Simulated Annealing-Particle Swarm Optimization (SA-PSO): Particle Distribution Study and Application in Neural Wiener-based NMPC

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Abstract— **Good nonlinear optimization plays a vital role in advanced controller such as nonlinear model predictive control (NMPC). Particle swarm optimization (PSO) is one of nonlinear optimization which has a good potential to be implemented in the NMPC. Even though PSO can determine global optimum value, it is less superior in determining a local minimum value. Meanwhile, another optimizer known as simulated annealing (SA), has an opposite capability of PSO in determining the local and global values. Consequently, in this work, the SA and PSO optimizers have been combined to form SA-PSO which expected to cater both local and optimum point searching. From the particle distribution study, the result shows that the SA-PSO has better particle distribution than the original PSO. The proposed SA-PSO optimizer has also successfully applied in NMPC to control temperatures in the MTBE reactive distillation. The set point tracking test show that the NMPC using SA-PSO have good performance with small amount of overshoot, low settling time and small amount of error.**

Keywords—Simulated annealing optimization, PSO, Nonlinear Model Predictive control, Neural Wiener

I. INTRODUCTION

Nonlinear Model predictive Control is one of advanced controller which able to tackle the nonlinear system and interaction between the variables in multiple input and multiple output (MIMO) controller. Neural Wiener based NMPC is one type of NMPC which has an ability to control highly nonlinear process [1-4]. Neural – Wiener (NW) model is known to be as one of the powerful block oriented model which utilizes low computational time and thus has been selected to be embedded in the MPC [2-4].

Optimizer is one of important component in NMPC which requires an iterative algorithm that can cause an additional computational time in order to minimize the objective function of controller. For the categories of slow response and noisy process such as temperature controller, the accurate optimizer with has acceptable computational time is more important than the faster optimizer with low accurate performance.

One of the optimizer which has good performance and can be applied in the NMPC is a particle swarm optimization (PSO) [5-9] and simulated annealing (SA) [10]. PSO is one of nonlinear optimization which has a strong ability in finding the global optimum value but less superior in finding local minimum. Meanwhile, simulated annealing (SA) optimizer has a strong ability to determine the local optimum value but less superior in finding the global optimum value. Hence, the combination of SA and PSO will enhance the convergence and accuracy of the optimizer. In this work, the particle distribution is evaluated for PSO and SA-PSO in solving a mathematic optimization problem. The SA-PSO is then applied in the Neural Wiener based MPC (NWMPC) to control the tray temperatures in MTBE reactive distillation.

II. DEVELOPMENT OF SA- PSO

PSO has been developed by Kennedy and Eberhart in 1995 [11]. Original PSO algorithm have a population (called a swarm) of candidate solution (called particles). These particles will move around to search the optimum value. Meanwhile, SA is one of nonlinear optimization which uses randomized algorithm or probabilistic algorithm to find the optimum value. The SA algorithm concept is based on annealing process in metallurgy which uses heating and then controlling the cooling of a material to increase the size of its crystals and to reduce their defects. This slow cooling process becomes the basic idea of SA optimization. In this optimization method, the slow decrease of the probability will accept the worse value of solutions in order to find the optimum value of solution at the exploration of solution space [12].

To increase the performance of optimizer, PSO optimizer has been used and has been combined with SA. If the costs function which must be minimized is f: □ $n \rightarrow$ □. The goal is to find a solution a for which $f(a) \leq f(b)$ for all b in the search-space. A basic PSO algorithm is then:

1. Create *a* ‘population’ of agents (called particles) uniformly distributed over the search-space.
2. Evaluate each particle’s position according to the objective function.
3. If a particle’s current position is better than its previous best position, update it.

