Thin Lenses

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1 Introduction

This experiment involves thin lenses and the understanding of their properties. Thin lenses can be found in many places and most importantly enable those with poor vision to see the world around them. The basic equation to consider in this exercise is known as the thin lens equation:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{1}$$

Where f is the focal length of the lens in question, o is the distance between the lens and a real object, and i is the distance between the lens and a real image. This equation works for both converging and diverging lenses and will be used extensively throughout this lab.

A practice repeated in this lab experiment is the process of ray-tracing. Ray-tracing is a method of determining the path that light travels from object to lens to image. Tracing the path of light is especially useful when lens systems become more complex. The most challenging setups in this lab are those involving virtual objects. Virtual objects can be found only through mathematical methods, but can produce real images as a later procedure will demonstrate.

This lab requires the use of a light source, two different converging lenses and a diverging lens. In addition to the lenses, a screen, mirror, black and white optics pins, and a optics track or table will be used to facilitate these exercises. A sphereometer is used in a later exercise and a precise metric ruler is helpful when gathering high resolution data points like the distance between the legs of the sphereometer.

2 Theory

Thin lenses experiments are more of an exercise of thought than the pencil and paper, as the calculations are quite trivial. The challenge lies in the concepts of real and virtual and their associated sign conventions that create the real experiment. The same concepts and conventions apply to both converging and diverging lenses and will be used throughout this lab experiment.

Thin lenses work by bending light as it passes into a denser medium of glass. At the interface between air and glass, light slows as it enters the glass lens. By carefully sculpting a rounded convex or concave surface, a lens crafter can force light to focus at a certain point known as the focal length of the lens. The focal length is dependent upon the curvature of the surface. As the surface becomes more convex, parallel light rays converge at a point after entering and exiting the lens, this surface is called a converging lens. When a surface is concave, light diverges after exiting the lens creating a diverging lens.

A mirror reflects light at an angle equal to the angle of incidence, because of this, mirrors exhibit many intriguing properties of their own. When working in conjunction with thin lenses, a mirror can change the expected behavior of a lens system. Mirrors tend to switch sign conventions for images.

If a lens produces a real image, it will always be found on the opposite side of the real object. Real objects and real images cannot exist on the same side of a lens if the correct sign conventions are used. This is important to remember in these exercises. A real object placed inside the focal length of a lens can create a virtual image on the same side of the lens as the object. This image is considered to be virtual because it cannot be shown on a screen, and mathematically it carries a negative value.

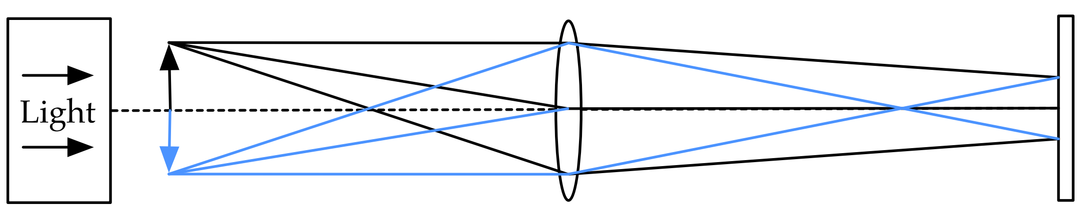
The same idea can be applied to objects. A virtual object is one that cannot be seen by the observer. It can produce a real image in the case of the double lens exercise seen in this lab. In this case, the virtual object is created by a real image of an auxiliary lens and mathematically appears to be on the same side of the main lens as the real image. Virtual images can be located roughly with parallax. Parallax is typically used in astronomy to estimate the distance to distant stars. It relies on a change in observation position, as the observer focuses on a central location between their eye and a static background.

The goal of data collection in this experiment is to capture enough data points to plot a graph of 1/image vs 1/object for both the converging and diverging lenses. Given the relationship between object distance and image distance implied by equation 1, we can expect the graph to have a negative slope and appear linear.

3 Apparatus and Procedures

I. Autocollimation

Autocollimation - Converging Lens

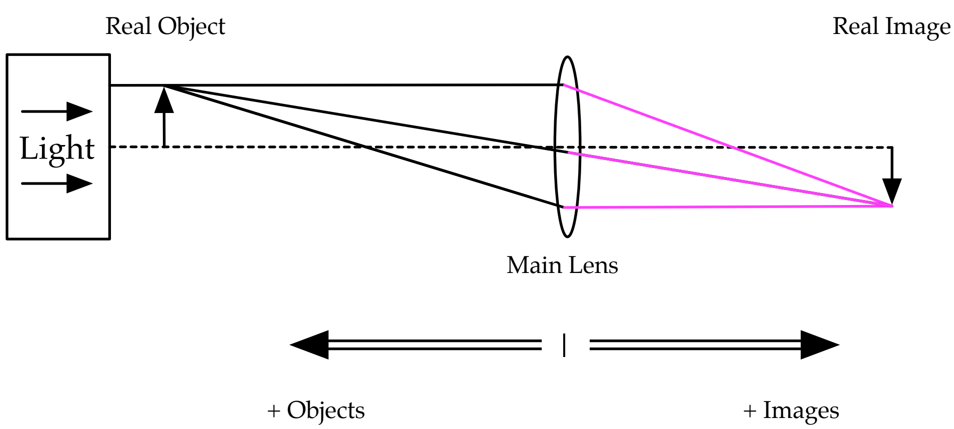


Autocollimation does not rely on two sets of numbers to determine a focal length. Instead, the observer is only required to record the distance between the object and the lens. The mirror reflects the light back through the lens and produces a real image at the same location as the real object for the lens. The distance between the lens and the real object determines the focal length of the lens.

