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Article · April 2011

DOI: 10.1109/SIS.2011.5952581

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Diversity Control in Particle Swarm Optimization

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***Abstract*—Population diversity of particle swarm optimization (PSO) is important when measuring and dynamically adjusting algorithm's ability of "exploration" or "exploitation". Population diversities of PSO based on L_1 norm are given in this paper. Useful information on search process of an optimization algorithm could be obtained by using this measurement. Properties of PSO diversity based on L_1 norm are discussed. Several methods for diversity control are tested on benchmark functions, and the method based on current position and average of current velocities has the best performance. This method could control the PSO diversity effectively and gets better performance than the standard PSO.**

I. Introduction

Particle Swarm Optimization (PSO), which is one of the evolutionary computation techniques, was invented by Russ Eberhart and James Kennedy in 1995 [1], [2]. It is a population-based stochastic algorithm modeled on the social behaviors observed in flocking birds. Each particle, which represents a solution, flies through the search space with a velocity that is dynamically adjusted according to its own and its companion's historical behaviors. The particles tend to fly toward better search areas over the course of the search process [3], [4].

In this paper, the basic PSO algorithm and the importance of diversity will be reviewed in Section II. In Section III, a comparison of element-wise PSO diversity and dimension-wise PSO diversity, which based on L_1 norm or L_2 norm are given, followed by experiments on diversity monitoring and analysis for some benchmark functions in Section IV. Several methods for diversity control are tested on benchmark functions in Section V, and a novel method based on current position and average of current velocities has the best performance. Finally, Section VI concludes with some remarks and future research directions.

II. Particle Swarm Optimization

The original PSO algorithm is simple in concept and easy in implementation [5], [6]. The basic equations are as follow:

$$\begin{aligned} v_{ij} &= v_{ij} + c_1\text{rand}()(p_{ij} - x_{ij}) \\ &\quad + c_2\text{Rand}()(p_{nj} - x_{ij}) \qquad (1) \\ x_{ij} &= x_{ij} + v_{ij} \qquad (2) \end{aligned}$$

where c_1 and c_2 are positive constants, and rand() and Rand() are two random functions in the range $[0, 1]$ and are different for each dimension and each particle.

The most important factor affecting an optimization algorithm's performance is its ability of "exploration" or "exploitation". Exploration means the ability of a search algorithm to explore different areas of the search space in order to have high probability finding good optimum. Exploitation, on the other hand, means the ability to concentrate the search around a promising region in order to refine a candidate solution. A good optimization algorithm optimally balances these conflicted objectives. Within the PSO, these objectives are addressed by the velocity update equation.

Velocity clamp was firstly used to adjust the ability between exploration and exploitation [7]. Like the equation below, current velocity will be equal to maximum velocity or minus maximum velocity if velocity is greater than the maximum velocity or less than the minus maximum velocity, respectively.

$$v_{ij} = \begin{cases} V_{max} & v_{ij} > V_{max} \\ v_{ij} & -V_{max} \leq v_{ij} \leq V_{max} \\ -V_{max} & v_{ij} < -V_{max} \end{cases} \qquad (3)$$

However, velocity clamp only makes algorithm less like to diverge, it cannot help algorithm "jump out" a local optimum or refine the candidate solution.

Shi and Eberhart introduced a new parameter, an inertia weight w to balance the exploration and exploitation [8] [9]. This inertia weight w is added to equation (1), and it can be a constant, linear decreasing value over time [10], or fuzzy value [11], [12]. The new velocity update equation is as follows:

$$\begin{aligned} v_{ij} &= wv_{ij} + c_1\text{rand}()(p_{ij} - x_{ij}) \\ &\quad + c_2\text{Rand}()(p_{nj} - x_{ij}) \qquad (4) \end{aligned}$$

Adding an inertia weight is more effective than velocity clamp for it is not only increasing the probability for algorithm to converge, but have a way to control the whole process of algorithm's searching. Generally speaking, algorithm should have a bigger exploration and lower exploitation ability at first, which has a high probability to find more local optima. Exploration should be decreased, and exploitation should be increased to refine candidate solutions over the time. Accordingly, the inertia weight w, should be linear decreased or even dynamically determined by a fuzzy system.

The whole process of PSO search could be adjusted by adding an inertia weight, however, it is difficult to change inertia weight in order to dynamically adjust the ability of exploration or exploitation during algorithm searching. Diversity, which can be a way to monitor an algorithm's state of

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exploration or exploitation, is important for helping adjust the ability of exploration and exploitation. Diversity can reveal internal characteristic of a search process.

III. Diversity Definition

Population diversity is a way to monitor the degree of convergence or divergence in PSO search process [13], [14]. In other words, the particles' current distribution and velocity tendency, whether it is in the state of "fly" to a large search space or refine in a local area, can be obtained from this measurement. Shi and Eberhart gave several definitions of PSO population diversity measurements in [15], [16], [17], and these definitions of population diversities could be divided into three parts: position diversity, velocity diversity, and cognitive diversity. The analysis of different definitions is as follows.

A. Position Diversity

Position diversity measures distribution of particles' current positions whether the particles are going to diverge or converge could be reflected from this measurement. Position diversity gives the current distribution information of particles.