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4. Determine the best particle (according to the particle's previous best positions).
5. Update particles' velocities according to
$$v_i^{t+1} = v_i^t + \varphi_1 U_1^t (Pb_i^t - x_i^t) + \varphi_2 U_2^t (gb_i^t - x_i^t) \quad (1)$$
where, v_i^{t+1} is velocity of agent i at iteration k, φ_1 and φ_2 is weighting factor, U_1^t and U_2^t is uniformly distributed random number between 0 and 1, x_i^t is current position of agent i at iteration k, Pb_i^t is the best known position of particle i. Meanwhile, gb_i^t is the best known position of the entire swarm.
6. Move particles to their new positions according to:
$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (2)$$
7. Go to step 2 until stopping criteria are satisfied.

In this work, the first modification that has been made is by adding the weighting function (*w*) in the inertia of velocity equations shown below:

$$v_i^{t+1} = w.v_i^t + \varphi_1 U_1^t (Pb_i^t - x_i^t) + \varphi_2 U_2^t (gb_i^t - x_i^t) \quad (3)$$

The value of the inertia weight (*w*) is decreased automatically during a run as a proposed by Shi and Eberhart [11]. The second modification made is by combining the simulated annealing (SA) optimization method on PSO as a proposed by Wang and Xiao [12]. The combination of SA and PSO algorithm's have similar searching process is also started from initializing a group of random particles. In SA calculation, the new individuals are given randomly around the original individuals and the changing range of original particles as a parameter r_1, to each particle reduces step by step as the generation increasing. The changing range of original particles as a parameter r_1 of each particle and the final result of *x*, can be defined as:

$$Present = present + r_1 - r_2.2.\ rand\ (1) \quad (4)$$
$$r_1 = \left(\frac{(lb-ub)(0-(0-w_{max})}{(it_{min}-it_{max})(it_{max}-it)}\right) \quad (5)$$
$$x_{(i\to n)} = x_{(i\to n)} + r_1 - r_1 * 2 * rand\ (1) \quad (6)$$

Where *rand (1)* is a random number between 0 and 1, the parameter r_1 here also reduces step by step as the generation increasing, *lb* is lower bound and *ub* is upper bound [12]. In this work, the value of *W* is decreasing during the iteration in PSO calculation. This SA method when activated is using 50% of total particle, meanwhile the PSO method use the remaining 50% of total particle.

III. Particle Distribution Test of SA-PSO

The combination of SA-PSO was tested to solve simple optimization to find the value of x_1 and x_2 for the problem shown below:

Minimize: $f(x) = (x_1 - 1)^2 + (x_2 - 2)^2$ (7)
Constraints: $-10 < x_1 < 10$; (8)
$-10 < x_2 < 10$ (9)

Using PSO without modification, where the value of W is 1, the particles have scattered in the distribution area as shown in Fig.1. The figure shows that the particle distributions are like multi sinusoidal pattern, which spread along the distribution area and convex to the one point than continuously repeated the same pattern until the end of iteration.

The modified PSO was developed by adding weighing coefficient (*W*) as shown to eq. 5. In order to solve this simple optimization, the particles started by distributing along the area and reach the convex form at the 60 of iteration. These particle distributions were very fast and effective to search global optimum as shown in Fig. 2. However, this technique is not good to determine local optimum.

