Dynamic Differential Evolution Strategy and Applications in Electromagnetic Inverse Scattering Problems

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***Abstract*—A novel evolution algorithm, the dynamic differential evolution strategy, is developed to solve optimization problems. It inherits the genetic operators from the differential evolution strategy. However, it differs itself from the differential evolution strategy by updating its population dynamically while the differential evolution strategy updates its population generation by generation. The dynamic updating of population leads to a larger virtual population size and quicker response to change of population status. Two trial functions have been minimized using the dynamic differential evolution strategy. Comparison with the differential evolution strategy has been carried out. It has been observed that the dynamic differential evolution strategy significantly outperforms the differential evolution strategy in efficiency, robustness, and memory requirement. It is then applied to solve a benchmark electromagnetic inverse scattering problem. The outstanding performance of the dynamic differential evolution strategy is further consolidated.**

***Index Terms*—Dynamic differential evolution strategy, electromagnetic inverse scattering.**

I. Introduction

THE differential evolution strategy (DES) [1] is a very simple but very powerful stochastic global optimizer. The crucial idea behind DES is a scheme for generating trial parameter vectors. It has been proven a very good global optimizer for function optimization [1], [2] and has been applied to electromagnetic inverse problems [2]–[4], composite materials [5], antennas [6], and a lot of other mathematical and engineering fields.

Like all genetic algorithms, DES is population based. It evolutes generation by generation until the termination conditions have been met. In DES as shown in Fig. 1, while delivering baby individuals of a new generation, the individuals (including the optimal individual) in the last generation serve as parents. The optimal individual in the last population acts as father while all the other nonoptimal individuals act as mother. The father is mutated to produce a trial vector which is then used to carry out the crossover operation with the mother to deliver a baby. The newborn baby competes with its mother for a position in the next generation.

In nature, from the point of view of population updating, DES is static, i.e., the whole population keeps unchanged until it is replaced by the next population. In each evolution loop, all genetic operations are executed over individuals generated in the last evolution loop. DES does not make use of any improvement taking place in the current evolution loop. The mutation operator keeps using the last optimal individual to produce trial vectors until the current evolution loop completes, even if the last optimal individual has already been replaced by a better individual. At the same time, all mothers come from the last generation regardless of the fact that some of the nonoptimal individuals have already been replaced by their more competitive children. Therefore, DES does not follow the progress of the population status immediately. Instead, it responds to the population progress after a time lag. Inevitably, it results in slower convergence.

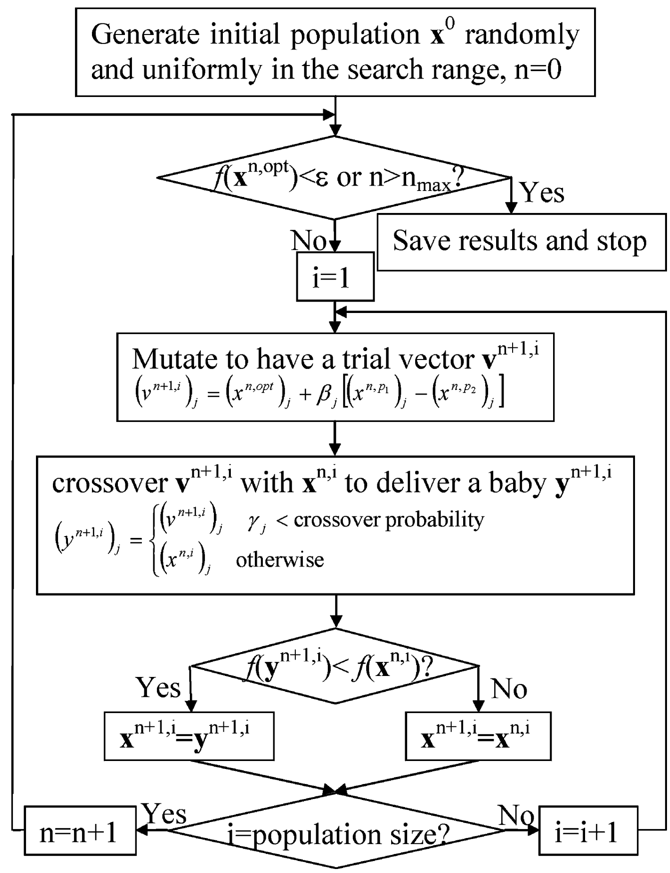

Fig. 1. Block diagram of differential evolution strategy (ε: convergence threshold for minimization problem, β_j: random number uniformly distributed in [0, 1], γ_j: random number uniformly distributed in [0, 1], $1 \leq p_1 \neq p_2 \neq i \leq$ population size).

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The evolution mechanism of DES also implies extra memory requirement since the parent generation has to be saved while generating a child population. In programming practice, memories have to be allocated for the present population and its immediate parent population.

It is well known that the Seidel iterative method and the relaxation method for linear equations might converge faster than ordinary iterative method. The key leading to the success of these methods is the dynamic updating of the parameters.

It is noticed that DES does not depend on any population quantities such as average objective function value. Therefore, all genetic operations are workable if one or more individuals in the population are changed. In addition, it does not affect the workability of the algorithm to add better individuals into the population, or to eliminate worse individuals from the population.

Accordingly, the evolution mechanism of DES is modified to update the population dynamically. A novel evolution algorithm, the dynamic differential evolution strategy (DDES), is consequently developed. It inherits the genetic operators from DES. However, its evolution mechanism is completely different with that of DES.

A trial function of unique minimum and a trial function of multiple minima have been minimized using DDES. Comparison with DES has been carried out. It has been observed that DDES significantly outperforms DES in efficiency, robustness, and memory requirement. It is then applied to solve a benchmark electromagnetic inverse scattering problem. The outstanding performance of DDES is further consolidated.

II. Dynamic Differential Evolution Strategy

The algorithm is shown in Fig. 2. It is very easy to notice that DDES inherits all of the basic genetic operators of DES, i.e., mutation, crossover, and mother–child competition. For details of the genetic operators involved, please refer to [2].

However, comparing Fig. 2 with Fig. 1, it is equally easy to notice that the evolution mechanism of DDES differs significantly from that of DES. The static evolution mechanism of DES is abandoned. Instead, a dynamic evolution mechanism is applied.

One of the significant differences between DDES and DES is the dynamic updating of the optimal individual. As shown in Fig. 2, there is an additional competition between a newborn competitive baby and the current optimal individual. The current optimal will be replaced by the newborn competitive baby if the newborn competitive baby is better than the current optimal. The updated optimal will be used in the following evolutions immediately.

In addition, the nonoptimal individuals are also updated dynamically. This is the other significant difference between DDES and DES. A nonoptimal mother will be replaced by her newborn baby if the newborn baby is better than her. The newborn baby will also be used immediately in the following evolutions.

Therefore, the trial vectors are always generated using the newly updated population including the newly updated optimal individual. Consequently, DDES updates the population dynamically and responds to any improvement immediately.

