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AN INTRODUCTION TO COMPUTER-AIDED DESIGN

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CONTENTS

chapter 5

PROBLEMS AS THEY CONFRONT THE ENGINEER

5.1 IOWA CADET: AN ALGORITHM FOR COMPUTER-AIDED DESIGN

The advent of computer hardware allowing the substitution of a time-sharing mode of operation for a batch system of program processing so dramatically reduces turnaround time in the debugging process that engineering designers can conveniently use the computer and obtain usable answers very rapidly. The extent to which engineering designers will avail themselves of this tool will be great if convenient alogrithms of use are available.

An algorithm which permits expeditious use of a computer facility as well as the fruits of previous labor is the previously mentioned IOWA CADET. The software elements are a battery of search programs, a library of useful engineering-analysis routines, and the convenient documentation of these programs.

In the beginning just a few search programs will suffice, the most important being the multidimensional searching program that will crawl over a merit hypersurface and discover the location of the largest ordinate to the degree of accuracy specified by the designer. Gradient, pattern, and grid-searching technique are useful. Such subprograms are documented in Appendix 2.

The multidimensional search programs can be used for one-dimensional searches; however, there exist much more effective one-dimensional search techniques. The golden section search technique is very efficient, and such a subprogram is documented in Appendix 2.

The design library consists of analysis routines which the designer finds useful in his day-to-day work. These subroutines are simply automated conditional responses to familiar situations. The battery of subroutines useful in a design room for agricultural implements differs markedly from that useful in a structural design office. Since useful routines vary greatly from job situation to job situation, it is false hope to become complacent and await the availability of proprietary routines (developed by computing machinery companies) that will solve your particular engineering design problems. Only broad, general-interest routines can find a market. Only the designer knows what is important to *him* in his work. Only the designer can envision the library that will be of value to him. *If it is to exist, only he can develop it or supervise its creation.*

Given the existence of search programs and a design library, how can they be utilized?

1. The designer writes an EXECUTIVE PROGRAM to read in all necessary information and to document the problem and its output information. This EXECUTIVE PROGRAM calls the SEARCH subroutine.
2. The SEARCH subroutine is a library program that will crawl over a multidimensional hypersurface and discover the location and magnitude of the largest ordinate to the hypersurface. The SEARCH program does this by calling the subprogram MERIT in such a way as to find the largest ordinate of merit.
3. The designer writes the MERIT subroutine which generates alternative solutions to his problem, tests them against the constraints of the problem, and, if they are satisfactory, calculates their merit according to the rules laid down by the designer. In generating alternatives and calculating the solution figure of merit, the MERIT subroutine calls design library subroutines.

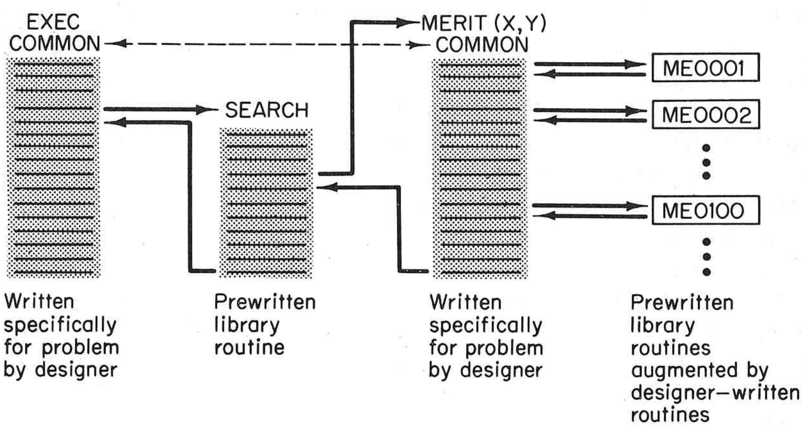

FIG. 63. IOWA CADET* algorithm for simple synthesis.

Computer Augmented Design Engineering Technique.

4. The DESIGN subroutines are really analysis subprograms that have been written, checked out, found useful, and documented in a convenient manner in order to facilitate ease of use. As problems require new subroutines, these are written by the designer, used, and then documented and deposited in the library for subsequent utilization. Thus a nucleus design library grows in useful directions and as time elapses, grows in value inasmuch as it is custom-built to serve the needs of a particular designer of a specific design office.

In the beginning of an operation, such as in an engineering school seeking to provide a computer-aided design experience for its students, the initial design library will be of modest scope and prewritten by the design faculty. As larger problems are solved and new routines added to the library, an impressive capability can be amassed for student and faculty use. Figure 63 illustrates in its simplest form the algorithm just described.

5.2 PROPER PROPORTIONS FOR A CAN

As our first example of the use of the IOWA CADET algorithm for the computer-aided solution of an engineering problem, let us consider the task of determining the proportions of the right circular cylindrical container fabricated of sheet metal that will have the least mass for a specified interior volume. For a given gauge of sheet metal this is equivalent to seeking a minimum of sheet metal area. This problem is posed simply in order to exemplify the use of the algorithm.

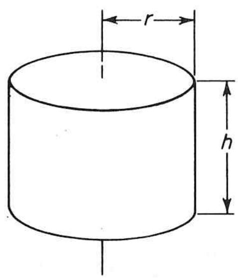

FIG. 64. The essential geometry of a right circular cylinder.

In order to obtain an order-of-magnitude solution, let us initially ignore the excess of metal that must be provided in order to dish the ends for strength and crimp the ends to the cylindrical sleeve. We shall also ignore the overlap necessary for the soldered seam, the thickness of the metal itself, and the edge allowances for crimping. Figure 64 indicates the essential geometry of the can. The volume is given by

$$V = r^2h$$

and for a specified volume V the altitude h is determined from

$$h = \frac{V}{\pi r^2}$$

The lateral area of the can is composed of the sleeve area plus the area of the two circular ends. This lateral area A is given by

$$A = 2\pi r^2 + 2\pi rh$$

The area may be expressed as a function of the single independent variable r by substituting for h in the previous equation

$$A = 2\pi r^2 + \frac{2V}{r}$$

In order to minimize the area with respect to the independent variable r, we differentiate the expression for A with respect to r and equate to zero. Thus

$$\frac{dA}{dr} = 4\pi r - \frac{2V}{r^2} = 0$$

from which

$$r^3 = \frac{V}{2\pi}$$

and the optimal radius is given by

$$r = \left(\frac{V}{2\pi}\right)^{1/3}$$

The associated height is

$$h = \frac{V}{\pi r^2} = \frac{V(2\pi)^{2/3}}{\pi V^{2/3}} = 2r \tag{5.1}$$

The optimal proportions provided by this simple mathematical model, which ignores fabrication allowances, indicate that an equality exists between the altitude h and the diameter d. We shall now improve the modeling of the physical situation by introducing the following additional parameters:

r_1 = dishing allowance on the container ends, in.
r_2 = crimping allowance on the container ends, in.
r_3 = seam allowance for forming the cylindrical sleeve, in.
h_1 = crimping allowance for attaching an end, in.

Figure 65 indicates the essential geometry of this more realistic model. Ignoring the corner chamfers on the sleeve blank and the thickness of the sheet metal itself, we find that for our revised model the interior volume is given by

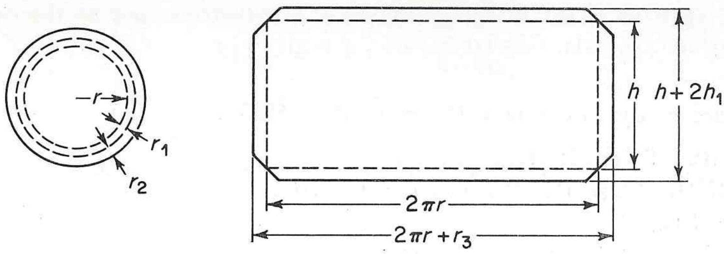

FIG. 65. The geometry of parts of a right circular cylindrical sheet-metal can.

$$V = \pi r^2 h$$

and the area of sheet metal provided to fabricate a can is predicted by

$$A = 2\pi(r + r_1 + r_2)^2 + (2\pi r + r_3)\,(h + 2h_1)$$

Let us now suppose that the numerical work associated with the problem is sufficiently complex to require computer assistance. We identify the tactical figure of merit function as the volume-to-area ratio V/A,

$$\frac{V}{A} = \frac{\pi r^2 h}{2\pi(r + r_1 + r_2)^2 + (2\pi r + r_3)(h + 2h_1)} \qquad \text{(merit function)}$$

which we will seek to maximize. For a given volume of container there is a functional constraint which does not allow the altitude of the can to be independent of the radius. Thus the equation

$$h = \frac{V}{\pi r^2} \qquad \text{(functional constraint)}$$

constitutes the functional constraint for this problem.

The search for the optimal V/A ratio is carried out over a one-dimensional domain wherein the radius r is the independent variable. There may or may not be regional constraints. We shall reject solutions with a negative radius and declare

$$0 \leq r \leq 10 \qquad \text{(regional constraint)}$$

as the regional constraint.

Inasmuch as we are faced with a one-dimensional search problem, we shall choose the efficient golden section search method and utilize the IOWA CADET library subroutine GOLD1. The documentation of GOLD1 in Appendix 2 indicates that the subroutine calls another subroutine MERIT1 (X, Y) in carrying out the process of reducing the interval of uncertainty from a span of 10 in., $0 \leq r \leq 10$, to a specified fraction of this initial interval.

We are ready to compose the subroutine MERIT1(R, Y)

```
      SUBROUTINE MERIT1(R, Y)
      COMMON VOL, R1, R2, R3, H1, H, AREA
      PI = 3.14159
      H = VOL/(PI*R*R)
      AREA = 2.*PI*(R + R1 + R2)*(R + R1 + R2)
     1         + (2.*PI*R + R3)*(H + 2.*H1)
      Y = VOL/AREA
      RETURN
      END
```

The subroutine simply calculates the figure of merit Y, and in order to do so requires knowledge of the radius R (which comes from the search routine GOLD1 through the call list) and the voulme VOL. The fabrication allowances R1, R2, R3, and H1 are supplied to MERIT1 from the executive program via the COMMON statement. Since for documentation reasons we desire knowledge of the altitude H and the surface area AREA in the executive program, the values of H and AREA are passed back to the executive program, via the COMMON statement. Hence the declarative statement

```
COMMON VOL, R1, R2, R3, H1, H, AREA
```

in the subroutine MERIT1.

We have remaining the task of composing the executive program for this problem. It should perform the following functions:

1. State the problem.
2. Define the variables.
3. Read in necessary information.
4. Pass information to MERIT1.
5. Call subroutine GOLD1, which searches the domain of the figure of merit, reducing the interval of uncertainty to desirable size and reporting results of the search.
6. Document input information.
7. Document output information.
8. The executive program should be written in such a way so that it can be used to solve many problems involving variation of the parameters, hence all parametric information should be read from cards.

An examination of the executive program which follows shows that these requirements have been met.

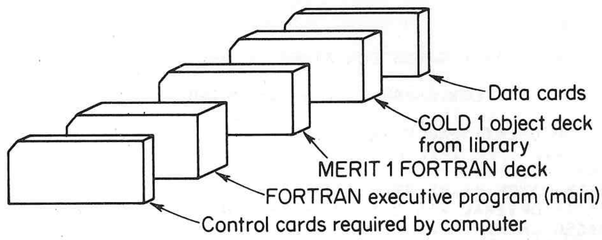

FIG. 66. Assembly of punch card decks to solve can problem.

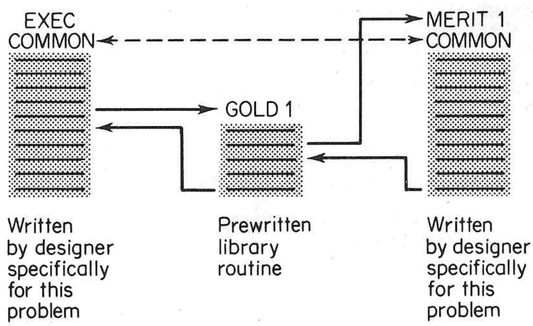

FIG. 67. IOWA CADET program structure for can problem.

The program deck is assembled as shown in Fig. 66, which represents in card form the program structure depicted in Fig. 67. The problem is now run on the computer.

```
C      OPTIMAL PROPORTIONS FOR A RIGHT CYLINDRICAL CONTAINER
C
C      NOMENCLATURE
C
C      VOL=VOLUME OF CONTAINER, IN3
C      H=HEIGHT OF CONTAINER, IN
C      R=RADIUS OF CONTAINER, IN
C      D=DIAMETER OF CONTAINER, IN
C      R1=DISHING ALLOWANCE ON CONTAINER ENDS, IN
C
C      R2=CRIMPING ALLOWANCE ON CONTAINER ENDS, IN
C      R3=SEAM ALLOWANCE FOR FORMING CYLINDRICAL SHELL, IN
C      H1=CRIMPING ALLOWANCE FOR ATTACHING END, IN
C      AREA=AREA OF SHEET METAL USED IN FABRICATION, IN2
C
       COMMON VOL,R1,R2,R3,H1,H,AREA
C
C      ..... READ INPUT DATA .....
C
     5 READ(1,2)VOL,R1,R2,R3,H1,RHIGH,RLOW,F
     2 FORMAT(8F10.5)
       I=1
C
C      ..... INITIATE GOLDEN SECTION SEARCH FOR MAXIMUM VOLUME TO     .....
C      ..... AREA RATIO WHICH IS THE FIGURE OF MERIT.  A MAXIMUM      .....
C      ..... VOLUME  TO AREA RATIO IMPLIES A MINIMUM SHEET METAL      .....
```

```
C       ..... AREA FOR A GIVEN CONTAINER VOLUME                              .....
C
        CALL GOLD1(I,RLOW,RHIGH,F,Y,X,XLOW,XHIGH,J5)
C
C       ..... DOCUMENT SEARCH RESULTS .....
C
        D=2.*X
        WRITE(3,1)VOL,R1,R2,R3,H1
      1 FORMAT('1OPTIMAL PROPORTIONS FOR RIGHT CIRCULAR CYLINDRICAL CAN',/
       1,' BASED ON MINIMUM WEIGHT',//,
       2' INPUT DATA',//,
       3' VOLUME OF CONTAINER, IN3, ............................',F10.5,/,
       4' DISHING ALLOWANCE ON CONTAINER ENDS, IN, .............',F10.8,/,
       5' CRIMPING ALLOWANCE ON CONTAINER ENDS, IN, ............',F10.8,/,
       6' SEAM ALLOWANCE FOR FORMING CYLINDRICAL SHELL, IN, ....',F10.8,/,
       7' CRIMPING ALLOWANCE FOR ATTACHING END, IN, ............',F10.8)
        WRITE(3,3)H,D,AREA
      3 FORMAT(/,' OUTPUT DATA',//,
       1' HEIGHT OF CAN, IN, ...................................',F10.5,/,
       2' DIAMETER OF CAN, IN, .................................',F10.5,/,
       3' AREA OF SHEET METAL USED, IN2,........................',F10.5)
C
C       ..... FOR COMPARISON INDICATE OPTIMAL PROPORTIONS USING THE      .....
C       ..... SIMPLIFIED THEORY AND DETERMINE THE SHEET METAL AREA       .....
C       ..... NEEDED TO FABRICATE A REAL CAN USING THESE PROPORTIONS     .....
C
        RAD=(VOL/(2.*3.14159))**0.333333
        HI=VOL/(3.14159*RAD*RAD)
        H=HI
        DIA=2.*RAD
C
C       ..... CALL MERIT1 TO COMPUTE SHEET METAL AREA, VALUE OF          .....
C       ..... WHICH IS RETURNED VIA COMMON.                              .....
C
        CALL MERIT1(RAD,YY)
        WRITE(3,4)HI,DIA,AREA
      4 FORMAT(/,' OPTIMAL PROPORTIONS IGNORING SEAMS, TABS AND DISHING',/
       1/,
       2' HEIGHT OF CAN,IN, ....................................',F10.5,/,
       3' DIAMETER OF CAN, IN, .................................',F10.5,/,
       4' AREA OF SHEET METAL USED, IN2, .......................',F10.5)
        GO TO 5
        END

       SUBROUTINE MERIT1(R,Y)
       COMMON VOL,R1,R2,R3,H1,H,AREA
       PI=3.14159
       H=VOL/(PI*R*R)
       AREA=     2.*PI*(R+R1+R2)*(R+R1+R2)+(2.*PI*R+R3)*(H+2.*H1)
       Y=VOL/AREA
       RETURN
       END
```

CONVERGENCE MONITOR SUBROUTINE GOLD1

N	Y1	Y2	X1	X2
2	0.6069737E 00	0.3283069E 00	0.3825839E 01	0.6184157E 01
3	0.7467008E 00	0.6069737E 00	0.2368318E 01	0.3825839E 01
4	0.6163253E 00	0.7467008E 00	0.1467520E 01	0.2368318E 01
5	0.7467008E 00	0.7216181E 00	0.2368318E 01	0.2925039E 01
6	0.7263275E 00	0.7467008E 00	0.2024242E 01	0.2368318E 01
7	0.7467008E 00	0.7441732E 00	0.2368318E 01	0.2580965E 01
8	0.7427955E 00	0.7467008E 00	0.2236891E 01	0.2368318E 01
9	0.7467008E 00	0.7469471E 00	0.2368318E 01	0.2449539E 01
10	0.7469471E 00	0.7463315E 00	0.2449539E 01	0.2499742E 01
11	0.7470390E 00	0.7469471E 00	0.2418517E 01	0.2449539E 01

```
12   0.7469819E 00    0.7470390E 00    0.2399343E 01    0.2418517E 01
13   0.7470390E 00    0.7470303E 00    0.2418517E 01    0.2430367E 01
14   0.7470275E 00    0.7470390E 00    0.2411193E 01    0.2418517E 01
15   0.7470390E 00    0.7470397E 00    0.2418517E 01    0.2423043E 01
16   0.7470397E 00    0.7470377E 00    0.2423043E 01    0.2425840E 01
17   0.7470397E 00    0.7470397E 00    0.2421314E 01    0.2423043E 01
```

```
LEFTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY .........    0.9999998E-02
RIGHTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY ........    0.1000000E 02
FRACTIONAL REDUCTION OF INTERVAL OF UNCERTAINTY ......    0.9999999E-03
EXTREME ORDINATE DISCOVERED DURING SEARCH ............    0.7470397E 00
ABSCISSA OF EXTREME ORDINATE .........................    0.2421314E 01
NEW LEFTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY .....    0.2418517E 01
NEW RIGHTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY ....    0.2425840E 01
NUMBER OF FUNCTION EVALUATIONS EXPENDED IN SEARCH ....               17
OPTIMAL PROPORTIONS FOR RIGHT CIRCULAR CYLINDRICAL CAN
BASED ON MINIMUM WEIGHT

INPUT DATA

VOLUME OF CONTAINER, IN3, ......................... 100.00000
DISHING ALLOWANCE ON CONTAINER ENDS, IN, ...........0.09999996
CRIMPING ALLOWANCE ON CONTAINER ENDS, IN, ..........0.19999999
SEAM ALLOWANCE FOR FORMING CYLINDRICAL SHELL, IN, ....0.29999995
CRIMPING ALLOWANCE FOR ATTACHING END, IN, ..........0.09999996

OUTPUT DATA

HEIGHT OF CAN, IN, ................................   5.42936
DIAMETER OF CAN, IN, ..............................   4.84263
AREA OF SHEET METAL USED, IN2,..................... 133.86168

OPTIMAL PROPORTIONS IGNORING SEAMS, TABS AND DISHING

HEIGHT OF CAN,IN, .................................   5.03081
DIAMETER OF CAN, IN, ..............................   5.03079
AREA OF SHEET METAL USED, IN2, .................... 134.04379
```

The convergence monitor for subroutine GOLD1 has been actuated by the call (I = 1). Figure 68 indicates the figure of merit curve for the particular case where

$V = 100$ cu in.
$R1 = 0.1$ in.
$R2 = 0.2$ in.
$R3 = 0.3$ in.
$H1 = 0.1$ in.

The height of the can is 5.429 in. and the diameter is 4.842 in. The area of sheet metal used is 133.86 sq in.

Note that the executive program also figured the optimal proportions for the container ignoring fabrication allowances. Observe also that the simplified model of Eq. (5.1) gives equal height and diameter. The rather mild discrepancy between the two models (the optimal value of diameter) is due to the peaking of the two merit functions near the same abscissa. The corresponding ordinates, however, differ to a greater degree. In study-

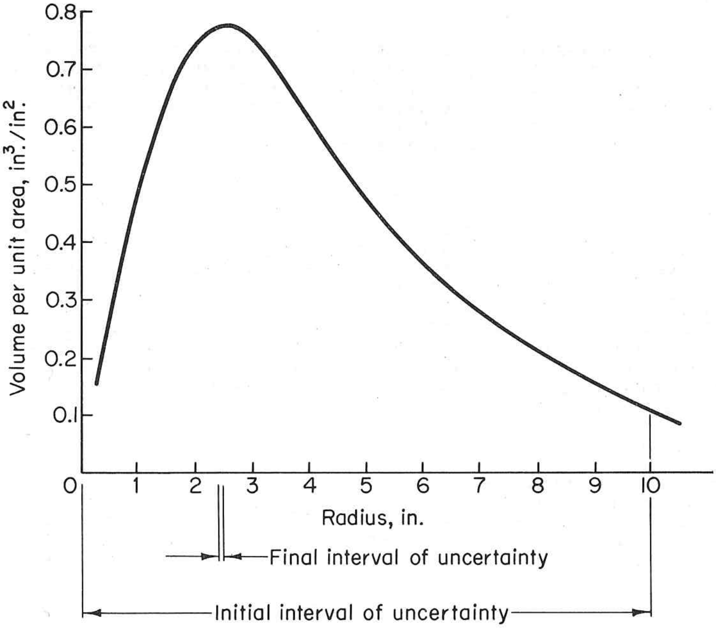

FIG. 68. The figure of merit for the can problem indicating the initial and final interval of uncertainty.

ing Fig. 69 notice that the displacement of the practical optimum from the simple optimum is consistently toward the smaller radius, giving the practical optimal configuration a slightly larger height. The good agreement between the optimal radius in the two models should not mislead one into expecting the necessary area to produce a can of a specified volume with similar accuracy, A study of Fig. 69 will reveal why.

An inspection of the documentation of the computer program reveals that numbers such as R1, R2, R3, and H1 are not expressed precisely. This occurs because the IBM 360 computer is a hexidecimal machine operating with base 16 arithmetic. There is no concise representation for a number like 0.1 and the machine carries it as 0.09999996. By controlling format differently we can cause roundup to occur and read the printout as 0.100. A nonengineering reader might be puzzled if confronted with the number 0.09999996 after reading 0.1.

5.3 THE PROPER GEAR RATIO FOR A RAPID TRANSIT CAR

Consider the following situation: A city contemplating the opening of a rapid transit line has engaged a consulting engineering firm to recommend equipment for the contemplated new line. The project engineer is con-

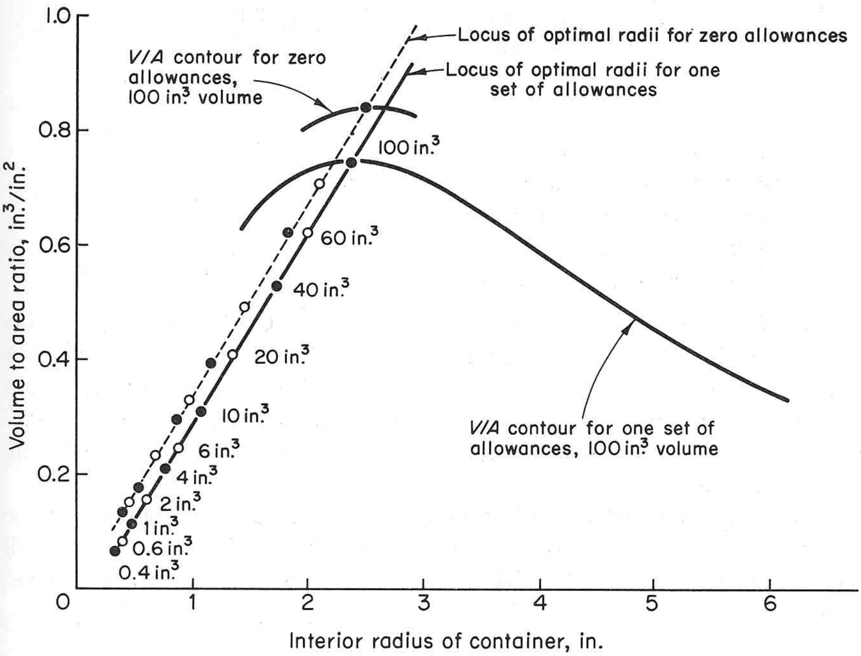

FIG. 69. Comparison of solutions obtained from simple mathematical model and the revised model incorporating fabrication allowances.

sidering the performance that can be obtained with rapid transit equipment recently built for another city. His thought is that costs will be considerably less if the supplier builds equipment using the principal jigs and plans already in existence. A simple variable with which to modify performance and not incur great increases in price is the gear ratio between the traction motors and the axles of the powered wheels. Failing to obtain a satisfactory resolution of problems in this manner, the project engineer has other alternatives, such as changing both the gear ratio and the traction motors.

