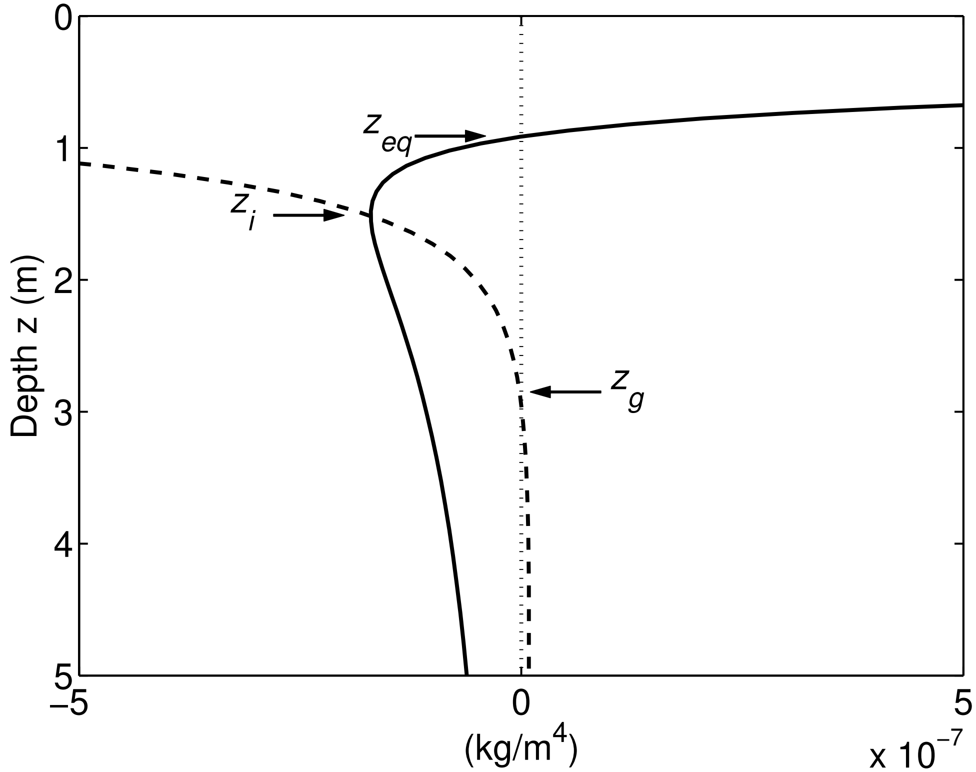
Gray area shows incremental ice growth. Ice fraction is relative to pore volume.

# Location of $z_{eq}$, $z_i$, and $z_g$

$z_{\text{eq}}$ ... equilibrium depth

$z_i$ ... interface depth

$z_g$ ... geothermal limit




solid line ... $\frac{\langle\rho_{sv}\rangle(z)-\langle\rho(0)\rangle}{z}$

dash line ... $\partial\left\langle\rho_{sv}\right\rangle/\partial z$

All three of these depths migrate with changing ice content.

for realistic Mars subsurface temperatures at 60°N.

# Numerics of Interface Retreat

Depth of uppermost ice $z_p$, might or might not cross grid-point during advance

Mass conservation:

$$\frac{dz_p}{dt} f(z_p) \Phi_0 \rho_{\text{ice}} = - \langle J \rangle = \eta D \frac{\Delta \langle \rho_v \rangle}{z_p}$$

$z_p$ ... shallowest depth with pore ice; $f$ ... fraction of pore filled with ice; $\Phi_0$ ... ice-free porosity; $0 \leq \eta \leq 1$ ... constriction of diffusion

Integrate over one time step $\Delta t_B$:

$$\int_{z_p(0)}^{z_p(\Delta t_B)} f(z) z dz = \int_0^{\Delta t_B} \frac{1}{\Phi_0} \frac{\eta D}{\rho_{\text{ice}}} \Delta \langle \rho_v \rangle dt$$

$$\sum_{j=0}^{z_p} f(z_j) z_j \Delta z_j = \frac{1}{\Phi_0} \frac{\eta D}{\rho_{\text{ice}}} \Delta \langle \rho_v \rangle \Delta t_B$$