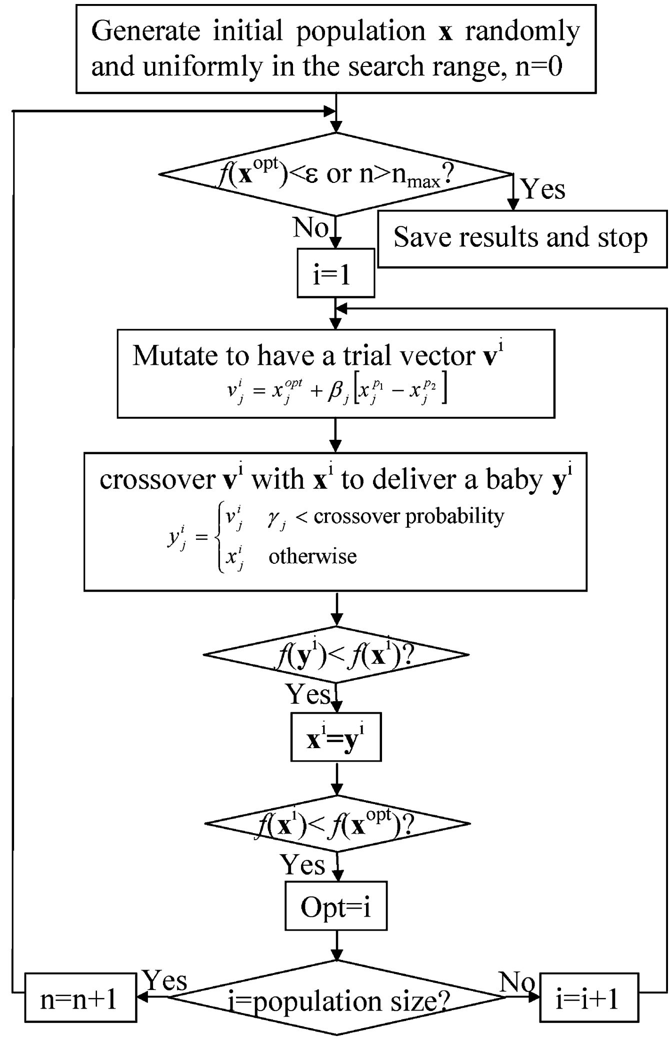

Fig. 2. Block diagram of dynamic differential evolution strategy (ε: convergence threshold for minimization problem, β_j: random number uniformly distributed in [0, 1], γ_j: random number uniformly distributed in [0, 1], $1 \leq p_1 \neq p_2 \neq i \leq$ population size).

As shown in Fig. 2, the generation indexes n and $n+1$ do not show up any more in DDES. This indicates that all genetic operations are executed over individuals in the current population. There is only one population whose individuals are continuously updated by their competitive children. Although there is still a flag n representing the current generation, it is only used for the purpose of result reporting.

Apparently, no buffer population is needed. Consequently, memory requirement of DDES is halved.

III. Minimization of Function of Unique Minimum

A. Problem Definition and Parameters Setting

In this section, DDES will be applied to minimize the following function:

$$f(\mathbf{x}) = \sum_{i=1}^{N}(x_i - i)^2, \qquad x_i \in [-500, 500]. \tag{1}$$

The unique global minimum of this function is $f(x_i^{\text{opt}} = i) = 0$. However, in our simulation, the search is regarded convergent if $f(\mathbf{x}) < \varepsilon = 0.01$. On the other hand, the search will be aborted if the number of objective function evaluations (NOFE) exceeds the allowed cap.

For the convenience of the following electromagnetic inverse scattering problem, three cases of number of optimization parameters are investigated first. The investigated number of optimization parameters is 8, 16, and 24. The number of optimization parameters matches that of the involved electromagnetic inverse scattering problems of a single cylinder, parallel cylinders, and triple cylinders.

To demonstrate the superior ability of DDES in solving optimization problems involving very large number of optimization parameters, two more cases are considered. The number of optimization parameters is 50 and 100, respectively.

In each case, different sets of genetic parameters (population size, mutation intensity, and crossover probability) are tested. The mutation intensity and the crossover probability change from 0.05 to 1 at a step 0.05. For the first three cases considered, the population size lies within 4 and 80 at a step of 4. The range of population size is 5 to 100 at a step of 5 for the fourth case. We can only try two population sizes 50 and 100 for the fifth case due to very limited computational resources.

For the convenience of the following description, we call the simulation of each set of genetic parameters a trial. Each trial contains 100 searches to eliminate the randomness of the statistical results. In each trial, the number of convergent searches, the minimum, maximum, and average NOFE of all convergent searches is recorded. For each population size, the number of all convergent searches, the number of full-success trials, and the minimum, maximum, and average NOFE of the best (in terms of average NOFE) full-success trial among all full-success trials are counted.

For comparison, the above function is also minimized using DES.

B. Eight-Parameter Function Minimization

Consider the first case where the number of optimization parameters is 8 with a cap of NOFE 16 000. The results are shown in Figs. 3 and 4. For DDES, the number of convergent searches jumps sharply from a very small value (132) at population size 4 to 17 167 at population size 8. After that, it keeps going up to a stable value (≅37 000). This means that about 92.5% of all searches locate the optimal solution successfully. Similarly, the number of full-success trials jumps from 0 at population size 4, to 6 at population size 8, and then to 164 at population 12. It continues to increase to the stable value ($\cong 350 = 87.5\%$). As to the NOFE of the best full-success trial, the average value remains almost constant (≅700). The minimum value also does not change much around 500. Therefore, DDES gives us freedom to apply bigger population size at no or very little cost of the success. However, the maximum value fluctuates weakly around 1000.

The results by DES are also shown in Figs. 3 and 4. The stable values of convergent searches and full-success trials drop

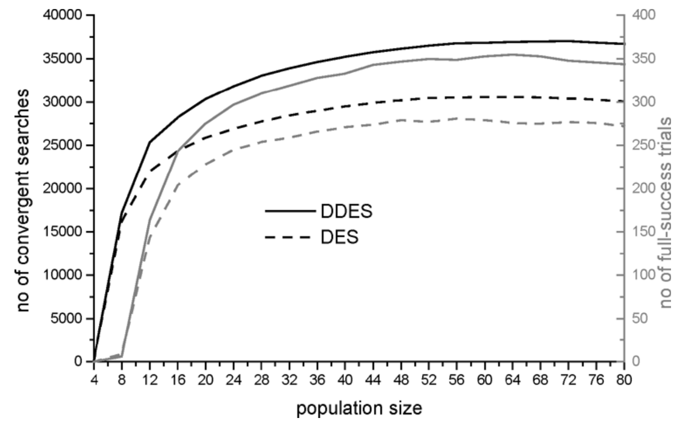

Fig. 3. Function minimization using DDES-robustness (eight optimization parameters, cap of NOFE: 16 000).