The project engineer assigns you the problem of determining the optimal gear ratio for a rapid transit car which results in a minimum of elapsed time between the start of motion at one station and the cessation of motion at the next station. Phase 1 of the problem is to determine this ratio for a single-car train running on level tangent track with equally spaced station stops. The solution to phase 1 will be considered here.

Resistance to motion of a railway car occurs because of rail-wheel rolling friction, axle journal-bearing friction, curve resistance, grade resistance, and air resistance. For our problem resistance due to grade and curvature of track is zero. Resistance in pounds per ton of car weight is given by the Davis formula modified for the fact that this car is not following others, but is the first car of a one-car train:

$$r = 1.3 + \frac{29}{w} + 0.03v + 0.0024 \frac{AV^2}{wn} \tag{5.2}$$

where

r = resistance, lbf/ton of car
w = average weight carried per axle, tons
V = train speed, miles per hour
n = number of axles
A = cross-sectional area, square feet

In Sec. 2.6 a preliminary analysis of the dynamics of this problem was made. The plan of attack begins with the recognition that several subroutines will be needed to conveniently write the merit function. Such subroutines would ordinarily be in the design library of a railway equipment manufacturer, but we shall compose simplified versions for the purpose of broadening our understanding.

The subroutine DRAG will be needed to determine the retarding force on the car corresponding to the present velocity of the vehicle. The call will be DRAG (V, A1, W1, RETARD), where the arguments in call list represent

V = vehicle speed, miles per hour
A1 = cross-sectional area, square feet
W1 = vehicle weight, tons
RETARD is the drag force in pounds.

A COMMON statement will be required, but its contents are presently unknown. We can expect the cross-sectional area A and the vehicle weight W to be in the common declaration, and to prevent the compiler from rejecting the program because the same quantity appears both in the subroutine call and in the COMMON statement, the dummy variables A1 and W1 are placed in the call list. Thus we write

```
SUBROUTINE DRAG (V, A1, W1, RETARD)
COMMON (to be completed at end of programming)
A = A1
W = W1
WT = W/4.
S = 1.3 + 29./WT + 0.03*V + 0.0024 *A*V*V/(4.*WT)
RETARD = S*W
RETURN
END
```

We note that we need not have any variables in COMMON which are used in the subroutine DRAG, inasmuch as all necessary values are entered via the call list.

The next subroutine of use would be one which, if given the speed of the vehicle, would return the distance necessary to stop the car and the elapsed time in braking the car to a halt. This subroutine will be given the name BRAKE, and the call will be BRAKE (V, BDIST, BTIME) where

V = vehicle speed in miles per hour
BDIST = distance necessary to stop the vehicle in feet
BTIME = time duration of braking in seconds

The service braking pressure will be insufficient to cause a wheel skid. The coefficient of adhesion utilized during service braking will be designated COEFF2. In this simplified example the braking will be presumed to be a constant deceleration event and the programming proceeds as follows:

```
SUBROUTINE BRAKE (V, BDIST, BTIME)
COMMON (to be completed at the end of programming)
VEL = V*88./60.
BDIST = VEL*VEL/(2.*COEFF2*32.174)
BTIME = SQRT(2.*BDIST/(COEFF2*32.174))
RETURN
END
```

We note that it is necessary to have the parameter COEFF2 in the common statement since it does not appear in the call list.

The next useful subroutine will be named TMOTOR, and its purpose is to return the magnitude of the traction motor torque corresponding to any vehicle speed. The motors utilized will be the approximate equivalent of General Electric 1240, which in their running characteristic exhibit the following relationship between motor torque (in inch-pounds) and the angular velocity (in radians per second) of the armature shaft:

$$T = \frac{29.9(10)^9}{\omega^{3.13}}$$

Below an angular velocity of 106 radians per second, current limiting devices will maintain the torque at a level of 12150 inch-pounds. The programming proceeds as follows:

```
  SUBROUTINE TMOTOR (V, R1, RATIO1, TM)
  COMMON (to be completed at the end of programming)
  R = R1
  RATIO = RATIO1
  VEL = V*88./60.
  OMEGA = 12.*VEL*RATIO/R
  IF(OMEGA - 106.)2, 1, 1
1 TM = 29.9E09/(OMEGA**3.13)
```

```
  RETURN
2 TM = 12150.
  RETURN
  END
```

We can expect the gear ratio RATIO and the wheel radius R to appear in the COMMON declaration, hence the dummy variables RATIO1 and R1 are employed in the call list to prevent the compiler from rejecting the program (for containing the same variable in the subroutine call list and the COMMON declaration list). We note that the COMMON declaration need not have been present in this subprogram.

The next useful subprogram will be named EFFORT and will be called by EFFORT (RATIO1, TM, R1, TE). The tractive effort can be expressed in terms of the motor torque and the gear ratio and wheel radius as

```
TE = 4.*RATIO*TM/R
```

and we program as follows:

```
  SUBROUTINE EFFORT (RATIO1, TM, R1, TE)
  COMMON (to be completed at the end of programming)
  RATIO = RATIO1
  R =R1
  TE = 4.*RATIO*TM/R
  IF(TE – W*2000.*COEFF1)1, 2, 2
2 TE = W*2000.*COEFF1
  L = 1
1 RETURN
  END
```

It is necessary to note that when the tractive effort exceeds the adhesive force available at the rail (about 20 per cent of car weight), the wheels will slip, unless anti-wheelslip devices are brought into play to limit the motor torque. The setting of integer L to unity alerts the engineer to the fact that anti-wheelslip devices are required by a particular gear ratio. The COMMON declaration must return L to the executive program, and will also contain RATIO and R.

Now that these incidental subroutines are written, we can plan the general arrangement of the programming elements to solve the problem. Fundamentally we expect the plot of the reciprocal of the elapsed time between the stations (start to stop) and the gear ratio to exhibit a maximum such as depicted by Fig. 70. When the gear ratio is very large, very low average and balancing speeds result, and the elapsed time is consequently very large. When the gear ratio is small, the acceleration is very small and again the elapsed time is very large. Somewhere within the domain of positive gear ratios there is one that corresponds to minimum elapsed time (and hence maximum ordinate in Fig. 70). A golden section search will be effective in this one-dimensional situation. We select library routine GOLD2

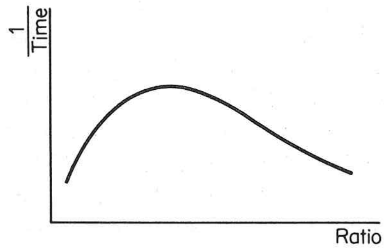

FIG. 70. The general trend of the merit function for the rapid transit car problem.

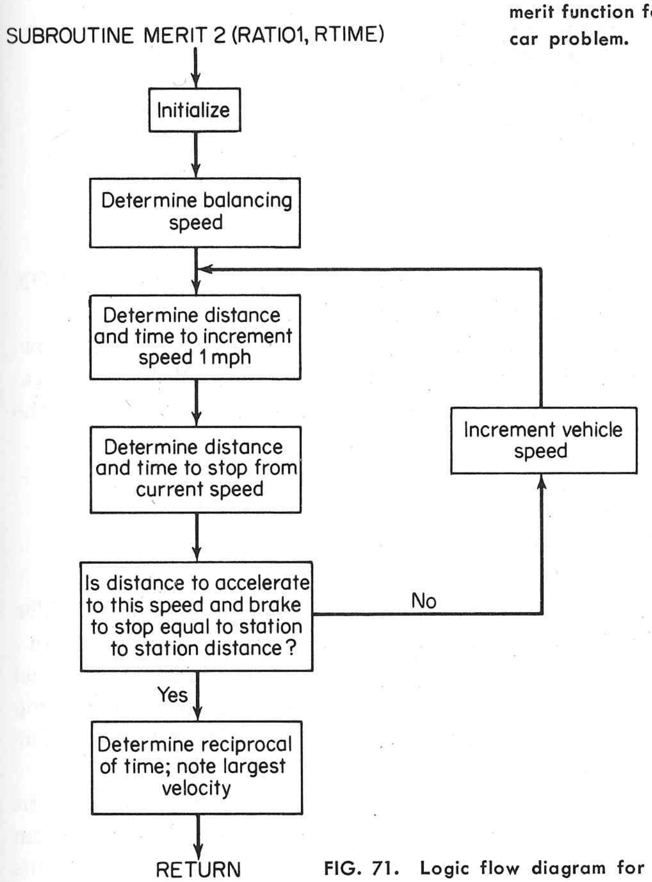

FIG. 71. Logic flow diagram for subroutine MERIT2.

which calls subroutine MERIT2. For every tendered value of the gear ratio, MERIT2 must return an ordinate RTIME, which is the reciprocal of the elapsed time between stations. The purpose of MERIT2 is therefore clear, and we structure it as indicated in Fig. 71.

The balancing speed corresponding to a given gear ratio must be determined. At this speed the tractive effort TE is equal to the drag force RETARD. If the accelerating force FACC = TE − RETARD, then clearly when FACC is zero, the associated velocity is the balancing speed. Treating FACC as a function of vehicle speed V, we utilize another library subroutine ROOT1 which will locate zero places in a function within a specified interval of uncertainty. In order to effectively search for the zero place, the subroutine ROOT1 calls another subroutine EQUAT1 which must return to ROOT1 the ordinate FACC corresponding to a tendered value of V. Such a subroutine could be structured as follows:

```
SUBROUTINE EQUAT1 (V, FACC)
COMMON (to be completed at the end of programming)
CALL DRAG (V, A, W, RETARD)
CALL TMOTOR (V, R, RATIO, TM)
CALL EFFORT (RATIO, TM, R, TE)
FACC = TE – RETARD
RETURN
END
```

For this subroutine values must be supplied to it through the COMMON declaration. These include A, W, R, RATIO.

The structure of the subroutine MERIT2 is displayed as written below. The programs are arranged as indicated in Fig. 72. The COMMON declaration is composed of the common needs of all these subroutines and the executive program.

```
  COMMON COEFF1, COEFF2, R, W, A, RATIO, L, VBIG,
1 BSPEED, VLOW, VHIGH, F, MPRINT, DIST
```

A complete program follows, with convergence monitor printout. The single-car train weighs 29 tons, has a frontal area of 80 ft^2, and uses 32-in.-diameter wheels. It is running on level tangent track with limiting rail-wheel adhesion of 0.20 and braking adjusted to demand 0.15 adhesion during service stops. There are 4 d-c traction motors similar to GE 1240. The distance between stations is 4,000 ft.

The optimal gear ratio is 2.61. The average speed between stations is 44.2 mph and involves an elapsed time of 61.8 seconds. The maximum speed obtained between stations is 65.0 mph. The balancing speed at this ratio is 70.7 mph.

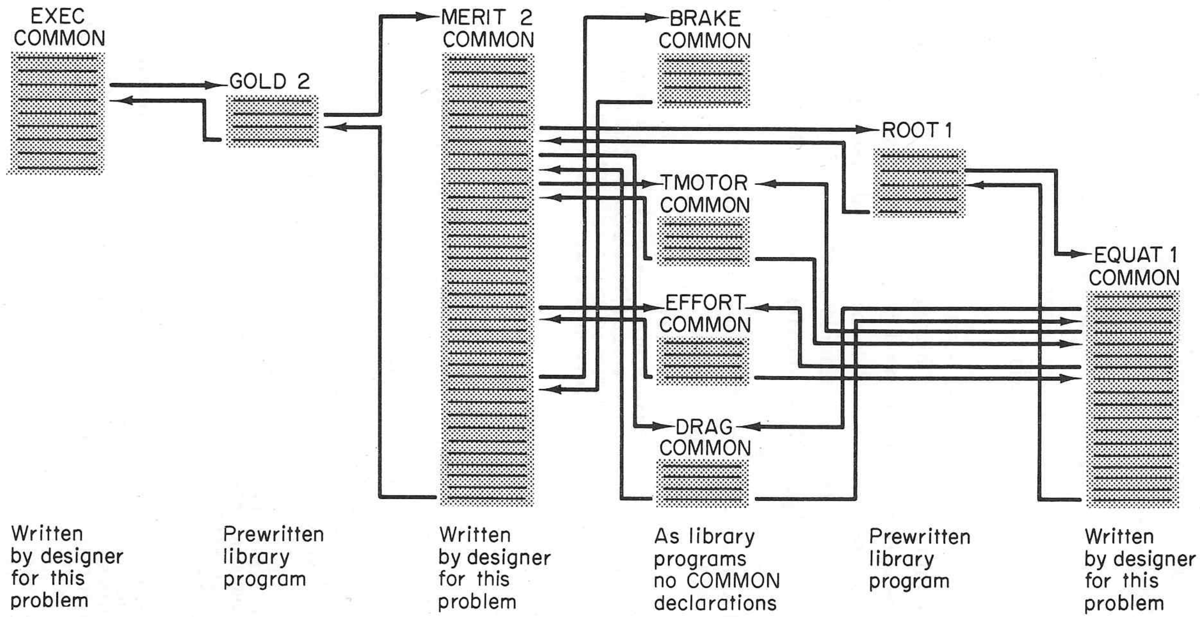

FIG. 72. Example of program arrangement for the solution of the rapid transit car gear-ratio problem using the IOWA CADET algorithm.

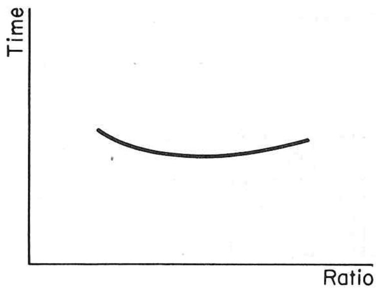

FIG. 73. Variation of start-to-stop time with gear ratio.

The opportunity is present to introduce multicar trains and curvature, grade, and speed restrictions, as well as variable interstation distances. In short it becomes possible to model the line and study performances. We are close to a simulation, a valuable contribution which the computer makes quite easily. Figure 73 can be constructed to determine how sensitive performance is to gear value.

This problem shows a one-dimensional optimization problem in which the computer is helpful to the engineer.

```
C      EXECUTIVE PROGRAM FOR DETERMINING THE OPTIMAL GEAR RATIO BETWEEN
C      THE TRACTION MOTOR AND THE WHEELS ON A ELECTRICALLY PROPELLED RAPID
C      TRANSIT CAR IN ORDER TO MINIMIZE START-TO-STOP TIME BETWEEN EQUALLY
C      SPACED STATIONS ON LEVEL TANGENT TRACK.                      MISCHKE
C
C      VARIABLES
C
C      V=VELOCITY OF VEHICLE, MILES/HR
C      W=WEIGHT OF VEHICLE; TONS
C      A=CROSS SECTIONAL AREA OF CAR, SQ FT
C      RATIO=GEAR RATIO BETWEEN TRACTION MOTOR AND WHEELS
C      R=RADIUS OF WHEELS
C
C      L=0  NO WHEEL SLIP DURING ACCELERATION
C       =1  WHEEL SLIP DURING ACCELERATION
C      VLOW=LOWER EXTENT OF V-DOMAIN
C      VHIGH=HIGHER EXTENT OF V-DOMAIN
C      F=FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY DESIRED
C
C      TM=TRACTION MOTOR TORQUE, IN-LBF
C      TE=TRACTIVE EFFORT OF VEHICLE, LBF
C      OMEGA=ANGULAR SPEED OF MOTOR, RAD/SEC
C      WT=VEHICLE WEIGHT, TONS/AXLE
C      RETARD=DRAG FORCE ON VEHICLE, LBF
C
C      COEFF1=ADHESION LIMIT BETWEEN RAILS AND WHEELS
C      COEFF2=WORKING ADHESION DURING SERVICE BRAKE APPLICATION
C      RLOW=LOWER EXTENT OF RATIO-DOMAIN
C      RHIGH=HIGHER EXTENT OF RATIO DOMAIN
C      RTIME=RECIPROCAL OF ELAPSED TIME STATION TO STATION,  1./SEC
C      DIST=DISTANCE BETWEEN STATIONS, FEET
C
       COMMON COEFF1,COEFF2,R,W,A,RATIO,L,VBIG,BSPEED,VLOW,VHIGH,F,
      1 MPRINT,DIST
       DIMENSION IABC(20)
C
C      ..... READ INPUT DATA .....
C
     1 READ(1,2)W,A,R,COEFF1,COEFF2,DIST
     2 FORMAT(5F10.5,F10.0)
```

```
    3 READ(1,4)VLOW,VHIGH,RLOW,RHIGH,F,MPRINT
    4 FORMAT(5F10.5,I5)
    5 READ(1,6)(IABC(I),I=1,20)
    6 FORMAT(20A4)
      L=0
C
C     ..... INITIATE GOLDEN SECTION SEARCH TO FIND THE GEAR RATIO  .....
C     ..... AT WHICH THE RECIPROCAL OF ELAPSED TIME BETWEEN STATIONS ...
C     ..... HAS AN EXTREME VALUE                                   .....
C
      CALL GOLD2(MPRINT,RLOW,RHIGH,F,RTIME,RATIO,RLO,RHI,J5)
C
C     ..... PREPARE OUTPUT AND PRINT DOCUMENTATION .....
C
      TIME=1./RTIME
      SSPEED=(DIST/5280.)/(TIME/3600.)
      WRITE(3,7)W,A,R,COEFF1,COEFF2,DIST
    7 FORMAT('1RAPID TRANSIT CAR OPTIMIZATION PROGRAM'//' INPUT DATA'//,
     1' WEIGHT OF CAR, TONS ................................',F15.2,/,
     2' FRONTAL AREA OF CAR, SQ FT ..........................',F15.2,/,
     3' RADIUS OF CAR WHEELS, INCHES ........................',F15.2,/,
     4' ADHESION LIMIT BETWEEN WHEEL AND RAIL ...............',F15.2,/,
     5' WORKING ADHESION VALUE DURING SERVICE BRAKING .......',F15.2,/,
     6' DISTANCE BETWEEN STATIONS, FEET  ....................',F15.3)
      WRITE(3,18)(IABC(I),I=1,20)
   18 FORMAT(/,20A4)
      IF(L)1,8,10
    8 WRITE(3,9)
    9 FORMAT(//,' NO WHEELSLIP DURING ACCELERATION'//)
      GO TO 12
   10 WRITE(3,11)
   11 FORMAT(//,' WHEELSLIP DURING ACCELERATION'//)
   12 WRITE(3,13)RATIO,TIME,SSPEED,VBIG,BSPEED
   13 FORMAT(
     1' OPTIMAL GEAR RATIO ..................................',F15.5,/,
     2' ELAPSED TIME BETWEEN STATIONS, SECONDS ..............',F15.2,/,
     3' AVERAGE SPEED BETWEEN STATIONS, MILES/HR ............',F15.2,/,
     4' MAXIMUM SPEED ATTAINED BETWEEN STATIONS, MILES/HR ....',F15.2,/,
     5' BALANCING SPEED AT THIS RATIO, MILES/HR .............',F15.2)
      GO TO 1
      END


      SUBROUTINE MERIT2(RATIO1,RTIME)
      COMMON COEFF1,COEFF2,R,W,A,RATIO,L,VBIG,BSPEED,VLOW,VHIGH,F,
     1 MPRINT,DIST
C
C     ..... INITIALIZE .....
C
      RATIO=RATIO1
      L=0
      ATIME=0.
      ADIST=0.
      VOLD=0.
      DELTA=1.
      VBIG=0.
      V=1.
C
C     ..... DETERMINE BALANCING SPEED FOR THIS RATIO USING ROOT1   .....
C
      MPRINT=0
      CALL ROOT1(MPRINT,VLOW,VHIGH,F,BSPEED,B2,B3,B4,J8,J9)
      MPRINT=1
C
C     ..... DETERMINE ACCELERATING FORCE, AND BY AN APPROXIMATE    .....
C     ..... INTEGRATION TECHNIQUE, ESTABLISH MERIT ORDINATE        .....
C
    6 IF(V-BSPEED)10,11,11
   11 DT=2.
      GO TO 12
   10 CALL DRAG(V,A,W,RETARD)
      CALL TMOTOR(V,R,RATIO,TM)
      CALL EFFORT(RATIO,TM,R,TE)
      CALL BRAKE(V,BDIST,BTIME)
```

```
      FACC=TE-RETARD
      DT=2000.*W/(32.174*FACC)
   12 AT=ATIME+DT
      AD=ADIST+(V+VOLD)*0.5*DT*5280./3600.
      DSTOS=AD+BDIST
      TSTOS=AT+BTIME
      IF(DIST-DSTOS)5,4,4
    5 IF(DELTA-0.01)9,7,7
    7 DELTA=DELTA/10.
      V=VOLD+DELTA
      GO TO 6
C
C     ..... ACCUMULATE ACCELERATION TIME AND ACCELERATION DISTANCE .....
C     ..... NOTE VOLD, THEN INCREMENT V                              .....
C
    4 ATIME=AT
      ADIST=AD
      VOLD=V
      V=V+DELTA
      GO TO 6
C
C     ..... CALCULATE MERIT ORDINATE AND NOTE LARGEST ATTAINED V     .....
C     ..... AT THIS RATIO.                                           .....
C
    9 RTIME=1./TSTOS
      VBIG=V
      RETURN
      END


      SUBROUTINE TMOTOR(V,R1,RATIO1,TM)
      COMMON COEFF1,COEFF2,R,W,A,RATIO,L,VBIG,BSPEED,VLOW,VHIGH,F,
     1 MPRINT,DIST
      R=R1
      RATIO=RATIO1
      VEL=V*88./60.
      OMEGA=12.*VEL*RATIO/R
      IF(OMEGA-106.)2,1,1
C
C     ..... MOTOR ON RUNNING CHARACTERISTIC .....
C
    1 TM=29.9E09/(OMEGA**3.13)
      RETURN
C
C     ..... MOTOR ON STARTING CHARACTERISTIC .....
C
    2 TM=12150.
      RETURN
      END


      SUBROUTINE EFFORT(RATIO1,TM,R1,TE)
      COMMON COEFF1,COEFF2,R,W,A,RATIO,L,VBIG,BSPEED,VLOW,VHIGH,F,
     1 MPRINT,DIST
      RATIO=RATIO1
      R=R1
C
C     ..... DETERMINE TRACTIVE EFFORT .....
C
      TE=4.*RATIO*TM/R
C
C     ..... DETERMINE IF WHEELS SLIP .....
C
      IF(TE-W*2000.*COEFF1)1,2,2
C
C     ..... WHEELSLIP EQUIPMENT OPERATIVE .....
C
    2 TE=W*2000.*COEFF1
      L=1
C
C     ..... WHEELS DO NOT SLIP .....
C
    1 RETURN
      END
```

```
      SUBROUTINE DRAG(V,A1,W1,RETARD)
      COMMON COEFF1,COEFF2,R,W,A,RATIO,L,VBIG,BSPEED,VLOW,VHIGH,F,
     1 MPRINT,DIST
      A=A1
      W=W1
      WT=W/4.
      S=1.3+29./WT+0.03*V+0.0024*A*V*V/(4.*WT)
      RETARD=S*W
      RETURN
      END


      SUBROUTINE EQUAT1(V,FACC)
      COMMON COEFF1,COEFF2,R,W,A,RATIO,L,VBIG,BSPEED,VLOW,VHIGH,F,
     1 MPRINT,DIST
C
C     ..... THIS SUBROUTINE DETERMINES THE ACCELERATING FORCE      .....
C     ..... FOR THE SUBROUTINE ROOT1.                              .....
C
      CALL DRAG(V,A,W,RETARD)
      CALL TMOTOR(V,R,RATIO,TM)
      CALL EFFORT(RATIO,TM,R,TE)
      FACC=TE-RETARD
      RETURN
      END


      SUBROUTINE BRAKE(V,BDIST,BTIME)
      COMMON COEFF1,COEFF2,R,W,A,RATIO,L,VBIG,BSPEED,VLOW,VHIGH,F,
     1 MPRINT,DIST
C
C     ..... THIS BRAKING SUBROUTINE BASED UPON A CONSTANT          .....
C     ..... DECELERATION APPROXIMATION.                            .....
C
      VEL=V*88./60.
      BDIST=VEL*VEL/(2.*COEFF2*32.174)
      BTIME=SQRT(2.*BDIST/(COEFF2*32.174))
      RETURN
      END
```