II. Testing the Thin Lens Equation with a Converging Lens

A. Real Objects and Real Images

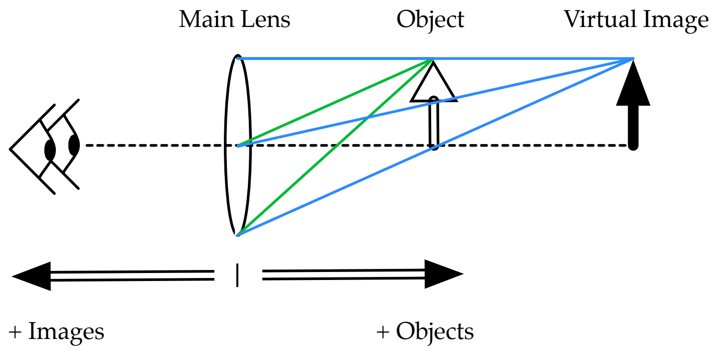
Real Object with Real Image

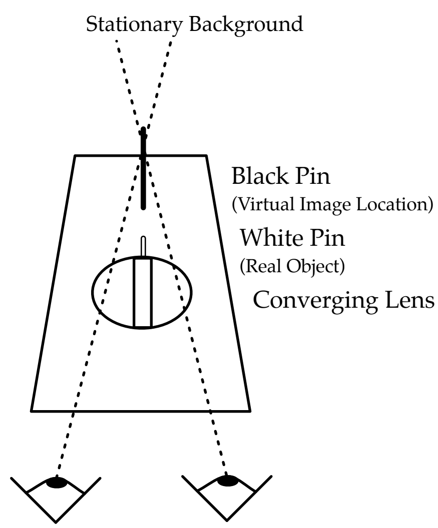


Once the focal length of the converging lens is known, we begin with a single converging lens system to begin our graphical testing of equation 1. Set the light source so that it passes through the real object and light rays then pass through the lens to produce a real image on the screen on the opposite side of the lens.

B. Real Objects and Virtual Images

Real Object with Virtual Image



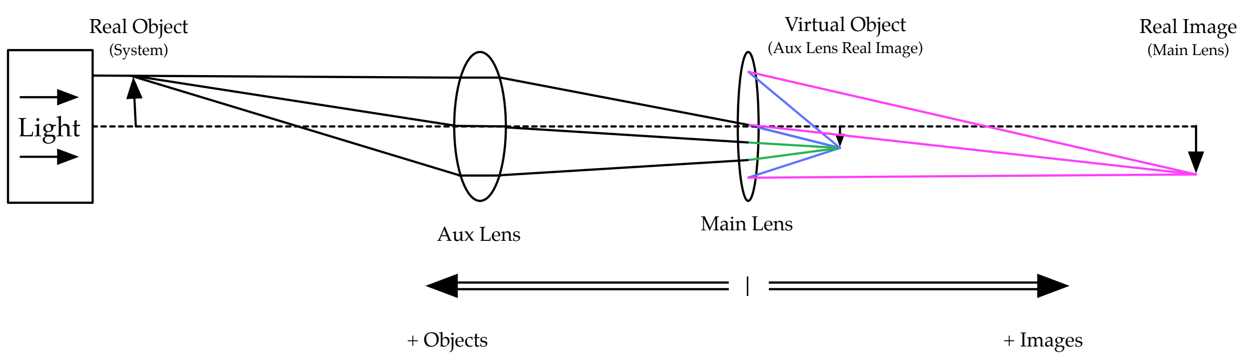


Virtual images appear when a real object is placed inside the focal length of the lens. Traditionally the sign convention used is positive (+) in the direction of the real object or lens, so a virtual image will have a negative (-) position value.

To determine the position of the virtual image with parallax, set up the converging lens so that an observer can look through it and see the white pin in the lens. Place the black pin so that the top of it can be seen over the top of the lens allowing the observer to see both the white pin image in the lens and the black pin above the lens. The observer should move side to side horizontally and watch for the relative distance between the white pin image and black pin tip. From the principles of parallax, the virtual image will be in approximately the same location as the black pin when the image stops moving relative to the black pin tip as the observer moves left to write changing their angle of observation in the system.

C. Virtual Objects and Real Images

Virtual Object with Real Image



To test for virtual object location with a real image, the apparatus should be set up with a light source shining through a real object, causing light to pass through an auxiliary lens followed by the main lens which will then produce a real image on a screen at the end of the track.

Here, we use an auxiliary lens with a greater curvature and therefore converging power than our main lens to help keep the virtual object confined to our track. We use a sign convention here with positive images to the right and positive objects to the left. Again, positive sign implies real object or image.

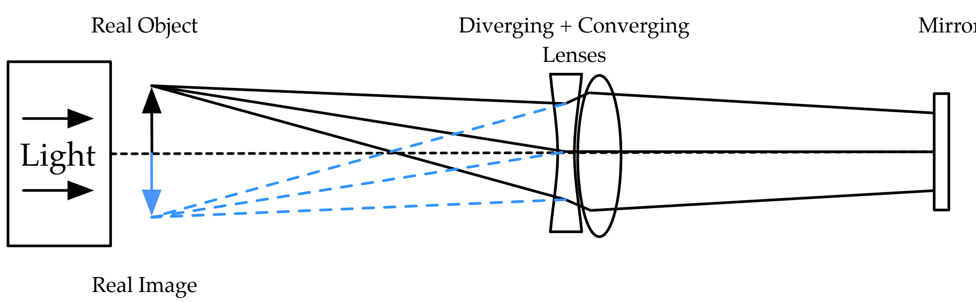
D. Graph 1/Obj vs 1/Image for ABC

After taking data with setups A, B, and C for the converging lens, plotting all three graphs on one large graph reveals a linear line with negative slope passing through quadrants I, II, and IV.

