Numerical Optimization of Computer Models

Hans-Paul Schwefel

JOHN WILEY & SONS
Chichester · New York · Brisbane · Toronto

First published under:
Hans-Paul Schwefel, Numerische Optimierung von Computer-Modellen mittels der Evolutionsstrategie
(Interdisciplinary Systems Research, Vol. 26)

British Library Cataloguing in Publication Data:

Schwefel, Hans Paul
Numerical optimization of computer models.
1. Evolution—Mathematical models
2. Mathematical optimization
I. Title II. Numerische Optimierung von Computer—Modellen mittels der Evolutionsstrategie.
English
575'.0072 QH362

ISBN 0 471 09988 0

Printed in Great Britain

APPENDIX 1

CATALOGUE OF PROBLEMS

The catalogue is divided into three groups of test problems corresponding to the three divisions of the numerical strategy comparison. The optimization problems are all formulated as minimum problems with a specified objective function F(x) and solution x^*. For the second set of problems, the initial conditions $x^{(0)}$ are also given. Occasionally relative or local minima and other stationary points of the objective function are also indicated. Inequality constraints are formulated such that the constraint functions $G_j(x)$ are all greater than zero within the allowed or feasible region. If a solution lies on the edge of the feasible region then the active constraints are mentioned, for which the values of the constraint functions must be just equal to zero. Where possible the structure of the minimum problem is depicted geometrically by means of a two-dimensional contour diagram. Together with this are specified the range of values of the variables which the diagram illustrates and the values of the objective function on the contour lines drawn. Constraints are shown as double lines with arrows indicating the direction of the feasible region. In some cases there is a brief mention of any especially characteristic behaviour shown by individual strategies during their iterative search for the minimum.

A1.1 Test problems for the first part of the strategy comparison

Problem 1.1 (Sphere model)

Objective function: $F(x) = \sum_{i=1}^{n} x_i^2$

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

For n = 2 a contour diagram is sketched under problem 2.17. For this, the simplest of all quadratic problems, none of the trial strategies fails.

Problem 1.2

Objective function: $F(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j \right)^2$

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

A contour diagram for n = 2 is given under problem 2.9. The objective function of this true quadratic minimum problem can be written in matrix notation as:

$$F(x) = x^T A\, x.$$

The n x n matrix of coefficients A is symmetric and positive definite. According to Schwarz, Rutishauser and Stiefel (1968) its condition number K is a measure of the numerical difficulty of the problem. Among other definitions, that of Todd (1949) is useful, namely:

$$K = \lambda_{max}/\lambda_{min} = a^2_{max}/a^2_{min}$$

where

$$\lambda_{max} = \underset{i}{\mathrm{Max}}\{|\lambda_i|;\ i=1(1)n\}$$

and similarly for λ_{min}. The λ_i are the eigenvalues of the matrix A and the a_i are the lengths of the semi-axes of an n-dimensional elliptic contour surface $F(x) = \text{const.}$

Condition numbers for the present matrix

$$A = \{a_{ij}\} = \begin{bmatrix} n & n-1 & n-2 & \dots & n-j+1 & \dots & 1 \\ n-1 & n-1 & n-2 & \dots & n-j+1 & \dots & 1 \\ n-2 & n-2 & n-2 & \dots & n-j+1 & \dots & 1 \\ \vdots & \vdots & \vdots & & & & \vdots \\ n-i+1 & \dots\ n-i+1 & \dots\ n-i+1 & \dots & n-i+1 & \dots & 1 \\ \vdots & \vdots & \vdots & & \vdots & & \\ 1 & 1 & 1 & \dots & 1 & \dots & 1 \end{bmatrix}$$

were calculated for various values of n by means of an algorithm of Greenstadt (1967b), which uses the Jacobi method of diagonalisation. As can be seen from the following table, K increases with the number of variables as $O(n^2)$.

n	K	K/n^2
1	1	1
2	6.85	1.71
3	16.4	1.82
6	64.9	1.80
10	175	1.75
20	678	1.69
30	1500	1.67
60	5930	1.65
100	16400	1.64

Not all the search methods achieved the required accuracy. For many variables the coordinate strategies and the complex method of Box terminated the search prematurely. Powell's method of conjugate gradients even got stuck without the termination criterion taking effect.

A1.2 Test problems for the second part of the strategy comparison

Problem 2.1 after Beale (1958)

Objective function: $F(x) = \{1.5 - x_1(1-x_2)\}^2$

$$+ \{2.25 - x_1(1-x_2^2)\}^2$$

$$+ \{2.625 - x_1(1-x_2^3)\}^2$$

Minimum: $x^* = (3, 0.5)$; $F(x^*) = 0$.