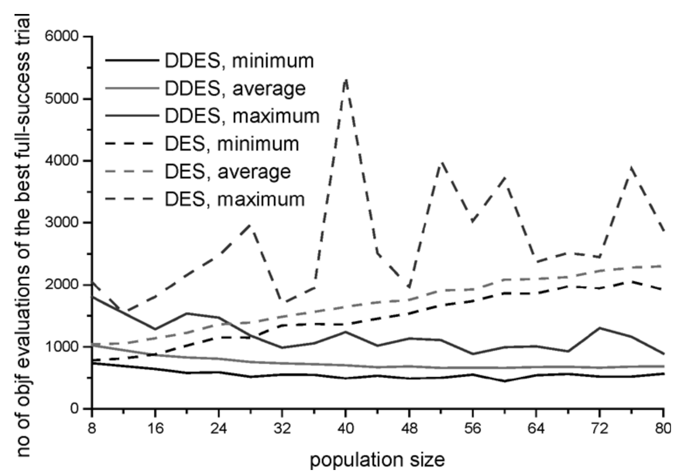

Fig. 4. Function minimization using DDES-NOFE versus population size (eight optimization parameters, cap of NOFE: 16 000).

significantly to 30 000(75%) and 270(67.5%), respectively. At the same time, the NOFE by DES is much larger than that of DDES and increases with the population size (almost linearly). In addition, all of the three values show much stronger fluctuation around their mean values. This observation shows us a dilemma: less NOFE, or higher possibility of success. Usually, people have to apply bigger population size at the cost of larger NOFE to ensure success. In fact, higher possibility of finding the global optimum is one of the main motivations for people to resort to global optimizers.

Further inspection of the simulation results shows that DDES converges at much more sets of genetic parameters. It is also noticed that the NOFE by DDES is much less sensitive to the mutation intensity and crossover probability. It changes very little in a much wider range of mutation intensity and crossover probability.

Less NOFE means higher efficiency. Higher success rate (number of convergent searches and full-success trials) and weaker sensitivity of efficiency and success rate with respect to control parameters (population size, mutation intensity, and crossover probability) means robustness. Apparently, DDES outperforms DES significantly from the point of view of robustness and efficiency.

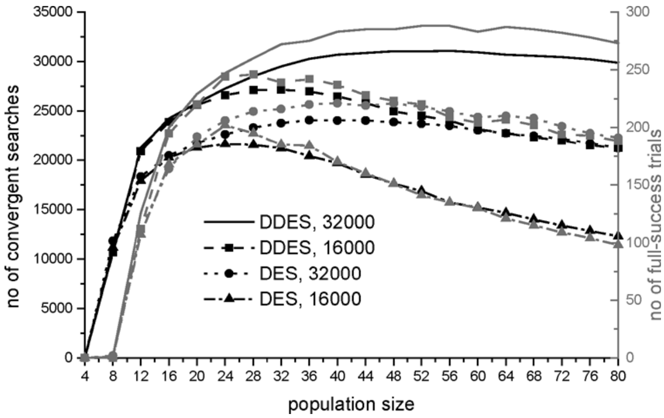

Fig. 5. Function minimization using DDES-robustness (16 optimization parameters, cap of NOFE: 16 000 or 32 000).

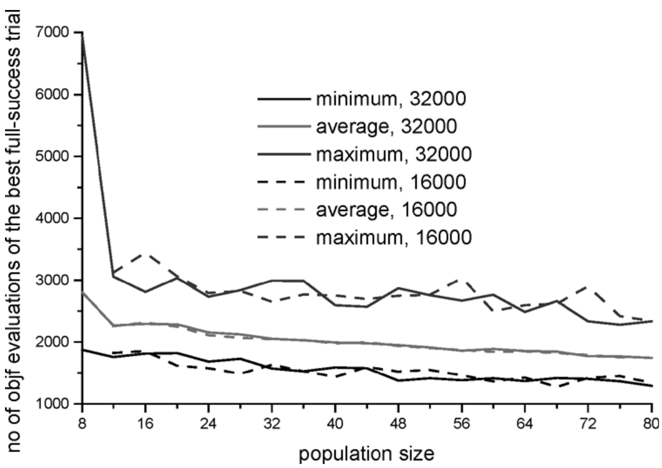

Fig. 6. Function minimization using DDES-NOFE versus population size (16 optimization parameters, cap of NOFE: 16 000 or 32 000).

C. Sixteen-Parameter Function Minimization

Next, the function with 16 optimization parameters is minimized. First, we limit the cap of NOFE at 16 000. The simulation results are shown in Figs. 5–7. As the population size increases, the number of successful searches and the number of full-success trials of both DDES and DES increase to their peak values at certain population size. They decrease thereafter. However, similar phenomena as that of the case of eight optimization parameters is observed on the minimum, average and maximum NOFE of the best full-success trial.

It is suspected that a lot of searches are terminated at quasi-convergent stage due to insufficient cap of NOFE. Therefore, the cap is lifted to 32 000. The corresponding results are also shown in Figs. 5 and 6. For DDES, both the number of successful searches and number of full-success trials approach their stable values as the population size increases. This clearly confirms our suspicion. However, it seems that the cap is still insufficient for DES since the respective values have yet not reached their stable values. This further consolidates the superiority of DDES. On the other hand, the cap 16 000 is more than enough for both DDES and DES to capture the best full-success trial.

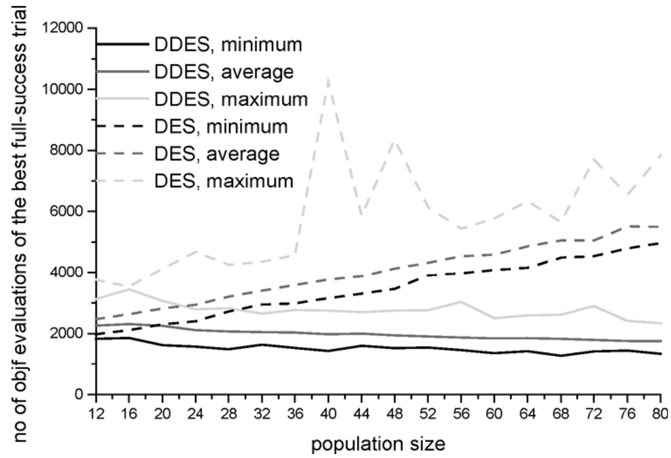

Fig. 7. Function minimization using DDES-NOFE versus population size (16 optimization parameters, cap of NOFE: 16 000).

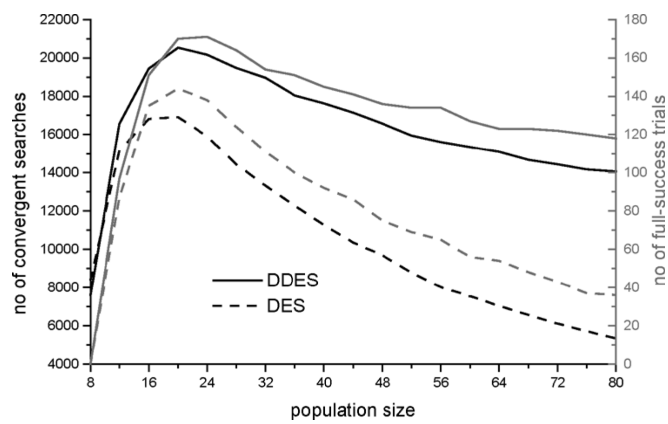

Fig. 8. Function minimization using DDES-robustness (24 optimization parameters, cap of NOFE: 16 000).