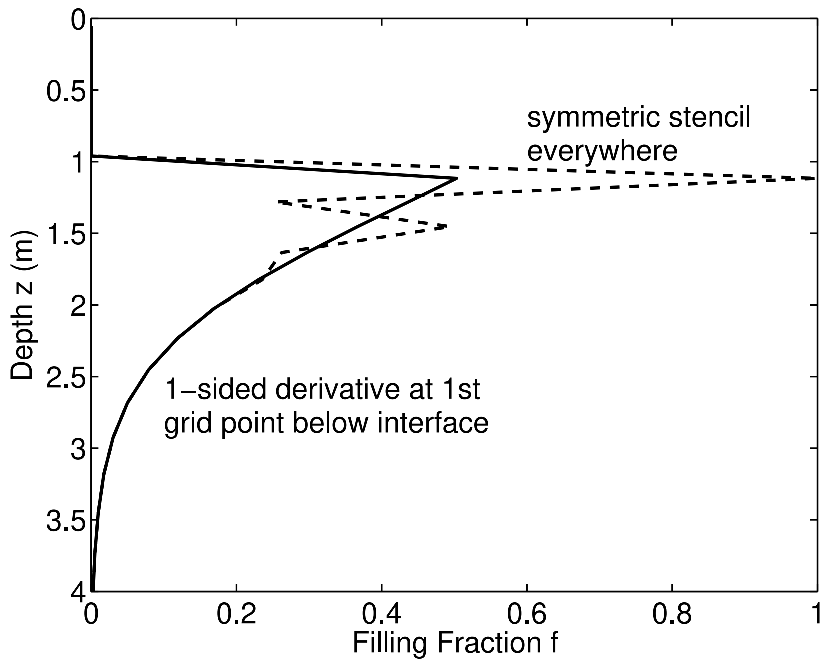
To find $z_p(\Delta t_B)$, sum until this equality is exceeded.

# Numerics with Partial Pore Filling

We have to deal with $\sigma(z,t)$, not just depth-to-ice-table $z_T(t)$.
$\sigma$ ... ice density; Mass change for interstitial ice:

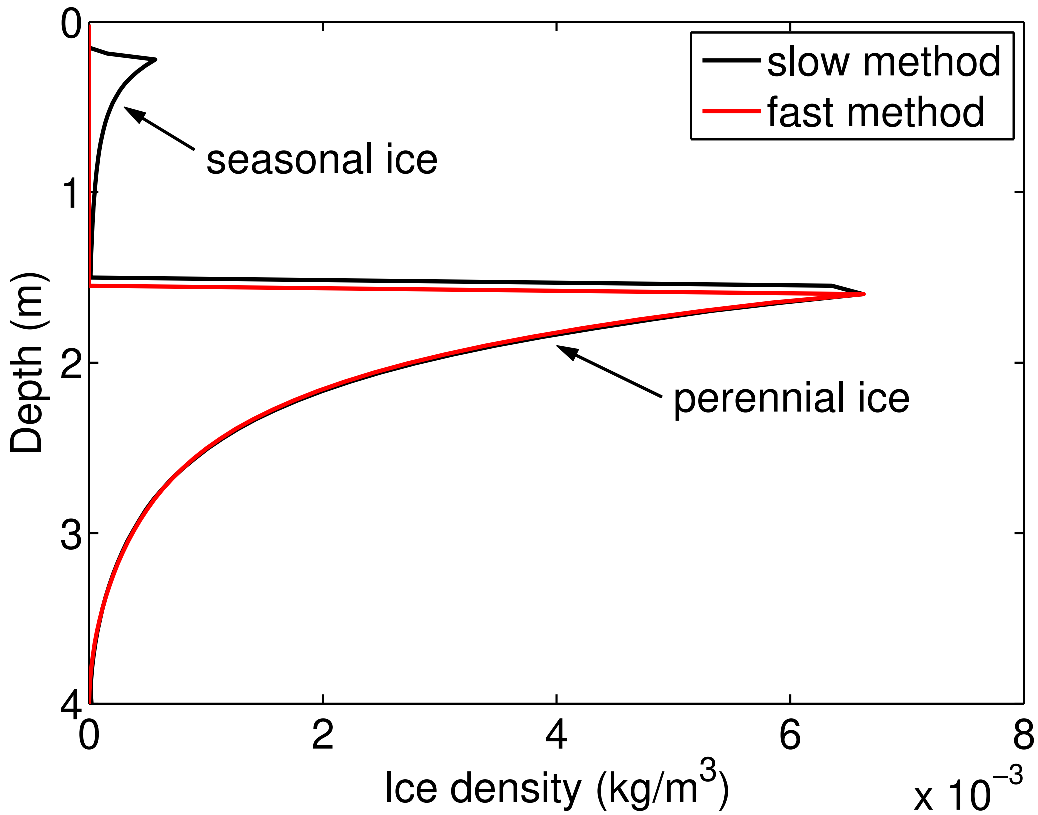
$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial z}\left(\eta D_{\text{icefree}} \frac{\partial \langle \rho_{sv} \rangle}{\partial z}\right)$$