Position diversity could be measured either element-wise or dimension-wise. For the purpose of generality and clarity, m represents the number of particles and n the number of dimensions. Each particle is represented as x_{ij}, i represents the ith particle, $i = 1, \cdots, m$, and j is the jth dimension, $j = 1, \cdots, n$.

Element-wise definition [15] is as follows:

$$\bar{x} = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \tag{5}$$

$$D_E^p = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} (x_{ij} - \bar{x})^2 \tag{6}$$

where $\bar{x}$ is the mean of current position for all particles in all dimensions, and D_E^p measures all particles position diversity in all dimensions.

Element-wise measurement sets all particles and all dimensions as an entirety to calculate the population diversity of PSO, therefore, this kind of definitions has some blemishes:

- Lack of each dimension's diversity information: D_E^p represents the swarm position diversity, no measurement of particles' position diversity on a single dimension.
- Confusion about the difference in dimensions: Consider a simple scenario, using two particles to solve a problem with two dimensions, particles at (1, 7) and (7, 1), respectively; or two particles converge to (1, 7), the results of element-wise diversity are same: $\bar{x} = 4$, $D_E^p = 9$. However, these two situations should have different results of population diversity measurement. The above example shows that element-wise diversity cannot give the useful information of problem's optimum with different values among all dimensions.

Dimension-wise definition [15] is as follows:

$$\bar{\mathbf{x}} = \frac{1}{m} \sum_{i=1}^{m} x_{ij} \tag{7}$$

$$\mathbf{D}^p = \frac{1}{m} \sqrt{\sum_{i=1}^{m} (x_{ij} - \bar{x}_j)^2} \tag{8}$$

where $\bar{\mathbf{x}} = [\bar{x}_1, \cdots, \bar{x}_j, \cdots, \bar{x}_n]$, $\bar{\mathbf{x}}$ represents the mean of particles' current positions on each dimension. $\mathbf{D}^p = [D_1^p, \cdots, D_j^p, \cdots, D_n^p]$, which measures particles' position diversity based on L_2 norm for each dimension, therefore, it lacks of whole swarm information during algorithm search process. Furthermore, L_1 norm can be used to instead of L_2 norm.

New definition of position diversity, which based on the L_1 norm, is as follows:

$$\bar{\mathbf{x}} = \frac{1}{m} \sum_{i=1}^{m} x_{ij} \tag{9}$$

$$\mathbf{D}^p = \frac{1}{m} \sum_{i=1}^{m} |x_{ij} - \bar{x}_j| \tag{10}$$

$$D^p = \frac{1}{n} \sum_{j=1}^{n} D_j^p \tag{11}$$

where $\bar{\mathbf{x}} = [\bar{x}_1, \cdots, \bar{x}_j, \cdots, \bar{x}_n]$, $\bar{\mathbf{x}}$ represents the mean of particles' current positions on each dimension. $\mathbf{D}^p = [D_1^p, \cdots, D_j^p, \cdots, D_n^p]$, which measures particles' position diversity based on L_1 norm for each dimension. D^p measures the whole swarm population diversity.

Besides, the definition of diversity on dimensions has clearer geometric means: $\bar{\mathbf{x}}$ is the center of all positions on each dimension, and D_j^p is the average distance of particles from the center in j dimension. In other word, if swarm moves to the center from current distribution, the distance of all particles need to move is mD_j^p in dimension j, and the total distance is $m \sum_{j=1}^{n} D_j^p = m \times n \times D^p = mnD^p$.

B. Velocity Diversity

Velocity diversity, which gives the dynamic information of particles, measures the distribution of particles' current velocities, In other words, velocity diversity measures the "activity" information of particles. Based on the measurement of velocity diversity, particle's tendency of expansion or convergence could be revealed.

The dimension-wise definition of velocity diversity, which was given by Shi and Eberhart [16], is based on L_2 norm.

These velocity diversity definitions are as follow.

$$v_{ij}^{nor} = \frac{v_{ij}}{\sqrt{\sum_{j=1}^{n} v_{ij}^2}} \quad (12)$$

$$\bar{v}_j^{nor} = \frac{1}{m}\sum_{i=1}^{m} v_{ij}^{nor} \quad (13)$$

$$\mathbf{D}^v = \frac{1}{m}\sqrt{\sum_{i=1}^{m}(v_{ij}^{nor} - \bar{v}_j^{nor})^2} \quad (14)$$

where $\mathbf{D}^v = [D_1^v, \cdots, D_j^v, \cdots, D_n^v]$, $\mathbf{D}^v$ measures velocity diversity based on L_2 norm for each single dimension.

Similar with definitions of position diversity, velocity diversity based on L_2 norm lacks of measurement on whole swarm velocity, $\mathbf{D}^v$ only measures velocity distribution on each dimension. L_1 norm can also be used to replace L_2 norm.