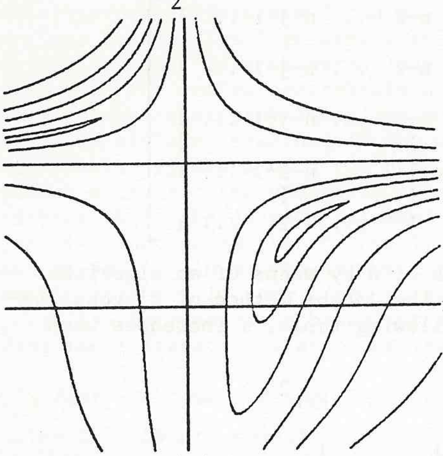
Besides the strong minimum x^* there is also a weak minimum at infinity:

$$x' \to (-\infty, 1+); \; F(x') \to 0.$$

Saddle point: $x'' = (0,1)$; $F(x'') \simeq 14.20$.

Start: $x^{(0)} = (0,0)$; $F(x^{(0)}) \simeq 14.20$.

Figure A.1 - Contour diagram for problem 2.1



$-5 \leqq x_1 \leqq 7$

$-1 \leqq x_2 \leqq 2$

$F(x) = /0.1, 1, 4, \simeq 14.20, 36, 100/$

For very large initial step lengths the (1+1) evolution strategy converged once to the weak minimum x'.

Problem 2.2

As problem 2.1, but with:

Start: $x^{(0)} = (0.1, 0.1)$; $F(x^{(0)}) \simeq 12.99$.

Problem 2.3

Objective function: $F(x) = -|x_1 \sin\{\surd(|x_1|)\}|$.

There are infinitely many local minima, the positions of which can be specified by a transcendental equation:

$$\surd(|x_1^*|) = 2 \tan\{\surd(|x_1^*|)\}.$$

For $|x_1^*| >> 1$ we have approximately:

$$x_1^* \simeq \pm\{\pi(0.5 + k)\}^2 \text{ for } k = 1,2,3, \ldots \text{ integer;}$$

and

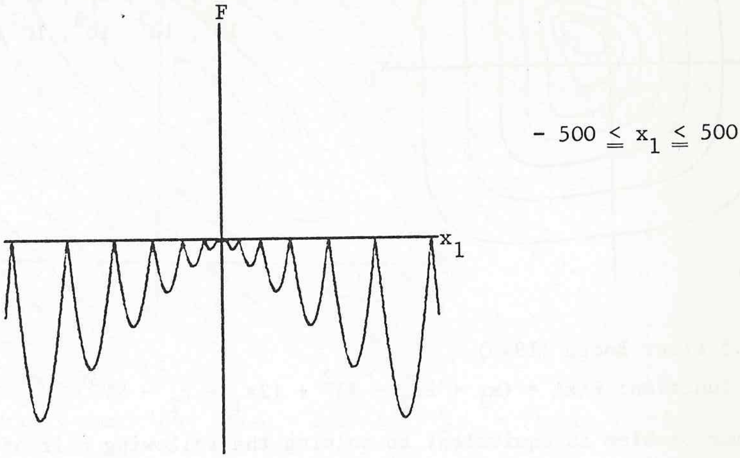
$$F(x_1^*) \simeq |x_1^*|.$$

Whereas in reality none of the finite local minima is at the same time a global minimum, the finite word length of the digital computer used together with the system-specific method of evaluating the sine function give rise to an apparent global minimum at

$$x_1^* = \pm\ 4.44487453 \cdot 10^{16};$$
$$F(x^*) = -4.44487453 \cdot 10^{16}.$$

Counting from the origin it is the 67,108,864th local minimum in each direction. If x_1 is increased above this value, the objective function value is always set to zero.

Figure A.2 - Representation of the objective function $F = F(x_1)$ for problem 2.3



Start: $x_1^{(0)} = 0$; $F(x^{(0)}) = 0$.

Most strategies located the first or highest local minimum left or right of the starting point (the origin). The two membered evolution method, depending on the sequence of random numbers, found (for example) the 2nd, 9th or 34th local minimum. Only the (10,100) evolution strategy almost always reached the apparent global minimum.

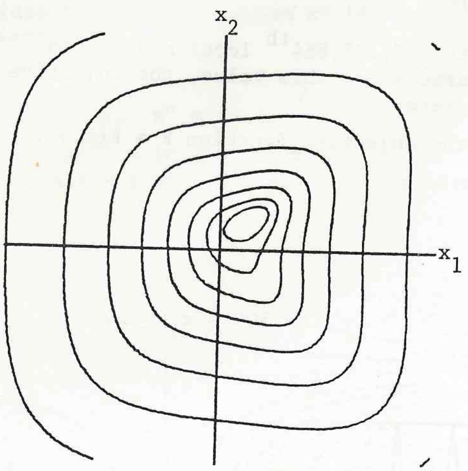
Problem 2.4

Objective function: $F(x) = \sum_{i=1}^{n} \{(x_1 - x_i^2)^2 + (x_i - 1)^2\}$ for $n = 5$.