D. Twenty-Four-Parameter Function Minimization

Now, increase the number of optimization parameters further to 24. With the experience gained in Section III-C, the cap is kept at 16 000 for the sake of simulation time. As the simulation results given in Figs. 8 and 9 show, the cap is insufficient for the quasi-convergent searches to converge to desirable optimal solution. Likewise, it is sufficient for the best full-success trial.

E. Minimization of Function With Large Number of Optimization Parameters

The number of optimization parameters is relatively small for the above three cases. It would be more convincing if the superior performance of DDES sustains for optimization problems involving very large number of optimization parameters. Therefore, the above trial function with 50 and 100 optimization parameters are minimized.

First, both DDES and DES are applied to minimize the trial function with 50 optimization parameters. The cap of NOFE is 100 000. The whole simulation takes about one and a half months (DDES and DES are executed on different workstations,

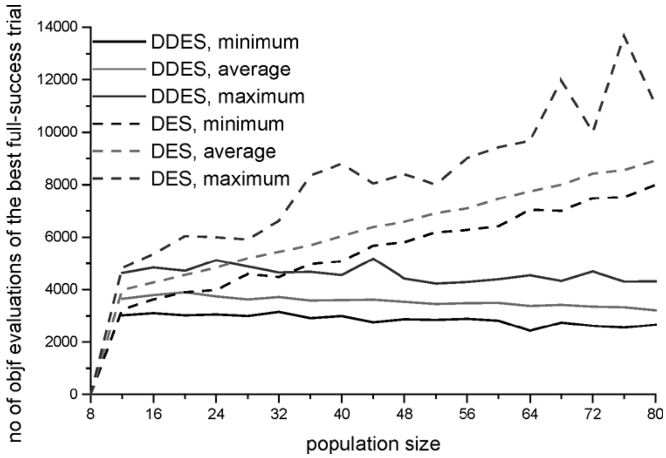

Fig. 9. Function minimization using DDES-NOFE versus population size (24 optimization parameters, cap of NOFE: 16 000).

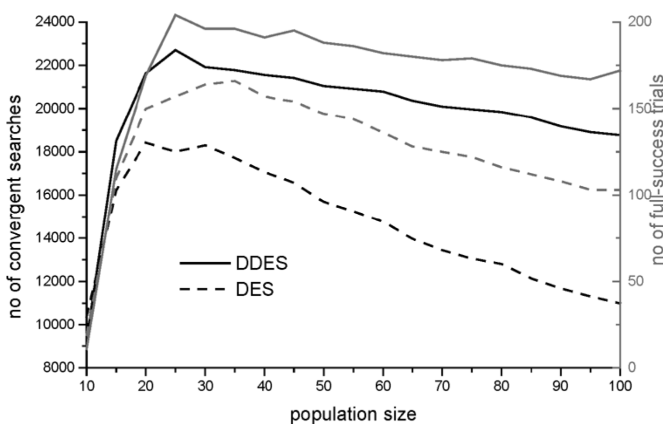

Fig. 10. Function minimization using DDES-robustness (50 optimization parameters, cap of NOFE: 100 000).

therefore, more than three months in total). The simulation results are shown in Figs. 10 and 11. Although the cap is still insufficient for both DDES and DES to reach their stable values of number of successful searches and number of full-success trials, both of them capture their best full-success trial. The comparison shown in Figs. 10 and 11 adds further evidence that DDES is much more superior than DES.

Finally, both DDES and DES are applied to minimize the trial function with 100 optimization parameters. The cap of NOFE is also 100 000. As mentioned above, the simulation time would be intolerably long if we try population size from 5 to 100 at the step of 5. Instead, we do simulation at population size 50 and 100 only. It takes us about one week to complete the simulations. The results are given in Table I. Obviously, DDES is more robust and converges much faster than DES.

IV. Minimization of Function of Multiminimum

The above trial function has a unique minimum. As global optimizations are targeting for multiextreme problems, it would be more convincing if DDES performs equally well for multiextreme problems. Most importantly, the concerned electromagnetic inverse problem is a multiextreme problem.

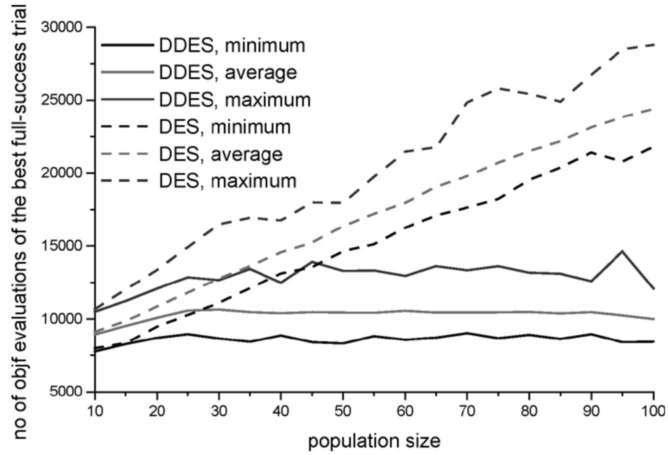

Fig. 11. Function minimization using DDES-NOFE versus population size (50 optimization parameters, cap of NOFE: 100 000).

TABLE I
Function Minimization Using DDES and DES (100 Optimization Parameters, Cap of NOFE: 100 000)

	Population size			
	50		100	
	DDES	DES	DDES	DES
No of successful searches (total 40000)	11136	7903	10327	4022
No of full-success trials (total 400)	96	69	77	35
Minimum NOFE of the best full-success trial	25411	32850	26939	51400
Average NOFE of the best full-success trial	29411.43	37757.5	31203.32	56965
Maximum NOFE of the best full-success trial	35110	43300	39812	65600

In this section, the superior performance of DDES in minimizing a function of multiextreme is investigated. The multiextreme function is

$$f(\mathbf{x}) = \sum_{i=1}^{N} \left(x_i^2 - i\right)^2, \qquad x_i \in [-500, 500]. \tag{2}$$

The minima for this function are $f(x_i^{\text{opt}} = \pm\sqrt{i}) = 0$.

For the sake of computation time, the case of eight optimization parameters is studied. Similarly, different sets of genetic parameters are tested. The mutation intensity and the crossover probability change from 0.05 to 1 at a step 0.05. The population size varies within 8 and 80 at a step of 8.

Two cases of the cap of NOFE, 4000 and 16 000 are studied. Fig. 12(a) and (b) shows the number of successful searches and the number of full-success trials for the two cases. For comparison, the corresponding simulation result by DES is also shown here. The superior performance of DDES over DES is quite evident.