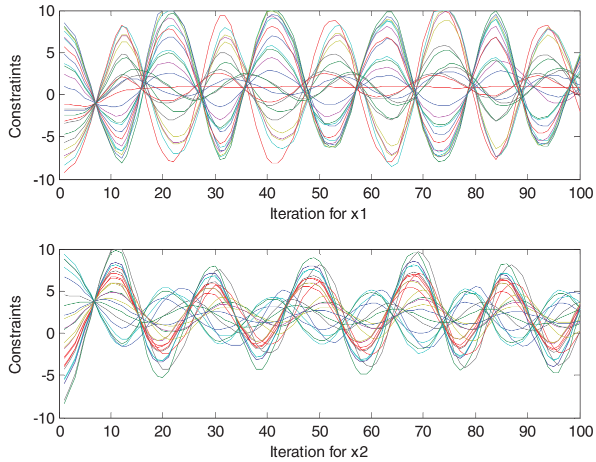

Figure 1. The particle distribution using Original PSO without modification of W

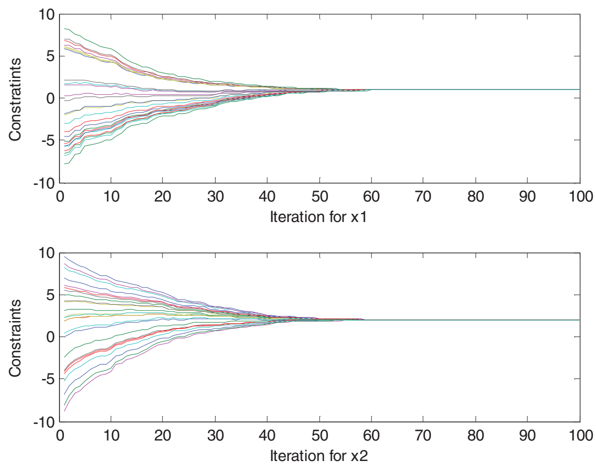

Figure 2. Particle distribution using PSO with value of W (0.001 – 1)

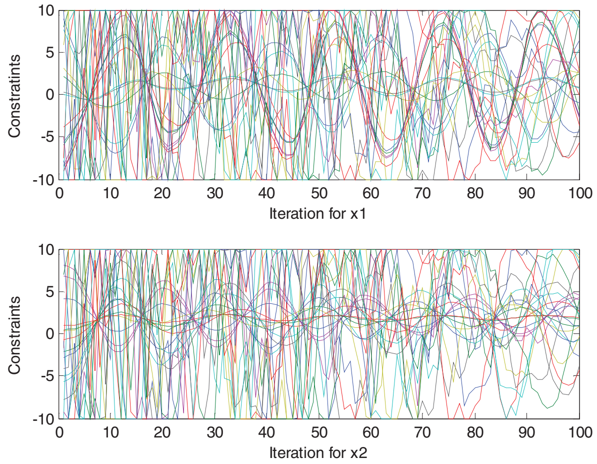

Figure 3. The particle distribution using SA - PSO without modification of W

The PSO which was combined with SA, which use pure PSO without modification of W was applied to solved the simple optimization problem, which the particle distribution profile were shown in Fig. 3. Half of number particle act using the pure PSO program which create multi-sinusoidal pattern same as in the Figure 1. Mean while the half of number particle doing SA pattern. This combination can increase the accuracy of optimization result as listed in Table 4.3. The result show that the errors are decrease than the result using the pure PSO and modified PSO.

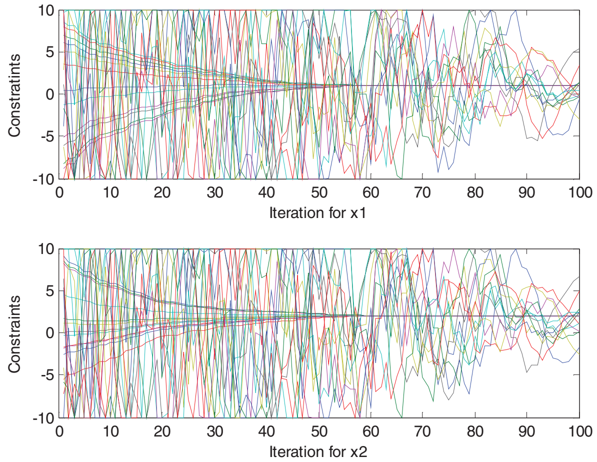

Figure 4. The particle distribution using SA - PSO with value of W (0.7 – 1)

The combination of modified PSO and SA was successfully applied to solve this simple optimization, the result was exactly correct as listed in Table 1, where **x* is the answer [13]. This result is better than the other methods have been used. The particles distribution are shown in Fig. 4, which the half of particle move around the area of distribution following the modified PSO (W around 0.001 – 1). These movements of particles have better result than then the other to find the objective, which the particle move from the highest magnitude of the distribution area and reach the convex at the 60 iteration.