Minimum: $x_i^* = 1$ for $i = 1(1)n$; $F(x^*) = 0$.

Start: $x_i^{(0)} = 10$ for $i = 1(1)n$; $F(x^{(0)}) = 40905$ for $n = 5$.

Figure A.3 - Contour diagram for problem 2.4 for n = 2



$-7 \leqq x_i \leqq 7$ for i = 1, 2

$F(x)=/10^0,\ 10^1,\ 10^2,\ 10^3,\ 10^4,\ 10^5,\ 10^6,\ 10^7/$

Problem 2.5 after Booth (1949)

Objective function: $F(x) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$.

This minimum problem is equivalent to solving the following pair of linear equations:

$$x_1 + 2x_2 = 7$$

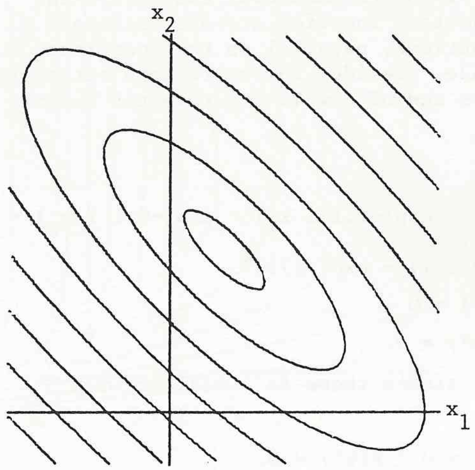
$$2x_1 + x_2 = 5.$$

An approach to the latter problem is to determine the x_1 and x_2 which minimise the error in the equations. The error is defined here in the sense of a Gauss approximation as the sum of the squares of the components of the residual vector.

Minimum: $x^* = (1,3)$; $F(x^*) = 0$.

Start: $x^{(0)} = (0,0)$; $F(x^{(0)}) = 74$.

Figure A.4 - Contour diagram for problem 2.5



$-3 \leq x_1 \leq 5$

$-1 \leq x_2 \leq 7$

F(x)=/1,9,25,49,81,121,
169,225/

Problem 2.6

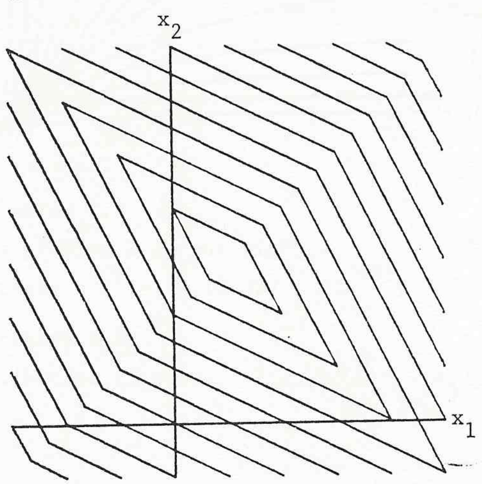
Objective function: $F(x) = \mathrm{Max}\{|x_1 + 2x_2 - 7|;|2x_1+x_2-5|\}$.

This represents an attempt to solve the previous system of linear equations of problem 2.5 in the sense of a Tchebycheff approximation. Accordingly the error is defined as the absolute maximum component of the residual vector.

Minimum: $x^* = (1,3)$; $F(x^*) = 0$.

Start: $x^{(0)} = (0,0)$; $F(x^{(0)}) = 7$.

Figure A.5 - Contour diagram for problem 2.6



$-3 \leq x_1 \leq 5$

$-1 \leq x_2 \leq 7$

F(x)=/1,2,3,4,5,6,7,8,9,
10,11/

Several of the search procedures were unable to find the minimum. They converged to a point on the line $x_1 + x_2 = 4$, which joins together the sharpest corners of the rhombohedral contours. The partial derivatives of the objective function are discontinuous there; in the unit vector directions, parallel to the coordinate axes, no improvement can be made. Besides the coordinate strategies, the methods of Hooke and Jeeves and of Powell are thwarted by this property.