Obviously, the cap 4000 is significantly short of sufficiency. It seems that the cap 16 000 is also insufficient for DDES to reach the stable value of the number of successful searches.

Nevertheless, from the NOFE of the best full-success trial of the two cases as shown in Fig. 13, it is observed that even the cap 4000 is sufficient to catch the optimal performance of DDES. Similar conclusion applies to DES.

We have made further comparison between the optimal genetic parameters of the two cases studied here, and that of the function of unique minimum. It is noted that the optimal genetic parameters, at least for the above two problems, are independent

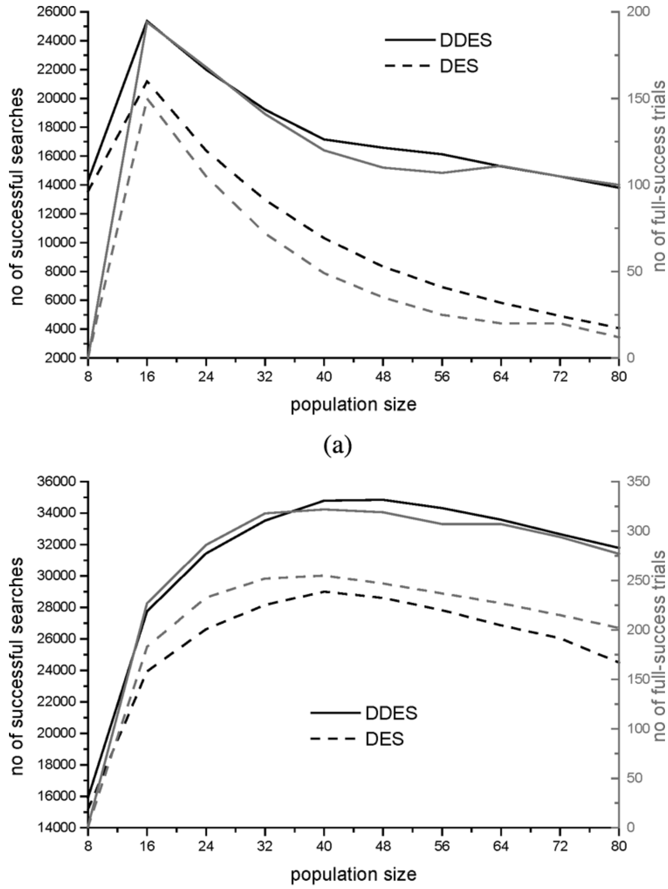

Fig. 12. Minimization of a multiextreme function using DDES-robustness (eight optimization parameters). (a) Cap of NOFE: 4000. (b) Cap of NOFE: 16 000.

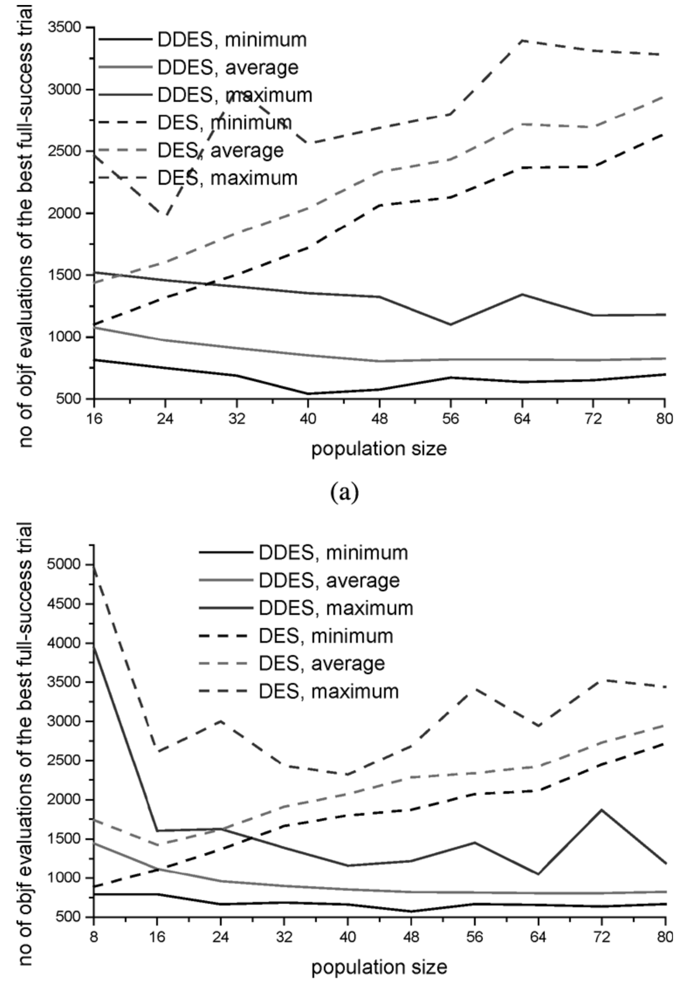

Fig. 13. Minimization of a multiextreme function using DDES-NOFE versus population size (eight optimization parameters, cap of NOFE: 4000). (a) Cap of NOPE: 4000. (b) Cap of NOPE: 16 000.

or very weakly dependent on the actual minimization problem we are solving. This observation is extremely helpful for us to quickly obtain the optimal, or at least quasi-optimal genetic parameters to solve time-consuming minimization problems more efficiently.

It is expected to minimize the above multiextreme function with more optimization parameters so that DDES can be further justified. Simulation is in progress. However, as has been said before, it is very time consuming. The simulation results will be reported once the simulation is completed.

V. Electromagnetic Inverse Scattering of Multiple Perfectly Conducting Cylinders Using Dynamic Differential Evolution Strategy

A. Introduction

Electromagnetic inverse problems [10] are of great interest to both scientific researchers and engineers. It is well known that electromagnetic inverse scattering problems are mathematically very difficult and computationally very expensive. Robust and efficient inversion algorithms are always in great demand.

In this section, the potential of DDES to serve as a robust and efficient inversion algorithm for electromagnetic inverse problems will be investigated by applying it to solve a benchmark electromagnetic inverse scattering problem, the electromagnetic inverse scattering of multiple perfectly conducting cylinders with transverse magnetic (TM) wave incidence.

This benchmark electromagnetic inverse scattering problem has been solved using Newton–Kontorovitch algorithm, simple genetic algorithm, real-coded genetic algorithm [7], DES [2], and differential evolution strategy with individuals in groups [8], among which DES are more superior. The advantages and disadvantages of these algorithms were summarized in [10].