III. Determine the focal length of a thin diverging lens

To determine the focal length of the diverging lens place the diverging lens next to a converging lens and set up the apparatus so that light passes through the object, diverging lens, converging lens, then reflects off the mirror, reverses, and produces an image on top of the object. In our case the image is sightly below the object due to the alignment of our components.

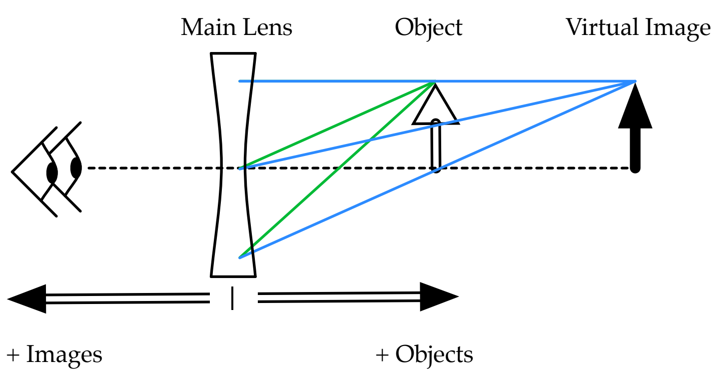
Focal Length of Diverging Lens

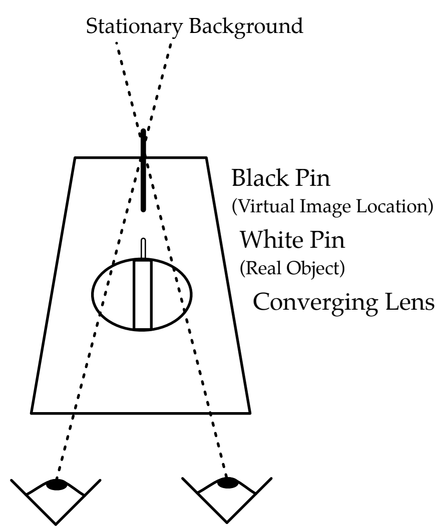


IV.Testing the Thin Lens Equation with a Diverging Lens.

A. Real Objects and Virtual Images using Parallax

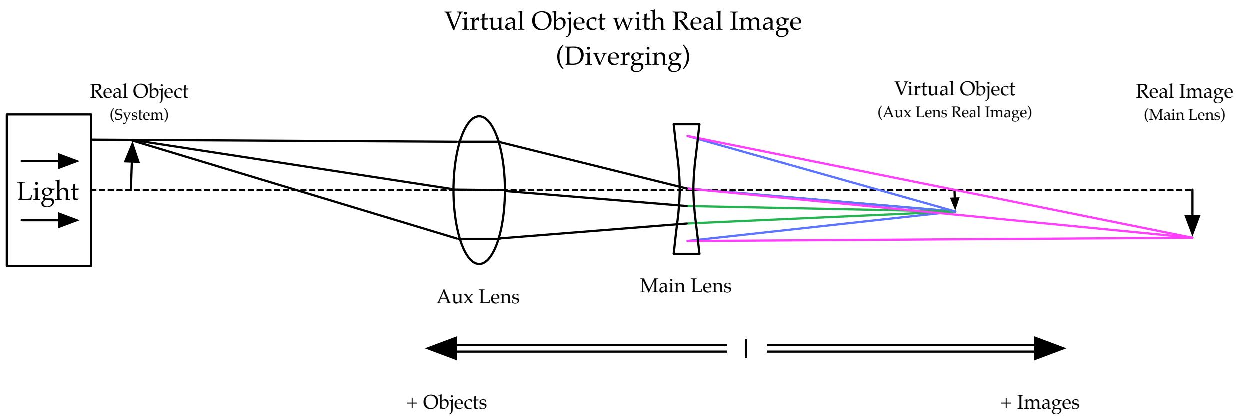
Real Object with Virtual Image





Using the parallax method the same way as with the converging lens, find the location of the white pin and black pin so that the virtual image of the white pin appears to be stationary relative to the tip of the black pin seen above the lens.

B. Virtual Objects and Real Images

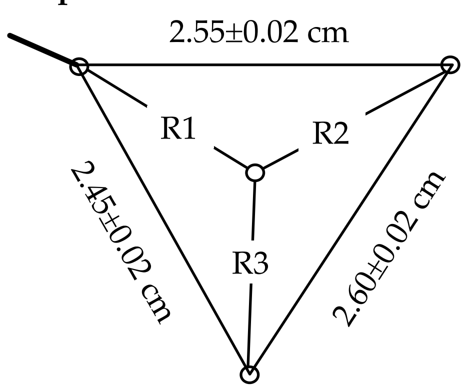


Test for virtual objects with real images by using an auxiliary converging lens in front of the main, diverging lens. Set up the light source so that it again travels through the object and through the two lenses and place the screen on the opposite side of the two lenses where a real image in formed by the system.

C. Graph 1.Obj vs 1/Image for Diverging Lens

After taking data with setups A and B with the diverging lens, plotting all three graphs on one large graph reveals a linear line with negative slope passing through quadrants II, III, and IV.

V. **Spherometer**



R1 = 1.42 ± 0.02mm
R2 = 1.45 ± 0.02mm
R3 = 1.49 ± 0.02mm
Zero Point: 26.650 ± 0.002 mm

To begin with the sphereometer, measure the distance between the legs and the distance between the legs and the traveling center measurement pin. Once the distances are known and recorded the device must be zeroed. To find the zero point, use a hard flat surface large enough to accommodate the sphereometer and carefully turn the wheel until the center pin rests on the flat surface.