New definition of velocity diversity based on L_1 norm is as follows:

$$\bar{\mathbf{v}} = \frac{1}{m}\sum_{i=1}^{m} v_{ij} \quad (15)$$

$$\mathbf{D}^v = \frac{1}{m}\sum_{i=1}^{m} |v_{ij} - \bar{v}_j| \quad (16)$$

$$D^v = \frac{1}{n}\sum_{j=1}^{n} D_j^v \quad (17)$$

where $\bar{\mathbf{v}} = [\bar{v}_1, \cdots, \bar{v}_j, \cdots, \bar{v}_n]$, $\bar{\mathbf{v}}$ represents the mean of particles' current velocities on each dimension; and $\mathbf{D}^v = [D_1^v, \cdots, D_j^v, \cdots, D_n^v]$, $\mathbf{D}^v$ measures velocity diversity of all particles on each dimension. D^v represents the whole swarm velocity diversity based on L_1 norm.

In addition, velocity diversity based on L_1 norm has clearer geometric meaning: $\bar{\mathbf{x}}$, which is the average velocity of particles on each dimension, was found at first; then the variance of velocity on each dimension j could be calculated. This variance gives the information of particles "vitality", even the swarm may have same average of velocity on each dimension during different searching period, the variance will be small when the swarm converges.

C. Cognitive Diversity

Cognitive diversity represents the distribution of all current moving targets found by particles. The measurement of cognitive diversity is the same as position diversity except for using each particle's current personal best position instead of current position. Therefore, the analysis for position diversity is also being applied to cognitive diversity. The diversity could be dimension-wise and based on the L_1 norm. The definition of PSO cognitive diversity is as follows:

$$\bar{\mathbf{p}} = \frac{1}{m}\sum_{i=1}^{m} p_{ij} \quad (18)$$

$$\mathbf{D}_j^c = \frac{1}{m}\sum_{i=1}^{m} |p_{ij} - \bar{p}_j| \quad (19)$$

$$D^c = \frac{1}{n}\sum_{j=1}^{n} D_j^c \quad (20)$$

where $\bar{\mathbf{p}} = [\bar{p}_1, \cdots, \bar{p}_j, \cdots, \bar{p}_n]$ and $\bar{\mathbf{p}}$ represents the average of all particles' personal best position in history (*pbest*) on each dimension; $\mathbf{D}^c = [D_1^p, \cdots, D_j^p, \cdots, D_n^p]$, which represents the particles' cognitive diversity for each dimension based on L_1 norm. D^c measures the whole swarm cognitive diversity.

The above discussion gives the definitions of population diversities from three parts: position, velocity, and cognitive. These diversity definitions are based on L_1 norm which have clearer geometric meaning. Three definitions have the same form and easy to understood or implemented. More searching information of optimization algorithms could be revealed from this measurement.

IV. Diversity Monitoring and Analysis

Wolpert and Macerady have proved that under certain assumptions no algorithm is better than other one on average for all problems [18]. Consider the generalization, twelve benchmark functions were used in our experimental studies [19],[20]. The aim of the experiment is not compare the ability or the efficacy of PSO algorithm with different parameter setting or structure, e.g., global star or local ring, but to compare the measurement of runtime information when PSOs are executed.

The 12 benchmark functions are given in Table I. Functions $f_1 - f_5$ are unimodal. f_5 is a noisy quadric function, where random$[0, 1)$ is a uniformly distributed random variable in $[0, 1)$. Functions $f_6 - f_{12}$ are multimodal. f_6 has 2 minima when dimensions $n = 4 \sim 30$. [21]

In all experiments, PSO has 50 particles, and parameters are set as the standard PSO [22]. Each algorithm runs 50 times, 10 000 iterations in each run.

Due to the limit of space, the simulation results of three representative benchmark functions are reported here. The functions include f_5, which is a unimodal function with random noise; f_9, which is a noncontinuous multimodal function, and f_{11}, a continuous multimodal function.

A. Comparison of Different PSO Diversity Definitions

Figure 1 shows a comparison of different PSO population diversity definitions. Firstly, for the PSO with global star structure, Fig.1 (a) shows the cognitive diversity of f_5 function, Fig.1 (b) shows the position diversity of f_9 function, and Fig.1 (c) shows the velocity diversity of f_{11} function; secondly, for the PSO with local ring structure, Fig.1 (d) shows the velocity diversity of f_5 function, Fig.1 (e) shows the cognitive diversity

TABLE I

THE 12 BENCHMARK FUNCTIONS USED IN OUR EXPERIMENTAL STUDY, WHERE n IS THE DIMENSION OF THE FUNCTION, $f_{\min}$ IS THE MINIMUM VALUE OF THE FUNCTION, AND SEARCH SPACE $\subseteq R^n$