Problem 2.7 after Box (1966)

Objective function: $F(x) = \sum_{i=1}^{10} [\exp(-0.1\ i\ x_1) - \exp(-0.1\ i\ x_2) - x_3\{\exp(-0.1\ i) - \exp(-i)\}]^2.$

Minima: $x^* = (1, 10, 1);\ F(x^*) = 0$

$x^* = (10, 1, -1);\ F(x^*) = 0.$

Besides these two equal strong minima there is a weak minimum along the line

$$x_1' = x_2'\ ;\ x_3' = 0\ ;\ F(x') = 0.$$

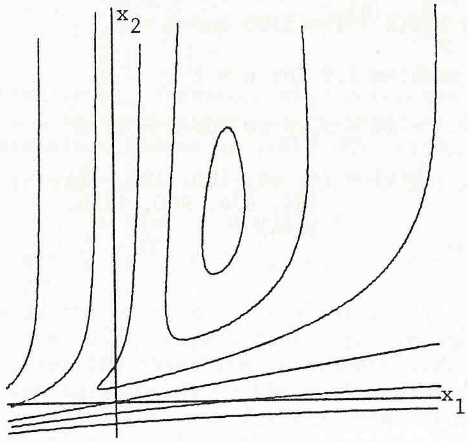
Because of the finite computational accuracy the weak minimum is actually broadened into a region:

$$x_1'' \simeq x_2''\ ;\ x_3'' \simeq 0\ ;\ F(x'') = 0,\ \text{if } x_1 >> 1.$$

Start: $x^{(0)} = (0, 20, 20)\ ;\ F(x^{(0)}) \simeq 1022.$

Many strategies only roughly located the first of the strong minima defined above. The evolution strategies tended to converge to the weak minimum, since the minima are at equal values of the objective function. The second strong minimum, which is never referred to in the relevant literature, was sometimes found by the multimembered evolution strategy.

Figure A.6 - Contour diagram for problem 2.7 on three planes:

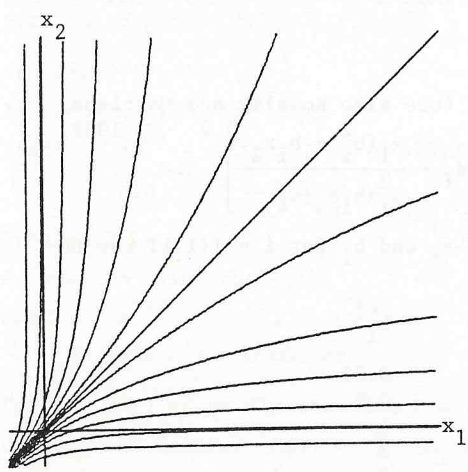


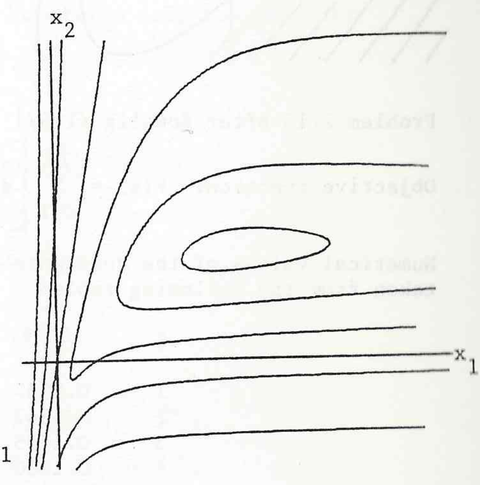
Top left: $x_3 = 1$
$-1 \leq x_1 \leq 3$
$-2 \leq x_2 \leq 22$

Bottom right: $x_3 = -1$
$-2 \leq x_1 \leq 22$
$-1 \leq x_2 \leq 3$

Bottom lef $x_3 = 0$
$-2 \leq x_i \leq 22$
for i=1, 2

$F(x)=/0.03,0.3,1, \simeq 3.064, 10,30/$





Problem 2.8

As problem 2.7, but with

Start: $x^{(0)} = (0,10,20)$; $F(x^{(0)}) \simeq 1031.$

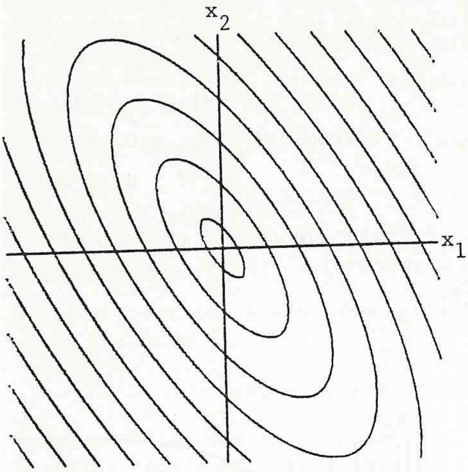
Problem 2.9

Objective function: $F(x) = \sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2$ for n = 5.