TABLE I. EFFECT OF WEIGHTING COEFFICIENT ON PSO AND SA-PSO

Optimizer	Weighting Coefficient	Particle	Iterations	Result	
				X1	*X2*
PSO	1	25	100	1.0068	2.0307
PSO	0.001 - 1	25	100	0.9996	2.0074
SAPSO	1	25	100	0.9797	1.9601
SAPSO	0.001 - 1	25	100	1.000	2.000
**x*		-		1.0000	2.000

IV. APPLICATION OF SA-PSO IN NEURAL WIENER BASED MPC (NWMPC)

A. Development of MTBE reactive distillation Model

The most promising technique of producing MTBE is from methanol and isobutene, where the liquid-phase reaction is catalyzed by ion exchange resin (heterogeneous reaction). The reaction scheme is:

$$i\text{-}C_4H_8 + CH_3\text{-}OH \rightleftharpoons C_5H_{12}O \qquad (10)$$

The specification of MTBE RD considered here can be found in Fig. 5. In this work, the MTBE reactive distillation model has been developed in Aspen dynamic.

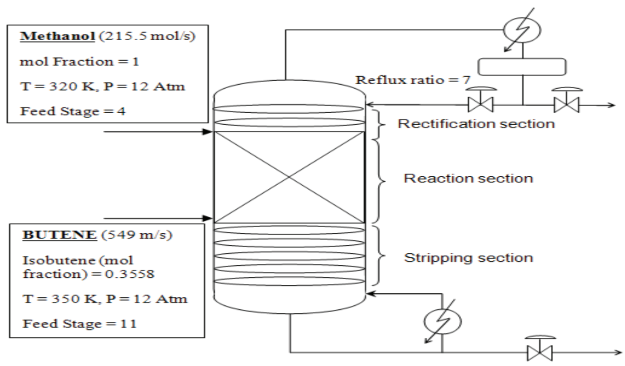

Figure 5. MTBE Reactive Distillation Column [14].

B. Neural Wiener model identification

Neural Wiener (N-W) model is consisting of linear block and nonlinear block as shown in Fig. 6. The N-W identification algorithm is begun by data generation in order to collect dynamic input-output data. Then, a linear model is identified to produce the intermediate variable of $v_{(k)}$ from the input data. The $v_{(k)}$ output is a linear dynamic part of state space model which consist of v_1 and v_2. Finally, neural network model is identified using intermediate variable as input variable. The N-W model block arrangement is shown in Fig. 6.

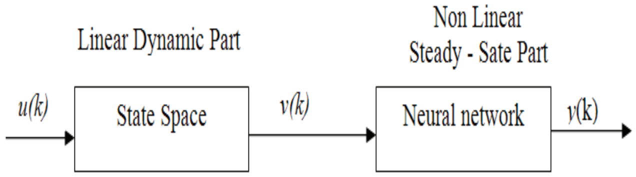

Figure 6. Neural Wiener model configuration [2]

In this work, state space model is used as the linear block and MIMO neural network model is used as the nonlinear block. The MIMO Neural network used is a feed forward neural network with 15 hidden nodes and 1 hidden layer. The output $y_{(k)}$ of the N-W model is described as below:

$$y(k) = w_0 + \sum_{i=1}^{K} w_i^2 \varphi\{w_{i,0}^1 + w_{i,1}^1[C\,x(k) + D\,u(k) + e(k)]\} \quad (11)$$

where w_0 is the bias, $w_{i,j}$ is the weight of first layer, and w_i is the weight of second layer, φ is a nonlinear transfer function such as hyperbolic tangent sigmoid transfer function and tansig, K is the number of hidden nodes.