The radius of curvature of a lens can be determined using the following equation:

$$r = \frac{s}{2} + \frac{d^2}{6s}$$

Where s is the displacement reading on the spherometer, and d is the distance between two of the legs of the sphereometer.

4 Results and Analysis

I. Autocollimation

Once our components are set up on our track, we proceed with determining the focal length of the converging lens using the method described in the procedure section under autocollimation. The focal length of the converging lens is 18.10 ± 0.03 cm.

50.05 ± 0.02 cm - 31.95 ± 0.02 cm = 18.10 ± 0.03 cm
(lens position) - (object/image position) = focal length

II. Testing the Thin Lens Equation with a Converging Lens

A. Real Objects and Real Images

With the focal length of the converging lens known, we proceed to graphically test equation 1 by setting up our apparatus to create real images from real objects. Our results are as follows.

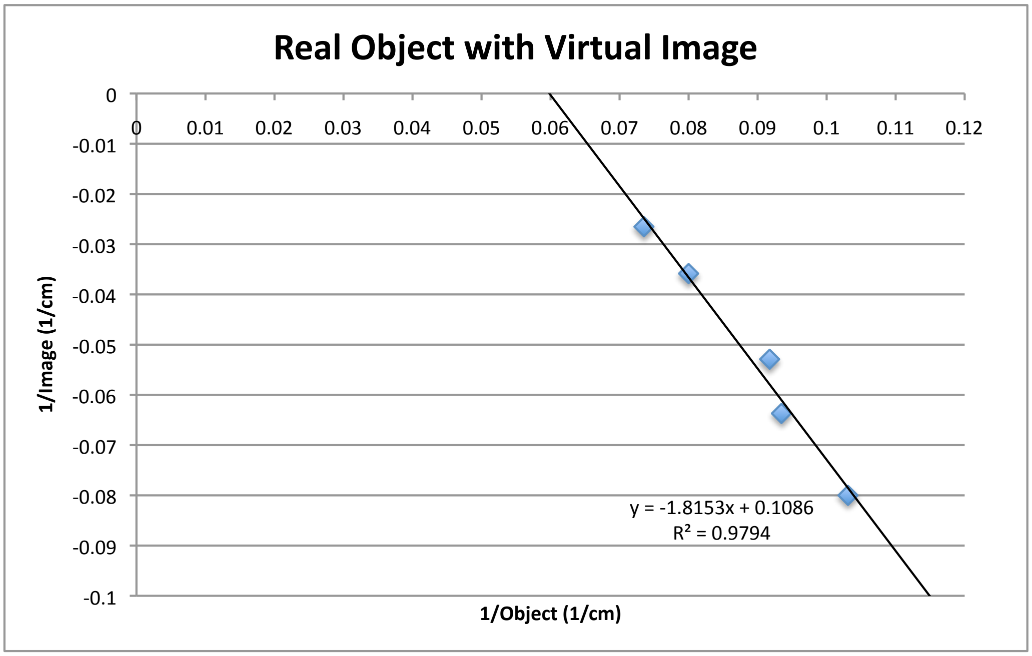
Real Object with Real Image Data Table				
Trial	**a object**	**a' image**	**1/object**	**1/image**
1	30.00±0.05	47.30±0.05	0.033333333	0.021141649
2	24.75±0.05	71.67±0.05	0.04040404	0.013952839
3	43.87±0.05	31.81±0.05	0.02279462	0.031436655
4	28.99±0.05	50.07±0.05	0.034494653	0.019972039
5	49.92±0.05	28.89±0.05	0.020032051	0.034614053
6	34.50±0.05	39.51±0.05	0.028985507	0.025310048
7	37.04±0.05	36.41±0.05	0.02699784	0.027464982

We find the real object vs real image data to be located in quadrant I in our cartesian coordinate graph. This agrees with our sign convention of positive sign implies real object or image. As we will see in our complete plot, the trend line is linear with negative slope and the y-intercept corresponds to 1/focal length of the lens.

B. Real Objects and Virtual Images

Next we look for virtual images created by our lens system from real objects. We expect to see data appear in quadrant IV once plotted.

Real Object - Virtual Image				
Trial	**W.P. Distance**	**Distance B.W.**	**1/object**	**1/image**
1	10.9	18.9	0.091743119	-0.052910053
2	12.5	27.9	0.08	-0.035842294
3	10.7	15.7	0.093457944	-0.063694268
4	9.7	12.5	0.103092784	-0.08
5	13.6	37.7	0.073529412	-0.026525199