CONVERGENCE MONITOR SUBROUTINE GOLD2

N	Y1	Y2	X1	X2
2	0.1325857E-01	0.1205516E-01	0.4437694E 01	0.6562305E 01
3	0.1481423E-01	0.1325857E-01	0.3124611E 01	0.4437694E 01
4	0.1565030E-01	0.1481423E-01	0.2313081E 01	0.3124611E 01
5	0.1564945E-01	0.1565030E-01	0.1811528E 01	0.2313081E 01
6	0.1565030E-01	0.1598705E-01	0.2313081E 01	0.2623056E 01
7	0.1598705E-01	0.1510083E-01	0.2623056E 01	0.2814632E 01
8	0.1541933E-01	0.1598705E-01	0.2504656E 01	0.2623056E 01
9	0.1598705E-01	0.1536908E-01	0.2623056E 01	0.2696231E 01
10	0.1528829E-01	0.1598705E-01	0.2577831E 01	0.2623056E 01
11	0.1598705E-01	0.1559307E-01	0.2623056E 01	0.2651006E 01
12	0.1533265E-01	0.1598705E-01	0.2605782E 01	0.2623056E 01
13	0.1598705E-01	0.1583791E-01	0.2623056E 01	0.2633732E 01
14	0.1607842E-01	0.1598705E-01	0.2616457E 01	0.2623056E 01
15	0.1613457E-01	0.1607842E-01	0.2612379E 01	0.2616457E 01
16	0.1616909E-01	0.1613457E-01	0.2609859E 01	0.2612379E 01
17	0.1619043E-01	0.1616909E-01	0.2608301E 01	0.2609859E 01

LEFTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY	0.1000000E 01
RIGHTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY	0.1000000E 02
FRACTIONAL REDUCTION OF INTERVAL OF UNCERTAINTY	0.9999999E-03
EXTREME ORDINATE DISCOVERED DURING SEARCH	0.1619043E-01
ABSCISSA OF EXTREME ORDINATE	0.2608301E 01
NEW LEFTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY	0.2605782E 01
NEW RIGHTHAND ABSCISSA OF INTERVAL OF UNCERTAINTY	0.2612379E 01
NUMBER OF FUNCTION EVALUATIONS EXPENDED IN SEARCH	17

```
RAPID TRANSIT CAR OPTIMIZATION PROGRAM

INPUT DATA

WEIGHT OF CAR, TONS .................................          29.00
FRONTAL AREA OF CAR, SQ FT ..........................          80.00
RADIUS OF CAR WHEELS, INCHES ........................          16.00
ADHESION LIMIT BETWEEN WHEEL AND RAIL ...............           0.20
WORKING ADHESION VALUE DURING SERVICE BRAKING .......           0.15
DISTANCE BETWEEN STATIONS, FEET .....................        4000.000

TRACTION MOTORS ARE SIMILAR TO GE 1240 WITH FIELD SHUNT ONE


NO WHEELSLIP DURING ACCELERATION


OPTIMAL GEAR RATIO ..................................        2.60830
ELAPSED TIME BETWEEN STATIONS, SECONDS ..............          61.76
AVERAGE SPEED BETWEEN STATIONS, MILES/HR ............          44.16
MAXIMUM SPEED ATTAINED BETWEEN STATIONS, MILES/HR ...          65.01
BALANCING SPEED AT THIS RATIO, MILES/HR .............          70.66
```

5.4 A COMPUTATIONAL CAM OF MINIMAL MASS

Computational schemes can utilize analog mechanical devices in order to perform computational operations within a device called a *control* or *analog computer*. In an analog device a quantity such as shaft rotational displacement is used to represent the change in a variable, and the angular position of the shaft itself can be compared to the magnitude of the variable. Under these circumstances it is easy to appreciate how a spur-gear pair connecting parallel shafts can perform multiplication by a fixed number.

There are circumstances in the course of computational evolutions that require an analog rotation to be converted to an equivalent analog translation, A device capable of performing this conversion is the pinion and rack. Another device is the radial follower cam. The second device has the advantage of not only changing the mode of representation from rotational to translational but incorporating a functional transformation in the process. Control of the functional transformation is accomplished by specifying a particular contour for the cam.

Figure 74 shows a knife-edge offset radial follower cam, which is a suitable type where the load driven by the cam is small. The XY coordinate system is fixed in the earth (or in the guides of the follower) with the origin coincident with the point of contact between the cam and follower when the shaft is in the position associated with zero of the rotational variable. The xy coordinate system is fixed in the cam with the origin at the center of cam rotation in XY. At the zero position of the rotational variable, the corresponding axes of the two coordinate systems are parallel. The abscissa angle between the x axis and the X axis is designated θ. The location of the origin of the XY frame in the xy frame is specified by (a, b).

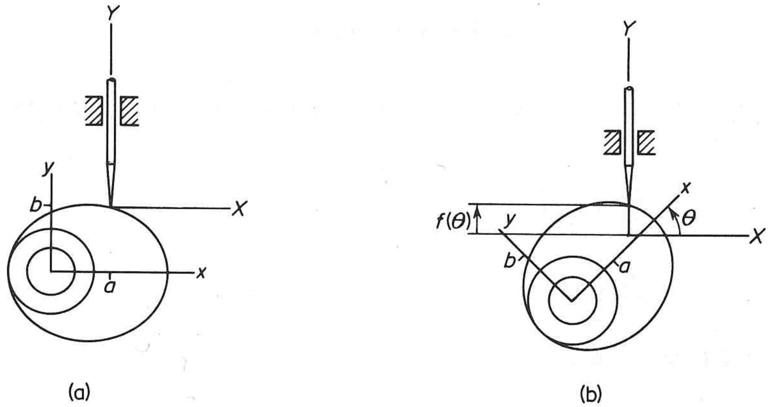

FIG. 74. A cam with an offset knife-edge follower used as a function generator in (a) its initial position and (b) in a subsequent position defined by angle θ.

We may think of a as representing the offset of the follower path from the center of the cam bearing. We may interpret b as the elevation of the initial point of contact above the x axis. Figure 74(b) indicates the geometry associated with a rotational displacement θ of the cam. The point of contact between the cam and follower moves up the Y axis and the distance is a function of theta $f(\theta)$ determined by the cam contour. The location of the point of contact in the XY frame determines values of $f(\theta)$ and the location of the point of contact in the xy frame locates points on the cam contour. The transformation equation can be shown to be

$$x = (Y + b) \sin \theta + (X + a) \cos \theta \tag{5.3}$$

$$y = (Y + b) \cos \theta - (X + a) \sin \theta \tag{5.4}$$

We note that the point of contact is constrained to lie on the Y axis, and therefore $X = 0$. The Y coordinate can be interpreted as a desired function of θ, and Eqs. (5.3) and (5.4) can be written as

$$x = (f(\theta) + b) \sin \theta + a \cos \theta \tag{5.5}$$

$$y = (f(\theta) + b) \cos \theta - a \sin \theta \tag{5.6}$$

A problem encountered with this type of cam is one of the cocking and binding of the follower in the follower guides when the knifeedge contact is too far from normal with the cam contour. Such deviation from normality is measured by an angle called the *pressure angle*, which is denoted by the symbol α in Fig. 75. From the geometry of the figure it is easy to determine that

$$\alpha = \theta + \arctan \frac{dy}{dx}$$

The value of dy/dx can be found by determining $dy/d\theta$ and $dx/d\theta$ in Eqs. (5.5) and (5.6)

$$\frac{dy}{d\theta} = -\,[f(\theta) + b] \sin \theta + f'(\theta) \cos \theta - a \cos \theta \tag{5.7}$$

$$\frac{dx}{d\theta} = [f(\theta) + b] \cos \theta + f'(\theta) \sin \theta - a \sin \theta \tag{5.8}$$

and it follows from Eqs. (5.7) and (5.8) that

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \tan (\psi - \theta)$$

where

$$\psi = \arctan \frac{f'(\theta) - a}{f(\theta) + b}$$

The pressure angle may be expressed as

$$\alpha = \theta + \arctan [\tan (\psi - \theta)] = \theta + \psi - \theta = \psi$$

or

$$\alpha = \arctan \frac{f'(\theta) - a}{f(\theta) + b} \tag{5.9}$$

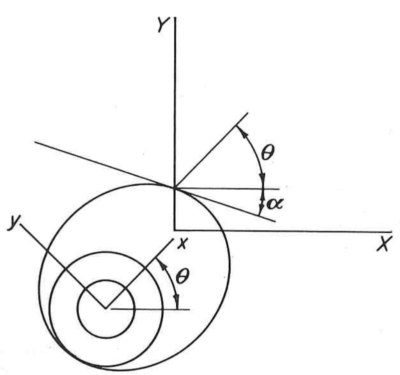

FIG. 75. The pressure angle α of the cam at position θ.

It is clear that points on the contour of the cam are determined by knowledge of the function $f(\theta)$ and of the parameters a and b using Eqs. (5.5) and (5.6). It is also clear that the local value of the pressure angle α is determined by $f(\theta)$, $f'(\theta)$, a, and b using Eq. (5.9). In an approximate way, the magnitude of the parameter b controls the physical size of the cam, and from Eq. (5.9) it is seen that a large value of b depresses the magnitude of the pressure angle α.

Let us consider the problem of providing the specifications for a computational cam that will generate the function $\ln \theta$ with the input variable θ varying between 60° and 160°. We shall add to the problem the limitation that the pressure angle α may not exceed 30° and the requirement that of all possible cams that could meet these specifications, the one selected shall involve a minimum plate area. This is somewhat akin to minimum weight specification. See Fig. 76.

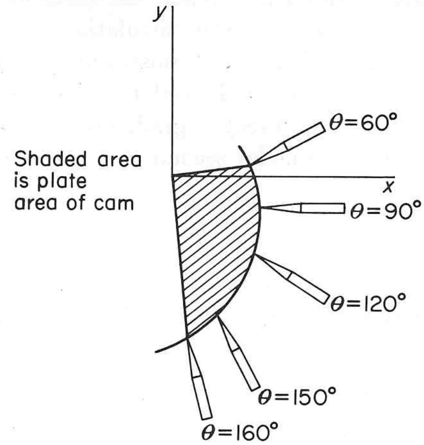

FIG. 76. The plate area of the cam.

In planning a machine computation program it is necessary to choose a figure of merit. The figure of merit selected to solve this problem is the reciprocal of the plate area of the cam. The functional constraint is the fact that the pressure angle may not exceed 30°. The regional constraints are the ranges of values permitted the parameters a and b. Given the function to be generated, it is clear that the independent variables at the designer's disposal are the parameters a and b. The search problem becomes two-dimensional, and for such a search we shall select library function GRID4, which is capable of carrying out grid-type searches in as many as eight design variables. The documentation of GRID4 indicates that this subroutine calls a subroutine MERIT4, which generates the ordinate to the merit surface for tendered values of the independent variables. This routine, MERIT4, we must write.

In writing the MERIT4 subroutine we shall face the problem of incorporating the functional constraint of the limit on the size of the pressure angle. We cannot simply identify the values of a and b which locate points in the feasible plane area to be excluded from consideration by virtue of excessive pressure angle. One approach to resolving this problem is to calculate the merit ordinate without regard to the violation of the pressure angle constraint, and then if the constraint is violated, *penalize* the merit ordinate in such a way as to make that point have a very poor merit—say, zero or less. Another way this can be done is to arrange it so that the slope above the infeasible area directs the search back to the feasible domain. Without prior knowledge of the configuration of the merit surface this may seem like a formidable task, but there are some things that we can do.

If we know the coordinates of one point in feasible space, a simple stratagem is available to us. The merit ordinate in feasible space is designed to be always positive, and it is calculated in accordance with the appropriate merit subroutine. When the merit ordinate of a point in infeasible space is required, the merit calculation is pursued to the point wherein the violation of a functional constraint is ascertained. A negative merit ordinate is then returned. In order to ensure that the slope of the negative merit surface will direct a gradient-sensitive search back into feasible space, a smooth unimodal second hypersurface is placed under the merit hypersur-

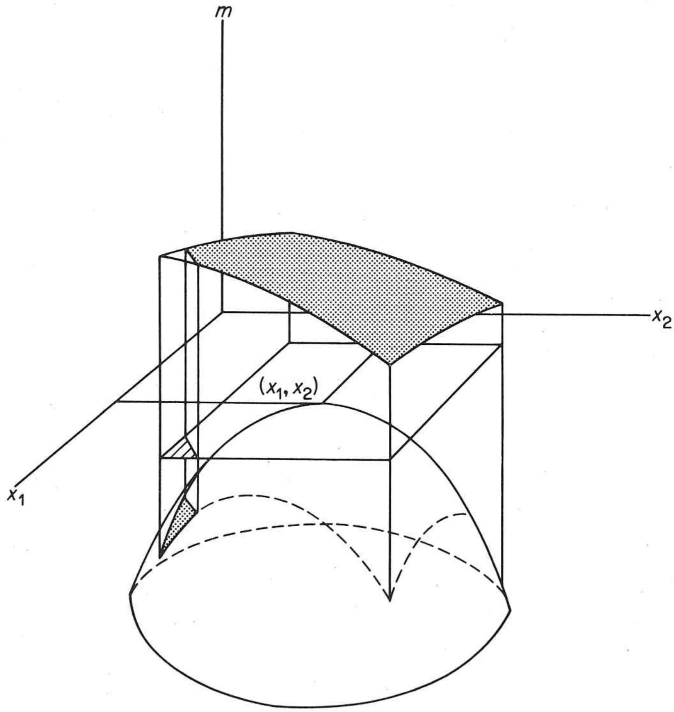

FIG. 77. The shaded surface is the merit surface with and without penalty. The location (x_1, x_2) is a known point in feasible space. The cross-hatched domain within regional constraints is that domain in violation of the functional constraints.

face. This second surface is called the *penalty* surface. The apex of the penalty surface is placed at the known point in feasible space, and the ordinate of the apex is zero. Thus the penalty surface has no extremes in infeasible space and a gradient-sensitive search straying into infeasible space is guided unerringly back toward feasible space. Figure 77 suggests the geometry involved.

A subroutine generating the penalty surface is part of the IOWA CADET design library, and its listing is as follows:

```
      SUBROUTINE PENAL (N, GOOD, X, Y)
      DIMENSION GOOD(9), X(9)
      Y = 0.
      DO 100 I = 1, N
      Y = Y + (GOOD(I) - X(I))*(GOOD(I) - X(I))
  100 CONTINUE
      Y = -Y
      RETURN
      END
```

The MERIT function for this problem will be one constructed to respond to interrogations of GRID4 multidimensional search routine. It must accept a tendered value of offsets a and b, determine the plate area of the cam and determine whether the functional constraint of the limiting value of the pressure angle is violated. If this proves to be the case, subroutine PENAL is called to supply a penalty negative ordinate for return to GRID4.

A subroutine FUNCT is provided to supply an ordinate and first derivative of the function to be generated by the cam when a value of rotational angle θ is supplied.

A complete listing of the program which implements the plan depicted in Fig. 78 follows.

```
C     EXECUTIVE PROGRAM TO DETERMINE THE CONTOUR AND OFFSET OF A
C     KNIFE-EDGED RADIAL FOLLOWER CAM OF MINIMUM SIZE TO GENERATE
C     A SPECIFIED FUNCTION.
C                                                          MISCHKE
C     THETA1=INITIAL ANGULAR POSITION OF CAM SHAFT, DEG.
C     THETA2=FINAL ANGULAR POSITION OF CAM SHAFT, DEG.
C     PLIMIT=HIGHEST PERMISSIBLE PRESSURE ANGLE, DEG.
C     ALOW=LOWER LIMIT OF FOLLOWER PATH OFFSET, IN.
C     AHIGH=UPPERLIMIT OF FOLLOWER PATH OFFSET, IN.
C
C     BLOW=LOWER LIMIT OF INITIAL FOLLOWER POSITION, IN.
C     BHIGH=UPPER LIMIT OF INITIAL FOLLOWER POSITION, IN.
C     AREA=AREA OF CAM PLATE, SQ.IN.
C     RAREA=RECIPROCAL OF AREA (FIGURE OF MERIT)
C
C     A=FOLLOWER PATH OFFSET, IN.
C     B=INITIAL FOLLOWER POSITION,IN.
C     XX=X-COORDINATE OF POINT ON CAM CONTOUR, IN.
C     YY=Y-COORDINATE OF POINT ON CAM CONTOUR, IN.
C     THETA=ABSCISSA ANGLE TO POINT (XX,YY), DEG.
C
      COMMON THETA1,THETA2,PLIMIT,GOOD(9),I1
      DIMENSION IABC(20),X(9),XR(9),XL(9),XLOW(9),XHIGH(9)
C
C     ..... READ IN DATA .....
C
    1 READ(1,2)THETA1,THETA2,PLIMIT,ALOW,AHIGH,BLOW,BHIGH
    2 FORMAT(7F10.5)
      READ(1,3)I1,I2,F,AA,BB,R
    3 FORMAT(2I2,4F10.5)
      READ(1,4)(IABC(I),I=1,20)
    4 FORMAT(20A4)
C
C     ..... INITIATE A GRID SEARCH FOR LARGEST FIGURE OF MERIT     .....
C     ..... WHICH IS THE RECIPROCAL OF THE PLATE AREA OF THE CAM.  .....
C
```

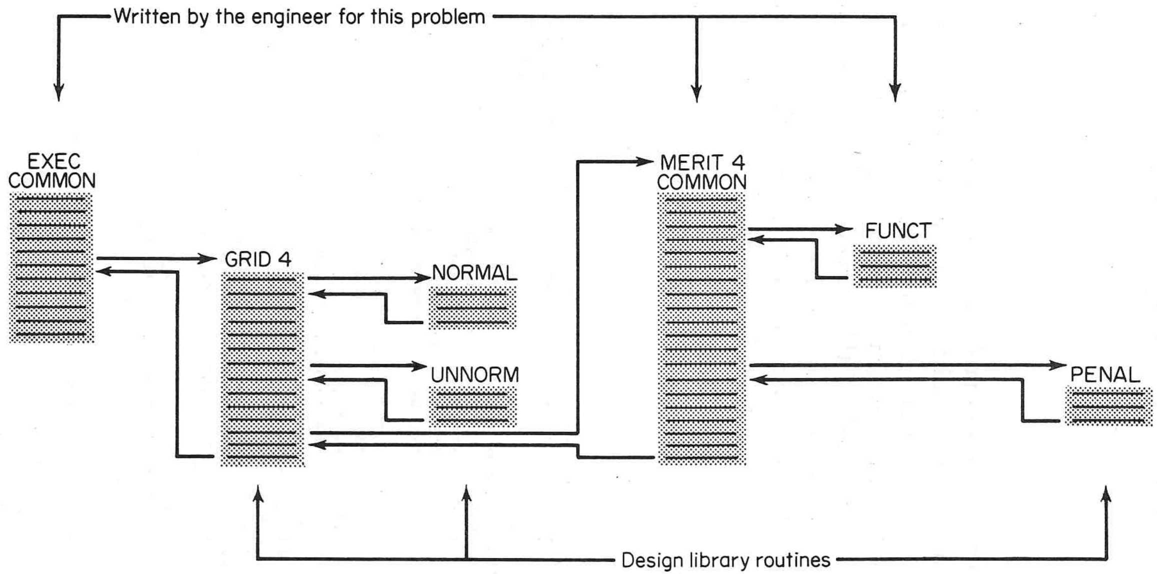

FIG. 78. Example of program arrangement for the solution of the cam problem using the IOWA CADET algorithm.

```
      GOOD(1)=AA
      GOOD(2)=BB
      XL(1)=ALOW
      XL(2)=BLOW
      XR(1)=AHIGH
      XR(2)=BHIGH
      CALL GRID4(I1,I2,XL,XR,F,R,RAREA,X,XLOW,XHIGH,NN)
C
C     ..... PREPARE OUTPUT .....
C
      AREA=1./RAREA
      A=X(1)
      B=X(2)
C
C     ..... DOCUMENT SEARCH .....
C
      WRITE(3,5)THETA1,THETA2,PLIMIT
    5 FORMAT('1EXECUTIVE PROGRAM PRINT OUT FOR MINIMUM AREA PLATE CAM'//
     1' INPUT DATA'//
     2' INITIAL ANGULAR POSITION OF CAM SHAFT, DEG ...........',F15.0,/,
     3' FINAL ANGULAR POSITION OF CAM SHAFT, DEG..............',F15.0,/,
     4' UPPER LIMIT ON PRESSURE ANGLE, DEG....................',F15.0//)
      WRITE(3,6)(IABC(I),I=1,20)
    6 FORMAT(20A4,//,' OUTPUT DATA',//)
      WRITE(3,7)A,B,AREA
    7 FORMAT(
     1' OFFSET OF PATH OF RADIAL FOLLOWER, IN ...............',F15.5,/,
     2' INITIAL FOLLOWER RADIAL DISPLACEMENT, IN ............',F15.5,/,
     3' PLATE AREA OF MINIMAL AREA CAM, SQ IN ...............',F15.5//,
     4' CAM CONTOUR FOLLOWS',//,
     5' X-COORDINATE    Y-COORDINATE    SHAFT ANGLE     PRESSURE ANGLE',/,
     6'     INCHES          INCHES        DEGREES            DEGREES    ',//)
C
C     ..... DETERMINE AND DOCUMENT CAM CONTOUR.....
C
      C=180./3.14159
      DEL=(THETA2-THETA1)/10.
      THETA=THETA1-10.
      DO 100 I=1,12
      CALL FUNCT(THETA,G,DY)
      XX=(G+B)*SIN(THETA/C)+A*COS(THETA/C)
      YY=(G+B)*COS(THETA/C)-A*SIN(THETA/C)
      PANGLE=C*ATAN(ABS((DY-A)/(G+B)))
      WRITE(3,9)XX,YY,THETA,PANGLE
    9 FORMAT(4F15.5)
      THETA=THETA+DEL
  100 CONTINUE
      WRITE(3,101)R,NN
  101 FORMAT(/,' GRID SEARCH DATA',//,
     1' FRACTIONAL GRID REDUCTION UTILIZED ..................',F15.8,/,
     2' NUMBER OF MERIT FUNCTION EVALUATIONS EXPENDED ........',I15)
      GO TO 1
      END


      SUBROUTINE MERIT4(X,Y)
      COMMON THETA1,THETA2,PLIMIT,GOOD(9),I1
      DIMENSION X(9)
C
C     ..... INITIALIZE .....
C
      A=X(1)
      B=X(2)
      PBIG=-360.
      DELTA=(THETA2-THETA1)/10.
      THETA=THETA1
      AREA=0.
      C=180./3.14159
C
C     ..... DETERMINE CAM PLATE AREA FOR THIS VALUE OF A & B.       .....
C
      DO 100 I=1,10
      CALL FUNCT(THETA,G,DY)
      XX=(G+B)*SIN(THETA/C)+A*COS(THETA/C)
```

```
      YY=(G+B)*COS(THETA/C)-A*SIN(THETA/C)
      RR=XX*XX+YY*YY
      AREA=AREA+0.5*RR*DELTA/C
      PANGLE=C*ATAN(ABS((DY-A)/(G+B)))
      IF(PANGLE-PBIG)2,2,1
    1 PBIG=PANGLE
    2 THETA=THETA+DELTA
  100 CONTINUE
C
C     ..... IF FUNCTIONAL CONSTRAINT OF LIMITING VALUE OF THE        .....
C     ..... PRESSURE ANGLE IS VIOLATED, PENALIZE THE MERIT ORDINATE.....
C
      IF(PBIG-PLIMIT)5,6,6
    5 Y=1./AREA
      RETURN
    6 CALL PENAL(I1,GOOD,X,Y)
      RETURN
      END

      SUBROUTINE FUNCT(THETA,Y,DY)
      C=180./3.14159
      Y=ALOG(THETA/C)
      DY=C/THETA
      RETURN
      END


      SUBROUTINE PENAL(N,GOOD,X,Y)
C
C     PENALTY ORDINATE SUBROUTINE
C     PENAL(I1,A2,A3,B1)                                        MISCHKF
C
C     THIS SUBROUTINE SUBSTITUTES A MERIT SURFACE OF NEGATIVE
C     ORDINATE AND HAVING SLOPES APPROPRIATE TO LEAD A GRADIENT SEARCH
C     BACK INTO FEASIBLE SPACE WITH UP TO NINE INDEPENDENT VARIABLES.
C
C     CALLING PROGRAM REQUIREMENTS
C
C     PROVIDE A DECLARATION STATEMENT AS FOLLOWS:
C
C          DIMENSION A2(100),A3(100)
C
C     NOMENCLATURE
C
C     I1=NUMBER OF INDEPENDENT COORDINATES IN MERIT HYPERSPACE
C     A2=COORDINATES OF A POINT IN FEASIBLE SPACE, COLUMN VECTOR
C     A3=TENDERED COORDINATES, COLUMN VECTOR
C     B1=PENALTY ORDINATE CORRESPONDING TO TENDERED COORDINATES, A3
C
      DIMENSION GOOD(9),X(9)
      Y=0.
      DO 100 I=1,N
      Y=Y+(GOOD(I)-X(I))*(GOOD(I)-X(I))
  100 CONTINUE
      Y=-Y
      RETURN
      END
```

The following are outputs from the preceding program utilizing the following basic data:

θ_1 = THETA1 = 60. deg
θ_2 = THETA2 = 160. deg
α = PLIMIT = 30. deg
ALOW = 0. in.
AHIGH = 1.5 in.
BLOW = 0. in.