B. Problem Formulation

The geometry of the problem is redrawn in Fig. 14 where O is the origin, Ω, a circle with radius $\mathrm{R}^{\mathrm{meas}}$, is the measuring (data) domain in which the scattered electric fields are measured, the black dots on Ω are receivers, D is the imaging (object) domain which is usually chosen to be circular or rectangular. K perfectly conducting cylinders within D are the objects to be reconstructed, $O_i(d_i, \psi_i)$ is the local origin of the ith cylinder which can be any point within the cylinder contour, C_i. The contour is represented by a local shape function $\rho_i = F_{\mathrm{i}}(\theta_i)$ in the

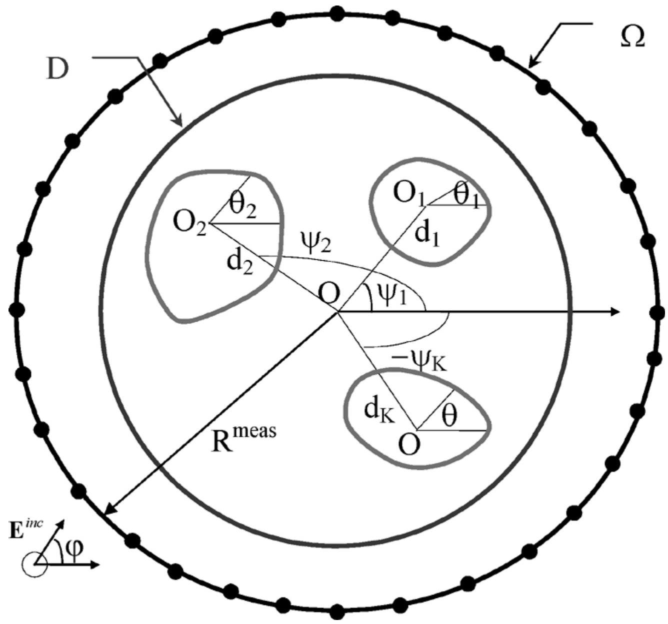

Fig. 14. Geometry of the electromagnetic inverse problem.

local polar coordinate system. In our practice, it is approximated by a closed cubic B-spline local shape function [11], $F_i^B(\theta_i)$, of N control points, C_{ij}, $0 \le j \le N-1$.

The inverse problem is to locate the unknown cylinders and reconstruct the cylinder contours, given the scattered electric fields measured on Ω, $\mathbf{E}_{\text{meas}}^{\text{scat}}(\mathbf{r} \in \Omega)$. In practice, $\mathbf{E}_{\text{meas}}^{\text{scat}}$ is measured at a finite number of frequency point (N_f) and a finite number of incident angles for the incident plane waves (N_a) by a finite number of receivers (N_r). Accordingly, $\mathbf{E}_{\text{meas}}^{\text{scat}}$ becomes an $N_f \times N_a \times N_r$-dimensional vector.

The object and data equations of the electromagnetic inverse problem are

$$E_z^{\text{inc}}(\mathbf{r}) = \sum_{j=1}^{K} \frac{\omega\mu_0}{4} \oint_{C_j} J_{zj}(\mathbf{r}') H_0^{(2)} \times (k_0|\mathbf{r}-\mathbf{r}'|)\, d\mathbf{r}', \qquad \mathbf{r} \in \bigcup_{i=1}^{K} C_i \tag{3}$$

$$E_z^{\text{scat}}(\mathbf{r}) = \sum_{j=1}^{K} -\frac{\omega\mu_0}{4} \oint_{C_j} J_{zj}(\mathbf{r}') H_0^{(2)} (k_0|\mathbf{r}-\mathbf{r}'|)\, d\mathbf{r}' \tag{4}$$

where $\mathbf{E}^{\text{inc}}(\mathbf{r}) = \hat{z}E_z^{\text{inc}} = \hat{z}\exp(-jk_0\hat{k}\cdot\mathbf{r})$ (time factor $e^{j\omega t}$ assumed and suppressed) is the incident field, $\omega = 2\pi f$ is the angular frequency, $\mathbf{r} = x\hat{x} + y\hat{y}$, $k_0 = \omega/c$ is the wavenumber in free space, c is the light speed, $\hat{k} = \cos\varphi\hat{x} + \sin\varphi\hat{y}$ is the incident wave unit vector and φ is the incident angle. $\hat{x}$, $\hat{y}$ and $\hat{z}$ are the unit vectors in the x, y, and z directions respectively, $\mathbf{J}_j(\mathbf{r}) = \hat{z}J_{zj}(\mathbf{r})$ $j = 1,K$ is the induced surface electric current on the surface of the jth cylinder, $H_0^{(2)}(\cdot)$ is the Hankel's function of second kind of zeroth order, $\mathbf{E}^{\text{scat}}(\mathbf{r}) = \hat{z}E_z^{\text{scat}}(\mathbf{r})$ is the scattered electric field.

This benchmark electromagnetic inverse problem is mathematically defined as

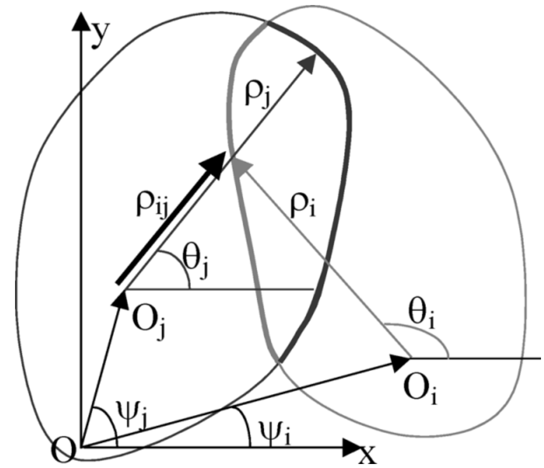

Fig. 15. Intersection of cylinders.

$$\min f(\mathbf{x}) = \frac{\sqrt{\sum_{j=1}^{N_f\times N_a\times N_r} \left|\left(\mathbf{E}_{\text{meas}}^{\text{scat}}\right)_j - \left(\mathbf{E}_{\text{simul}}^{\text{scat}}\right)_j\right|^2}}{\sqrt{\sum_{j=1}^{N_f\times N_a\times N_r} \left|\left(\mathbf{E}_{\text{meas}}^{\text{scat}}\right)_j\right|^2}} + \alpha \sum_{i=1}^{K} \sum_{j=1, j\neq i}^{K} \oint_{C_i} F_i(\theta_i)\, U\left[F_j(\theta_j) - \rho_{ij}(\theta_j)\right] d\theta_i \tag{5}$$

where $\mathbf{x} = [\mathbf{x}_1 \ \cdots \ \mathbf{x}_K]$ is the vector of optimization parameters, $\mathbf{x}_i$ is the subvector containing the geometrical parameters of cylinder i, $\mathbf{x}_i = [d_i \ \ \psi_i \ \ C_{i0} \ \ \cdots \ \ C_{iN-1}]$, $\mathbf{E}_{\text{simul}}^{\text{scat}}$ is an $N_f \times N_a \times N_r$-dimensional vector containing the scattered electric fields simulated with the latest reconstruction results of cylinder locations O_i and local shape functions F_i, α is the intensity of penalty imposed on infeasible reconstruction results with intersecting cylinders as shown in Fig. 15

$$U(t) = \begin{cases} 0, & t < 0 \\ 1, & t \ge 0. \end{cases}$$

The first term of the objective function is the relative reconstruction error which gives a measurement on how close the reconstruction results approach the true profile, while the second term is an artificial penalty applied only in the case of intersecting cylinders.