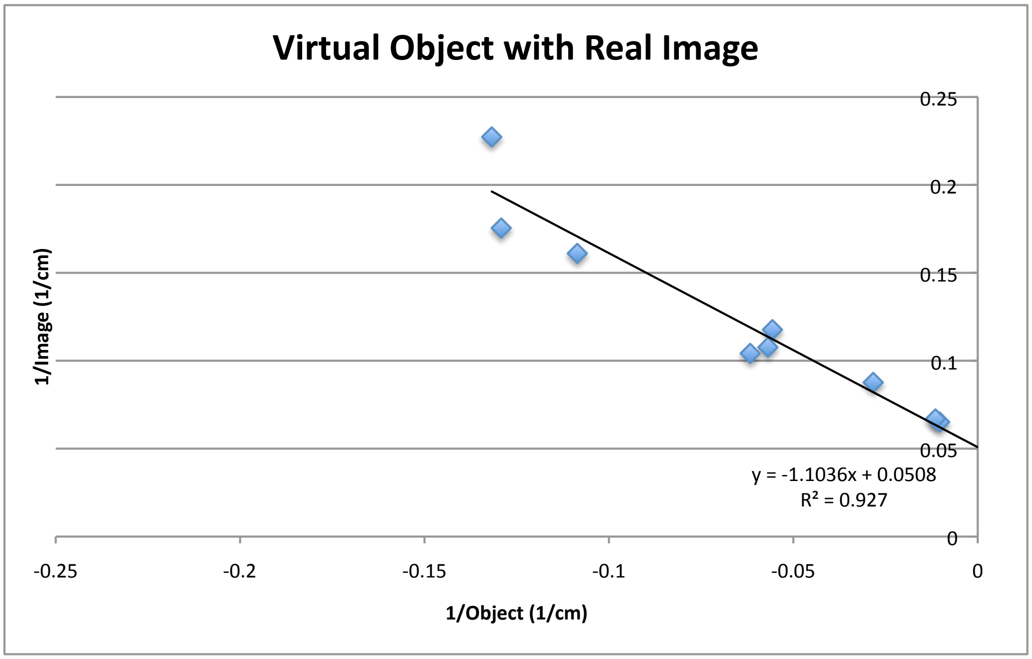


Again, our plot matches our sign convention. Since our image is virtual it has a negative sign causing our data to appear in quadrant IV. Again, we see a linear, negative slope with similar y-intercept.

C. Virtual Objects and Real Images

Based on our first two data sets, we expect to see a virtual object, real image data set to appear in quadrant II of the cartesian coordinate plot since virtual objects carry a negative sign.

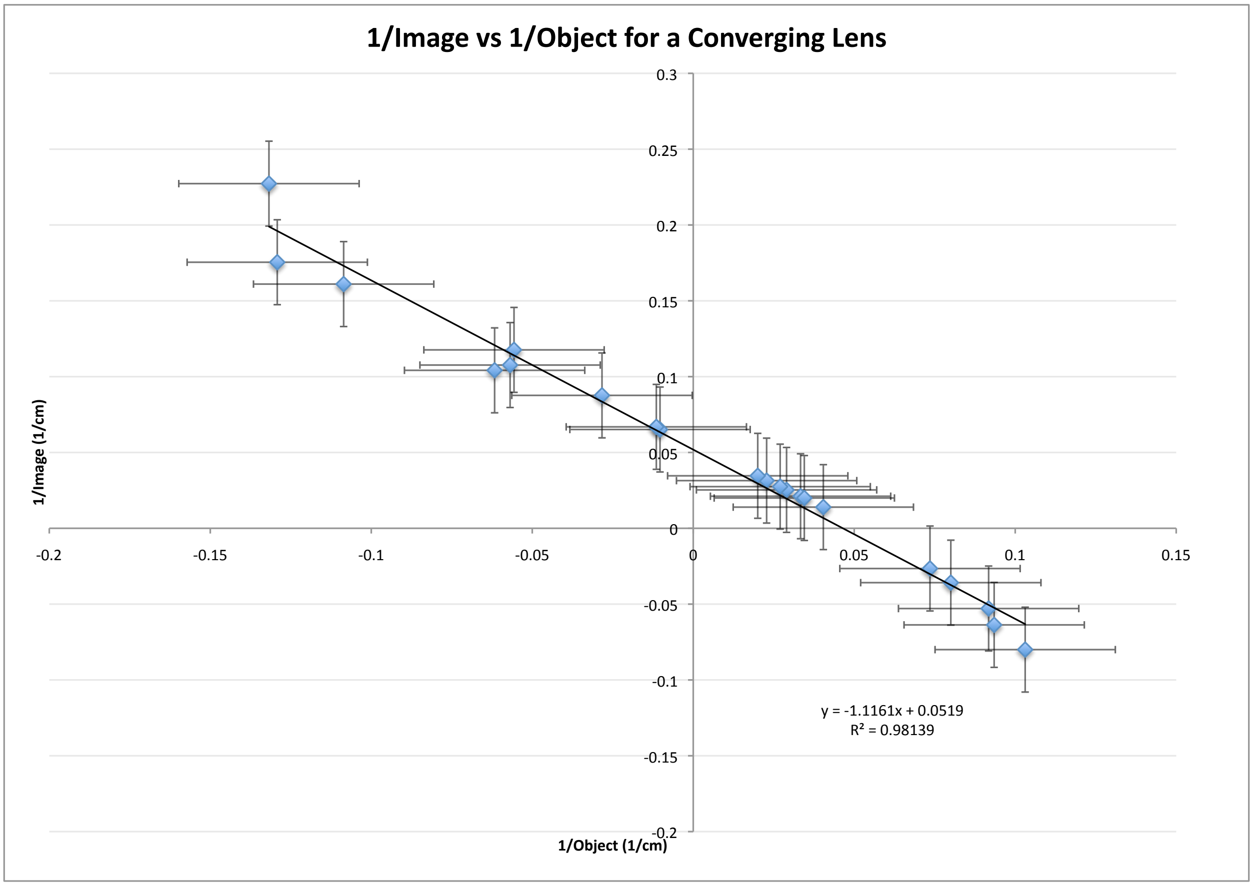
Virtual Object - Real Image								
Trial	**Aux Lens**	**Main Lens**	**Real Image**	**Virtual Object (calc)**	**Focal Length (aux)**	**Virtual Object calc2**	**1/Vo**	**1/Ri**
1	15.1	14.66	15.35	111.57	13.3	-96.91	-0.0103186	0.06514658
2	15.1	24.35	14.95	111.57	13.3	-87.22	-0.011465	0.06688963
3	26.11	19.52	4.4	27.11	13.3	-7.59	-0.1317741	0.22727273
5	19.97	4.53	11.41	39.82	13.3	-35.29	-0.0283364	0.08764242
6	19.97	22.24	9.29	39.82	13.3	-17.58	-0.056882	0.10764263
7	19.97	32.08	5.7	39.82	13.3	-7.74	-0.129195	0.1754386
8	29.58	6.2	8.5	24.17	13.3	-17.97	-0.0556623	0.11764706
9	19.97	30.61	6.21	39.82	13.3	-9.21	-0.1085748	0.1610306
10	19.97	23.61	9.6	39.82	13.3	-16.21	-0.0616894	0.10416667



The virtual object with real image plot confirms our expectation by appearing in quadrant II, implying that our sign convention still holds under this setup.