BHIGH = 1.5 in.
I1 = 2 independent variables
I2 = 1 convergence monitor print
F = 0.01
AA = 1.
BB = 1.

```
CONVERGENCE MONITOR SUBROUTINE GRID4

     NN   SIDE        Y         X(1)        X(2)

      5 0.100E 01 0.516E 00 0.100E 01 0.500E 00
      9 0.700E 00 0.698E 00 0.650E 00 0.500E 00
     13 0.490E 00 0.730E 00 0.772E 00 0.377E 00
     17 0.343E 00 0.730E 00 0.772E 00 0.377E 00
     21 0.240E 00 0.736E 00 0.833E 00 0.317E 00
     25 0.168E 00 0.810E 00 0.833E 00 0.233E 00
     29 0.118E 00 0.810E 00 0.803E 00 0.263E 00
     33 0.824E-01 0.850E 00 0.803E 00 0.222E 00
     37 0.576E-01 0.850E 00 0.803E 00 0.222E 00
     41 0.404E-01 0.850E 00 0.803E 00 0.222E 00
     45 0.282E-01 C.850E 00 0.803E 00 0.222E 00
     49 0.198E-01 0.850E 00 0.803E 00 0.222E 00
     53 0.138E-01 0.850E 00 0.800E 00 0.225E 00

LARGEST MERIT ORDINATE FOUND DURING SEARCH ........... 0.84989303E 00
NUMBER OF FUNCTION EVALUATIONS USED DURING SEARCH ....             58
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY EXTANT 0.96888803E-02


XLOW(1)= 0.79238486E 00   X(1)= 0.79965162E 00   XHIGH(1)= 0.80691826E 00
XLOW(2)= 0.21786785E 00   X(2)= 0.22513461E 00   XHIGH(2)= 0.23240125E 00


EXECUTIVE PROGRAM PRINT OUT FOR MINIMUM AREA PLATE CAM

INPUT DATA

INITIAL ANGULAR POSITION OF CAM SHAFT, DEG ...........           60.
FINAL ANGULAR POSITION OF CAM SHAFT, DEG..............          160.
UPPER LIMIT ON PRESSURE ANGLE, DEG....................           30.


FUNCTION GENERATED IS Y=ALOG(THETA) FROM 60. TO 160. DEGREES

OUTPUT DATA


OFFSET OF PATH OF RADIAL FOLLOWER, IN ................       0.79965
INITIAL FOLLOWER RADIAL DISPLACEMENT, IN .............       0.22513
PLATE AREA OF MINIMAL AREA CAM, SQ IN ................       1.17662

CAM CONTOUR FOLLOWS

X-COORDINATE    Y-COORDINATE    SHAFT ANGLE     PRESSURE ANGLE
   INCHES          INCHES         DEGREES          DEGREES


      0.58213        -0.55541        50.00000        75.59627
      0.63474        -0.55689        60.00000        29.78920
      0.67324        -0.60593        70.00000         2.53857
      0.68930        -0.69044        80.00000         8.49204
      0.67672        -0.79965        90.00000        13.54532
      0.63134        -0.92331       100.00000        16.16481
      0.55098        -1.05151       110.00000        17.62712
      0.43537        -1.17472       120.00000        18.47346
      0.28609        -1.28392       130.00000        18.96492
      0.10642        -1.37086       140.00000        19.24001
     -0.09874        -1.42827       150.00000        19.37770
     -0.32319        -1.45007       160.00000        19.42549
```

```
GRID SEARCH DATA

FRACTIONAL GRID REDUCTION UTILIZED .................       0.69999999
NUMBER OF MERIT FUNCTION EVALUATIONS EXPENDED ........             58
```

In the first case the fractional grid reduction utilized was R = 0.7. This is just a little higher than its minimum allowable value. Some attributes of the grid-type search can be seen from the inspection of the output of this program.

When R = $\frac{2}{3}$ the grid search abandons some of the feasible basis area irrevocably when it centers on a new ordinate. Thus if unsearched ground is abandoned on the basis of scanty information in the five (in this particular case) function evaluations, then the search is forever barred from regaining the ground even while in hot pursuit of growing ordinates.

The first output used 58 function evaluations to establish approximately that $a = 0.79965$, $b = 0.22513$, and that the plate area of the cam was 1.17662 sq in.

```
CONVERGENCE MONITOR SUBROUTINE GRID4

  NN   SIDE          Y            X(1)         X(2)

   5 0.100E 01  0.516E 00  0.1C0E 01  0.500E 00
   9 0.750E 00  0.516E 00  0.100E 01  0.500E 00
  13 0.563E 00  0.683E 00  0.859E 00  0.359E 00
  17 0.422E 00  0.860E C0  0.859E 00  0.148E 00
  21 0.316E 00  0.860E 0C  0.859E 00  0.148E 00
  25 0.237E 00  C.860E 00  0.859E 00  0.148E 00
  29 0.178E 00  C.860E 00  0.859E 00  0.148E 00
  33 0.133E 00  0.860E 00  0.859E 00  0.148E 00
  37 0.100E 00  0.864E 00  0.834E 00  0.173E 00
  41 0.751E-01  0.864E 00  0.834E 00  0.173E 00
  45 0.563E-01  0.866E 0C  0.820E 00  0.188E 00
  49 0.422E-01  0.866E 00  0.820E 00  0.188E 00
  53 0.317E-01  0.866E 00  0.820E 00  0.188E 00
  57 0.238E-01  0.866E 00  0.820E 00  0.188E 00
  61 0.178E-01  0.866E 00  0.820E 00  0.188E 00
  65 0.134E-01  0.866E 00  0.820E 00  0.188E 00
  69 0.100E-01  0.866E 00  0.820E 00  0.188E 00

LARGEST MERIT ORDINATE FOUND DURING SEARCH ........... 0.86570883E 00
NUMBER OF FUNCTION EVALUATIONS USED DURING SEARCH ....             74
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY EXTANT 0.75169429E-02


XLOW(1)= 0.81463039E 00   X(1)= 0.82026815E 00   XHIGH(1)= 0.82590580E 00
XLOW(2)= 0.18190622E 00   X(2)= 0.18754399E 00   XHIGH(2)= 0.19318163E 00


EXECUTIVE PROGRAM PRINT OUT FOR MINIMUM AREA PLATE CAM

INPUT DATA

INITIAL ANGULAR POSITION OF CAM SHAFT, DEG ...........              60.
FINAL ANGULAR POSITION OF CAM SHAFT, DEG.............             160.
UPPER LIMIT ON PRESSURE ANGLE, DEG..................               30.


FUNCTION GENERATED IS Y=ALOG(THETA) FROM 60. TO 160. DEGREES

OUTPUT DATA
```

OFFSET OF PATH OF RADIAL FOLLOWER, IN	0.82027
INITIAL FOLLOWER RADIAL DISPLACEMENT, IN	0.18754
PLATE AREA OF MINIMAL AREA CAM, SQ IN	1.15512

CAM CONTOUR FOLLOWS

X-COORDINATE INCHES	Y-COORDINATE INCHES	SHAFT ANGLE DEGREES	PRESSURE ANGLE DEGREES
0.56659	-0.59536	50.00000	81.04099
0.61249	-0.59354	60.00000	29.95560
0.64497	-0.63816	70.00000	0.25946
0.65586	-0.71727	80.00000	11.28900
0.63913	-0.82027	90.00000	16.03162
0.59074	-0.93708	100.00000	18.37592
0.50860	-1.05803	110.00000	19.62175
0.39251	-1.17378	120.00000	20.29817
0.24404	-1.27555	130.00000	20.65388
0.06647	-1.35532	140.00000	20.81830
-0.13539	-1.40602	150.00000	20.86401
-0.35542	-1.42179	160.00000	20.83415

GRID SEARCH DATA

FRACTIONAL GRID REDUCTION UTILIZED	0.75000000
NUMBER OF MERIT FUNCTION EVALUATIONS EXPENDED	74

The second output is based on R = 0.75, and 74 function evaluations established $a = 0.82027$ in., $b = 0.18754$ in., and the plate area of the cam as 1.15512 sq in. The number of function evaluations increased, as we might have expected, but the plate area is not much smaller. This is an attribute of multidimensional searches such as GRID4. The abandonment of portions of the basis area is not done with certainty that the extreme is outside the abandoned region. Secondly, the search pattern is sparse and an element of luck is involved in locating the constrained extremum along a slowly rising cliff, such as is involved here.

CONVERGENCE MONITOR SUBROUTINE GRID4

NN	SIDE	Y	X(1)	X(2)
5	0.100E 01	0.516E 00	0.100E 01	0.500E 00
9	0.800E 00	0.516E 00	0.100E 01	0.500E 00
13	0.640E 00	0.712E 00	0.840E 00	0.340E 00
17	0.512E 00	0.712E 00	0.840E 00	0.340E 00
21	0.410E 00	0.712E 00	0.840E 00	0.340E 00
25	0.328E 00	0.856E 00	0.840E 00	0.176E 00
29	0.262E 00	0.856E 00	0.840E 00	0.176E 00
33	0.210E 00	0.856E 00	0.840E 00	0.176E 00
37	0.168E 00	0.856E 00	0.840E 00	0.176E 00
41	0.134E 00	0.856E 00	0.840E 00	0.176E 00
45	0.107E 00	0.858E 00	0.813E 00	0.203E 00
49	0.859E-01	0.858E 00	0.813E 00	0.203E 00
53	0.687E-01	0.858E 00	0.813E 00	0.203E 00
57	0.550E-01	0.858E 00	0.813E 00	0.203E 00
61	0.440E-01	0.858E 00	0.813E 00	0.203E 00
65	0.352E-01	0.858E 00	0.813E 00	0.203E 00
69	0.281E-01	0.858E 00	0.813E 00	0.203E 00
73	0.225E-01	0.858E 00	0.813E 00	0.203E 00
77	0.180E-01	0.858E 00	0.809E 00	0.208E 00
81	0.144E-01	0.858E 00	0.809E 00	0.208E 00
85	0.115E-01	0.858E 00	0.809E 00	0.208E 00

```
LARGEST MERIT ORDINATE FOUND DURING SEARCH ........... 0.85822439E 00
NUMBER OF FUNCTION EVALUATIONS USED DURING SEARCH ....              90
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY EXTANT 0.92233382E-02


XLOW(1)= 0.80173469E 00   X(1)= 0.80865222E 00   XHIGH(1)= 0.81556964E 00
XLOW(2)= 0.20058888E 00   X(2)= 0.20750642E 00   XHIGH(2)= 0.21442384E 00


EXECUTIVE PROGRAM PRINT OUT FOR MINIMUM AREA PLATE CAM

INPUT DATA

INITIAL ANGULAR POSITION OF CAM SHAFT, DEG ...........            60.
FINAL ANGULAR POSITION OF CAM SHAFT, DEG..............           160.
UPPER LIMIT ON PRESSURE ANGLE, DEG....................            30.

FUNCTION GENERATED IS Y=ALOG(THETA) FROM 60. TO 160. DEGREES

OUTPUT DATA


OFFSET OF PATH OF RADIAL FOLLOWER, IN ................       0.80865
INITIAL FOLLOWER RADIAL DISPLACEMENT, IN .............       0.20751
PLATE AREA OF MINIMAL AREA CAM, SQ IN ................       1.16520

CAM CONTOUR FOLLOWS

X-COORDINATE     Y-COORDINATE     SHAFT ANGLE      PRESSURE ANGLE
   INCHES           INCHES          DEGREES           DEGREES


      0.57441        -0.57363        50.00000         78.06279
      0.62397        -0.57350        60.00000         29.97446
      0.65976        -0.62042        70.00000          1.38511
      0.67350        -0.70237        80.00000          9.69254
      0.65909        -0.80865        90.00000         14.62869
      0.61242        -0.92911       100.00000         17.13553
      0.53134        -1.05394       110.00000         18.50659
      0.41560        -1.17370       120.00000         19.28021
      0.26680        -1.27948       130.00000         19.71301
      0.08820        -1.36314       140.00000         19.93997
     -0.11535        -1.41750       150.00000         20.03748
     -0.33767        -1.43658       160.00000         20.05121

GRID SEARCH DATA

FRACTIONAL GRID REDUCTION UTILIZED ...................    0.79999995
NUMBER OF MERIT FUNCTION EVALUATIONS EXPENDED ........            90
```

The third output is based upon R = 0.8 and used 90 function evaluations to establish $a = 0.80865$ in., $b = 0.20751$ in., and the plate area of the cam as 1.16520 sq in. Figure 79 indicates the path of progress of the grid center during one of these searches. The plate areas obtained are within approximately 5 per cent of one another. A mixture of strategies is useful in gaining the little remaining altitude of the merit surface, if desired.

5.5 A GEAR TRAIN OF MINIMAL INERTIA

There are applications in servomechanisms and control devices wherein an angular displacement is transmitted through a gear reduction to another

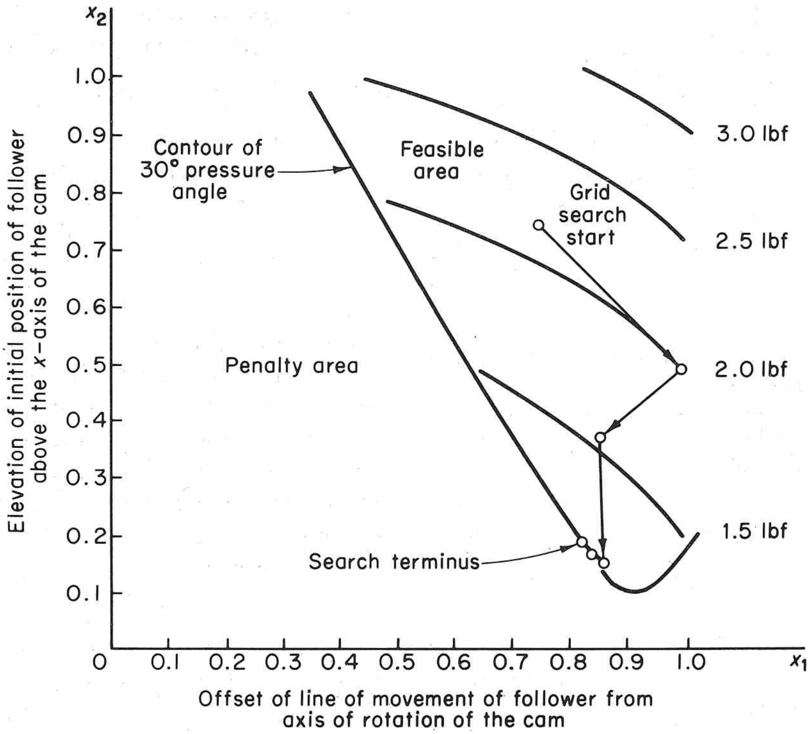

FIG. 79. The search path in attaining the constrained extremum in the cam problem.

shaft. In the interest of accomplishing the angular displacement as quickly as possible, minimization of the inertia of the intervening gear train is attempted when the load inertia is small enough that the gear-train inertia represents the principal resistance to acceleration. When the overall gear ratio is specified by other considerations, should the reduction be accomplished as a single reduction, a double reduction, a triple reduction, or more?

While pursuit of the answers to these questions is interesting, we will confine out attention to the problem of determining, for a triple reduction, the appropriate distribution of the individual steps so as to minimize the gear-train inertia. Figure 80 depicts the spur gear train in which the gears and pinions are disklike. The native inertias (polar moments of inertia about principal axis of revolution) are represented by

I_M = armature inertia
I_P = pinion inertia
I_{G1} = gear 1 inertia
I_{G2} = gear 2 inertia

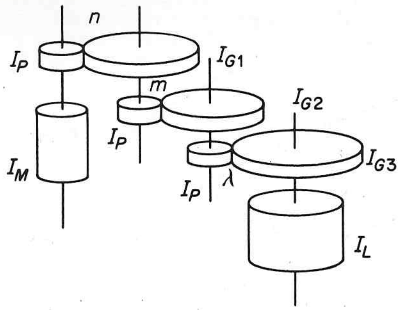

FIG. 80. A triple reduction spur-gear train with an overall step-down ratio $R = nm\lambda$. I_M and I_L refer to the motor armature and load rotary inertias respectively.

I_{G3} = gear 3 inertia
I_L = inertia of the driven load

and the other parameters are

n = first step-down ratio
m = second step-down ratio
λ = third step-down ratio
R = overall gear reduction, which equals $nm\lambda$

We shall approach this problem from a traditional analytical viewpoint in order to see diminishing returns as the problem complexity grows, then turn to a computer solution as an example of a search carried out over two independent variables.

In Fig. 81, all the inertias have been reflected to the motor shaft and the magnitude of these equivalent inertias are specified by the following rules:*

Rule I: Inertia on a second shaft is reflected onto the motor shaft as the native inertia divided by the step-down ratio squared.

Rule II: The inertia of a disk gear on a second shaft in mesh with a disk pinion on the the motor shaft is reflected to the pinion shaft as the *pinion* inertia multiplied by the step-down ratio squared.

Applying these rules to the shish kebob equivalent of Fig. 81, we obtain

$$I_e = I_M + I_P - n^2 I_p + \frac{I_P}{n^2} + \frac{m^2 I_P}{n^2} + \frac{I_P}{n^2 m^2} + \frac{\lambda^2 I_P}{n^2 m^2} + \frac{I_L}{n^2 m^2 \lambda^2}$$

Introduction of the overall ratio $R = nm\lambda$ and substitution for λ results in

$$I_e = I_M + (1 + n^2)I_P + \left(\frac{1}{n^2} + \frac{m^2}{n^2}\right)I_P + \left(\frac{1}{n^2 m^2} + \frac{R^2}{n^4 m^4}\right)I_P + \frac{I_L}{R^2} \quad (5.10)$$

In order to minimize the equivalent inertia in a formal fashion it is necessary to differentiate Eq. (5.10) with respect to n and m and equate the derivatives to zero.

*For development, see Charles R. Mischke, *Elements of Mechanical Analysis* (Reading, Mass.: Addison-Wesley Publishing Company, Inc., 1963), pp. 59 ff.

$$\frac{\partial I_e}{\partial n} = \left(2n - \frac{2}{n^3} - \frac{2m^2}{n^3} - \frac{2}{n^3m^2} - \frac{4R^2}{n^5m^4}\right)I_P = 0$$

$$\frac{\partial I_e}{\partial m} = \left(\frac{2m}{n^2} - \frac{2}{n^2m^3} - \frac{4R^2}{n^4m^5}\right)I_P = 0$$

In order to determine n and m, it is necessary to solve simultaneously the equations

$$n^6m^4 - n^2m^4 - n^2m^6 - n^2m^2 - 2R^2 = 0 \tag{5.11}$$

and

$$n^2m^6 - n^2m^2 - 2R^2 = 0 \tag{5.12}$$

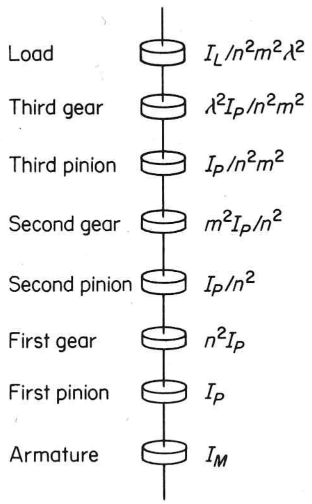

FIG. 81. The shish-kebab equivalent of all inertias reflected onto the motor armature shaft.

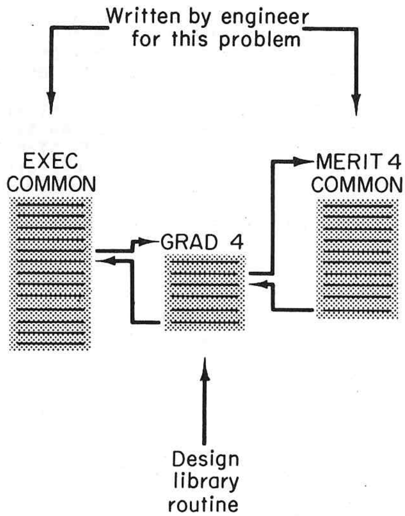

FIG. 82. Example of program arrangement for solution of the gear-train problem using the IOWA CADET algorithm.

If there had been a fourth reduction, there would be three simultaneous equations similar to Eqs. (5.11) and (5.12) to be solved. We are clearly close to the limit of tractability of this form of solution.