C. Settings of Parameters

For the convenience of comparison, the settings of parameters in [2] are followed here unless specified otherwise. Therefore, the frequency of the incident field is 300 MHz, or equivalently, $\lambda = 1$ m. It illuminates the unknown object from eight different angles, i.e., $\varphi_i = i\pi/4$, $i = 0,7$. $R^{\text{meas}} = 10\lambda$. Thirty-two receivers located uniformly on Ω are used for each illumination to record the scattered fields. No noise is included in the synthetic data. The number of control points for each closed cubic B-spline local shape function is 6. The search range of control points is [0, 1] λ. The local origins are searched in a

TABLE II
RECONSTRUCTION OF PROFILE 1 (A CIRCULAR CYLINDER WITH CENTER AT ORIGIN AND RADIUS 0.3λ) USING DDES AND DES

	Population size			
	40		80	
	DDES	DES	DDES	DES
No of successful searches (total 40000)	16151	9101	14112	7057
No of full-success trials (total 400)	54	20	54	13
Minimum NOFE of the best full-success trial	361	720	407	1200
Average NOFE of the best full-success trial	625.07	1248.8	750.6	1865.6
Maximum NOFE of the best full-success trial	1280	2240	1573	3360

circle around the origin with radius 1λ. In addition, the number of cylinders is assumed to be known exactly in advance.

D. Single Circular Cylinder

The first sample target is a circular cylinder with center at the origin and radius $0.3\lambda_0$. To ensure that the optimal genetic parameters we obtained before while minimizing the two trial functions are applicable to other minimization problems with identical number of optimization parameters, we conduct similar study to find the optimal genetic parameters corresponding to the current electromagnetic inverse problem. In accordance, the mutation intensity and the crossover probability vary within 0.05 and 1 at a step of 0.05. However, since the concerned electromagnetic inverse problem is extremely time-consuming, we are unable to try too many cases of population size. Instead, we try two typical population sizes, 40 and 80. In addition, 4000 is set to be the cap of NOFE to shorten the simulation time further since we are more interested in the best performance of the algorithm.

The simulation results are shown in Table II where the corresponding results of DES are also given. For both population sizes, the number of successful searches of DDES is almost double of that of DES. The discrepancy of the number of full-success trials is even much bigger (three times for population size 40, and four times for population size 80).

As to the NOFE of the best full-success trial, the minimum, average and the maximum values of DDES are much smaller than the corresponding values of DES. More importantly, the average NOFE of DDES shows slight increase from population size 40 to 80 which might be the weak fluctuation we have observed before. However, the corresponding value of DES increases dramatically. This gives further justification to the superior performance of DDES over DES.

Consistency of the optimal genetic parameters corresponding to the best full-success trials is noted in the recorded optimal genetic parameters corresponding to the unique-minimum function minimization problem of eight optimization parameters, the multiminimum function minimization problem of eight optimization parameters, and the current electromagnetic inverse scattering problem. This consistency is of great practical importance since it provides us a much easier way to find the optimal genetic parameters to solve the very time-consuming electromagnetic inverse problems and other optimization problems more efficiently.

E. Two Parallel Circular Cylinders

The next profile consists of two identical and parallel circular cylinders. The radius of each cylinder is 0.3λ, $d_1 = 0.8\lambda\ \psi_1 = 0^0$; $d_2 = 0.8\lambda\ \psi_2 = 180^0$. This profile is more difficult to handle since we have to deal with the intersection between cylinders as shown in Fig. 15. As mentioned earlier, the intersection is considered by imposing a penalty term in the objective function. However, we have no idea how much penalty is most appropriate to refrain the intersection at no or little cost of the algorithm's convergence behavior. Therefore, we can only choose the penalty intensity according to our past experience. For the profiles considered here, we choose $\alpha = 0.1$.

TABLE III
RECONSTRUCTION OF PROFILE 2 (TWO PARALLEL CIRCULAR CYLINDERS) USING DDES AND DES. (a) DDES, CAP OF NOFE: 4000; (b) DES, CAP OF NOFE: 4000

Successful searches	Minimum NOFE	Average NOFE	Maximum NOFE	Mutation intensity	Crossover probability
3	1187	2518.66667	3527	0.20	0.80
2	1707	2240.50000	2774	0.20	1.00
8	2493	3419.12500	3791	0.30	0.60
8	2115	3023.25000	3714	0.30	0.70
19	1702	3089.05263	3942	0.30	0.80
16	1652	2741.00000	3872	0.30	0.90
12	1939	2962.16667	3762	0.30	1.00
4	3581	3868.75000	3983	0.40	0.60
13	2904	3494.23077	3967	0.40	0.70
27	2071	3199.40741	3993	0.40	0.80
35	1798	2932.40000	3952	0.40	0.90
25	1140	2801.08000	3997	0.40	1.00
11	2892	3580.90909	3954	0.50	0.80
41	2231	3366.17073	3929	0.50	0.90
34	1933	2873.73529	3825	0.50	1.00
11	3375	3773.63636	3971	0.60	1.00

(a)

Successful searches	Minimum NOFE	Average NOFE	Maximum NOFE	Mutation intensity	Crossover probability
1	3920	3920.00000	3920	0.40	0.60
1	3520	3520.00000	3520	0.40	0.70
1	3920	3920.00000	3920	0.50	0.80
4	3360	3660.00000	4000	0.50	0.90
1	2800	2800.00000	2800	0.50	1.00

(b)

Similar investigation as above on the mutation intensity and crossover probability is conducted. However, since the concerned electromagnetic inverse problem is much more time-consuming, we can only try one population size, 80. In addition, we double the step of mutation intensity and crossover probability to 0.1. Accordingly, the starting point of the mutation intensity and crossover probability is 0.1.

Two cases of the cap of NOFE, 4000 and 16 000, are studied. The statistical result is shown in Table III. It is quite clear that DDES outperforms DES significantly. However, it is noticed that no trial is fully successfully for both cases. This may be due to inappropriate choice of the penalty intensity and/or insufficient cap of NOFE. Consequently, we have to lower the criteria of success so that a quantitative comparison can be conducted. Without loss of the general characteristics of the algorithms, we regard 90% as full-success for the present problem. The corresponding comparison result is shown in Table IV which quantitatively shows the superior performance of DDES clearly.