D. Graph 1/Obj vs 1/Image for ABC

Given the data we have collected to this point in the lab, plotting all three data sets on one graph will help us determine the characteristics of our converging lens.



Here we show error bars based on a standard deviation of 0.028 cm. This is calculated by taking the square root of the sum of the squares of the estimated error on each data point.

$$\sigma_{tot} = \sqrt{\sigma_o^2 + \sigma_i^2}$$

The slope of the estimated trend line is shown to be -1.1161, this is near our expected inverse linear relationship between 1/image and 1/object. The y-intercept is shown to be 0.0519 which is 1/focal length of our lens. Taking the reciprocal of the y-intercept gives us our experimental value of the focal length.

1/y-intercept = focal length

This number is 19.27 cm which is 1.16 cm off from our 18.10 cm focal length determined by autocollimation. It is safe to assume that the true focal length of the lens lies somewhere between these two experimental values. Additional data points would help narrow this difference. In future experiments, where more time is available, additional data points will be necessary.

III. Determine the focal length of a thin diverging lens

After setting up our lens system with the diverging and converging lens together, we again place a mirror on the track to complete our autocollimation apparatus in order to determine the focal point of our diverging lens. After careful alignment and observation of the real image the system produces next to the real object, we find our lenses are sitting 35.23±0.02 cm from the object. From the principles of autocollimation, we know that the focal length of our lens is 35.23±0.02cm.

$$f_{diverging} = 35.23 \pm 0.02 \text{ cm}$$

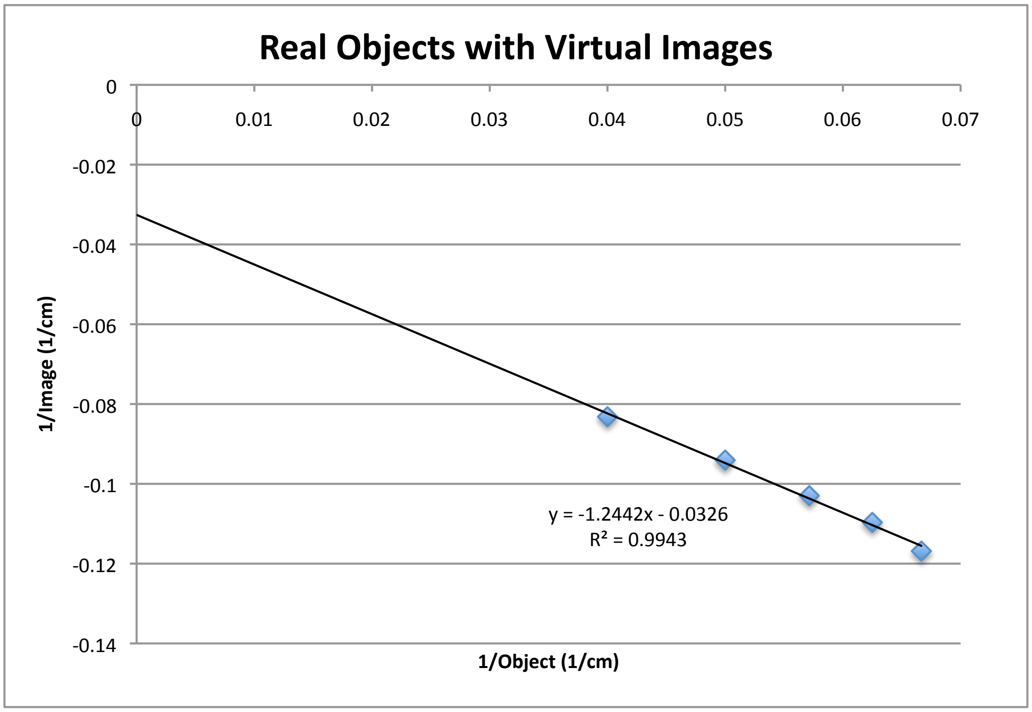
IV. Testing the Thin Lens Equation with a Diverging Lens

With the focal length of the diverging lens now known, we graphically test equation 1 using the diverging lens.

A. Real Objects and Virtual Images using Parallax

To test for virtual images with real objects, we use a method similar to the parallax method used with the converging lens. The principles will be the same, however we replace the converging lens with the diverging lens.