Function name	Test function	Dimension	Search space	$f_{\min}$
Sphere [19]	$f_1(\mathbf{x}) = \sum_{i=1}^{n} x_i^2$	50	$[-100, 100]^n$	0
Schwefel's P2.22 [19]	$f_2(\mathbf{x}) = \sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	50	$[-10, 10]^n$	0
Schwefel's P1.2 [19]	$f_3(\mathbf{x}) = \sum_{i=1}^{n}(\sum_{k=1}^{i} x_k)^2$	50	$[-100, 100]^n$	0
Step [19]	$f_4(\mathbf{x}) = \sum_{i=1}^{n}(\lfloor x_i + 0.5\rfloor)^2$	50	$[-100, 100]^n$	0
Quadric Noise [19]	$f_5(\mathbf{x}) = \sum_{i=1}^{n} i x_i^4 + \text{random}[0, 1)$	50	$[-1.28, 1.28]^n$	0
Generalized Rosenbrock [19]	$f_6(\mathbf{x}) = \sum_{i=1}^{n}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	50	$[-10, 10]^n$	0
Schwefel [19]	$f_7(\mathbf{x}) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i\rvert}) + 418.9829n$	50	$[-500, 500]^n$	0
Generalized Rastrigin [19]	$f_8(\mathbf{x}) = \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i) + 10]$	50	$[-5.12, 5.12]^n$	0
Noncontinuous Rastrgin [20]	$f_9(\mathbf{x}) = \sum_{i=1}^{n}[y_i^2 - 10\cos(2\pi y_i) + 10]$ $y_i = \begin{cases} x_i & \lvert x_i\rvert < \frac{1}{2} \\ \frac{\text{round}(2x_i)}{2} & \lvert x_i\rvert \geq \frac{1}{2} \end{cases}$	50	$[-5.12, 5.12]^n$	0
Ackley [19]	$f_{10}(\mathbf{x}) = -20e^{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}}$ $-e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)} + 20 + e$	50	$[-32, 32]^n$	0
Griewank [19]	$f_{11}(\mathbf{x}) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$	50	$[-600, 600]^n$	0
Generalized Penalized [19]	$f_{12}(\mathbf{x}) = \frac{\pi}{n}\{10\sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2$ $\times[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\}$ $+\sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{1}{4}(x_i + 1)$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a, \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	50	$[-50, 50]^n$	0

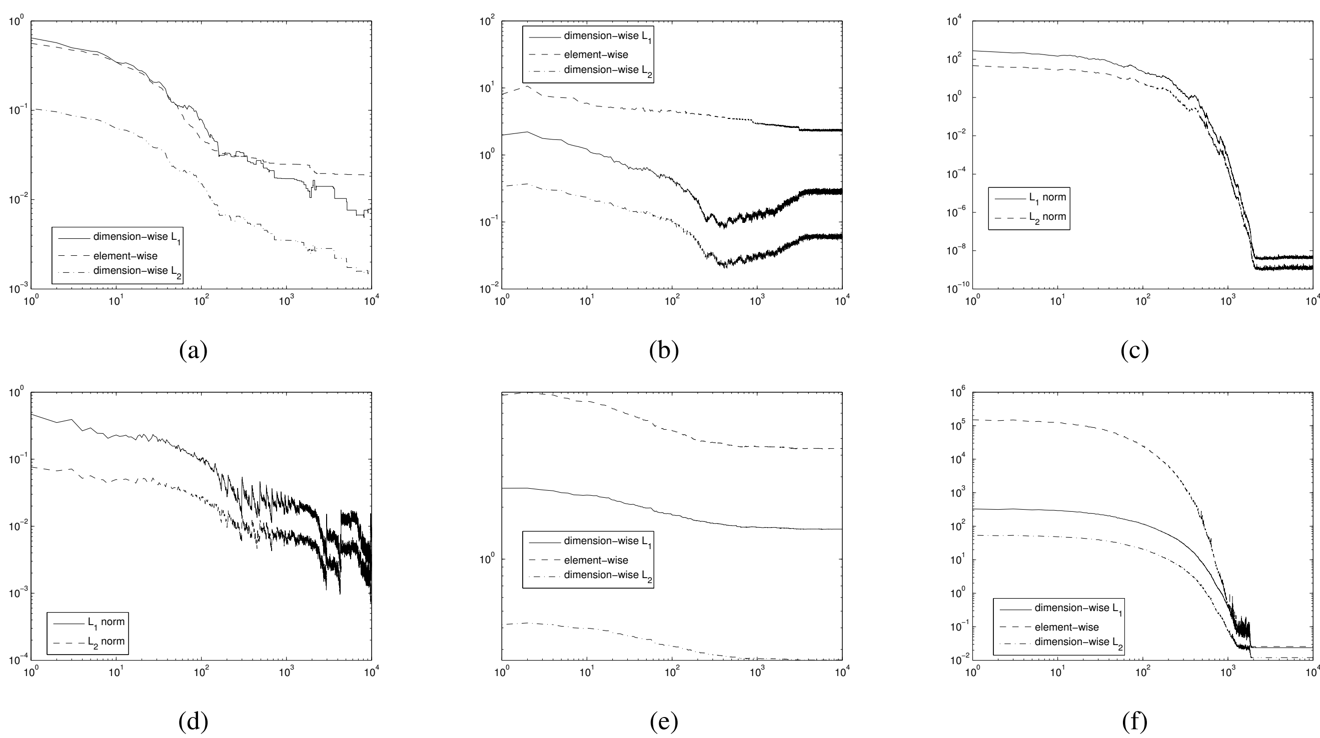

Fig. 1. Different definitions of PSO population diversity. Global star structure: (a) f_5 cognitive, (b) f_9 Position, (c) f_{11} velocity; Local ring structure: (d) f_5 velocity, (e) f_9 cognitive, (f) f_{11} position

of f_9 function, and Fig.1 (f) shows the position diversity of f_{11} function.