$\eta$ ... constriction of diffusion (discontinuous at interface)
This involves 2nd spatial derivative of $\rho_{sv}$ (difficult near interface
$\Rightarrow$ one-sided finite-differences from below)



# Comparison between Fast and Slow Method

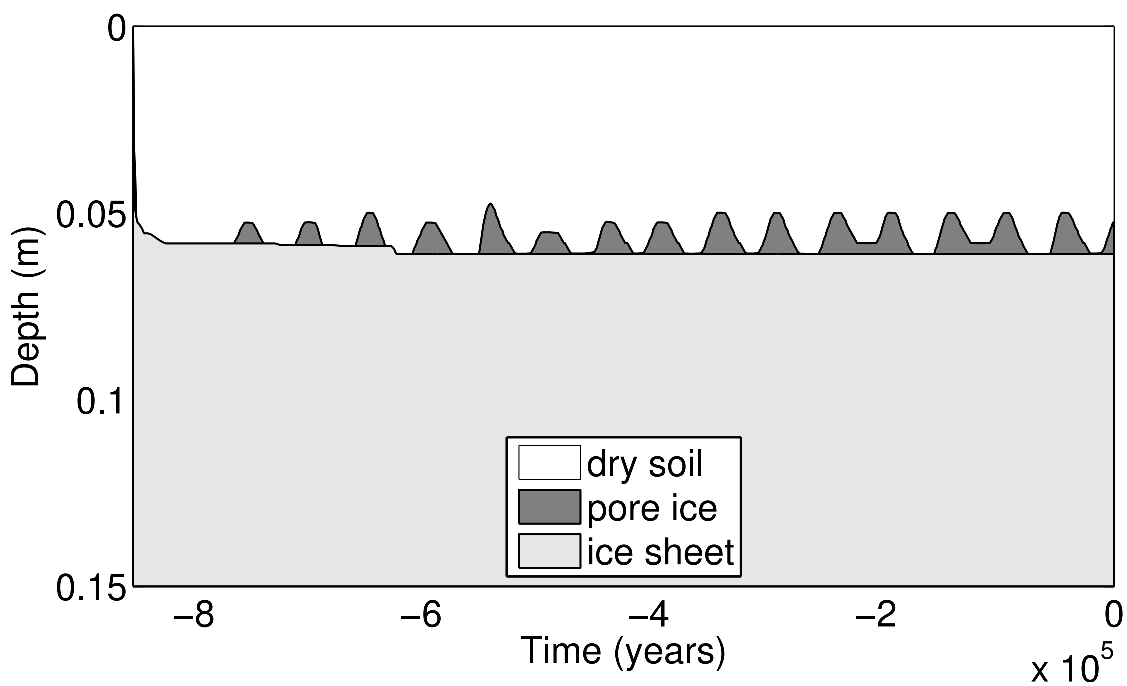
*slow* ... microphysical model
*fast* ... time-averaged transport equations




Speedup is $\sim 10^4$-fold! About same speed as a thermal model.
Fast method does not resolve diurnal and seasonal cycle.