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

Start: $x_i^{(0)} = 10$ for $i = 1(1)n$; $F(x^{(0)}) = 5500$ for $n = 5$.

Figure A.7 - Contour diagram for problem 2.9 for n = 2



$-10 \leqq x_i \leqq 10$ for $i = 1, 2$

$F(x) = /4, 36, 100, 196, 324, 484, 676, 900, 1156, 1444/$

Problem 2.10 after Kowalik (1967) (see also Kowalik and Morrison, 1968)

Objective function: $$F(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2 .$$

Numerical values of the constants a_i and b_i for $i = 1(1)11$ can be taken from the following table:

i	a_i	b_i^{-1}
1	0.1957	0.25
2	0.1947	0.5
3	0.1735	1
4	0.1600	2
5	0.0844	4
6	0.0627	6
7	0.0456	8
8	0.0342	10
9	0.0323	12
10	0.0235	14
11	0.0246	16

In this non-linear fitting problem, formulated as a minimum problem, the free parameters α_j ; $j = 1(1)4$ of a function

$$y(z) = \frac{\alpha_1(z^2+\alpha_2 z)}{z^2+\alpha_3 z+\alpha_4}$$

have to be determined with reference to 11 data points $\{y_i;z_i\}$ such that the error, as measured by the euclidian norm, is minimised (Gauss or least squares approximation).

Minimum: $x^* \simeq (0.1928, 0.1908, 0.1231, 0.1358)$;

$F(x^*) \simeq 0.0003075$.

Start: $x^{(0)} = (0, 0, 0, 0)$; $F(x^{(0)}) \simeq 0.1484$.

Near the optimum, if the variables are changed in the last decimal place (with respect to the machine accuracy), then rounding error causes the objective function to behave almost stochastically. The best solution is by the multimembered evolution strategy with recombination. It deviates significantly from the optimal solution as defined by Kowalik and Osborne (1968). Since this best value has a quasi-singular nature, it is repeatedly lost by the population of a (10,100) evolution strategy, with the result that the termination criterion of the search sometimes only takes effect after a long time if at all.

Problem 2.11

As problem 2.10, but with:

Start: $x^{(0)} = (0.25, 0.39, 0.415, 0.39)$;

$F(x^{(0)}) \simeq 0.005316$.

Problem 2.12

As problem 2.10, but with:

Start: $x^{(0)} = (0.25, 0.40, 0.40, 0.40)$;

$F(x^{(0)}) \simeq 0.005566$.

Problem 2.13 after Fletcher and Powell (1963)

Objective function: $F(x) = \sum_{i=1}^{n} (A_i-B_i)^2$ for n = 5,

where
$$\left.\begin{aligned} A_i &= \sum_{j=1}^{n} (a_{ij} \sin \alpha_j + b_{ij} \cos \alpha_j) \\ B_i &= \sum_{j=1}^{n} (a_{ij} \sin x_j + b_{ij} \cos x_j) \end{aligned}\right\} \text{for } i=1(1)n.$$

a_{ij} and b_{ij} are integer random numbers in the range (-100,100) and α_i are random numbers in the range $(-\pi, \pi)$. A minimum of this problem is simultaneously a solution of the equivalent system of n simultaneous non-linear (transcendental) equations:

$$\sum_{j=1}^{n} (a_{ij} \sin x_j + b_{ij} \cos x_j) = A_i \text{ for } i=1(1)n.$$

The solution is again approximated in the least squares sense.

Minimum: $x_i^* = \alpha_i$ for $i = 1(1)n$; $F(x^*) = 0$.

Because the trignometric functions are multi-valued there are infinitely many equivalent minima (real solutions of the system of equations) of which up to 2^n lie in the interval

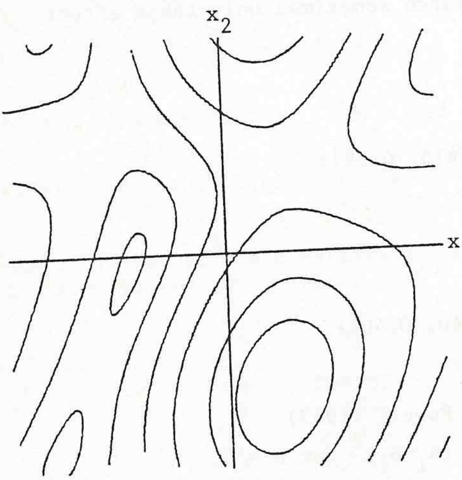
$$\alpha_i - \pi < x_i < \alpha_i + \pi \; ; \; i = 1(1)n.$$

Start: $x_i^{(0)} = \alpha_i + \delta_i$ for $i = 1(1)n$,

where δ_i = random numbers in the range $[-\pi/10, \pi/10]$. To provide the same conditions for all the search methods the same sequence of random numbers was used in each case, and hence

$$F(x^{(0)}) \simeq 1182 \text{ for } n = 5.$$

Figure A.8 Contour diagram for problem 2.13 for n = 2.