From the Figure 1, conclusions could be made that dimension-wise population diversities is better than element-wise. The measurement of element-wise diversity cannot give useful information of particles' distribution. The diversity based on L_1 norm and L_2 norm have the same changing curve. L_1 norm is better than L_2 norm for two reasons:

- In general terms, L_1 norm have higher computational efficiency.
- The value of L_1 is larger than L_2 norm, (Figure 1: (a), (b), (e), (f)), or the value of L_1 has larger variation range (Figure 1: (c), (d)). These show that diversity based on L_1 norm can reveal more significant information at least for the tested benchmark functions under dimension-wise population diversity.

B. PSO Diversity Analysis

Figure 2 displays the population diversities which includes position diversity, cognitive diversity, and velocity diversity. Firstly, Fig.2 (a), (b), (c) display the diversities of f_5 function, f_9 function, and f_{11} function for PSO with global star structure, respectively; secondly, Fig.2 (d), (e), (f) display the diversities of f_5 function, f_9 function, and f_{11} function for PSO with local ring structure, respectively.

From the figure, some conclusions could be made that position diversity and cognitive diversity have the same changing tendency, and cognitive diversity curve is a simplified position diversity curve without vibrate. Velocity diversity will fall to a tiny value after algorithm find a "good enough" value or "stuck" in a local optimum.

It is observed from running PSO that particles "fly" from one side of optimum to another side on each dimension continually [23]. Velocity diversity and position diversity usually have a continuous vibrate.

V. DIVERSITY CONTROL

Diversity is the measurement of exploration and exploitation, however, the goal is not only to observe, but to control the diversity, that is state of the exploration or exploitation could be dynamically adjusted. Since adding random noise may increases population diversity, in the first experiment below, we add noise in PSO to see its impacts.

A. Based on Random Noise

Noise is added to equation (2), new equation is as follows:

$$x_{ij} = x_{ij} + v_{ij} + c_3\text{RAND}() \tag{21}$$

where c_3 could be a positive or negative constant, or adjusted during the algorithm search process. RAND() is a random function in the range $[0, 1]$ and the value is different for each dimension and each particle. Some representative results are given in Table II. This method does not performs better than the standard PSO.

B. Based on Average of Current Velocities

Velocity measures the "flying" tendency of particles. The average of current velocities may affect the population diversity. The next experiment, which adds the average velocity to equitation (2), is as follows:

$$x_{ij} = x_{ij} + v_{ij} + c_3\text{RAND}()\bar{v}_j \tag{22}$$

where c_3 could be a positive or negative constant, or adjusted during the algorithm search process. RAND() is a random function in the range $[0, 1]$ and the value is different for each dimension and each particle; $\bar{v}_j$ is the average of current velocities for dimension j.

Some representative results are shown in table III. This method does not have significant improvement over the standard PSO.

C. Based on Current Position and Average of Current Velocities

In the third experiment, we utilize current position in addition to the current average velocity. The new equation is as follows:

$$x_{ij} = x_{ij} + v_{ij} + c_3\text{RAND}()(x_{ij} - \bar{v}_j) \tag{23}$$

where c_3 could be a positive or negative constant, or adjusted during the algorithm searching. RAND() is a random function in the range $[0, 1]$ and is different for each dimension and each particle; $\bar{v}_j$ is the average of current velocities for dimension j.

Several representative results are shown in table IV. Some significant improvements are achieved from this method, the best or mean of results are much better than the standard PSO. Figure 3 measures the difference of the standard PSO and PSO with diversity control: firstly, for the PSO with global star structure, Fig.3 (a) shows the position diversity of f_5 function, Fig.3 (b) shows the cognitive diversity of f_9 function, and Fig.3 (c) shows the velocity diversity of f_{11} function; secondly, for the PSO with local ring structure, Fig.3 (d) shows the cognitive diversity of f_5 function, Fig.3 (e) shows the velocity diversity of f_9, and Fig.3 (f) shows the position diversity of f_{11} function.

From the Table IV and Figure 3, conclusions could be made that if c_3 is positive, the diversity will be increased, particles search wider area than the standard PSO does, the results have a large variance. If c_3 is negative, the diversity will be decreased, particles converge faster than the standard PSO, the results have a small variance. If c_3 was initialized with a positive value and decreases to negative values during PSO search process, both the best and mean result will be improved. The relationship between this new parameter and diversity control is still required to be further researched. How to set this parameter c_3, or how to adjust this parameter for different problem during different state of PSO searching is our future work.