# Example: Phoenix Landing Site

massive ice sheet formed 863 ka ago (local obliquity maximum)



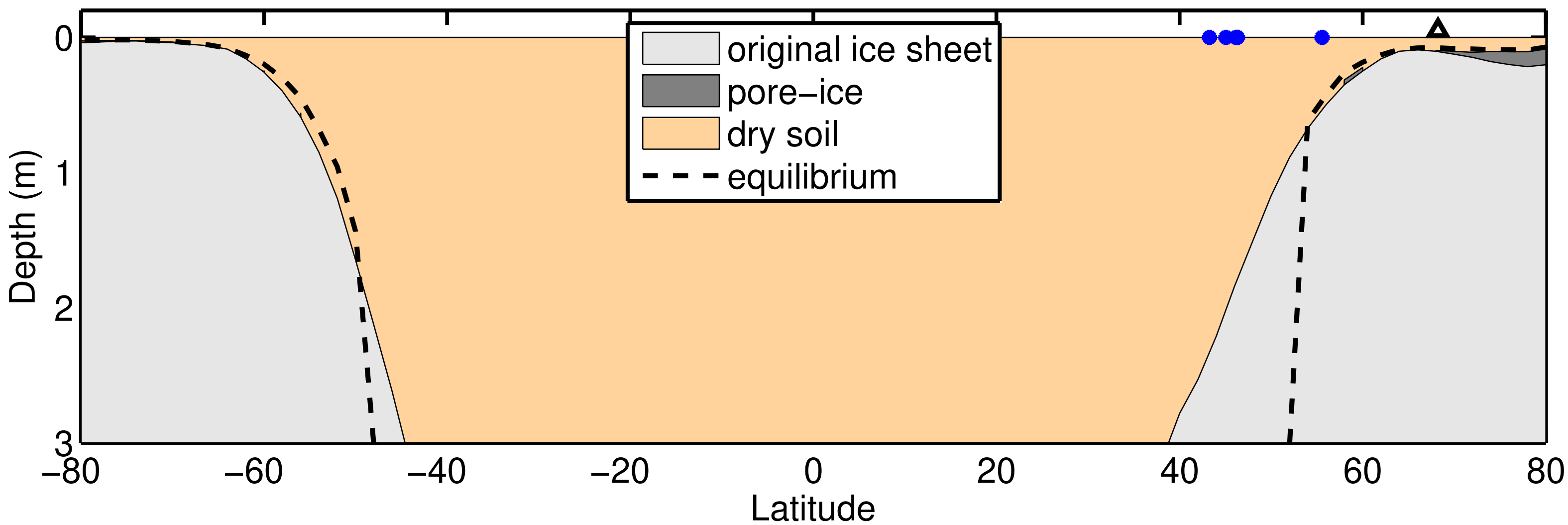

Filling fraction is always 100% (vertical growth mode).

Present-day thickness of pore ice layer is only 9 mm. Phoenix observed mostly pore-ice, while neutron spectroscopy reveals massive ice.

*(Schorghofer & Forget 2012)*

# Example: Present-day Ice Distribution

Scenario: Massive ice sheet formed 1 Ma ago




consistent with: MONS geographic boundary, MONS burial depths, MONS ice content, pore ice at Phoenix Landing Site, mid-latitude icy impact sites *(Schorghofer & Forget 2012)*

# Summary

1. MSIM includes a powerful subsurface thermal model (semi-implicit, $2^{nd}$ order accurate, stabilized)

2. MSIM also includes microphysical model for vapor transport and adsorption, but it is computationally slow.

3. Method for fast subsurface ice modeling: **Solve time-averaged transport equations**

4. Non-equilibrium model with partial pore filling is fast but complex (whereas retreat or growth with full pores is easy)

5. MSIM is publicly available (but it includes a few third-party routines) `https://github.com/nschorgh/MSIM/`

# References

**MSIM is publicly available at** https://github.com/nschorgh/MSIM/ (but not open-source yet, because of third-party routines)

**Methods:**

1. https://github.com/nschorgh/MSIM/blob/main/MSIM_Methods.pdf
2. Thermal model: *Schörghofer & Khatiwala, PSJ (2024)*
   (most of it was written 2001–2003)
3. Fast non-equilibrium model: *Schörghofer, Icarus (2010)*

**Science Applications:**

1. Vapor diffusion: *Schorghofer & Aharonson, JGR (2005)*
2. Equilibrium ice table: *Schorghofer & Aharonson, JGR (2005)*, *Aharonson & Schorghofer, JGR (2006)*
3. Non-equilibrium ice evolution: Schorghofer, Nature (2007), Schorghofer & Forget, Icarus (2012), Vos et al., JGR (2022, 2023)