Real Objects with Virtual Images (Diverging)				
Trial	**White Pin**	**Black Pin**	**1/Object**	**1/image**
1	17.5±0.5	-36.0±0.5	0.057142857	-0.102965748
2	25.0±0.5	-125.0±0.5	0.04	-0.08318797
3	20.0±0.5	-53.0±0.5	0.05	-0.094055894
4	15.0±0.5	-24.0±0.5	0.066666667	-0.116854637
5	16.0±0.5	-29.0±0.5	0.0625	-0.109670729

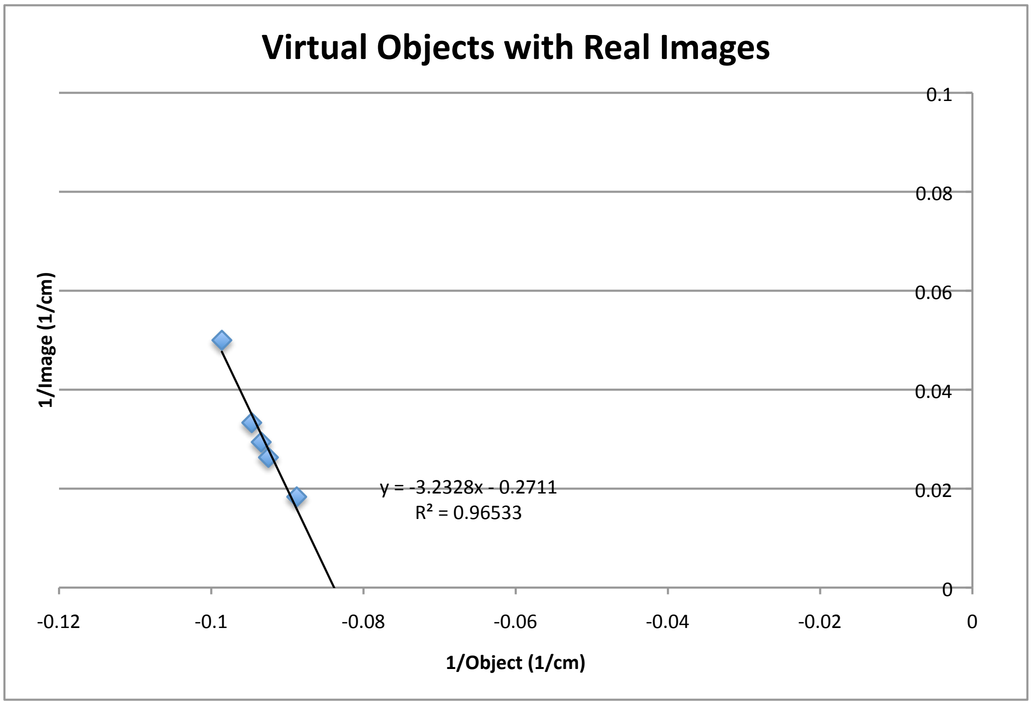


We again see a negative, linear slope near -1 with a y-intercept equal to 1/focal length of our diverging lens. This plot is fore real objects with virtual images, and has data points in quadrant IV, just as our converging lens showed.

B. Virtual Objects and Real Images

We expect to gather data points with a negative object position and positive image position placing the plot in quadrant II.

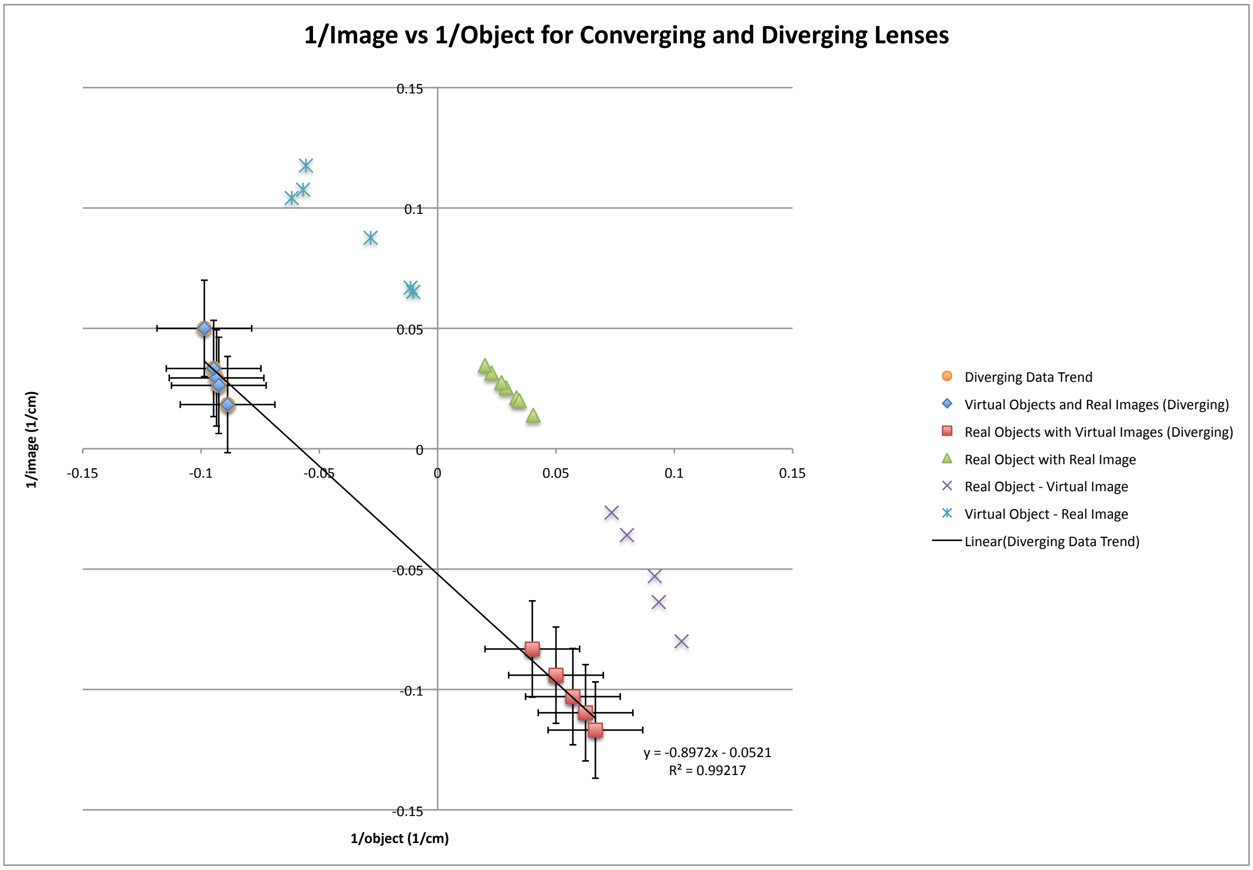
Virtual Objects and Real Images (Diverging)				
Trial	**Main Lens**	**Image**	**1/Object**	**1/Image**
1	20.80±0.05	34.0±0.5	-0.093436145	0.029411765
2	19.10±0.05	54.5±0.5	-0.088774926	0.018348624
3	19.80±0.05	38.0±0.5	-0.092489008	0.026315789
4	21.30±0.05	30.0±0.5	-0.094681147	0.033333333
5	22.70±0.05	20.0±0.5	-0.098607174	0.05