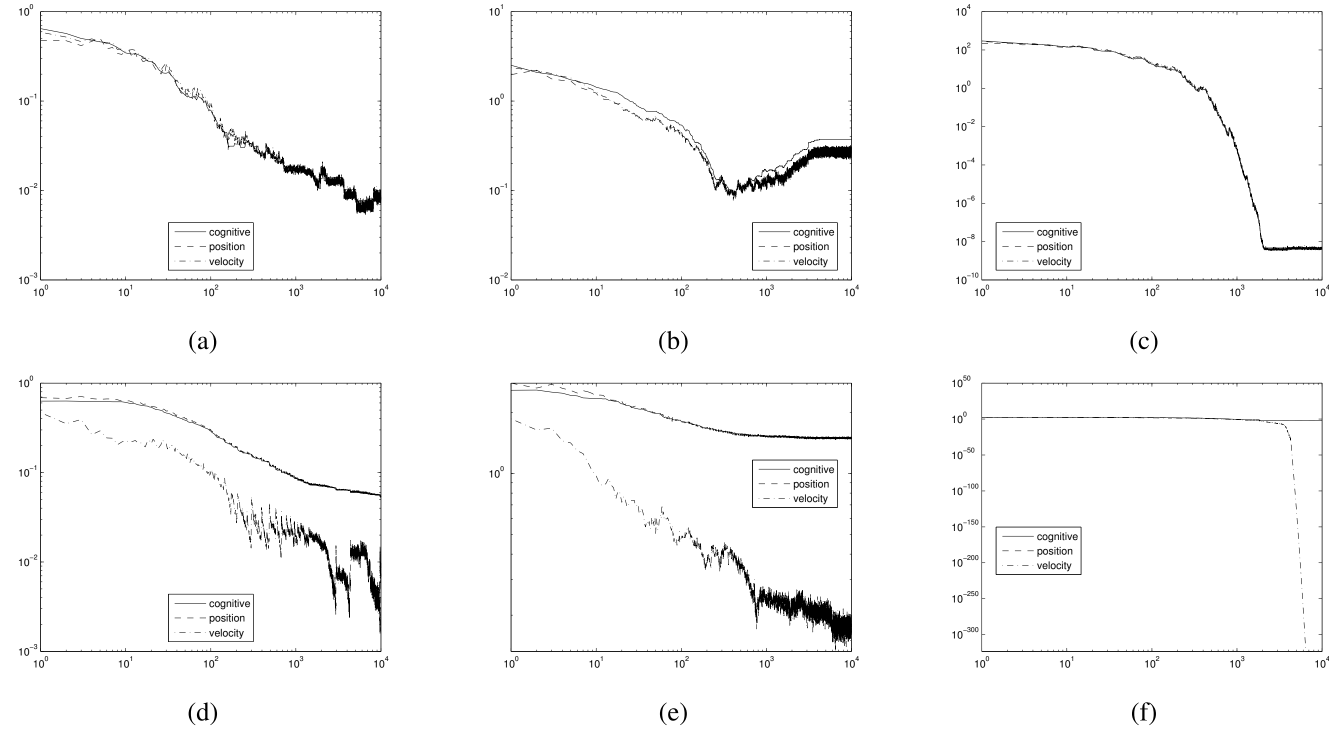

Fig. 2. PSO population diversity analysis. Global star structure: (a) f_5 population, (b) f_9 population, (c) f_{11} population; Local ring structure: (d) f_5 population, (e) f_9 population, (f) f_{11} population

VI. Conclusion

This paper proposed a definition of population diversity based on L_1 norm, compared the difference between these different definitions of population diversities, discussed the blemishes of existing definitions, and then analyzed population diversities changing during the algorithm search process.

Based on above analysis, a novel position update equation to modify the diversity during PSO search process is presented. New equation has an extra parameter c_3, the diversity could be increased or decreased by setting c_3 to be a positive or negative value, respectively.

The idea of diversity measuring and controlling can also be applied to other evolutionary algorithms, e.g., genetic algorithm, differential evolution. Because evolutionary algorithms have the same concepts of current population solutions, the position diversity could be measured and adjusted. It can be beneficial to dynamically adjust the algorithm's ability of exploration or exploitation, especially when the problem to be solved is a difficult or large-scale problem.

Acknowledgment

The authors' work is supported by National Natural Science Foundation of China under grant 60975080, and Suzhou Science and Technology Project under Grant SYJG0919.