F. Triple Circular Cylinders

The sample target here is three parallel circular cylinders. The radius of each cylinder is $0.3\lambda_0$, $d_1 = 0.8\lambda_0\ \psi_1 = 0^0$, $d_2 = 0.8\lambda_0\ \psi_2 = 120^0$, $d_3 = 0.8\lambda_0\ \psi_2 = 240^0$. Obviously,

TABLE III
RECONSTRUCTION OF PROFILE 2 (TWO PARALLEL CIRCULAR CYLINDERS) USING DDES AND DES. DDES, CAP OF NOFE: 16 000

Successful searches	Minimum NOFE	Average NOFE	Maximum NOFE	Mutation intensity	Crossover probability
1	8182	8182	8182	0.3	0.1
2	3157	3347.5	3538	0.3	0.3
3	3338	6170	11123	0.3	0.4
2	3391	6044.5	8698	0.3	0.5
5	1908	7218.8	11715	0.3	0.6
18	2592	7245.889	14334	0.3	0.7
40	1911	5800.05	15893	0.3	0.8
44	1467	4933.136	13226	0.3	0.9
30	1193	4653.767	11726	0.3	1
1	7906	7906	7906	0.4	0.1
3	6808	11895.33	14686	0.4	0.2
11	3638	7078.273	9653	0.4	0.3
20	2811	7260	14889	0.4	0.4
59	2675	7605.39	15884	0.4	0.5
97	3742	5984.258	11410	0.5	0.6
90	3096	4885.233	11811	0.5	0.7
84	2250	4072.226	8986	0.5	0.8
79	1835	3699.025	9024	0.5	0.9
69	1800	3279.942	11141	0.5	1
5	7811	9918.2	12588	0.6	0.1
31	4621	10817.87	15891	0.6	0.2
81	7145	11633.99	15984	0.6	0.3
92	7614	11699.65	15821	0.6	0.4
98	7091	10999.42	15821	0.6	0.5
97	6717	9823.381	13482	0.6	0.6
98	5443	8342.929	14943	0.6	0.7
97	4201	6594.876	13751	0.6	0.8
87	3385	5198.874	12357	0.6	0.9
85	2883	4490.941	8785	0.6	1
5	7821	10756	14879	0.7	0.1
38	6772	11056.29	15947	0.7	0.2
63	8408	13110.37	15866	0.7	0.3
16	12046	14077.44	15911	0.7	0.4
6	14059	14757.83	15838	0.7	0.5
4	13846	14867.25	15671	0.7	0.6
9	13030	14370.78	15754	0.7	0.7
38	10815	14489.08	15945	0.7	0.8
99	6997	10581.63	14173	0.7	0.9
95	6034	7950.937	12299	0.7	1
2	12191	14055	15919	0.8	0.1
21	6846	12022.9	15086	0.8	0.2
33	10470	14143.61	15982	0.8	0.3
17	13390	14866	15972	0.8	1
18	9259	12867.72	15718	0.9	0.2
10	13308	15005.3	15922	0.9	0.3
1	13100	13100	13100	1	0.2

(c)

TABLE III
RECONSTRUCTION OF PROFILE 2 (TWO PARALLEL CIRCULAR CYLINDERS) USING DDES AND DES. DES, CAP OF NOFE: 16 000

Successful searches	Minimum NOFE	Average NOFE	Maximum NOFE	Mutation intensity	Crossover probability
24	7600	12883.33	15520	0.4	0.4
64	5840	10983.75	15600	0.4	0.5
58	4240	9046.90	14960	0.4	0.6
15	2960	6464	12640	0.4	0.7
3	2640	4080	6080	0.4	0.8
15	12960	14976	16000	0.5	0.5
72	7760	11983.33	15920	0.5	0.6
84	5200	8905.71	13520	0.5	0.7
67	4000	6844.18	15200	0.5	0.8
13	3520	4873.85	6400	0.5	0.9
28	11280	14442.86	16000	0.6	0.7
95	6960	10998.74	15680	0.6	0.8
86	4720	7512.56	12960	0.6	0.9
75	4240	6226.13	12960	0.6	1
52	11440	14112.31	16000	0.7	0.9
94	7120	11498.72	15040	0.7	1

(d)

TABLE IV
RECONSTRUCTION OF PROFILE 2 (TWO PARALLEL CIRCULAR CYLINDERS) USING DDES AND DES (CAP OF NOFE: 16 000)

	DDES	DES
No of successful searches (total 10000)	1904	845
No of 90%-success trials (total 100)	9	2
Minimum no of objective function evaluations of the best 90%-success trial	3096	6960
Average no of objective function evaluations of the best 90%-success trial	4885.23	10998.74
Maximum no of objective function evaluations of the best 90%-success trial	11811	15680

TABLE V
RECONSTRUCTION OF PROFILE 3 (THREE PARALLEL CIRCULAR CYLINDERS) USING DDES AND DES. (a) DDES. (b) DES

(a)

Successful searches	Minimum NOFE	Average NOFE	Maximum NOFE	Mutation intensity	Crossover probability
20	7165	18336.35	30793	0.40	0.40
7	4910	7837.14	11599	0.40	0.70
2	5225	6122	7019	0.40	1.00
10	14828	18599.3	23619	0.70	1.00

(b)

Successful searches	Minimum NOFE	Average NOFE	Maximum NOFE	Mutation intensity	Crossover probability
5	24480	28368	31920	0.4	0.4
9	21720	24560	27480	0.7	1

it is much more difficult to handle than the previous two targets. As the reconstruction of this target is extremely time-consuming, we are not able to try too many sets of genetic parameters. Instead, the population size is chosen to be 120. Four cases of mutation intensity, 0.1, 0.4, 0.7, and 1.0 and four cases of crossover probability, 0.1, 0.4, 0.7, and 1.0 are examined. The cap of NOFE is 32 000.

The simulation result is given in Table V. In this case, the success percentage of both DDES and DES is very low. This low success percentage is due to incomplete sampling of the mutation intensity and crossover probability, insufficient cap of NOFE, and inappropriate choice of penalty intensity. From previous experience, we are very confident that the success percentage can be increased given sufficient computational resources. On the other hand, we are still able to observe the superior performance of DDES over DES from the limited successful cases.

VI. CONCLUSION AND FUTURE WORK

A novel evolution algorithm, DDES, is developed to solve optimization problems. It inherits the genetic operators from

DES. However, it differs itself from DES by updating its population dynamically while DES updates its population generation by generation. The dynamic updating of population leads to a larger virtual population size and quicker response to change of optimal search direction. DDES has been applied to minimize a trial function of unique minimum, to minimize a trial function of multiminimum, and to solve a benchmark electromagnetic inverse scattering problem. Comparison with DES has been carried out. It has been observed that DDES significantly outperforms DES in efficiency, robustness, and memory requirement.

As noted in our simulation, there seems to be consistency of optimal genetic parameters for different optimization problems with the same number of optimization parameters. We will conduct further investigation to consolidate this observation and report any interesting findings.

While applying DDES to solve the benchmark electromagnetic inverse scattering problem, it is noticed that the cylinder intersection has to be properly considered. However, our current approach to deal with the cylinder intersection depends on our past experience on choosing the penalty intensity. Our empirical experience is seriously insufficient for us to deal with the cylinder intersection most appropriately. We are now trying to develop an algorithm which handles the intersection more efficiently.

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