A computer solution using the IOWA CADET algorithm would involve a search over the domain of n and m for the configuration of minimal equivalent inertia. We choose as the figure of merit the reciprocal of the equivalent inertia. We observe no functional constraint and declare the regional constraints to be

$$1 \leq n \leq 3$$

$$1 \leq m \leq 3$$

Inasmuch as there are no functional constraints to introduce “cliffs,” a gradient search will be effective and we select the gradient search library subroutine GRAD4. According to the documentation it calls the subroutine MERIT4. The programs to solve this problem are structured as shown in Fig. 82. We begin by composing subroutine MERIT4, and then write the executive program. A listing of these programs follows.

```
C
C     EXECUTIVE PROGRAM FOR MINIMIZING THE INERTIA, AS SEEN BY MOTOR,
C     OF A MULTI-STEP DISK SPUR GEAR TRAIN, WITH OVERALL REDUCTION
C     RATIO SPECIFIED
C
C     I1=NUMBER OF GEAR REDUCTIONS IN TRAIN
C     RATIO=DESIRED OVERALL GEAR REDUCTION
C     XIM=ROTARY INERTIA OF MOTOR ARMATURE
C     XIP=ROTARY INERTIA OF PINIONS
C     XIL=ROTARY INERTIA OF LOAD ON LAST SHAFT
C
C     I2=CONVERGENCE MONITOR PRINT SIGNAL
C     I3=COMMENCE SEARCH LOCATION SIGNAL
C     DELTA=INITIAL EXPLORATION STEP SIZE
C     DMULT=STEPSIZE GROWTH MULTIPLIER
C     F=FRACTIONAL REDUCTION IN DOMAIN OF UNCERTAINTY DESIRED
C
C     EPS=SURVEY PATTERN INCREMENT
C     RLOW(I)=LOWER BOUNDS OF INTERVAL OF UNCERTAINTY
C     RHIGH(I)=UPPER BOUNDS OF INTERVAL OF UNCERTAINTY
C     X(I)=GEAR RATIOS FROM MOTOR TO LOAD
C
      COMMON I1,XIM,XIP,XIL,XIEQ,RATIO
      DIMENSION RLOW(9),RHIGH(9),X(9),P(9),Q(9)
C
C     ..... READ INPUT DATA .....
C
    9 READ(1,1)I1,RATIO,XIM,XIP,XIL
    1 FORMAT(I5,4F10.5)
      READ(1,2)I2,I3,DELTA,DMULT,EPS,F
    2 FORMAT(2I2,2F10.5,2F10.8)
      I1=I1-1
      DO 3 I=1,I1
      READ(1,4)XLOW,XHIGH
    4 FORMAT(2F10.5)
      RLOW(I)=XLOW
      RHIGH(I)=XHIGH
    3 CONTINUE
C
C     ..... INITIATE GRADIENT SEARCH FOR EXTREME FIGURE OF MERIT      .....
C     ..... WHICH IS THE RECIPROCAL OF THE EQUIVALENT INERTIA         .....
C     ..... AS SEEN ON THE MOTOR SHAFT.                               .....
C
      CALL GRAD4(I1,I2,I3,DELTA,DMULT,F,EPS,RLOW,RHIGH,Y,X,J3,J4,P,Q)
      I1=I1+1
C
C     ..... DOCUMENT SEARCH .....
C
      WRITE(3,5)I1,XIM,XIP,XIL,RATIO
    5 FORMAT('1EXECUTIVE PROGRAM OUTPUT FOR MINIMUM GEARTRAIN INERTIA'//
     1' INPUT DATA'//,
     2' NUMBER OF REDUCTION STEPS IN GEARTRAIN ...............',I10  ,/,
     3' ROTARY INERTIA OF MOTOR ARMATURE, SLUG FT2, ..........',F10.5,/,
     4' ROTARY INERTIA OF MOTOR PINION, SLUG FT2, ............',F10.5,/,
     5' ROTARY INERTIA OF LOAD ON LAST SHAFT, SLUG FT2, ......',F10.5,/,
     6' OVERALL GEAR RATIO SPECIFIED .........................',F10.5,/)
```

```
      WRITE(3,6)XIEQ
    6 FORMAT(/,' OUTPUT DATA',//,
     1' EQUIVALENT INERTIA OF OPTIMAL GEAR TRAIN,SLUG-FT2.....',F10.5,/,
     2' OPTIMAL GEAR RATIOS FROM MOTOR TO LOAD',/)
      DO 8 I=1,I1
      XX=X(I)
      WRITE(3,7)XX
    7 FORMAT(' RATIO(S) ',45X,F10.5)
    8 CONTINUE
      GO TO 9
      END

      SUBROUTINE MERIT4(X,Y)
      COMMON I1,XIM,XIP,XIL,XIEQ,RATIO
      DIMENSION X(9)
      N=I1
      PROD=1.
      SUM=0.
      BOTTOM=1.
      DO 2 I=1,N
      PROD=PROD*X(I)
    2 CONTINUE
      X(N+1)=RATIO/PROD
      N=N+1
      DO 1 I=2,N
      TOP=1.+X(I)*X(I)
      BOTTOM=BOTTOM*X(I-1)*X(I-1)
      SUM=SUM+TOP/BOTTOM
    1 CONTINUE
      XIEQ=XIM+XIP*(1.+X(1)*X(1))+XIP*SUM+XIL/(RATIO*RATIO)
      Y=1./XIEQ
      RETURN
      END
```

The following output was obtained using the data:

$$
\begin{aligned}
I1 &= 3 \text{ dimensions}\\
R &= \text{RATIO} = 10.\\
I_M &= \text{XIM} = 10. \text{ slug ft}^2\\
I_P &= \text{XIP} = \ 1. \text{ slug ft}^2\\
I2 &= 1 \text{ convergence monitor print}\\
I3 &= 1 \text{ commence search centrally}\\
\text{DELTA} &= 0.1\\
\text{DMULT} &= 1.2\\
\text{EPS} &= 0.0001\\
F &= 0.001
\end{aligned}
$$

RLOW (1) = 1. RHIGH (1) = 3.
RLOW (2) = 1. RHIGH (2) = 3.

```
CONVERGENCE MONITOR SUBROUTINE GRAD4

   N1        DELTA          Y           X(1)         X(2)         X(3)


    3    0.100E 00  0.565E-01  0.200E 01  0.200E 01  0.250E 01
    6    0.120E 00  0.571E-01  0.188E 01  0.200E 01  0.266E 01
    9    0.144E 00  0.573E-01  0.174E 01  0.200E 01  0.288E 01
   12    0.173E 00  0.571E-01  0.186E 01  0.212E 01  0.254E 01
   15    0.173E-01  0.573E-01  0.172E 01  0.199E 01  0.292E 01
   18    0.173E-02  0.573E-01  0.174E 01  0.200E 01  0.288E 01
```

```
21     0.207E-02 0.573E-01 0.174E 01 0.200E 01 0.288E 01
24     0.249E-02 0.573E-01 0.174E 01 0.200E 01 0.287E 01
27     0.299E-02 0.573E-01 0.174E 01 0.201E 01 0.287E 01
30     0.358E-02 0.573E-01 0.174E 01 0.201E 01 0.286E 01
33     0.430E-02 0.573E-01 0.174E 01 0.201E 01 0.285E 01
36     0.516E-02 0.573E-01 0.175E 01 0.201E 01 0.284E 01
39     0.619E-02 0.573E-01 0.175E 01 0.202E 01 0.283E 01
42     0.743E-02 0.573E-01 0.175E 01 0.202E 01 0.283E 01
45     0.892E-02 0.573E-01 0.175E 01 0.202E 01 0.283E 01
48     0.107E-01 0.573E-01 0.174E 01 0.202E 01 0.285E 01
51     0.107E-02 0.573E-01 0.175E 01 0.202E 01 0.283E 01
54     0.107E-03 0.573E-01 0.175E 01 0.202E 01 0.283E 01

LARGEST MERIT ORDINATE ................................ 0.57333771E-01
NUMBER OF FUNCTION EVALUATIONS ........................             60
FINAL SEARCH STEPSIZE ..................................    0.00010699
STEPSIZE GROWTH MULTIPLIER .............................    1.19999981
SURVEY PATTERN INCREMENT ............................... 0.99999990E-04
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY ......      0.00100000
SPECIE OF LARGEST MERIT ORDINATE .......................             1

X(  1)=    0.17486E 01      P(  1)=    -0.14901E-03      Q(  1)=    -0.10058E-03
X(  2)=    0.20209E 01      P(  2)=     0.10058E-03      Q(  2)=     0.10058E-03

MERIT EXTREME IS AN EXTREMUM


EXECUTIVE PROGRAM OUTPUT FOR MINIMUM GEARTRAIN INERTIA

INPUT DATA

NUMBER OF REDUCTION STEPS IN GEARTRAIN ................          3
ROTARY INERTIA OF MOTOR ARMATURE, SLUG FT2, ..........   10.00000
ROTARY INERTIA OF MOTOR PINION, SLUG FT2, ............    1.00000
ROTARY INERTIA OF LOAD ON LAST SHAFT, SLUG FT2, ...... 100.00000
OVERALL GEAR RATIO SPECIFIED ..........................   10.00000


OUTPUT DATA

EQUIVALENT INERTIA OF OPTIMAL GEAR TRAIN,SLUG-FT2.....   17.44173
OPTIMAL GEAR RATIOS FROM MOTOR TO LOAD

RATIO(S)                                                  1.74862
RATIO(S)                                                  2.02090
RATIO(S)                                                  2.82983
```

The convergence monitor indicates the gradient search terminated in a region of very small slope. The forward slopes were

$$P(1) = -0.14901E-03$$
$$P(2) = 0.10058E-03$$

and the slopes in the negative abscissa directions were

$$Q(1) = -0.10058E-03$$
$$Q(2) = +0.10058E-03$$

Since the sign of the slope did not change when passing over the central ordinate to the merit surface, the search reported as terminating at an extreme rather than at a "domelike" summit. The extreme ordinate reported is very close to the summit ordinate as evidenced by the small slopes. The search reports the gear ratios as 1.74862, 2.02090, 2.82983, in that order from the motor shaft.

The following output is from the same program but considering I1 = 2 dimensions—i.e., a double reduction drive of the load. In this case the search terminated at a maximum reporting the proper gear ratios to be 2.42924 and 4.11651, in that order from the motor shaft. Note that the equivalent inertia has increased from 17.44173 slug ft^2 to 20.94221 slug ft^2 despite the removal of a pinion and gear from the train.

```
CONVERGENCE MONITOR SUBROUTINE GRAD4

   N1        DELTA        Y            X(1)          X(2)          X(3)

    2     0.100E 00 0.444E-01 0.200E 01 0.500E 01 0.283E 01
    4     0.120E 00 0.462E-01 0.212E 01 0.472E 01 0.283E 01
    6     0.144E 00 0.473E-01 0.226E 01 0.442E 01 0.283E 01
    8     0.173E 00 0.477E-01 0.244E 01 0.410E 01 0.283E 01
   10     0.207E 00 0.471E-01 0.223E 01 0.449E 01 0.283E 01
   12     0.207E-01 0.477E-01 0.246E 01 0.407E 01 0.283E 01
   14     0.207E-02 0.478E-01 0.243E 01 0.411E 01 0.283E 01
   16     0.249E-02 0.478E-01 0.243E 01 0.411E 01 0.283E 01
   18     0.299E-02 0.478E-01 0.243E 01 0.412E 01 0.283E 01
   20     0.358E-02 0.478E-01 0.243E 01 0.412E 01 0.283E 01
   22     0.358E-03 0.478E-01 0.243E 01 0.412E 01 0.283E 01

LARGEST MERIT ORDINATE ..............................  0.47750439E-01
NUMBER OF FUNCTION EVALUATIONS ......................              26
FINAL SEARCH STEPSIZE ...............................      0.00035832
STEPSIZE GROWTH MULTIPLIER ..........................      1.19999981
SURVEY PATTERN INCREMENT ............................  0.99999990E-04
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY .....      0.00100000
SPECIE OF LARGEST MERIT ORDINATE ....................               2

X(  1)=    0.24292E 01      P(  1)=    -0.33528E-04      Q(  1)=    0.67055E-04

MERIT EXTREME IS A MAXIMUM


EXECUTIVE PROGRAM OUTPUT FOR MINIMUM GEARTRAIN INERTIA

INPUT DATA

NUMBER OF REDUCTION STEPS IN GEARTRAIN ..............            2
ROTARY INERTIA OF MOTOR ARMATURE, SLUG FT2, .........     10.00000
ROTARY INERTIA OF MOTOR PINION, SLUG FT2, ...........      1.00000
ROTARY INERTIA OF LOAD ON LAST SHAFT, SLUG FT2, .....    100.00000
OVERALL GEAR RATIO SPECIFIED ........................     10.00000


OUTPUT DATA

EQUIVALENT INERTIA OF OPTIMAL GEAR TRAIN,SLUG-FT2....     20.94221
OPTIMAL GEAR RATIOS FROM MOTOR TO LOAD

RATIO(S)                                                   2.42924
RATIO(S)                                                   4.11651
```

This last exercise of the program could have been done using a golden section one-dimensional search.

5.6 A FUNCTION GENERATOR OF MAXIMUM ACCURACY

The natural input to an automatic control system is often either a mechanical rotation or a mechanical translation. It is a prudent concession to operators' tastes to provide control station input as a rotation of a knob scanning a *linear* scale, thereby providing both confidence in interpolation and ease of use. The necessary command input may be a translational movement, as in the compression of a coil spring, motion of a valve stem, motion of a hydraulic amplifier piston, the movement of an inductance core, etc. The necessary input command may be a rotational movement, as in a potentiometer shaft, condenser shaft, rotary valve stem, etc. In terms of the characteristics of the system under control, and of the automatic control itself, these inputs are not linear with the performance of the system. In order to provide a *linear* input dial it is necessary to interpose a mechanism that will accept a uniform angular displacement and convert it into a nonuniform translational or rotational response of appropriate characteristics—i.e., it is necessary to interpose a function generator between the operator and the "real" mechanical input command.

The problem reduces itself to the synthesis of the kinematic proportions of a mechanism to be used as a function generator. The simplest devices include the offset slider-crank mechanism, the flat-faced radial follower cam, and the flat-faced radial follower disk cam for rotational-input–translational-output function generators. The plane four-bar mechanism may be used as a rotational-input–rotational-output function generator.

As an illustration of the use of the digital computer and the IOWA CADET algorithm we shall consider the three-point synthesis of a four-bar mechanism to perform as a function generator in the command module of an automatic feedback control. The plane four-bar chain can accept a rotational input and provide a rotational output with approximate fidelity to a given functional relationship. The schematic of a four-bar function generator is shown in Fig. 83. Four configurations can be identified as illustrated in Fig. 84.

The linkage exists when the vector chain $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$, $\boldsymbol{\rho}_3$, $\boldsymbol{\rho}_4$ is closed—i.e., when

$$\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2 + \boldsymbol{\rho}_3 + \boldsymbol{\rho}_4 = 0$$

In order to obtain relationships which involve link lengths and the angles α and β, the dot product $\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_2$ is evaluated ($\boldsymbol{\rho}_4 = -\hat{\mathrm{i}}$).

$$\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_2 = (\hat{\mathrm{i}} - \boldsymbol{\rho}_1 - \boldsymbol{\rho}_3) \cdot (\hat{\mathrm{i}} - \boldsymbol{\rho}_1 - \boldsymbol{\rho}_3)$$

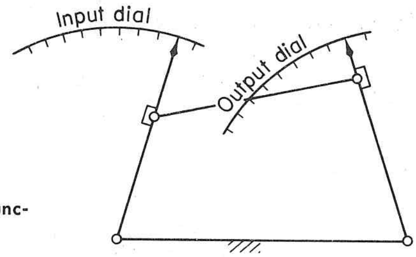

FIG. 83. The four-bar linkage as a function generator.

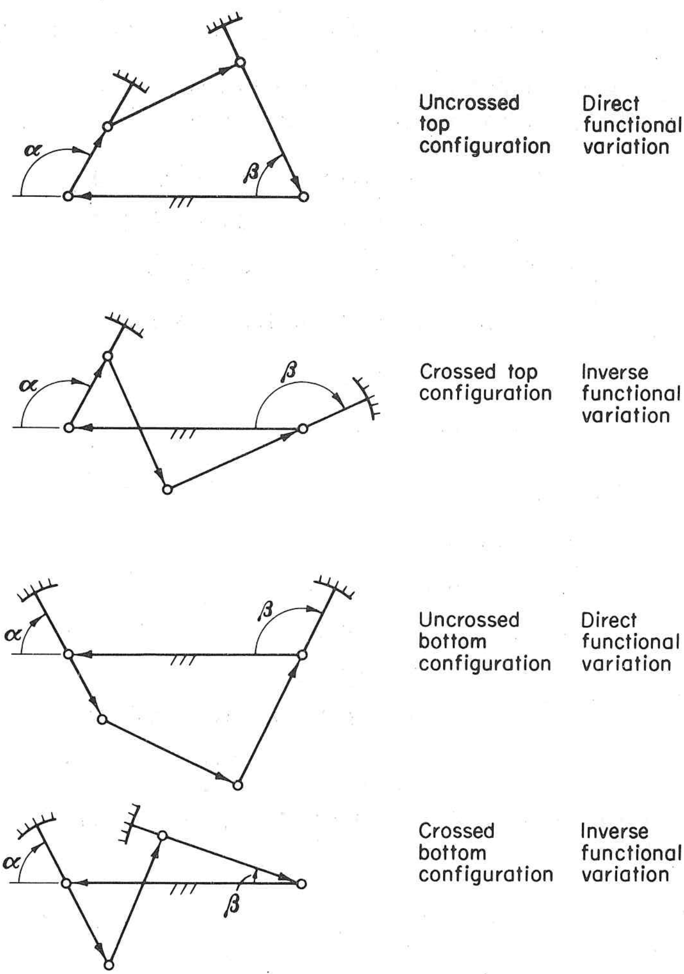

FIG. 84. Four possible configurations of the four-bar function generator.

This leads to the equation

$$\cos(\alpha - \beta) = \frac{(1 + \rho_1^2 - \rho_2^2 + \rho_3^2)}{2|\boldsymbol{\rho}_1||\boldsymbol{\rho}_3|} + \frac{1}{|\boldsymbol{\rho}_3|}\cos\alpha - \frac{1}{|\boldsymbol{\rho}_1|}\cos\beta \quad \text{(uncrossed top conf.)}$$

Denoting

$$\Delta_1 = \frac{1}{|\boldsymbol{\rho}_1|}, \quad \Delta_2 = \frac{(1 + \rho_1^2 - \rho_2^2 + \rho_3^2)}{2|\boldsymbol{\rho}_1||\boldsymbol{\rho}_3|}, \quad \Delta_3 = \frac{1}{|\boldsymbol{\rho}_3|}$$

it follows that

$$\cos(\alpha - \beta) = \Delta_3 \cos\alpha - \Delta_1 \cos\beta + \Delta_2 \qquad \text{(uncrossed top)}$$

A three-precision-point synthesis will be undertaken. The angular coordinates of the points of precision (zero error) are denoted (α_1, β_1), (α_2, β_2), (α_3, β_3).

$$\cos(\alpha_1 - \beta_1) = \Delta_3 \cos\alpha_1 - \Delta_1 \cos\beta_1 + \Delta_2 \tag{5.13}$$

$$\cos(\alpha_2 - \beta_2) = \Delta_3 \cos\alpha_2 - \Delta_1 \cos\beta_2 + \Delta_2 \tag{5.14}$$

$$\cos(\alpha_3 - \beta_3) = \Delta_3 \cos\alpha_3 - \Delta_1 \cos\beta_3 + \Delta_2 \tag{5.15}$$

Subtracting Eq. (5.14) from Eq. (5.13) and Eq. (5.15) from Eq. (5.13) yields

$$\delta_5 = \delta_1\Delta_3 - \delta_3\Delta_1$$

$$\delta_6 = \delta_2\Delta_3 - \delta_4\Delta_1$$

where

$$\delta_1 = \cos\alpha_1 - \cos\alpha_2$$

$$\delta_2 = \cos\alpha_1 - \cos\alpha_3$$

$$\delta_3 = \cos\beta_1 - \cos\beta_2$$

$$\delta_4 = \cos\beta_1 - \cos\beta_3$$

$$\delta_5 = \cos(\alpha_1 - \beta_1) - \cos(\alpha_2 - \beta_2)$$

$$\delta_6 = \cos(\alpha_1 - \beta_1) - \cos(\alpha_3 - \beta_3)$$

Solving the pair of simultaneous equations in Δ_1 and Δ_3 yields for the various configurations

Uncrossed top

$$\Delta_1 = \frac{\delta_2\delta_5 - \delta_1\delta_6}{\delta_1\delta_4 - \delta_2\delta_3}$$

$$\Delta_3 = \frac{\delta_4\delta_5 - \delta_3\delta_6}{\delta_1\delta_4 - \delta_2\delta_3}$$

$$\Delta_2 = \cos(\alpha_i - \beta_i) + \Delta_1 \cos\beta_i - \Delta_3 \cos\alpha_i, \quad i = 1, 2, 3$$

Uncrossed bottom

$$\Delta_1 = \frac{\delta_1\delta_6 - \delta_2\delta_5}{\delta_1\delta_4 - \delta_2\delta_3}$$

$$\Delta_3 = \frac{\delta_3\delta_6 - \delta_4\delta_5}{\delta_1\delta_4 - \delta_2\delta_3}$$

$$\Delta_2 = \cos(\alpha_i - \beta_i) - \Delta_1 \cos\beta_i + \Delta_3 \cos\alpha_i, \quad i = 1, 2, 3$$

For inverse functional variation the equations are

Crossed top

$$\Delta_1 = \frac{\delta_2\delta_5 - \delta_1\delta_6}{\delta_1\delta_4 - \delta_2\delta_3}$$

$$\Delta_3 = \frac{\delta_3\delta_6 - \delta_4\delta_5}{\delta_1\delta_4 - \delta_2\delta_3}$$

$$\Delta_2 = -\cos(\alpha_i - \beta_i) - \Delta_1 \cos\beta_i - \Delta_3 \cos\alpha_i, \quad i = 1, 2, 3$$

Crossed bottom

$$\Delta_1 = \frac{\delta_1\delta_6 - \delta_2\delta_5}{\delta_1\delta_4 - \delta_2\delta_3}$$

$$\Delta_3 = \frac{\delta_4\delta_5 - \delta_3\delta_6}{\delta_1\delta_4 - \delta_2\delta_3}$$

$$\Delta_2 = -\cos(\alpha_i - \beta_i) + \Delta_1 \cos\beta_i + \Delta_3 \cos\alpha_i, \quad i = 1, 2, 3$$

Configurations may be discerned by observing the algebraic signs of Δ_1 and Δ_3, both of which must be positive.

A suitable linkage may be synthesized by choosing precision points (α_1, β_1), (α_2, β_2), (α_3, β_3), evaluating δ's, discovering configuration by testing Δ_1 and Δ_3 for positiveness, solving appropriate equations for Δ_1, Δ_2, Δ_3 from which ρ_1, ρ_2, and ρ_3 follow ($\rho_4 = 1$).

The four-bar linkage synthesized by the previously described method will be completely faithful to the function desired only at three precision points. Elsewhere, error will be detectable. Such built-in errors are referred to as *structural* errors. In this type of application we may be seeking that linkage in which the largest structural error has the least possible magnitude. A sketch of an error for a three-precision-point synthesis is shown as Fig. 85. Notice that the three precision points within the usable range of the device break the error curve into four domains, each of which contains an extremum. The largest extreme of error can be reduced by moving the flanking precision points toward each other or by moving a precision point toward a boundary. Although this change results in lessening the extreme between the precision points, the adjustment will increase the error on flanking domains. It is clear that the choice of precision points resulting in the largest error of least size is the one which creates extremes in each of the four domains which are equal in size.

Recognition of this idea is central to a strategy for uncovering the optimal positioning of precision points. We shall envision our merit space

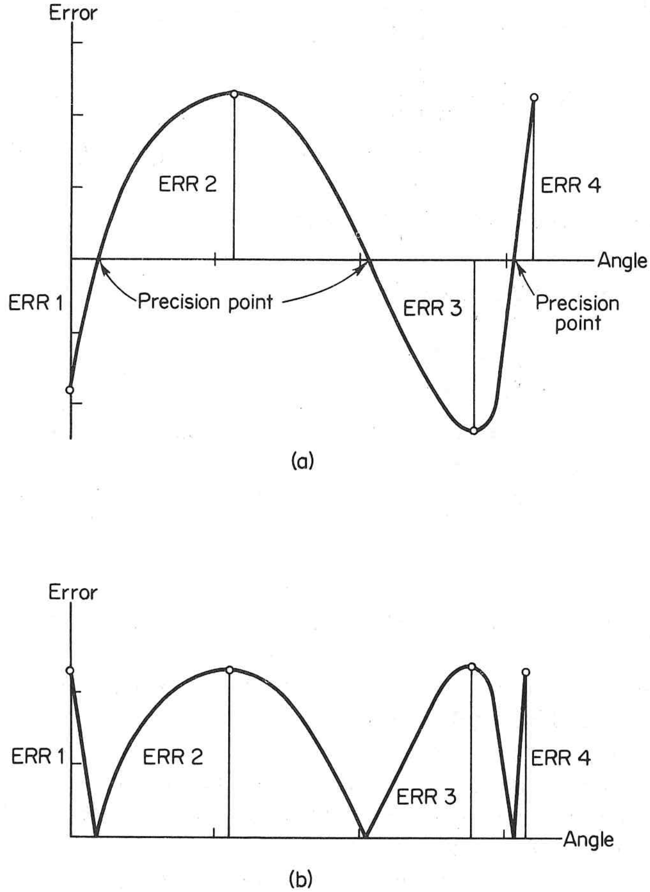

FIG. 85. Error curves for a three-point synthesis (a) unrectified and (b) rectified.

as having the independent coordinates which are the locations of the precision points (expressed conveniently as input dial readings) and use as the ordinate to the merit hypersurface the reciprocal of the largest structural error present in the mechanism.