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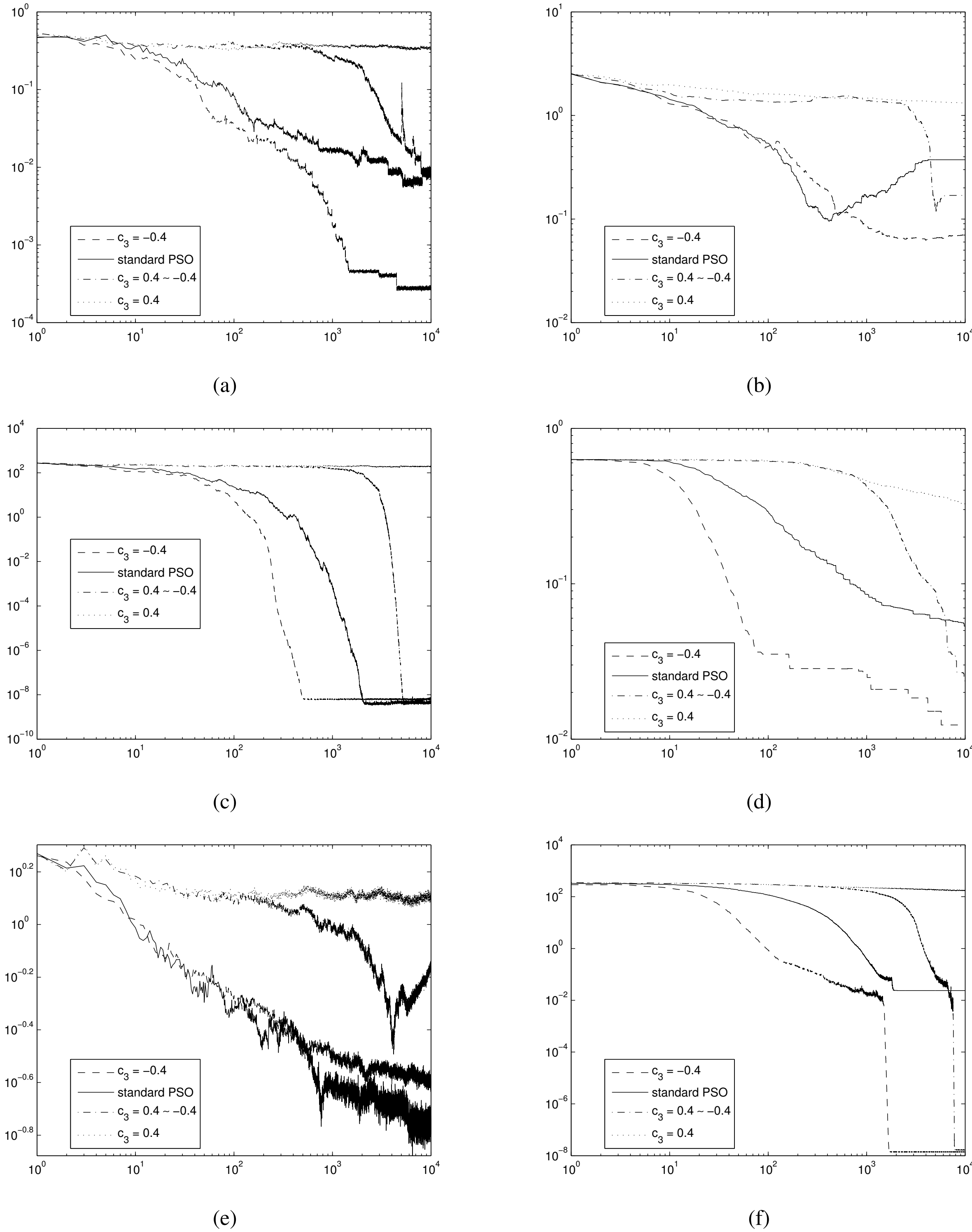

Fig. 3. PSO population diversity control based on current position and average of current velocities. Global star structure: (a) f_5 position, (b) f_9 cognitive, (c) f_{11} velocity; Local ring structure: (d) f_5 cognitive, (e) f_9 velocity, (f) f_{11} position

TABLE II
REPRESENTATIVE RESULTS OF PSO WITH DIVERSITY CONTROL BASED ON RANDOM NOISE. ALL ALGORITHMS HAVE BEEN RUN OVER 50 TIMES, WHERE "MEAN" INDICATES THE MEAN BEST FUNCTION VALUES FOUND IN THE LAST GENERATION.$[-0.05, 0.05]$ INDICATES THAT $c_3 RAND()$ IN THE RANGE OF -0.05 TO 0.05 DURING PSO SEARCH PROCESS

Result		Global star structure			Local ring structure		
		f_5	f_9	f_{11}	f_5	f_9	f_{11}
Standard PSO	best	**0.003306**	**98.0000**	**0**	**0.021183**	139.2511	**9.7699E-15**
	mean	3.763743	**170.2000**	18.0558	**0.078473**	**181.7243**	**1.7852E-14**
$[0, 0.05]$	best	0.027495	151.4569	0.001945	0.062837	139.5583	0.002694
	mean	3.324122	193.7475	14.4905	0.092465	189.9442	0.003257
$[0, 0.1]$	best	0.192631	171.7475	0.006434	0.251453	162.0773	0.008715
	mean	5.108825	243.0230	3.639894	0.389412	212.5552	0.012889
$[-0.05, 0]$	best	0.030778	125.2904	0.001924	0.055127	**135.7548**	0.002388
	mean	2.465845	177.0750	7.250727	0.089502	188.1026	0.003799
$[-0.1, 0]$	best	0.199754	162.2819	0.007742	0.320260	193.1383	0.010421
	mean	6.448594	242.4670	5.450799	0.400614	222.3157	0.012960
$[-0.05, 0.05]$	best	0.092398	187.5481	0.006408	0.203883	155.0355	0.007811
	mean	**0.206316**	237.1176	**0.022319**	0.283535	197.6311	0.010637
$[-0.1, 0.1]$	best	0.914132	248.5174	0.023818	1.022184	216.1435	0.027316
	mean	1.488689	328.5418	0.046167	1.777710	279.9250	0.041168

TABLE III
REPRESENTATIVE RESULTS OF PSO WITH DIVERSITY CONTROL BASED ON AVERAGE OF CURRENT VELOCITIES. ALL ALGORITHMS HAVE BEEN RUN OVER 50 TIMES, WHERE "MEAN" INDICATES THE MEAN BEST FUNCTION VALUES FOUND IN THE LAST GENERATION. $c_3 \sim [-0.05, 0.05]$ INDICATES THAT c_3 LINEAR INCREASES FROM -0.05 TO 0.05 DURING PSO SEARCH PROCESS