$-\pi/2 \leqq x_i \leqq \pi/2$ for i=1,2

F(x)=/300,3000,10000, 20000,35000,45000/

Because of the proximity of the starting point to the one solution $x_i^* = \alpha_i$ for i=1(1)n, all the strategies approached this minimum only.

Problem 2.14 after Powell (1962)

Objective function: $F(x) = (x_1 + 10x_2)^2 + 5(x_3-x_4)^2 + (x_2-2x_3)^4 + 10(x_1 - x_4)^4.$

Minimum: $x^* = (0,0,0,0)$; $F(x^*) = 0$.

Start: $x^{(0)} = (3,-1,0,1)$; $F(x^{(0)}) = 215$.

The matrix of second partial derivatives of the objective function goes singular at the minimum. Thus it is not surprising that a quasi-Newton method like the variable metric method of Davidon, Fletcher and Powell (applied here in Stewart's derivative-free form) got stuck a long way from the minimum. Looked at geometrically, there is a valley which becomes extremely narrow as it approaches the minimum. The evolution strategies therefore ended up by converging very slowly with a minimum step length, and the search had to be terminated for reasons of time.

Problem 2.15

As problem 2.14, except:

Start: $x^{(0)} = (1,2,3,4)$; $F(x^{(0)}) = 1512$.

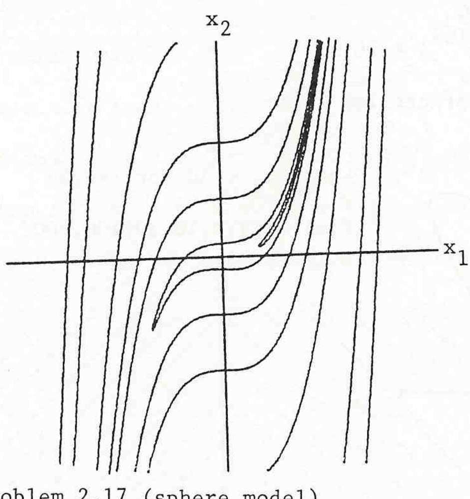
Problem 2.16 after Leon (1966a)

Objective function: $F(x) = 100(x_2 - x_1^3)^2 + (x_1 - 1)^2$.

Minimum: $x^* = (1,1)$; $F(x^*) = 0$.

Start: $x^{(0)} = (-1.2,1)$; $F(x^{(0)}) \simeq 749.0$.

Figure A.9 - Contour diagram for problem 2.16



$-3 \leqq x_i \leqq 3$ for $i = 1,2$

$F(x) = /0.25,4,64,250,$
$1000,5000,10000/$

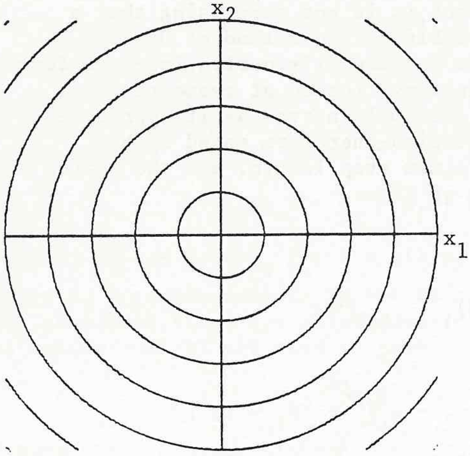
Problem 2.17 (sphere model)

Objective function: $F(x) = \sum_{i=1}^{n} x_i^2$ for $n = 5$.

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

Start: $x_i^{(0)} = 10$ for $i = 1(1)n$; $F(x^{(0)}) = 500$ for $n = 5$.

Figure A.10 - Contour diagram for problem 2.17 for n = 2.



$-10 \leq x_i \leq 10$ for i = 1,2

F(x) = /4,16,36,64,100,144,
196/

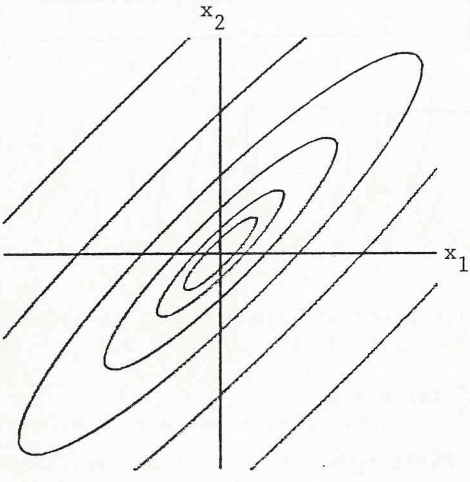
Problem 2.18 after Matyas (1965)

Objective function: $F(x) = 0.26(x_1^2 + x_2^2) - 0.48\, x_1\, x_2$.