In solving this problem we shall call upon several library routines already present in the IOWA CADET design library. These are ME0034, ME0035, and ME0036, which are documented in Appendix 2. The routine ME0034 accepts linkage configuration information, input dial angle, link lengths, and returns the output dial angle. ME0035 performs a three-point synthesis when given the input and output angles corresponding to the precision points, and returns the link lengths and the linkage configuration. ME0036 accepts the function values of the independent variable at the precision points and sector angles of input and output dials, and returns the input and output angles corresponding to the precision points as well as the dependent variable values at the precision points. Another useful subroutine is YASORT, which orders a column vector placing the largest value of Y in the first entry, and subsequent entries are in descending order of magnitude. The associated abscissas are ordered to correspond to the new arrangement of the column vector Y.

The plan for using IOWA CADET is indicated in Fig. 86. Inasmuch as no ridges are expected and the summit of merit function will be domelike, a gradient search will be utilized. The gradient search plan selected will be GRAD4, and the necessary name of the merit subroutine (which is called by GRAD4) is MERIT4, as indicated in the documentation of GRAD4. The subroutine MERIT4 will first call ME0036 to obtain important angles to use in calling ME0035. The subroutine ME0035 will synthesize a linkage and return the link lengths. Since the left-hand and right-hand extremes of error will be at the ends of the dial intervals, a simple call to MERIT1 at an extreme will give the largest errors in the extreme domains outside the precision points. For determining the largest errors in the two interior domains (a one-dimensional search), it is necessary to call GOLD1, which manipulates MERIT1 and returns the largest errors in the interior domains. The four largest domain errors are placed in a column vector, and the largest is found by subroutine YASORT. The reciprocal of the largest structural error present in the current linkage is returned as the merit ordinate to gradient search GRAD4.

The subroutine MERIT1 determines the absolute value of the absolute structural error in a linkage. In doing this, MERIT1 calls ME0034 and another subroutine FUNCT, which defines the function to be generated and returns an ordinate for a tendered abscissa.

```
C
C     EXECUTIVE PROGRAM FOR OPTIMAL PROPORTIONS OF A FOURBAR LINKAGE
C     WITH THREE PRECISION POINTS TO ANALOGICALLY GENERATE A FUNCTION.
C     C. MISCHKE   IOWA STATE UNIVERSITY
C
C     A1=INPUT ANGLE AT FIRST PRECISION POINT, RAD
C     A2=INPUT ANGLE AT SECOND PRECISION POINT, RAD
C     A3=INPUT ANGLE AT THIRD PRECISION POINT, RAD
C     A4=INPUT ANGLE AT EXTREME LEFT OF DIAL, RAD
C     A5=INPUT ANGLE AT EXTREME RIGHT OF DIAL, RAD
C
C     B1=OUTPUT ANGLE AT FIRST PRECISION POINT, RAD
C     B2=OUTPUT ANGLE AT SECOND PRECISION POINT, RAD
C     B3=OUTPUT ANGLE AT THIRD PRECISION POINT, RAD
C     B4=OUTPUT ANGLE AT EXTREME LEFT OF DIAL, RAD
C     B5=OUTPUT ANGLE AT EXTREME RIGHT OF DIAL, RAD
C
C     X1=VALUE OF INDEPENDENT VARIABLE AT FIRST PRECISION POINT
C     X2=VALUE OF INDEPENDENT VARIABLE AT SECOND PRECISION POINT
C     X3=VALUE OF INDEPENDENT VARIABLE AT THIRD PRECISION POINT
C     X4=VALUE OF INDEPENDENT VARIABLE AT EXTREME LEFT OF INPUT DIAL
C     X5=VALUE OF INDEPENDENT VARIABLE AT EXTREME RIGHT OF INPUT DIAL
C
C     Y1=VALUE OF DEPENDENT VARIABLE AT FIRST PRECISION POINT
```

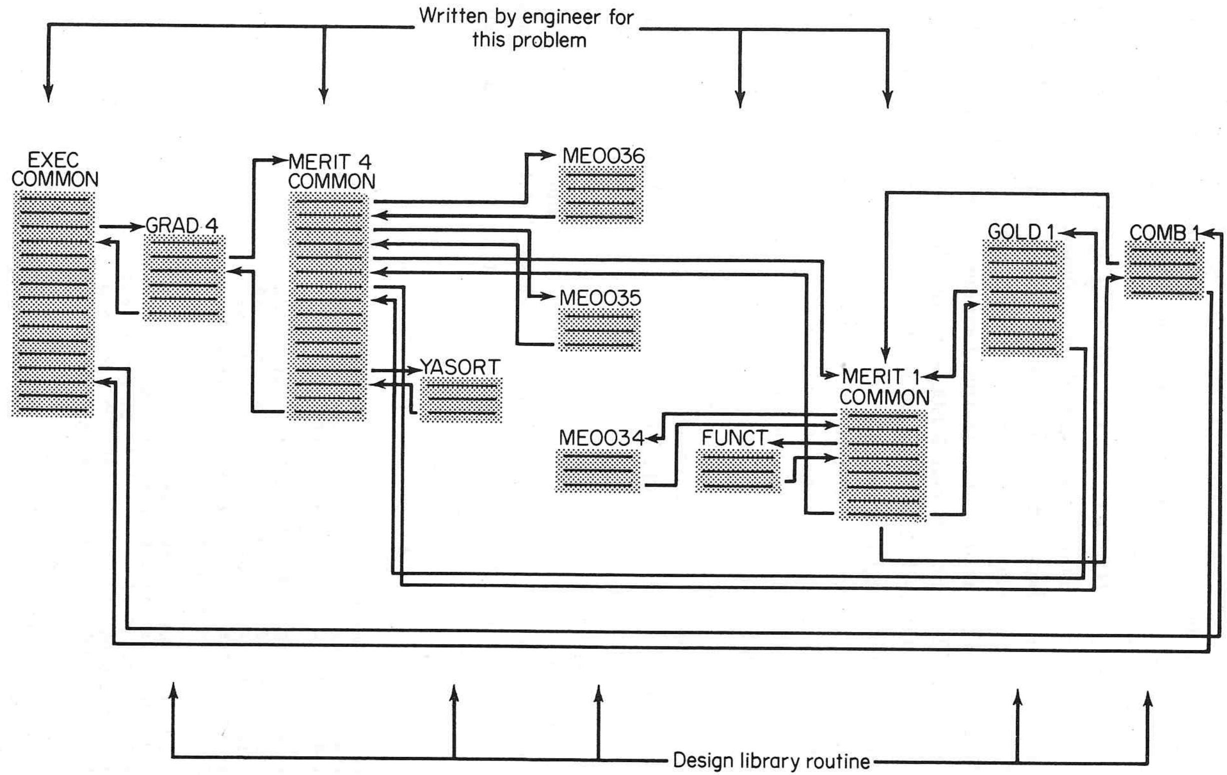

FIG. 86. Example of program arrangement for solution of the four-bar function generator problem using the IOWA CADET algorithm.

```
C        Y2=VALUE OF DEPENDENT VARIABLE AT SECOND PRECISION POINT
C        Y3=VALUE OF DEPENDENT VARIABLE AT THIRD PRECISION POINT
C        Y4= VALUE OF DEPENDENT VARIABLE AT EXTREME LEFT OF OUTPUT DIAL
C        Y5=VALUE OF DEPENDENT VARIABLE AT EXTREME RIGHT OF OUTPUT DIAL
C
C        XX=INDEPENDENT VARIABLE
C        YY=DEPENDENT VARIABLE
C        A=INPUT ANGLE CORRESPONDING TO XX
C        B=OUTPUT ANGLE CORRESPONDING TO YY
C
C        R1=LENGTH OF CRANK VECTOR
C        R2=LENGTH OF COUPLER VECTOR
C        R3=LENGTH OF FOLLOWER VECTOR
C        R4=LENGTH OF GROUNDED LINK VECTOR
C
         COMMON X4,X5,A4,A5,Y4,Y5,B4,B5,R1,R2,R3,R4,F,I4,J1
         DIMENSION XL(9),XR(9),X(9),P(9),Q(9),BB3(2001),BB4(2001),ABC(20)
       1 READ(1,2)A6,A7,B6,B7,X4,X5,Y4,Y5
       2 FORMAT(8F10.4)
         C=3.14159/180.
         A4=A6*C
         A5=A7*C
         B4=B6*C
         B5=B7*C
         READ(1,3)I1,I2,I3,DELTA,DMULT,EPS,F,I4
       3 FORMAT(3I2,2F10.5,2F10.8,I5)
         READ(1,4)(ABC(I),I=1,20)
       4 FORMAT(20A4)
         XL(1)=1.
         XL(2)=1.
         XL(3)=1.
         XR(1)=10.
         XR(2)=10.
         XR(3)=10.
         X(1)=1.1
         X(2)=7.
         X(3)=9.9
         CALL GRAD4(I1,I2,I3,DELTA,DMULT,F,EPS,XL,XR,RERR,X,K3,K4,P,Q)
         ERROR=1./RERR
         WRITE(3,5)(ABC(I),I=1,20),X4,X5,Y4,Y5,A6,A7,B6,B7
       5 FORMAT(1H1,'EXECUTIVE PROGRAM PRINTOUT',/,/,' INPUT DATA',//,20A4,
        1/,
        2' NUMBER APPEARING EXTREME LEFT OF INPUT DIAL ..........',F15.5,/,
        3' NUMBER APPEARING EXTREME RIGHT OF INPUT DIAL .........',F15.5,/,
        4' NUMBER APPEARING EXTREME LEFT OF OUTPUT DIAL .........',F15.5,/,
        5' NUMBER APPEARING EXTREME RIGHT OF OUTPUT DIAL ........',F15.5,/,
        6/,
        7' INPUT ANGLE ALPHA OF LOWEST INPUT DIAL READING .......',F15.5,/,
        8' INPUT ANGLE ALPHA OF HIGHEST INPUT DIAL READING ......',F15.5,/,
        9' OUTPUT ANGLE BETA OF LOWEST OUTPUT DIAL READING ......',F15.5,/,
        1' OUTPUT ANGLE BETA OF HIGHEST OUTPUT DIAL READING .....',F15.5)
         X1=X(1)
         X2=X(2)
         X3=X(3)
         CALL ME0036(X1,X2,X3,X4,X5,Y4,Y5,A4,A5,B4,B5,
        1 A1,A2,A3,B1,B2,B3,Y1,Y2,Y3)
         CALL ME0035(A1,B1,A2,B2,A3,B3,R1,R2,R3,R4,J1)
         WRITE(3,6)R1,R2,R3,R4
       6 FORMAT(/,' OUTPUT DATA',//,
        1' LENGTH OF CRANK VECTOR ..............................',E15.8,/,
        2' LENGTH OF COUPLER VECTOR ............................',E15.8,/,
        3' LENGTH OF FOLLOWER VECTOR ...........................',E15.8,/,
        4' LENGTH OF GROUNDED LINK VECTOR ......................',E15.8)
         GO TO(7,9,11,13),J1
       7 WRITE(3,8)
       8 FORMAT(/,' LINKAGE CONFIGURATION UNCROSSED TOP')
         GO TO 15
       9 WRITE(3,10)
      10 FORMAT(/,' LINKAGE CONFIGURATION CROSSED TOP')
         GO TO 15
      11 WRITE(3,12)
      12 FORMAT(/,' LINKAGE CONFIGURATION UNCROSSED BOTTOM')
         GO TO 15
```

```
13 WRITE(3,14)
14 FORMAT(/,' LINKAGE CONFIGURATION CROSSED BOTTOM')
15 I=1
   FF=0.1
   CALL COMB1(I,A4,A5,FF,BB1,BB2,BB3,BB4,BB5,BB6,L)
   GO TO 1
   END

   SUBROUTINE MERIT4(X,Y)
   COMMON X4,X5,A4,A5,Y4,Y5,B4,B5,R1,R2,R3,R4,F,I4,J1
   DIMENSION U(100),WYE(100),X(9)
   X1=X(1)
   X2=X(2)
   X3=X(3)
   CALL ME0036(X1,X2,X3,X4,X5,Y4,Y5,A4,A5,B4,B5,
  1A1,A2,A3,B1,B2,B3,Y1,Y2,Y3)
   CALL ME0035(A1,B1,A2,B2,A3,B3,R1,R2,R3,R4,J1)
   CALL MERIT1(A4,ERR1)
   CALL GOLD1(I4,A1,A2,F,ERR2,XX2,XL2,XR2,J4)
   CALL GOLD1(I4,A2,A3,F,ERR3,XX3,XL3,XR3,J6)
   CALL MERIT1(A5,ERR4)
   WYE(1)=ERR1
   WYE(2)=ERR2
   WYE(3)=ERR3
   WYE(4)=ERR4
   N=4
   U(1)=XX1
   U(2)=XX2
   U(3)=XX3
   U(4)=XX4
   CALL YASORT(N,U,WYE)
   Y=1./WYE(1)
   RETURN
   END

   SUBRCUTINE MERIT1(A,ERROR)
   COMMON X4,X5,A4,A5,Y4,Y5,B4,B5,R1,R2,R3,R4,F,I4,J1
   CALL ME0034(J1,A,R1,R2,R3,R4,B)
   YGEN=Y5-(B5-B)*(Y5-Y4)/(B5-B4)
   XX=X5-(A5-A)*(X5-X4)/(A5-A4)
   CALL FUNCT(XX,YTRUE)
   ERROR=ABS(YGEN-YTRUE)
   RETURN
   END

   SUBROUTINE FUNCT(XX,YTRUE)
   YTRUE=XX*XX
   RETURN
   END
```

The following output was obtained from this program with input data

A6 = 0. deg
A7 = 90. deg
B6 = 60. deg
B7 = 180. deg
X4 = 1.
X5 = 10.
Y4 = 1.
Y5 = 100.
I1 = 3 dimensions
I2 = 1 convergence monitor print
I3 = 4 search start at initial value of column vector X, established by FORTRAN declarative statement in executive program

DELTA = 0.1
DMULT = 1.2
EPS = 0.0001
F = 0.01
I4 = 0 GOLD1 convergence monitor do not print

The search began with the precision points located at input dial readings 1.1, 7.0, 9.9. The search terminated at an extreme due to the "lumpy" nature of the merit hypersurface. This surface cannot be examined too closely, since the merit ordinates used were established by golden section searches of not too fine a resolution. The proportions of the four-bar linkage were established as

$$\rho_1 = 0.77127433$$

$$\rho_2 = 0.16044645$$

$$\rho_3 = 0.39842039$$

$$\rho_4 = 1.00000000$$

but not to the number of significant figures implied by the computer output sheet. The linkage configuration is uncrossed top.

CONVERGENCE MONITOR SUBROUTINE GRAD4

N1	DELTA	Y	X(1)	X(2)	X(3)
4	0.100E 00	0.384E 00	0.110E 01	0.700E 01	0.990E 01
8	0.120E 00	0.389E 00	0.114E 01	0.689E 01	0.988E 01
12	0.144E 00	0.418E 00	0.122E 01	0.697E 01	0.978E 01
16	0.173E 00	0.444E 00	0.139E 01	0.694E 01	0.980E 01
20	0.207E 00	0.383E 00	0.141E 01	0.709E 01	0.966E 01
24	0.207E-01	0.436E 00	0.139E 01	0.694E 01	0.982E 01
28	0.207E-02	0.445E 00	0.139E 01	0.694E 01	0.980E 01
32	0.249E-02	0.445E 00	0.139E 01	0.694E 01	0.979E 01
36	0.299E-02	0.446E 00	0.139E 01	0.694E 01	0.979E 01
40	0.358E-02	0.446E 00	0.139E 01	0.694E 01	0.979E 01
44	0.430E-02	0.447E 00	0.139E 01	0.694E 01	0.979E 01
48	0.516E-02	0.448E 00	0.140E 01	0.693E 01	0.979E 01
52	0.619E-02	0.449E 00	0.140E 01	0.693E 01	0.979E 01
56	0.743E-02	0.449E 00	0.140E 01	0.693E 01	0.978E 01
60	0.892E-02	0.449E 00	0.140E 01	0.693E 01	0.978E 01
64	0.107E-01	0.452E 00	0.140E 01	0.693E 01	0.977E 01
68	0.128E-01	0.454E 00	0.141E 01	0.693E 01	0.977E 01
72	0.154E-01	0.454E 00	0.143E 01	0.692E 01	0.977E 01
76	0.154E-02	0.454E 00	0.141E 01	0.693E 01	0.977E 01

```
LARGEST MERIT ORDINATE ............................... 0.45404536E 00
NUMBER OF FUNCTION EVALUATIONS ......................              84
FINAL SEARCH STEPSIZE ...............................      0.00154069
STEPSIZE GROWTH MULTIPLIER ..........................      1.19999981
SURVEY PATTERN INCREMENT ............................  0.99999990E-04
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY .....      0.00100000
SPECIE OF LARGEST MERIT ORDINATE ....................               1

X(  1)=    0.14137E 01     P(  1)=     0.14350E 00     Q(  1)=     0.14032E 00
X(  2)=    0.69252E 01     P(  2)=    -0.20301E 00     Q(  2)=     0.94281E-01
X(  3)=    0.97692E 01     P(  3)=    -0.26008E 00     Q(  3)=    -0.13579E 00

MERIT EXTREME IS AN EXTREMUM
```

EXECUTIVE PROGRAM PRINTOUT

INPUT DATA

FUNCTION GENERATED IS Y=X*X

NUMBER APPEARING EXTREME LEFT OF INPUT DIAL	1.00000
NUMBER APPEARING EXTREME RIGHT OF INPUT DIAL	10.00000
NUMBER APPEARING EXTREME LEFT OF OUTPUT DIAL	1.00000
NUMBER APPEARING EXTREME RIGHT OF OUTPUT DIAL	100.00000
INPUT ANGLE ALPHA OF LOWEST INPUT DIAL READING	0.0
INPUT ANGLE ALPHA OF HIGHEST INPUT DIAL READING	90.00000
OUTPUT ANGLE BETA OF LOWEST OUTPUT DIAL READING	60.00000
OUTPUT ANGLE BETA OF HIGHEST OUTPUT DIAL READING	180.00000

OUTPUT DATA

LENGTH OF CRANK VECTOR	0.77127433E 00
LENGTH OF COUPLER VECTOR	0.16044645E 01
LENGTH OF FOLLOWER VECTOR	0.39842039E 00
LENGTH OF GROUNDED LINK VECTOR	0.10000000E 01

LINKAGE CONFIGURATION UNCROSSED TOP

CONVERGENCE MONITOR SUBROUTINE COMB1

ORDINATE	ABSCISSA	EXTREME
0.6261444E 00	0.0	-0.9999997E 51
0.8524799E-01	0.8267337E-01	0.6261444E 00
0.7083435E 00	0.1653467E 00	0.6261444E 00
0.1236617E 01	0.2480201E 00	0.7083435E 00
0.1660911E 01	0,3306935E 00	0.1236617E 01
0.1970069E 01	0.4133669E 00	0.1660911E 01
0.2153262E 01	0.4960402E 00	0.1970069E 01
0.2200516E 01	0.5787136E 00	0.2153262E 01
0.2105133E 01	0.6613870E 00	0.2200516E 01
0.1864670E 01	0.7440603E 00	0.2200516E 01
0.1482315E 01	0.8267337E 00	0.2200516E 01
0.9694214E 00	0.9094071E 00	0.2200516E 01
0.3475037E 00	0.9920805E 00	0.2200516E 01
0.3476715E 00	0.1074754E 01	0.2200516E 01
0.1059937E 01	0.1157427E 01	0.2200516E 01
0.1701859E 01	0.1240100E 01	0.2200516E 01
0.2132019E 01	0.1322773E 01	0.2200516E 01
0.2108536E 01	0.1405446E 01	0.2200516E 01
0.1160721E 01	0.1488119E 01	0.2200516E 01
0.1884399E 01	0.1570792E 01	0.2200516E 01

EXTREME ORDINATE FOUND IN EXHAUSTIVE SEARCH	0.2200516E 01
ABSCISSA CORRESPONDING TO EXTREME ORDINATE	0.5787136E 00
ORIGINAL LOWER BOUND ON INTERVAL OF UNCERTAINTY	0.0
ORIGINAL UPPER BOUND ON INTERVAL OF UNCERTAINTY	0.1570794E 01
FINAL LOWER BOUND ON INTERVAL OF UNCERTAINTY	0.4960402E 00
FINAL UPPER BOUND ON INTERVAL OF UNCERTAINTY	0.6613870E 00
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY	0.9999996E-01
NUMBER OF FUNCTION EVALUATION EXPENDED	20

The convergence monitor of subroutine COMB1 was allowed to print out the error in the Y indication using the input dial angle as the abscissa. The largest error in the Y indication occurs in the neighborhood of $\alpha = 0.5787136$ radians and is of magnitude 2.2200516. The next improvement can result from the use of a relative error criterion rather than an absolute error criterion in developing the merit ordinate.

Further improvement in accuracy, using the input sector of 90° and the output sector of 120°, can be sought by making A4 and B4 design variables in addition to the location of the precision points X(1), X(2), and X(3), raising the number of design variables to five.

5.7 A CURVED BEAM OF MINIMAL WEIGHT

A C clamp is to be designed to provide clearances as indicated in Fig. 87(a). The design load is to be 900 lbf with a design factor of 2, based upon ultimate strength as it relates to the stresses developed in the inner and outer fibers of the curved beam. The cross section is to be a "T" made of ASTM 40 cast iron. Because of casting limitations, the lower limit on the magnitude of the thickness dimensions x_2 and x_3 in Fig. 87(b) will be one quarter inch. The proportions of the tee section exhibiting the minimum weight are required.

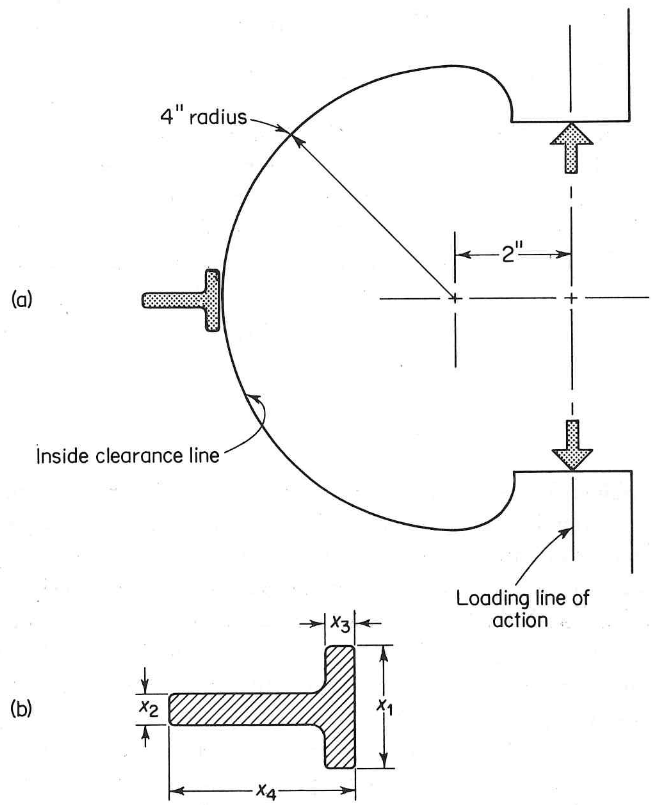

FIG. 87. The clearance geometry of a required C-clamp design and the geometry of the contemplated curved beam section.