C. Tuning of Neural Wiener Based MPC (NWMPC)

The N-W model developed and the SA-PSO optimizer proposed are embedded in the Neural Wiener NMPC as shown in Fig. 7. The NWMPC objective function for the MIMO case is consists of the quadratic error between each controlled variable and its set-point and the quadratic change of each manipulated variable. The MPC objective function for the 2 × 2 system is defined as follows:

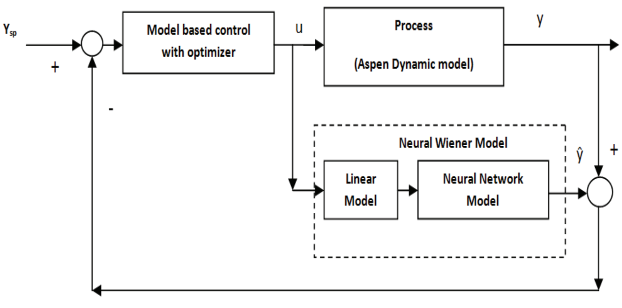

Figure 7. General structure of Neural-Wiener Based model predictive control

$$j_k = \sum_{i=1}^{P} \left(\left(y_{f1|k+i} - y_{sp1|k+i}\right)^2 Q_1 + \left(y_{f2|k+i} - y_{sp2|k+i}\right)^2 . Q_2 \right) + (\Delta u_{f1|k+i})^2 . R_1 + (\Delta u_{f2|k+i})^2 . R_2 \quad (13)$$

where y_f is predicted future output, y_{sp} is set point, Q is error penalty, R is input change penalty, Δu_f is future input change and k is current sampling time. The control variables chosen are tray temperature no.3 (CV_1) and no.8 (CV_2). Meanwhile the manipulated variables are reboiler duty (MV_1) and reflux flowrate (MV_2). The control and manipulated variables are chosen based on SVD test [15].

NWMPC using SA-PSO as a nonlinear optimizer was tuned to obtain the best control configuration. The NWMPC tuned parameters obtained are tabulated in Table II.

TABLE II. NWMPC AND PSO PARAMETER AFTER TUNING

MPC Parameter	Value		PSO Parameter	Value
Prediction horizon	5		Number of particle	110
Control horizon	2		weight function minimum	0.4
Error penalty	1	1	weight function maximum	0.9
Input change penalty	98000	4500	Number of iteration	60
input constraint upper bounds	0.05	0.05	Rho1 (weighing factor 1)	1.25
input constraint lower bounds	-0.05;	-0.05	Rho2 ((weighing factor 2)	1.25

D. Control study of NWMPC using NWMPC

In this set point test, the efficiency of reactive distillation column is set at 80% using the NMPC parameters and SA-PSO parameters as listed in Table II. In this test, set point steps were varied from 0, 7, 4, and 7 for CV_1, meanwhile for CV_2 they were varied at 0, 0.75, 0.39 and 0.75 which occur at every 2 hours. At the steady state condition, the MTBE purity was 95.24%, while the temperature of tray number 3 and 8 were 93.92 ^oC and 126.96 ^oC, respectively. NMPC Neural wiener using SA-PSO was able to bring the both CV to follow the set point as shown in Fig. 8. From the figure, it is shows that CV_1 and CV_2 reach the set point after approximately after 15 minutes after the set point has been introduced. The value of integral of time absolute error (ITAE) for CV_1 and CV_2 are 1.7938 and 0.3631, respectively. The small overshoot for CV_2 is only 0.18 $^\circ C$ from the set point, and rapidly goes back to the original value as shown in Fig. 8. These values are very small and prove the good performance of the proposed controller.

Fig. 9 shows the profile of manipulated variables MV_1 and MV_2 with respect of CV_1 and CV_2. The figure shows that MV_1 and MV_2 still within the allowed limit values. Meanwhile, Fig. 10 shows the profile of MTBE purity and isobutene conversion. The increase of MV_2 caused the increase of CV_1 and CV_2, while the increase of CV can be correlated with the increase of MTBE purity. Using NWMPC with SA-PSO, reactive distillation was able to achieve the desired MTBE purity of 95%, 99% and 97.5%, respectively, as shown in Fig. 10. Meanwhile, the isobutene conversion was more than 99.99% when the set point of MTBE purity was 95%. On the other hand, 99.86% and 99.88% conversion achieved when the MTBE purity was set point was at 99 % and 97.5%, respectively as shown in Fig. 10.