Result		Global star structure			Local ring structure		
		f_5	f_9	f_{11}	f_5	f_9	f_{11}
Standard PSO	best	0.003306	98.0000	**0**	0.021183	139.2511	9.7699E-15
	mean	3.763743	170.2000	18.0558	0.078473	181.7243	**1.7852E-14**
$c_3 = 0.05$	best	0.000300	91.2810	**0**	0.010046	**120.4208**	8.8817E-16
	mean	4.835266	154.0616	21.7101	0.028780	**162.1770**	0.000177
$c_3 = 0.1$	best	0.000213	**51.3985**	**0**	0.008539	138.2817	**7.7715E-16**
	mean	3.653916	**139.9491**	21.7121	0.028096	181.1114	6.4194E-05
$c_3 = -0.05$	best	0.000314	83.1481	**0**	0.008869	125.3705	1.4432E-15
	mean	3.976869	144.6972	16.3120	0.030871	182.1733	3.2111E-05
$c_3 = -0.1$	best	0.000209	66.5491	**0**	0.007992	144.7393	8.8817E-16
	mean	**1.990022**	153.8094	18.0800	**0.027247**	180.8293	9.6985E-05
$c_3 \sim [-0.05, 0.05]$	best	**0.000113**	65.2646	**0**	0.009003	146.1426	2.3314E-15
	mean	3.707627	153.0410	**14.4772**	0.032105	180.4098	0.000148
$c_3 \sim [-0.1, 0.1]$	best	0.000420	90.0021	**0**	**0.007660**	144.0002	9.9920E-16
	mean	2.365738	165.4357	14.4980	0.034546	174.4129	0.000149

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TABLE IV

REPRESENTATIVE RESULTS OF PSO WITH DIVERSITY CONTROL BASED ON CURRENT POSITION AND AVERAGE OF CURRENT VELOCITIES. ALL ALGORITHMS HAVE BEEN RUN OVER 50 TIMES, WHERE "MEAN" INDICATES THE MEAN BEST FUNCTION VALUES FOUND IN THE LAST GENERATION. $c_3 \sim [0.05, -0.05]$ INDICATES THAT c_3 LINEAR DECREASES FROM 0.05 TO -0.05 DURING PSO SEARCH PROCESS

Result		Global star structure			Local ring structure		
		f_5	f_9	f_{11}	f_5	f_9	f_{11}
Standard PSO	best	0.003306	98.0000	**0**	0.021183	139.2511	9.7699E-15
	mean	3.763743	170.2000	18.0558	0.078473	181.7243	1.7852E-14
$c_3 = 0.4$	best	15.0484	502.3478	156.2566	10.8490	474.6455	105.0195
	mean	44.2900	533.4721	233.7385	26.8031	550.2269	183.7515
$c_3 = 0.2$	best	0.019737	377.3507	0.000415	0.015253	366.2699	0.166868
	mean	15.8242	413.9585	54.3862	0.029428	390.5112	0.416948
$c_3 = 0.1$	best	0.001361	210.4438	**0**	0.006296	247.5167	**0**
	mean	7.681045	272.0944	52.3284	0.013490	272.3373	0.000193
$c_3 = 0.05$	best	0.001999	132.0845	**0**	0.012057	163.8312	**0**
	mean	8.593309	195.9888	54.2096	0.020866	194.9040	0.000201
$c_3 = -0.05$	best	9.2408E-05	27.1167	**0**	0.000468	99.77662	**0**
	mean	0.001132	62.5534	0.005293	0.002397	134.5600	2.2764E-06
$c_3 = -0.1$	best	3.8991E-05	36.2792	**0**	3.0822E-05	90.1938	**0**
	mean	0.000707	83.3589	0.004473	0.000527	117.1118	8.8288E-06
$c_3 = -0.2$	best	1.9932E-05	28.0569	**0**	8.8024E-06	75.1486	**0**
	mean	0.000549	59.0719	0.004092	0.000486	102.8416	**0**
$c_3 = -0.4$	best	5.1783E-05	**15.2198**	**0**	**4.0767E-06**	**84.2474**	**0**
	mean	**0.000358**	**37.4257**	0.003220	**0.000362**	**102.2737**	**0**
$c_3 = -0.8$	best	0.070678	347.6730	0.022260	0.006103	324.0427	**0**
	mean	2.289617	392.8169	42.3895	0.029982	358.4001	0.213375
$c_3 = 0.05 \sim -0.05$	best	0.000418	32.5417	**0**	0.002921	153.0000	**0**
	mean	0.002872	82.3480	0.012007	0.012222	185.6194	0.000347
$c_3 = 0.1 \sim -0.1$	best	0.000453	19.3198	**0**	0.002940	137.6087	**0**
	mean	0.002000	60.7679	0.012391	0.007120	192.0868	1.2374E-06
$c_3 = 0.2 \sim -0.2$	best	0.000216	21.2966	**0**	0.001008	159.4919	**0**
	mean	0.001909	124.3974	0.009576	0.003633	184.7151	3.0085E-06
$c_3 = 0.4 \sim -0.4$	best	4.7276E-05	84.0947	**0**	0.001177	180.9459	**0**
	mean	0.001344	187.7145	0.012821	0.001970	255.3435	1.9579E-05
$c_3 = 0.8 \sim -0.8$	best	**1.1010E-06**	247.4993	**0**	7.3074E-05	258.3948	**0**
	mean	0.001127	307.9561	**0.001575**	0.001389	320.8334	0.000225