Minimum: $x^* = (0,0)$; $F(x^*) = 0$.

Start: $x^{(0)} = (15, 30)$; $F(x^{(0)}) = 76.5$

Figure A.11 - Contour diagram for problem 2.18



$-30 \leq x_i \leq 30$ for i=1,2

F(x) = /1,3,10,30,100,300/

The coordinate strategies terminated the search prematurely because of the lower bounds on the step length (as determined by the machine), which precluded making any more successful line searches in the coordinate directions.

Problem 2.19 by Wood, after Colville (1968)

Objective function: $F(x) = 100(x_1 - x_2^2)^2 + (x_2 - 1)^2$

$$+ 90(x_3 - x_4^2)^2 + (x_4 - 1)^2$$

$$+ 10.1\{(x_1 - 1)^2 + (x_3 - 1)^2\}$$

$$+ 19.8(x_1 - 1)(x_3 - 1).$$

Minimum: $x^* = (1,1,1,1)$; $F(x^*) = 0$.

There is another stationary point near:

$$x' \simeq (1, -1, 1, -1) \ ; \ F(x') \simeq 8.$$

According to Himmelblau (1972) there are still further relative minima.

Start: $x^{(0)} = (-1, -3, -1, -3)$; $F(x^{(0)}) = 19192$.

A very narrow valley appears to run from the stationary point x' to the minimum. All the coordinate strategies together with the methods of Hooke and Jeeves and of Powell ended the search in this region.

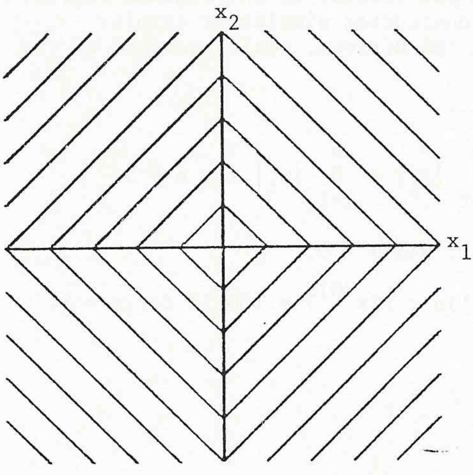
Problem 2.20

Objective function: $F(x) = \sum_{i=1}^{n} |x_i|$ for $n = 5$.

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

Start: $x_i^{(0)} = 10$ for $i = 1(1)n$; $F(x^{(0)}) = 50$ for $n = 5$.

Figure A.12 - Contour diagram for problem 2.20 for n = 2



$-10 \leq x_i \leq 10$ for $i = 1, 2$

$F(x) = /2,4,6,8,10,12,14,16,18,20/$

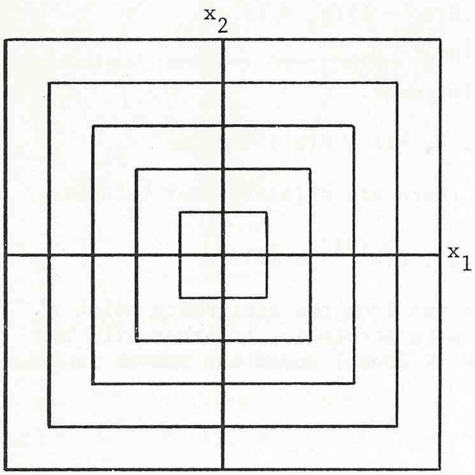
Problem 2.21

Objective function: $F(x) = \underset{i}{\mathrm{Max}} \{|x_i|;\ i = 1(1)n\}$ for $n = 5$.

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

Start: $x_i^{(0)} = 10$ for $i = 1(1)n$; $F(x^{(0)}) = 10$.

Figure A.13 - Contour diagram for problem 2.21 for n = 2.



$-10 \leq x_i \leq 10$ for $i = 1,2$

$F(x) = /2,4,6,8,10/$

Since the starting point is at a corner of the cubic contour surface, none of the coordinate strategies could find a point with a lower value of the objective function. The method of Powell also ended the search without making any significant improvement on the initial condition. Both the simplex method of Nelder and Mead and the complex method of Box also had trouble in the minimum search; in their cases the initially constructed simplex or complex collapsed long before reaching the minimum, again near one of the corners.