Our data set falls within the expected quadrant and agrees with our sign convention and previous approximations of y-intercept and slope.

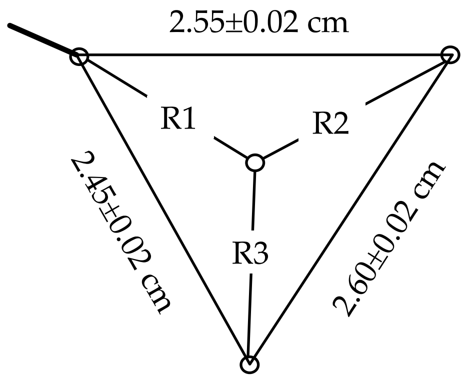
C. Graph 1/Object vs 1/Image for Diverging Lens

Now that we have data for both real objects with virtual images and virtual objects with real images for the diverging lens, we plot the new diverging data sets on top of our converging graph. We should now see data in three quadrants and estimated data in quadrant III. This trend line represents the relationship between virtual objects and virtual images for a diverging lens. There is no way to test for this in the lab, and therefore will only be a mathematical phenomenon in this scope of this lab.



The y-intercept again represents 1/focal length of the lens, but this time it is the focal length of the diverging lens. It is again around 19.19 cm, our focal length by autocollimation was found to be 35.2 cm. This larger difference between the intercept and autocollimation methods can be attributed to the increased difficulty in determining the location of virtual and real images due to the use of a converging lens in the system to enable measurement. If the diverging and converging lens do not form a near perfect fit, errors in measurement and calculation may become a contributing factor.

V. Spherometer



R1 = 1.42 ± 0.02mm
R2 = 1.45 ± 0.02mm
R3 = 1.49 ± 0.02mm
Average R:
1.45 ± 0.03mm

Zero Point: 26.650 ± 0.002 mm	
Converging Lens: 27.150 ± 0.002 mm	0.500 ± 0.003 mm
Converging Lens (II): 27.325 ± 0.002 mm	0.675 ± 0.003 mm
Diverging Lens: 26.143 ± 0.002 mm	-0.507 ± 0.003 mm

Given that:

$$r = \frac{s}{2} + \frac{d^2}{6s}$$

The radius of curvature (r) for each of our lenses are as follows.

Radius of Curvature for Lenses			
Lens	**s (mm)**	**d (cm)**	**r (cm)**
Converging I	0.500 ± 0.003	1.45 ± 0.03	0.95 ± 0.83
Converging II	0.675 ± 0.003	1.45 ± 0.03	0.85 ± 6.63
Diverging	-0.507 ± 0.003	1.45 ± 0.03	-0.94 ± 4.73

5 Discussion and Conclusions

In this lab we have examined the characteristics of thin lenses and tested their compliance to the thin lens equation (equation 1) by graphical methods. By studying and applying concepts of real and virtual images and objects, we have uncovered many secrets of optical instruments and the physics of light as it passes between materials of different densities causing its path to bend.

Uncertainties were initially estimated as the accuracy of our measuring tools and carried through calculations by the following method:

$$\sigma_f^2 = \sum_i^N \left(\frac{\partial f(x)}{\partial x_i} \right)^2 \sigma_i^2$$

Our optics track had a defined metric scale on the side to aid in our position measurements. We recorded measurements based on the assumption that we could estimate the location to the nearest 0.02 cm or 0.2 mm. While this is a reasonable estimate given the fine threading and positioning of the components on the track, it is simply the measurement of the position of the component holder. Each lens, mirror, and object are held in place by graspers which we can only assume are close enough to what the pointer displays on the side of the track. If this offset is different between each clamp, it will introduce additional error for which we are unable to account in our data points.

In general, we have much room for improvement in our labs, both in data taking and in general organization. In the future, we will take additional data points in an effort to reduce error in our data sets, both measured and calculated. We have learned that graphing data points as we record them is especially helpful to determine if one of our points is radically different than our others in either sign convention or incorrect calculations. We were able to catch any errors in our data before the final write up, however it was not before graphing them quickly either by hand or in Excel.

Another step that must be taken before beginning the lab is to read the full handout. While we did look through it all, we did not read it in as great of detail as is necessary to fully understand the purpose of the individual exercises in the greater lab. This is something that we intend to fix in future labs. Knowing exactly what we are to do in each step as it fits into the whole lab will be necessary in any lab experiment.

Thin lenses are quite fascinating components of an optics system. While they tend to not be challenging mathematically, their concepts do require some diligent thought as to while light behaves the way it does as it passes through a lens. Once the basic concepts are understood, and the proper sign convention is applied, thin lens experiments are quite enjoyable and informative.

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