Information on ASTM 40 cast iron is as follows:

Ultimate tensile stress	40,000 psi
Ultimate compressive stress	120,000 psi
Specific weight	0.26 lbf/in.^{3}

The question of the merit of alternatives will be considered first. The conditions in the statement of the problem limit the designer to finding the minimum weight configuration using a specified section and a stipulated material. In order to determine the weight of the toroidal curved beam, the distance from the center of curvature to the centroid of the cross section must be found. The figure of merit must be sensitive to the weight of the curved beam, and so the reciprocal of the beam weight is chosen to be the figure of merit. The regional constraints could be

$$0.25 \leq x_1 \leq 1.50$$
$$0.25 \leq x_2 \leq 0.50$$
$$0.25 \leq x_3 \leq 0.50$$
$$0.25 \leq x_4 \leq 3.00$$

The functional constraints involve the limiting stresses in the outermost fibers. The calculation of stresses from information on sectional geometry in a curved beam is straightforward, but the determination of geometry corresponding to a given stress level is very complex. Again, if the configuration tendered to the merit subroutine violates a functional constraint by exceeding a stress level, the merit ordinate will be penalized by calling subroutine PENAL. The functional constraints are

$$s_i \leq (s_t) \quad \text{allowable}$$
$$s_o \leq (s_c) \quad \text{allowable}$$

The problem becomes that of finding the configuration (specified by x_1, x_2, x_3, x_4) wherein the reciprocal of the beam weight is largest, subject to the regional and functional constraints. The problem geometry is five-dimensional with the four independent design variables of merit hyperspace being x_1, x_2, x_3, and x_4. If we could be certain that the minimum weight configuration is associated with $x_2 = x_3 = 0.25$, then we could operate in geometrically interpretable 3 space. Since this is not obviously true at this stage, we begin to program the problem for 5 space.

The IOWA CADET design library has a subroutine ME0053 which calculates the fiber stresses in a curved bar with a tee cross section. The documentation of this routine is in Appendix 2. We note that the subroutine has ten arguments in its call list, accepting seven and returning three. If

PP = load, lbf
RI = radius of curvature of inner fiber, in.
D = offset of center of curvature from load line of action, in.
BI = section-width at inner fiber, in.
BO = section-width at outer fiber, in.
C1 = thickness of cap of tee, in.
C2 = thickness of stem of tee, in.
SI = inner fiber stress, psi
SO = outer fiber stress, psi
R = radius of curvature of section centroidal locus, in.

then the proper call for the subroutine is

CALL ME0053 (PP, RI, D, BI, BO, C1, C2, SI, SO, R)

We are now in a position to construct the merit subroutine. Choosing grid-search library subroutine GRID4, we discover, from its documentation in Appendix 2, that it calls MERIT4 in searching for the extreme ordinate. The name of the merit subroutine is required to be MERIT4 (X, Y). Figure 88 reveals the structure of our strategy to solve the problem. The function of MERIT4 is to generate the figure of merit ordinate, with the assistance of subroutines ME0053 and PENAL, and respond to interrogations of GRID4 in its search for the extreme merit ordinate.

The executive program supplies data to GRID4 via its call list, and to

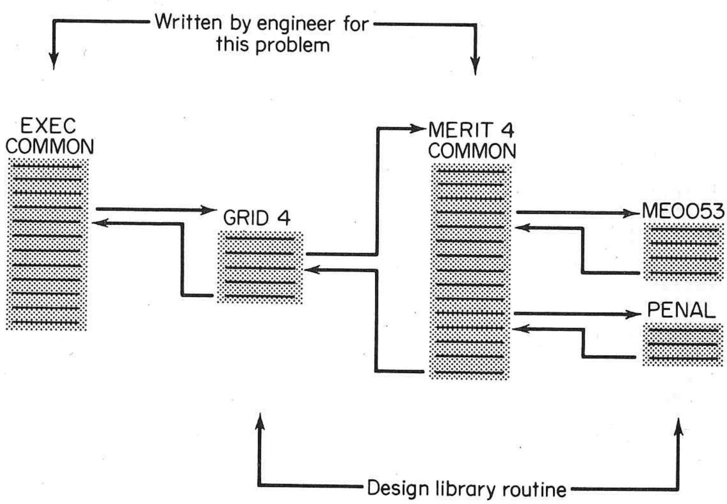

FIG. 88. Example of program arrangement for solution of the curved beam problem using the IOWA CADET algorithm.

MERIT4 via the COMMON declaration. Output information held by MERIT4 that cannot be passed back via GRAD4 is returned through the COMMON statement. We now write the MERIT4 subprogram and the executive program. A possible composition follows.

```
C      EXECUTIVE PROGRAM FOR C-CLAMP FRAME OF A 'T' SECTION OF
C      SMALLEST WEIGHT.
C
C      STATEMENT OF PROBLEM
C
C      A C-CLAMP IS TO BE DESIGNED HAVING A T-CROSSSECTION WITH CAP OF
C      THE T ON INSIDE OF FRAME. DEVISE A PROGRAM TO DETERMINE THE
C      PARAMETERS OF THE CONFIGURATION OF MINIMAL WEIGHT OF C-FRAME,
C      GIVEN ESSENTIAL GEOMETRY, MATERIAL SPECIFIC WEIGHT, ALLOWABLE
C      STRESSES IN TENSION AND COMPRESSION, LOAD, AND SUBJECT TO SECTION
C      MINIMUM THICKNESSES DUE TO CASTING CONSIDERATIONS
C
C      NOMENCLATURE
C
C      PP=FORCE ON C-CLAMP,POSITIVE OPENS 'C',LBF
C      D=DISTANCE BETWEEN CENTER OF CURVATURE AND LINE OF ACTION OF
C        FORCE PP, POSITIVE IF OUTSIDE, INCHES
C      RI=RADIUS OF CURVATURE OF INNER FIBER, INCHES
C      BI=SECTION WIDTH AT INNER FIBER, INCHES
C      BO=SECTION WIDTH AT OUTER FIBER, INCHES
C
C      C1=THICKNESS OF CAP OF 'T', INCHES
C      C2=LENGTH OF STEM OF 'T', INCHES
C      N=NUMBER OF INDEPENDENT VARIABLES IN OPTIMIZATION
C      I2=CONVERGENCE MONITOR PRINT SIGNAL
C      F=FRACTIONAL REDUCTION IN DOMAIN OF UNCERTAINTY DESIRED
C      R=FRACTIONAL REDUCTION IN GRID SIZE UTILIZED
C
C      XL(I)=LOWER BOUND OF SEARCH DOMAIN, COLUMN VECTOR
C      XR(I)=UPPER BOUND OF SEARCH DOMAIN, COLUMN VECTOR
C      GAMMA=SPECIFIC WEIGHT OF FRAME MATERIAL, LBF/IN3
C      GOOD=COLUMN VECTOR OF CROSS SECTION DIMENSIONS NOT VIOLATING
C      FUNCTIONAL CONSTRAINTS OF STRESS LEVEL AT OUTER AND INNER SURFACE.
C
       COMMON PP,D,RI,GAMMA,ST,SC,W,SI,SO,GOOD(9),N,XL(9),XR(9)
       DIMENSION X(9),IABC(20),XLOW(9),XHIGH(9)
C
C      ..... READ IN DATA .....
C
     1 READ(1,2)PP,D,RI,BI,BO,GAMMA
     2 FORMAT(6F10.5)
       READ(1,3)C1,C2,ST,SC,N
     3 FORMAT(2F10.5,2F10.0,I5)
       READ(1,4)I2,F,R
     4 FORMAT(I5,2F10.5)
       DO 6 I=1,N
       READ(1,5)XLO,XHI
     5 FORMAT(2F10.5)
       XL(I)=XLO
       XR(I)=XHI
     6 CONTINUE
       READ(1,9)(IABC(I),I=1,20)
     9 FORMAT(20A4)
       READ(1,10)(GOOD(I),I=1,4)
    10 FORMAT(4F10.5)
C
C      ..... INITIATE A GRID SEARCH TO FIND THE DIMENSIONS OF        .....
C      ..... LEAST WEIGHT SECTION.                                   .....
C
       CALL GRID4(N,I2,XL,XR,F,R,Y,X,XLOW,XHIGH,NN)
C
C      ..... DOCUMENT SEARCH RESULTS .....
C
```

```
      WRITE(3,7)PP,D,RI,C1,BO,ST,SC,GAMMA,(IABC(I),I=1,20)
    7 FORMAT('1EXECUTIVE PROGRAM OUTPUT MINIMUM WEIGHT C-CLAMP FRAME',/,
     1' FORCE ON C-CLAMP (POSITIVE OPENS  C ), LBF ...........',F10.0,/,
     2' DISTANCE FROM LOA OF FORCE TO CENTER OF CURVATURE,IN..',F10.5,/,
     3' RADIUS OF CURVATURE OF INNER FIBER, INCHES ...........',F10.5,/,
     4' MINIMUM SECTION WIDTH IN CAP OF  T , INCHES ..........',F10.5,/,
     5' MINIMUM SECTION WIDTH IN STEM OF  T , INCHES .........',F10.5,/,
     6' MAXIMUM ALLOWABLE TENSILE STRESS, PSI ................',F10.0,/,
     7' MAXIMUM ALLOWABLE COMPRESSIVE STRESS, PSI ............',F10.0,/,
     8' SPECIFIC WEIGHT OF FRAME MATERIAL, LBF/IN3 ...........',F10.5,/,
     920A4)
      WRITE(3,8)W,(X(I),I=1,4),SI,SO
    8 FORMAT(//,' CHARACTERISTICS OF OPTIMAL CONFIGURATION',//,
     1' WEIGHT OF FRAME, LBF .................................',F10.5,/,
     2' WIDTH OF SECTION AT INNER RADIUS,INCHES...............',F10.5,/,
     3' WIDTH OF SECTION AT OUTER RADIUS,INCHES...............',F10.5,/,
     4' THICKNESS OF CAP OF  T , INCHES ......................',F10.5,/,
     5' RADIAL DEPTH OF SECTION, INCHES.......................',F10.5,/,
     6' NORMAL STRESS AT INNER FIBER, PSI ....................',F10.0,/,
     7' NORMAL STRESS AT OUTER  FIBER, PSI ...................',F10.0)
      GO TO 1
      END

      SUBROUTINE MERIT4(X,Y)
      COMMON PP,D,RI,GAMMA,ST,SC,W,SI,SO,GOOD(9),N,XL(9),XR(9)
      DIMENSION X(9)
C
C     ..... CHECK TO SEE IF POINT VIOLATES ANY REGIONAL CONSTRAINT, ....
C     ..... IF SO, ASSIGN PENALTY ORDINATE AND RETURN.              ....
C
      DO 10 I=1,N
      IF(X(I)-XL(I))4,9,9
    9 IF(X(I)-XR(I))10,10,4
   10 CONTINUE
C
C     ..... INITIALIZE .....
C
      BI=X(1)
      BO=X(2)
      C1=X(3)
      C2=X(4)-X(3)
C
C     ..... CALL CURVED BEAM TEE-SECTION SUBROUTINE TO DETERMINE    .....
C     ..... STRESS LEVELS AT THE INNER AND OUTER SURFACES, AND TO   .....
C     ..... DETERMINE THE RADIUS OF CURVATURE OF LOCUS OF THE CENTER....
C     ..... OF MASS OF CROSS SECTIONAL AREA.                        .....
C
      CALL ME0053(PP,RI,D,BI,BO,C1,C2,SI,SO,R)
C
C     ..... PREPARE MERIT ORDINATE .....
C
      A=BI*C1+BO*C2
      W=R*A*GAMMA*3.14159
      Y=1./W
C
C     ..... DETERMINE IF FUNCTIONAL CONSTRAINTS HAVE BEEN VIOLATED .....
C     ..... BY THIS PARTICULAR CROSS SECTIONAL CONFIGURATION, IF SO,....
C     ..... RETURN PENALTY ORDINATE; IF NOT, RETURN MERIT ORDINATE .....
C
      IF(SI)6,1,1
    1 IF(ABS(ST)-ABS(SI))4,2,2
    2 IF(ABS(SC)-ABS(SO))4,3,3
    6 IF(ABS(SC)-ABS(SI))4,7,7
    7 IF(ABS(ST)-ABS(SO))4,3,3
    4 CALL PENAL(N,GOOD,X,Y)
    3 RETURN
      END

      SUBROUTINE PENAL(N,GOOD,X,Y)
      DIMENSION GOOD(9),X(9)
```

```
      Y=0.
      DO 100 I=1,N
      Y=Y+(GOOD(I)-X(I))*(GOOD(I)-X(I))
  100 CONTINUE
      Y=-Y
      RETURN
      END
```

When asymmetric cross sections are used for curved beams, the informed literature often points out that when materials are used that have different allowable fiber stresses in tension and compression, the opportunity is present to proportion the section so that the limiting stresses in tension and compression are reached simultaneously. One would believe that the weight of the section under these circumstances would be about as small as possible for that family of sections, since there is no "underworked" material present. The expectation of the outcome of this programming is, at this point, considered to be a set of section proportions which has the material working up to allowable limits on the outer and inner fibers.

In order to gain some confidence, advantage should be taken of the program flexibility to use data that would proportion a cross section for a material with equal allowable tensile and compressive stress limits. The first data used is for a hypothetical material with 20,000 psi working stress in tension and compression. The input data used will place the regional constraints as

$$0.50 \leq x_1 \leq 2.00$$
$$0.25 \leq x_2 \leq 0.50$$
$$0.25 \leq x_3 \leq 0.50$$
$$1.00 \leq x_4 \leq 3.00$$

because we feel certain that the solution lies within the above basis space. Since a point in feasible space must be provided for the orientation of the penalty hypersurface, the point chosen is the conservative location

$$x_1 = 2.00$$
$$x_2 = 0.50$$
$$x_3 = 0.50$$
$$x_4 = 2.00$$

The fractional reduction in the grid-search pattern was chosen as $r = 0.8$.

```
CONVERGENCE MONITOR SUBROUTINE GRID4

  NN   SIDE        Y          X(1)        X(2)        X(3)        X(4)

 17 0.100E 01 0.248E 00 0.100E 01 0.333E 00 0.333E 00 0.233E 01
 25 0.800E 00 0.248E 00 0.100E 01 0.333E 00 0.333E 00 0.233E 01
```

```
 41 0.640E 00 0.308E 00 0.840E 00 0.307E 00 0.307E 00 0.212E 01
 49 0.512E 00 0.308E 00 0.840E 00 0.307E 00 0.307E 00 0.212E 01
 65 0.410E 00 0.315E 00 0.738E 00 0.290E 00 0.290E 00 0.226E 01
 73 0.328E 00 0.315E 00 0.738E 00 0.290E 00 0.290E 00 0.226E 01
 89 0.262E 00 0.333E 00 0.803E 00 0.279E 00 0.279E 00 0.217E 01
 97 0.210E 00 0.333E 00 0.803E 00 0.279E 00 0.279E 00 0.217E 01
113 0.168E 00 0.336E 00 0.761E 00 0.272E 00 0.272E 00 0.223E 01
121 0.134E 00 0.336E 00 0.761E 00 0.272E 00 0.272E 00 0.223E 01
137 0.107E 00 0.336E 00 0.761E 00 0.272E 00 0.272E 00 0.223E 01
145 0.859E-01 0.336E 00 0.761E 00 0.272E 00 0.272E 00 0.223E 01
161 0.687E-01 0.340E 00 0.778E 00 0.269E 00 0.275E 00 0.220E 01
169 0.550E-01 0.340E 00 0.778E 00 0.269E 00 0.275E 00 0.220E 01
185 0.440E-01 0.340E 00 0.778E 00 0.269E 00 0.275E 00 0.220E 01
193 0.352E-01 0.340E 00 0.778E 00 0.269E 00 0.275E 00 0.220E 01
209 0.281E-01 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
217 0.225E-01 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
233 0.180E-01 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
241 0.144E-01 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
257 0.115E-01 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
265 0.922E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
281 0.738E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
289 0.590E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
305 0.472E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
313 0.378E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
329 0.302E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
337 0.242E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
353 0.193E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
361 0.155E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01
377 0.124E-02 0.340E 00 0.771E 00 0.268E 00 0.276E 00 0.221E 01

LARGEST MERIT ORDINATE FOUND DURING SEARCH ........... 0.34038961E 00
NUMBER OF FUNCTION EVALUATIONS USED DURING SEARCH ....           386
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY EXTANT 0.14854670E-02


XLOW(1)= 0.76983702E 00   X(1)= 0.77057981E 00   XHIGH(1)= 0.77132249E 00
XLOW(2)= 0.26740086E 00   X(2)= 0.26752466E 00   XHIGH(2)= 0.26764840E 00
XLOW(3)= 0.27572483E 00   X(3)= 0.27584863E 00   XHIGH(3)= 0.27597237E 00
XLOW(4)= 0.22115622E 01   X(4)= 0.22125530E 01   XHIGH(4)= 0.22135429E 01


EXECUTIVE PROGRAM OUTPUT MINIMUM WEIGHT C-CLAMP FRAME
FORCE ON C-CLAMP (POSITIVE OPENS  C ), LBF ...........        900.
DISTANCE FROM LOA OF FORCE TO CENTER OF CURVATURE,IN..     2.00000
RADIUS OF CURVATURE OF INNER FIBER, INCHES ...........     4.00000
MINIMUM SECTION WIDTH IN CAP OF  T , INCHES ..........     0.25000
MINIMUM SECTION WIDTH IN STEM OF  T , INCHES .........     0.25000
MAXIMUM ALLOWABLE TENSILE STRESS, PSI ................      20000.
MAXIMUM ALLOWABLE COMPRESSIVE STRESS, PSI ............     -20000.
SPECIFIC WEIGHT OF FRAME MATERIAL, LBF/IN3 ...........     0.26000
MATERIAL IS HYPOTHETICAL WITH EQUAL ALLOWABLE NORMAL STRESSES.


CHARACTERISTICS OF OPTIMAL CONFIGURATION

WEIGHT OF FRAME, LBF .................................     2.93781
WIDTH OF SECTION AT INNER RADIUS,INCHES...............     0.77058
WIDTH OF SECTION AT OUTER RADIUS,INCHES...............     0.26752
THICKNESS OF CAP OF  T , INCHES ......................     0.27585
RADIAL DEPTH OF SECTION, INCHES.......................     2.21255
NORMAL STRESS AT INNER FIBER, PSI ....................      20000.
NORMAL STRESS AT OUTER  FIBER, PSI ...................     -18963.
```

Examination of the output for this data indicates that the optimal proportions are

$$x_1 = 0.77058 \text{ in.}$$

$$x_2 = 0.26752 \text{ in.}$$

$$x_3 = 0.27585 \text{ in.}$$

$$x_4 = 2.21255 \text{ in.}$$

Inspection of the convergence monitor of GRID4 indicates that the section thicknesses x_2 and x_3 are approaching nearly their smallest allowable value of $\frac{1}{4}$ in. The working stresses at the outer fibers are 20,000 psi and $-18{,}963$ psi. Notice also that x_2 and x_3 decreased nearly concurrently during the search. We are given to suspect that the optimal section configuration is one in which the section thicknesses are $\frac{1}{4}$ in. and the program could have been made for two independent design variables. Further work along this line may be considered.

The input data for the ASTM 40 cast iron is now supplied to the program. The basis space is defined as

$$0.50 \leq x_1 \leq 2.00$$

$$0.25 \leq x_2 \leq 0.50$$

$$0.25 \leq x_3 \leq 0.50$$

$$1.00 \leq x_4 \leq 3.00$$

The point in feasible space is retained as $x_1 = 2.00$, $x_2 = 0.5$, $x_3 = 0.5$, $x_4 = 2.00$, and the fractional reduction in the grid-search pattern is chosen as 0.7.

```
CONVERGENCE MONITOR SUBROUTINE GRID4

  NN    SIDE         Y          X(1)         X(2)         X(3)         X(4)

  17 0.100E 01 0.284E 00 0.150E 01 0.333E 00 0.333E 00 0.167E 01
  25 0.700E 00 0.284E 00 0.150E 01 0.333E 00 0.333E 00 0.167E 01
  41 0.490E 00 0.311E 00 0.162E 01 0.313E 00 0.313E 00 0.150E 01
  49 0.343E 00 0.311E 00 0.162E 01 0.313E 00 0.313E 00 0.150E 01
  65 0.240E 00 0.315E 00 0.156E 01 0.303E 00 0.303E 00 0.158E 01
  73 0.168E 00 0.347E 00 0.139E 01 0.275E 00 0.275E 00 0.170E 01
  89 0.118E 00 0.355E 00 0.142E 01 0.270E 00 0.270E 00 0.166E 01
  97 0.824E-01 0.355E 00 0.142E 01 0.270E 00 0.270E 00 0.166E 01
 113 0.576E-01 0.359E 00 0.144E 01 0.268E 00 0.268E 00 0.164E 01
 121 0.404E-01 0.359E 00 0.144E 01 0.268E 00 0.268E 00 0.164E 01
 137 0.282E-01 0.360E 00 0.143E 01 0.266E 00 0.266E 00 0.165E 01
 145 0.198E-01 0.360E 00 0.143E 01 0.266E 00 0.266E 00 0.165E 01
 161 0.138E-01 0.361E 00 0.143E 01 0.266E 00 0.266E 00 0.164E 01
 169 0.969E-02 0.361E 00 0.143E 01 0.266E 00 0.266E 00 0.164E 01
 185 0.678E-02 0.362E 00 0.144E 01 0.266E 00 0.266E 00 0.164E 01
 193 0.475E-02 0.362E 00 0.144E 01 0.266E 00 0.266E 00 0.164E 01
 209 0.332E-02 0.362E 00 0.144E 01 0.265E 00 0.266E 00 0.164E 01
 217 0.233E-02 0.362E 00 0.144E 01 0.265E 00 0.266E 00 0.164E 01
 233 0.163E-02 0.362E 00 0.144E 01 0.265E 00 0.266E 00 0.164E 01
 241 0.114E-02 0.362E 00 0.144E 01 0.265E 00 0.266E 00 0.164E 01

LARGEST MERIT ORDINATE FOUND DURING SEARCH ........... 0.36198169E 00
NUMBER OF FUNCTION EVALUATIONS USED DURING SEARCH ....          258
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY EXTANT 0.59795380E-03

XLOW(1)= 0.14371367E 01  X(1)= 0.14373341E 01  XHIGH(1)= 0.14377346E 01
XLOW(2)= 0.26532203E 00  X(2)= 0.26538855E 00  XHIGH(2)= 0.26542181E 00
XLOW(3)= 0.26569873E 00  X(3)= 0.26573193E 00  XHIGH(3)= 0.26579845E 00
XLOW(4)= 0.16376114E 01  X(4)= 0.16381435E 01  XHIGH(4)= 0.16384096E 01
```

```
EXECUTIVE PROGRAM OUTPUT MINIMUM WEIGHT C-CLAMP FRAME
FORCE ON C-CLAMP (POSITIVE OPENS  C ), LBF ...........        900.
DISTANCE FROM LOA OF FORCE TO CENTER OF CURVATURE,IN..    2.00000
RADIUS OF CURVATURE OF INNER FIBER, INCHES ...........    4.00000
MINIMUM SECTION WIDTH IN CAP OF  T , INCHES ..........    0.25000
MINIMUM SECTION WIDTH IN STEM OF  T , INCHES .........    0.25000
MAXIMUM ALLOWABLE TENSILE STRESS, PSI ................     20000.
MAXIMUM ALLOWABLE COMPRESSIVE STRESS, PSI ............    -60000.
SPECIFIC WEIGHT OF FRAME MATERIAL, LBF/IN3 ...........    0.26000
MATERIAL IS ASTM 40 CAST IRON WITH DESIGN FACTOR OF TWO ON ULTIMATE.


CHARACTERISTICS OF OPTIMAL CONFIGURATION

WEIGHT OF FRAME, LBF .................................    2.76257
WIDTH OF SECTION AT INNER RADIUS,INCHES...............    1.43733
WIDTH OF SECTION AT OUTER RADIUS,INCHES...............    0.26539
THICKNESS OF CAP OF  T , INCHES ......................    0.26573
RADIAL DEPTH OF SECTION, INCHES.......................    1.63814
NORMAL STRESS AT INNER FIBER, PSI ....................     19998.
NORMAL STRESS AT OUTER  FIBER, PSI ...................    -30760.
```

The program using ASTM 40 stress limits surprisingly does not return a set of sectional proportions which works the material up to both its tensile and compressive stress levels of 20,000 psi and −60,000 psi, but reports intensities of 19,998 psi and −30,760 psi. The proportions reported are

$$x_1 = 1.43733 \text{ in.}$$

$$x_2 = 0.26539 \text{ in.}$$

$$x_3 = 0.26573 \text{ in.}$$

$$x_4 = 1.63814 \text{ in.}$$

and again we note that the section thicknesses x_2 and x_3 are close to the minimal allowable value of $\frac{1}{4}$ in. We are led to expect that the problem may have only two independent design variables. Since we are puzzled by the low value of the compressive stress at the outer fiber, −30,760 instead of −60,000, in the optimal section, we will now undertake the necessary work to modify the program we have for two independent design variables. This will place the problem in 3 space and visualization as well as geometric interpretation will be possible.