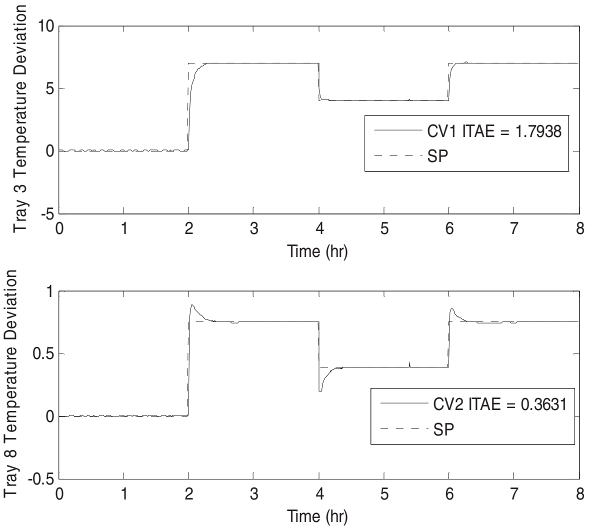

Figure 8. CV $_1$ and CV_2 of NWMPC using SA-PSO in set point test

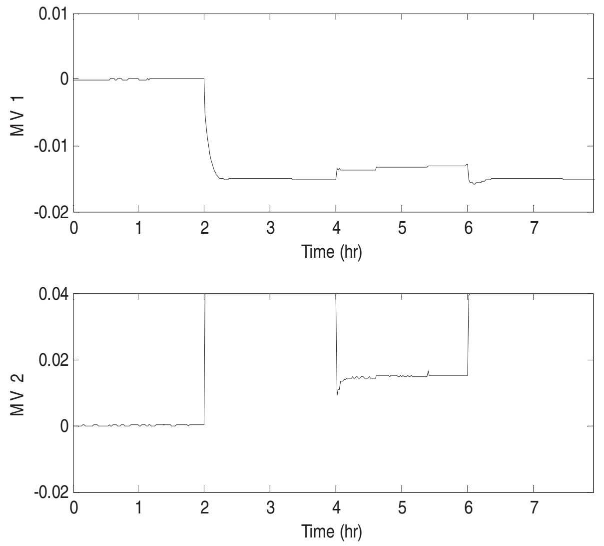

Figure 9. MV $_1$ and MV_2 of NWMPC using SA-PSO in set point test

V. Conclusion

SA-PSO has been successfully developed and applied to solve simple optimisation. The particles distribution on SA-PSO had shown a better distribution than original PSO. The SA-PSO has also successfully applied to control tray temperatures in the MTBE reactive distillation. The performance test of set point change showed that the proposed controller have good performance with small amount of overshoot, low settling time and small amount of error.

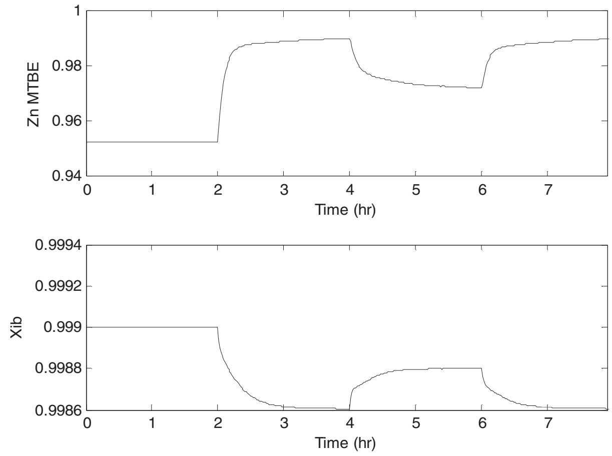

Figure 10. MTBE Purity and Isobutene Conversion of NWMPC using SA-PSO

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