Collaborative Research: Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS)

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You open your textbook, and in the chapter on topology, after a page of introduction, you see this:

Definition. A ***topology*** on a set X is a collection $\mathcal{T}$ of subsets of X having the following properties:

(1) $\varnothing$ and X are in $\mathcal{T}$.

(2) The union of the elements of any subcollection of $\mathcal{T}$ is in $\mathcal{T}$.

(3) The intersection of the elements of any finite subcollection of $\mathcal{T}$ is in $\mathcal{T}$.

A set X for which a topology $\mathcal{T}$ has been specified is called a ***topological space***.

Properly speaking, a topological space is an ordered pair $(X, \mathcal{T})$ consisting of a set X and a topology $\mathcal{T}$ on X, but we often omit specific mention of $\mathcal{T}$ if no confusion will arise.

If X is a topological space with topology $\mathcal{T}$, we say that a subset U of X is an ***open set*** of X if U belongs to the collection $\mathcal{T}$. Using this terminology, one can say that a topological space is a set X together with a collection of subsets of X, called *open sets*, such that $\varnothing$ and X are both open, and such that arbitrary unions and finite intersections of open sets are open.

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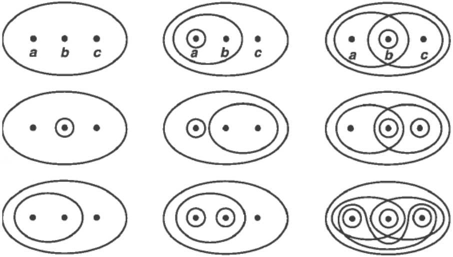
EXAMPLE 1. Let X be a three-element set, $X = \{a, b, c\}$. There are many possible topologies on X, some of which are indicated schematically in Figure 12.1. The diagram in the upper right-hand corner indicates the topology in which the open sets are X, $\varnothing$, $\{a, b\}$, $\{b\}$, and $\{b, c\}$. The topology in the upper left-hand corner contains only X and $\varnothing$, while the topology in the lower right-hand corner contains every subset of X. You can get other topologies on X by permuting a, b, and c.



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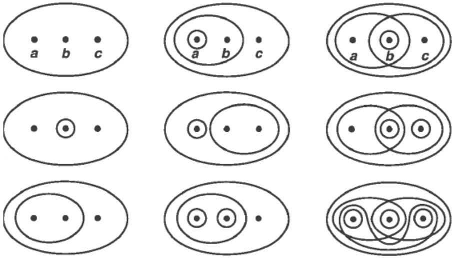

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How can we avoid this?

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> *Mathematicians. . . tend to absorb the writings of their predecessors directly into their own work. They do not comment on the writings of earlier mathematicians, even if they have been very much influenced by them. They simply make use of the material that they find in the authors they read. When advances are made in mathematics, later thinkers condense the findings and move on. Few mathematicians study works from past centuries; compared with contemporary mathematics, such older writings seem to them almost like the work of children.*

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- Evaluation (just beginning)

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- At least 3 people involved in final product: writer, primary reviewer, secondary reviewer.

List of Completed Projects

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Analysis/Calculus:

- Bolzano's Definition of Continuity, his Bounded Set Theorem, and an Application to Continuous Functions
- Investigations Into d'Alembert's Definition of Limit
- An Introduction to a Rigorous Definition of Derivative
- The Mean Value Theorem
- Abel and Cauchy on a Rigorous Approach to Infinite Series
- The Definite Integrals of Cauchy and Riemann
- Euler's Rediscovery of e (miniPSP)
- Euler's derivatives of the sine and cosine functions (miniPSP)
- Rigourous Debates over Debatable Rigor (Darboux's Monster Function)
- Why be so critical? (miniPSP)
- Henri Lebesgue and the Development of Integration(miniPSP)

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Topology:

- Topology from Analysis (miniPSP)
- Connecting Connectedness (miniPSP)
- The Cantor set before Cantor (miniPSP)

Example: Rigourous Debates over Debatable Rigor

2 Monsters in the Darboux-Houël Correspondence

The impetus for the ten-year debate concerning rigor in analysis in the Darboux-Houël correspondence was Houël's request for feedback on preliminary drafts of his intended textbook on differential calculus, eventually published as *Cours de Calcul infinitésimal* in 1878. Throughout this debate, Darboux offered

various counterexamples in a (vain) attempt to convince Houël of the need for greater care in certain of his (Houël's) proofs. The following excerpt from a letter written by Darboux on 24 January 1875 [as quoted in (Gispert, 1987, p. 101)]reveals one such example:

> Go on then and explain to me a little, I beg you, why it is that when one uses the rule for composition functions, the derivative of $y = x^2 \sin\frac{1}{x}$ is found to be $-\cos\frac{1}{x} + 2x\sin\frac{1}{x}$, which is indeterminate for $x = 0$ even though the true value is $\lim\frac{y}{x} = 0, \ldots$

$\boxed{\text{Task 1}}$

(a) What is the name that is usually used in a current US calculus or analysis textbook for what Darboux called 'the rule for composition functions'?
Use this rule to verify Darboux's claim about the derivative of $y = x^2 \sin\frac{1}{x}$ for $x \neq 0$.
Why is this derivative function indeterminate for $x = 0$?

(b) Notice that the function $y = x^2 \sin\frac{1}{x}$ given by Darboux is undefined at $x = 0$.
What did Darboux say in the preceding excerpt that gives us reason to believe that he was implicitly assuming that y is continuous at $x = 0$?

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- Talks: Seattle JMM, Colorado MAA Section, Calgary Alberta Canada, Montpellier, Hamberg, Columbus, JMM Atlanta
- Papers: *Monsters in the classroom: Learning analysis through the works of Gaston Darboux* (HPM 2016), *Primary source projects in an undergraduate mathematics classroom: A pilot case in a topology course* (CERME 10)

Denver Workshop

September 8 - 10, 2016 in Denver Colorado with 41 participants

Mock classroom

Students were recruited to attend 2 full length class periods in which PSPs would be utilized.


Janet Barnett
Siyi

Participant feedback

I very very very much enjoyed this workshop! I found it very informative and exciting, and has not only furthered my interest in teaching with PSPs, but I now find myself very excited to start working on writing some PSPs!

I honestly had no idea that so many people out there had already developed the kind of materials that I was hoping to develop for my couress. Having this huge starting point will greatly reduce how long it takes me to improve my courses along these lines.

I'm very excited to use it this semester and possible next semester

I am completely excited about using PSPs in my classroom.

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- Evaluation and assessment

All completed projects freely available and downloadable from
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Thank you very much!