In reprogramming, the design variable x_1 will now become the width of the section at the inner radius and x_2 will become the radial depth of the section. This is necessary because the column vectors in the library grid-search routine will be only two deep. With this thought in mind, the existing program is modified the small amount that is necessary.

```
C     EXECUTIVE PROGRAM FOR C-CLAMP FRAME OF A 'T' SECTION OF
C     SMALLEST WEIGHT.
C
C     STATEMENT OF PROBLEM
C
C     A C-CLAMP IS TO BE DESIGNED HAVING A T-CROSSSECTION WITH CAP OF
C     THE T ON INSIDE OF FRAME. DEVISE A PROGRAM TO DETERMINE THE
C     PARAMETERS OF THE CONFIGURATION OF MINIMAL WEIGHT OF C-FRAME,
C     GIVEN ESSENTIAL GEOMETRY, MATERIAL SPECIFIC WEIGHT, ALLOWABLE
```

```
C     STRESSES IN TENSION AND COMPRESSION, LOAD, AND SUBJECT TO SECTION
C     MINIMUM THICKNESSES DUE TO CASTING CONSIDERATIONS
C
C     NOMENCLATURE
C
C     PP=FORCE ON C-CLAMP,POSITIVE OPENS 'C',LBF
C     D=DISTANCE BETWEEN CENTER OF CURVATURE AND LINE OF ACTION OF
C       FORCE PP, POSITIVE IF OUTSIDE, INCHES
C     RI=RADIUS OF CURVATURE OF INNER FIBER, INCHES
C     BI=SECTION WIDTH AT INNER FIBER, INCHES
C     BO=SECTION WIDTH AT OUTER FIBER, INCHES
C
C     C1=THICKNESS OF CAP OF 'T', INCHES
C     C2=LENGTH OF STEM OF 'T', INCHES
C     N=NUMBER OF INDEPENDENT VARIABLES IN OPTIMIZATION
C     I2=CONVERGENCE MONITOR PRINT SIGNAL
C     F=FRACTIONAL REDUCTION IN DOMAIN OF UNCERTAINTY DESIRED
C     R=FRACTIONAL REDUCTION IN GRID SIZE UTILIZED
C
C     X(1)=INTERNAL WIDTH OF SECTION, IN.
C     X(2)=RADIAL DEPTH OF SECTION, IN.
C     XL(I)=LOWER BOUND OF SEARCH DOMAIN, COLUMN VECTOR
C     XR(I)=UPPER BOUND OF SEARCH DOMAIN, COLUMN VECTOR
C     GAMMA=SPECIFIC WEIGHT OF FRAME MATERIAL, LBF/IN3
C     GOOD=COLUMN VECTOR OF CROSS SECTION DIMENSIONS NOT VIOLATING
C     FUNCTIONAL CONSTRAINTS OF STRESS LEVEL AT OUTER AND INNER SURFACE.
C
      COMMON PP,D,RI,GAMMA,ST,SC,W,SI,SO,GOOD(9),N,XL(9),XR(9),T
      DIMENSION X(9),IABC(20),XLOW(9),XHIGH(9)
C
C     ..... READ IN DATA .....
C
    1 READ(1,2)PP,D,RI,BI,BO,GAMMA
    2 FORMAT(6F10.5)
      READ(1,3)C1,C2,ST,SC,N
    3 FORMAT(2F10.5,2F10.0,I5)
      READ(1,4)I2,F,R
    4 FORMAT(I5,2F10.5)
      DO 6 I=1,N
      READ(1,5)XLO,XHI
    5 FORMAT(2F10.5)
      XL(I)=XLO
      XR(I)=XHI
    6 CONTINUE
      READ(1,9)(IABC(I),I=1,20)
    9 FORMAT(20A4)
      READ(1,10)(GOOD(I),I=1,2)
   10 FORMAT(2F10.5)
      T=BO
C
C     ..... INITIATE A GRID SEARCH TO FIND THE DIMENSIONS OF          .....
C     ..... LEAST WEIGHT SECTION.                                     .....
C
      CALL GRID4(N,I2,XL,XR,F,R,Y,X,XLOW,XHIGH,NN)
C
C     ..... DOCUMENT SEARCH RESULTS .....
C
      WRITE(3,7)PP,D,RI,C1,BO,ST,SC,GAMMA,(IABC(I),I=1,20)
    7 FORMAT('1EXECUTIVE PROGRAM OUTPUT MINIMUM WEIGHT C-CLAMP FRAME',/,
     1' FORCE ON C-CLAMP (POSITIVE OPENS  C ), LBF ............',F10.0,/,
     2' DISTANCE FROM LOA OF FORCE TO CENTER OF CURVATURE,IN..',F10.5,/,
     3' RADIUS OF CURVATURE OF INNER FIBER, INCHES ..........',F10.5,/,
     4' MINIMUM SECTION WIDTH IN CAP OF  T , INCHES ..........',F10.5,/,
     5' MINIMUM SECTION WIDTH IN STEM OF  T , INCHES .........',F10.5,/,
     6' MAXIMUM ALLOWABLE TENSILE STRESS, PSI ...............',F10.0,/,
     7' MAXIMUM ALLOWABLE COMPRESSIVE STRESS, PSI ............',F10.0,/,
     8' SPECIFIC WEIGHT OF FRAME MATERIAL, LBF/IN3 ...........',F10.5,/,
     920A4)
      WRITE(3,8)W,X(1),BO,C1,X(2),SI,SO
    8 FORMAT(//,' CHARACTERISTICS OF OPTIMAL CONFIGURATION',//,
     1' WEIGHT OF FRAME, LBF .....................................',F10.5,/,
     2' WIDTH OF SECTION AT INNER RADIUS,INCHES.................',F10.5,/,
     3' WIDTH OF SECTION AT OUTER RADIUS,INCHES.................',F10.5,/,
```

```
     4' THICKNESS OF CAP OF  T , INCHES ....................',F10.5,/,
     5' RADIAL DEPTH OF SECTION, INCHES.....................',F10.5,/,
     6' NORMAL STRESS AT INNER FIBER, PSI ..................',F10.0,/,
     7' NORMAL STRESS AT OUTER  FIBER, PSI .................',F10.0)
      GO TO 1
      END

      SUBROUTINE MERIT4(X,Y)
      COMMON PP,D,RI,GAMMA,ST,SC,W,SI,SO,GOOD(9),N,XL(9),XR(9),T
      DIMENSION X(9)
C
C     ..... CHECK TO SEE IF POINT VIOLATES ANY REGIONAL CONSTRAINT, ....
C     ..... IF SO, ASSIGN PENALTY ORDINATE AND RETURN.              ....
C
      DO 10 I=1,N
      IF(X(I)-XL(I))4,9,9
    9 IF(X(I)-XR(I))10,10,4
   10 CONTINUE
C
C     ..... INITIALIZE .....
C
      BI=X(1)
      BO=T
      C1=T
      C2=X(2)-T
C
C     ..... CALL CURVED BEAM TEE-SECTION SUBROUTINE TO DETERMINE    .....
C     ..... STRESS LEVELS AT THE INNER AND OUTER SURFACES, AND TO   .....
C     ..... DETERMINE THE RADIUS OF CURVATURE OF LOCUS OF THE CENTER....
C     ..... OF MASS OF CROSS SECTIONAL AREA.                        .....
C
      CALL ME0053(PP,RI,D,BI,BO,C1,C2,SI,SO,R)
C
C     ..... PREPARE MERIT ORDINATE .....
C
      A=BI*C1+BO*C2
      W=R*A*GAMMA*3.14159
      Y=1./W
C
C     ..... DETERMINE IF FUNCTIONAL CONSTRAINTS HAVE BEEN VIOLATED .....
C     ..... BY THIS PARTICULAR CROSS SECTIONAL CONFIGURATION, IF SO,....
C     ..... RETURN PENALTY ORDINATE; IF NOT, RETURN MERIT ORDINATE .....
C
      IF(SI)6,1,1
    1 IF(ABS(ST)-ABS(SI))4,2,2
    2 IF(ABS(SC)-ABS(SO))4,3,3
    6 IF(ABS(SC)-ABS(SI))4,7,7
    7 IF(ABS(ST)-ABS(SO))4,3,3
    4 CALL PENAL(N,GOOD,X,Y)
    3 RETURN
      END

      SUBROUTINE PENAL(N,GOOD,X,Y)
      DIMENSION GOOD(9),X(9)
      Y=0.
      DO 100 I=1,N
      Y=Y+(GOOD(I)-X(I))*(GOOD(I)-X(I))
  100 CONTINUE
      Y=-Y
      RETURN
      END
```

CONVERGENCE MONITOR SUBROUTINE GRID4

NN	SIDE	Y	X(1)	X(2)
5	0.100E 01	0.323E 00	0.100E 01	0.233E 01
9	0.800E 00	0.323E 00	0.100E 01	0.233E 01
13	0.640E 00	0.323E 00	0.100E 01	0.233E 01
17	0.512E 00	0.323E 00	0.100E 01	0.233E 01
21	0.410E 00	0.353E 00	0.898E 00	0.220E 01

```
 25 0.328E 00 0.353E 00 0.898E 00 0.220E 01
 29 0.262E 00 0.353E 00 0.898E 00 0.220E 01
 33 0.210E 00 0.353E 00 0.898E 00 0.220E 01
 37 0.168E 00 0.353E 00 0.898E 00 0.220E 01
 41 0.134E 00 0.353E 00 0.898E 00 0.220E 01
 45 0.107E 00 0.353E 00 0.898E 00 0.220E 01
 49 0.859E-01 0.353E 00 0.898E 00 0.220E 01
 53 0.687E-01 0.353E 00 0.898E 00 0.220E 01
 57 0.550E-01 0.353E 00 0.898E 00 0.220E 01
 61 0.440E-01 0.354E 00 0.909E 00 0.218E 01
 65 0.352E-01 0.354E 00 0.909E 00 0.218E 01
 69 0.281E-01 0.354E 00 0.909E 00 0.218E 01
 73 0.225E-01 0.354E 00 0.909E 00 0.218E 01
 77 0.180E-01 0.354E 00 0.909E 00 0.218E 01
 81 0.144E-01 0.354E 00 0.894E 00 0.219E 01
 85 0.115E-01 0.355E 00 0.891E 00 0.219E 01
 89 0.922E-02 0.355E 00 0.891E 00 0.219E 01
 93 0.738E-02 0.356E 00 0.889E 00 0.219E 01
 97 0.590E-02 0.356E 00 0.884E 00 0.219E 01
101 0.472E-02 0.356E 00 0.882E 00 0.219E 01
105 0.378E-02 0.356E 00 0.879E 00 0.219E 01
109 0.302E-02 0.356E 00 0.878E 00 0.219E 01
113 0.242E-02 0.356E 00 0.878E 00 0.219E 01
117 0.193E-02 0.356E 00 0.877E 00 0.219E 01
121 0.155E-02 0.356E 00 0.877E 00 0.219E 01
125 0.124E-02 0.356E 00 0.877E 00 0.219E 01

LARGEST MERIT ORDINATE FOUND DURING SEARCH ........... 0.35647225E 00
NUMBER OF FUNCTION EVALUATIONS USED DURING SEARCH ....            130
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY EXTANT 0.14854670E-02


XLOW(1)= 0.87661237E 00  X(1)= 0.87735516E 00  XHIGH(1)= 0.87809783E 00
XLOW(2)= 0.21876745E 01  X(2)= 0.21886654E 01  XHIGH(2)= 0.21896553E 01


EXECUTIVE PROGRAM OUTPUT MINIMUM WEIGHT C-CLAMP FRAME
FORCE ON C-CLAMP (POSITIVE OPENS  C ), LBF ...........       900.
DISTANCE FROM LOA OF FORCE TO CENTER OF CURVATURE,IN..    2.00000
RADIUS OF CURVATURE OF INNER FIBER, INCHES ...........    4.00000
MINIMUM SECTION WIDTH IN CAP OF  T , INCHES ..........    0.25000
MINIMUM SECTION WIDTH IN STEM OF  T , INCHES .........    0.25000
MAXIMUM ALLOWABLE TENSILE STRESS, PSI ................     20000.
MAXIMUM ALLOWABLE COMPRESSIVE STRESS, PSI ............    -20000.
SPECIFIC WEIGHT OF FRAME MATERIAL, LBF/IN3 ...........    0.26000
MATERIAL IS HYPOTHETICAL WITH EQUAL ALLOWABLE NORMAL STRESSES.


CHARACTERISTICS OF OPTIMAL CONFIGURATION

WEIGHT OF FRAME, LBF .................................    2.80527
WIDTH OF SECTION AT INNER RADIUS,INCHES...............    0.87736
WIDTH OF SECTION AT OUTER RADIUS,INCHES...............    0.25000
THICKNESS OF CAP OF  T , INCHES ......................    0.25000
RADIAL DEPTH OF SECTION, INCHES.......................    2.18867
NORMAL STRESS AT INNER FIBER, PSI ....................     19674.
NORMAL STRESS AT OUTER  FIBER, PSI ...................    -19998.
```

The first trial of the two-independent-variable program is with the equal allowable stress material using the basis plane defined by

$$0.50 \leq x_1 \leq 2.00$$

$$1.00 \leq x_2 \leq 3.00$$

and the point in the feasible area as $x_1 = 2.00$, $x_2 = 2.00$. The fractional reduction in the grid-search pattern size is chosen as $r = 0.8$. The optimal configuration reported in the 3 space entertains an inner-fiber stress of

19,674 psi and an outer-fiber stress of $-19{,}998$ psi. The width of the inner section and the radial depth of the section of $\frac{1}{4}$-in. thicknesses are reported as

$$x_1 = 0.87736 \text{ in.}$$

$$x_2 = 2.18867 \text{ in.}$$

The section weight reported with the two-independent-variable program was 2.80527 lbf and with the four-independent variable program 2.93781 in the case of the equal-strength model. One is led to believe that the optimal configuration is indeed the one with the $\frac{1}{4}$-in. section thicknesses. The irregularities in the pattern-search results may be due to the adverse effects of dimensionality and the higher incidence of false moves.

```
CONVERGENCE MONITOR SUBROUTINE GRID4

     NN    SIDE          Y           X(1)         X(2)

      5 0.100E 01 0.371E 00 0.150E 01 0.167E 01
      9 0.800E 00 0.371E 00 0.150E 01 0.167E 01
     13 0.640E 00 0.371E 00 0.150E 01 0.167E 01
     17 0.512E 00 0.371E 00 0.150E 01 0.167E 01
     21 0.410E 00 0.371E 00 0.150E 01 0.167E 01
     25 0.328E 00 0.373E 00 0.117E 01 0.189E 01
     29 0.262E 00 0.373E 00 0.117E 01 0.189E 01
     33 0.210E 00 0.373E 00 0.117E 01 0.189E 01
     37 0.168E 00 0.373E 00 0.117E 01 0.189E 01
     41 0.134E 00 0.373E 00 0.117E 01 0.189E 01
     45 0.107E 00 0.376E 00 0.120E 01 0.185E 01
     49 0.859E-01 0.376E 00 0.120E 01 0.185E 01
     53 0.687E-01 0.376E 00 0.120E 01 0.185E 01
     57 0.550E-01 0.376E 00 0.120E 01 0.185E 01
     61 0.440E-01 0.376E 00 0.120E 01 0.185E 01
     65 0.352E-01 0.376E 00 0.120E 01 0.185E 01
     69 0.281E-01 0.377E 00 0.121E 01 0.184E 01
     73 0.225E-01 0.377E 00 0.121E 01 0.184E 01
     77 0.180E-01 0.377E 00 0.121E 01 0.184E 01
     81 0.144E-01 0.377E 00 0.121E 01 0.184E 01
     85 0.115E-01 0.377E 00 0.121E 01 0.184E 01
     89 0.922E-02 0.377E 00 0.121E 01 0.184E 01
     93 0.738E-02 0.377E 00 0.121E 01 0.184E 01
     97 0.590E-02 0.377E 00 0.120E 01 0.184E 01
    101 0.472E-02 0.377E 00 0.120E 01 0.184E 01
    105 0.378E-02 0.377E 00 0.120E 01 0.184E 01
    109 0.302E-02 0.377E 00 0.120E 01 0.184E 01
    113 0.242E-02 0.377E 00 0.120E 01 0.184E 01
    117 0.193E-02 0.377E 00 0.120E 01 0.184E 01
    121 0.155E-02 0.377E 00 0.120E 01 0.184E 01
    125 0.124E-02 0.377E 00 0.120E 01 0.184E 01

LARGEST MERIT ORDINATE FOUND DURING SEARCH ............ 0.37683195E 00
NUMBER OF FUNCTION EVALUATIONS USED DURING SEARCH ....              130
FRACTIONAL REDUCTION IN INTERVAL OF UNCERTAINTY EXTANT 0.14848709E-02


XLOW(1)= 0.11995468E 01   X(1)= 0.12002897E 01   XHIGH(1)= 0.12010317E 01
XLOW(2)= 0.18428879E 01   X(2)= 0.18438787E 01   XHIGH(2)= 0.18448687E 01


EXECUTIVE PROGRAM OUTPUT MINIMUM WEIGHT C-CLAMP FRAME
FORCE ON C-CLAMP (POSITIVE OPENS  C ), LBF ..........        900.
DISTANCE FROM LOA OF FORCE TO CENTER OF CURVATURE,IN..    2.00000
RADIUS OF CURVATURE OF INNER FIBER, INCHES ...........    4.00000
MINIMUM SECTION WIDTH IN CAP OF  T , INCHES ..........    0.25000
MINIMUM SECTION WIDTH IN STEM OF  T , INCHES .........    0.25000
```

```
MAXIMUM ALLOWABLE TENSILE STRESS, PSI ...............   20000.
MAXIMUM ALLOWABLE COMPRESSIVE STRESS, PSI ............  -60000.
SPECIFIC WEIGHT OF FRAME MATERIAL, LBF/IN3 ...........  0.26000
MATERIAL IS ASTM 40 CAST IRON WITH DESIGN FACTOR OF TWO ON ULTIMATE.

CHARACTERISTICS OF OPTIMAL CONFIGURATION

WEIGHT OF FRAME, LBF ................................   2.65370
WIDTH OF SECTION AT INNER RADIUS,INCHES..............   1.20029
WIDTH OF SECTION AT OUTER RADIUS,INCHES..............   0.25000
THICKNESS OF CAP OF  T , INCHES .....................   0.25000
RADIAL DEPTH OF SECTION, INCHES......................   1.84388
NORMAL STRESS AT INNER FIBER, PSI ...................    20000.
NORMAL STRESS AT OUTER  FIBER, PSI ..................   -26339.
```

The second trial of the two-independent-variable program for ASTM 40 cast iron was made with the basis plane defined by

$$0.5 \leq x_1 \leq 2.00$$

$$1.00 \leq x_2 \leq 3.00$$

and the point in the feasible area as $x_1 = 2.00$, $x_2 = 2.00$. The fractional reduction in the grid-size pattern was chosen as $r = 0.8$. The optimal configuration reported entertained an inner-fiber stress of 20,000 psi and an outer-fiber stress of $-26{,}339$ psi. The weight of the section was reported as 2.65370 lbf as compared to the previous report of 2.76257 lbf for ASTM 40. "Underworked" material is still present, but now the geometry is sufficiently simple so as to encourage a little more investigation.

Figure 89 shows some inner- and outer-stress loci traced upon the basis plane. Displayed also are contours of constant beam weight. The functional constraints on the problem are such as to confine the feasible points to lying above the limiting outer-fiber stress locus and above the limiting inner-fiber stress locus. In the case of the equal-strength material feasible points lie above the $-20{,}000$ psi locus and above the 20,000 psi locus. A point moving in the direction of decreasing weight of section is moving toward the lower-left-hand corner of the figure. The minimum-weight configuration is found at the intersection of the dashed $-20{,}000$ psi locus and the solid 20,000 psi locus.

In the case of the ASTM 40 cast iron, feasible points must lie above the dashed $-60{,}000$ psi locus and above the solid 20,000 psi locus. These loci intersect somewhere off the figure to the right. Clearly this intersection is not the one of minimal weight. The section working the material to its tensile and compressive limits is wide at the inner radius and of small radial depth. The optimal weight section is narrower at the inner radius and of greater depth. This is due to the curvature. These results also suggest that the tee section is not the best section to choose for this design if the very least possible beam weight is desired. However the statement of the problem limited our consideration to just the tee section.

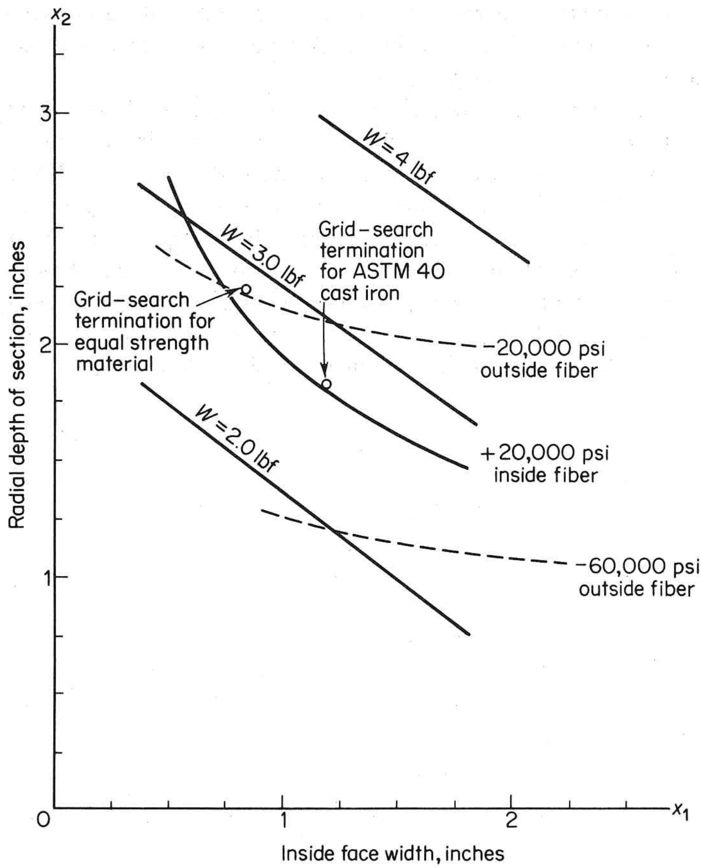

FIG. 89. Search domain for the C clamp with two independent design variables.

Note also from Fig. 89 that in the case of ASTM 40 how much the configuration can change in moving a point along the 20,000 psi stress locus without materially affecting the section weight. It would take a very sensitive searching technique to terminate on the constrained optimum. As an engineering solution the results are useful. Why?

5.8 EPILOGUE

This brief excursion into a part of the world of computer-aided design was planned, with the reader's cooperation and involvement, to acquaint him with the nature, triumphs, and frustrations associated with such sorties. The reader should now have an appreciation of the potential aid that the computer is capable of providing, and that its contribution is limited only by the resourcefulness, imagination, creativeness, boldness, and determination that he brings to the experience.

That only little is known and that pioneering work is still being done should neither surprise nor deter the engineer. This volume concerns itself with engineer-machine interaction as exemplified by the IOWA CADET algorithm. A list of references and additional reading in the general area of machine computation is provided in Appendix 4. Reading can begin in topical areas of immediate interest or need and broaden from that point.

The computer will make some dramatic changes in the engineer's world. Be alert to recognize them in their early stages so that you can incorporate additional skills and concepts into your professional knowledge.

appendix

4

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