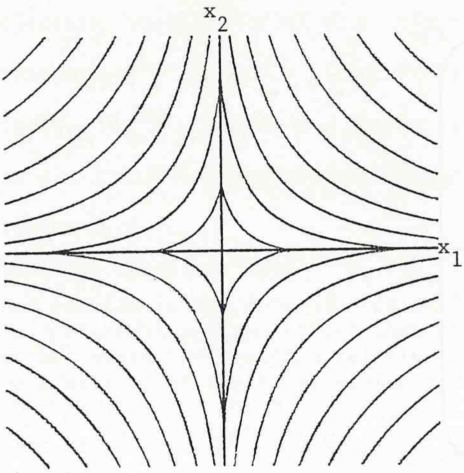
Problem 2.22

Objective function: $F(x) = \sum_{i=1}^{n} |x_i| + \prod_{i=1}^{n} |x_i|$ for $n = 5$.

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

Start: $x_i^{(0)} = 10$ for $i = 1(1)n$; $F(x^{(0)}) = 100050$ for $n = 5$.

Figure A.14 - Contour diagram for problem 2.22 for n = 2.



$-10 \leq x_i \leq 10$ for $i = 1,2$

$F(x) = /3,8,15,24,35,48,63,80,99/$

The simplex and complex methods did not find the minimum. As in the previous problem 2.21, this is due to the sharply pointed corners of the contours. The variable metric strategy also finally got stuck at one of these corners and converged no further. In this case the discontinuity in the partial derivatives of the objective function at the corners is to blame for its failure.

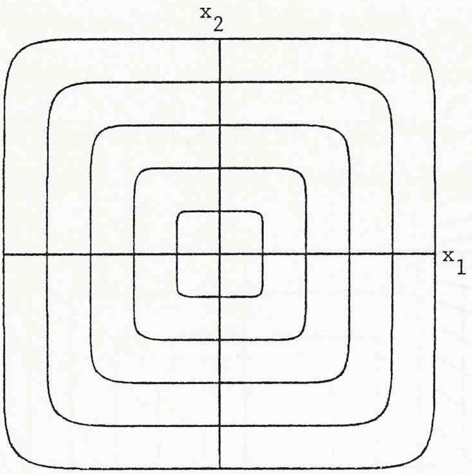
Problem 2.23

Objective function: $F(x) = \sum_{i=1}^{n} x_i^{10}$ for $n = 5$.

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

Start: $x_i^{(0)} = 10$ for $i = 1(1)n$; $F(x^{(0)}) = 5 \cdot 10^{10}$ for $n = 5$.

Figure A.15 - Contour diagram for problem 2.23 for n = 2



$-10 \leq x_i \leq 10$ for i = 1,2

$F(x) = /2^{10}, 4^{10}, 6^{10}, 8^{10}, 10^{10}/$

Only the two strategies which have a quadratic internal model of the objective function, namely the variable metric and conjugate directions methods, failed to converge, because the function F(x) is of much higher (tenth) order.

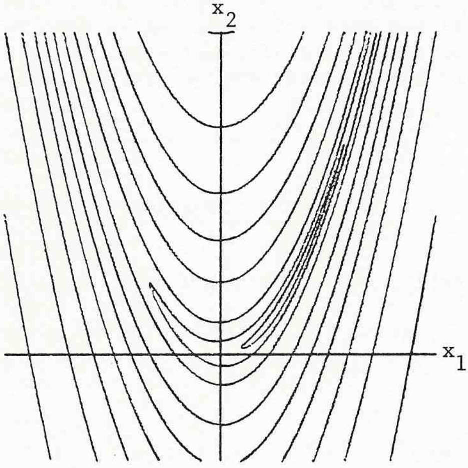
Problem 2.24 after Rosenbrock (1960)

Objective function: $F(x) = 100(x_2 - x_1^2)^2 + (x_1 - 1)^2$.

Minimum: $x^* = (1,1)$; $F(x^*) = 0$.

Start: $x^{(0)} = (-1.2, 1)$; $F(x^{(0)}) = 24.2$.

Figure A.16 - Contour diagram for problem 2.24



$-3 \leq x_1 \leq 3$

$-1.5 \leq x_2 \leq 4.5$

$F(x) = /0.5, 4, 20, 100, 250, 500, 1000, 2000, 5000/$

Problem 2.25

Objective function: $F(x) = \sum_{i=2}^{n} \{(x_1 - x_i^2)^2 + (x_i - 1)^2\}$ for n = 5.

Minimum: $x_i^* = 1$ for i = 1(1)n; $F(x^*) = 0$.

Start: $x_i^{(0)} = 10$ for i = 1(1)n; $F(x^{(0)}) = 32724$ for n = 5.

For n = 2 this becomes the same as problem 2.24.

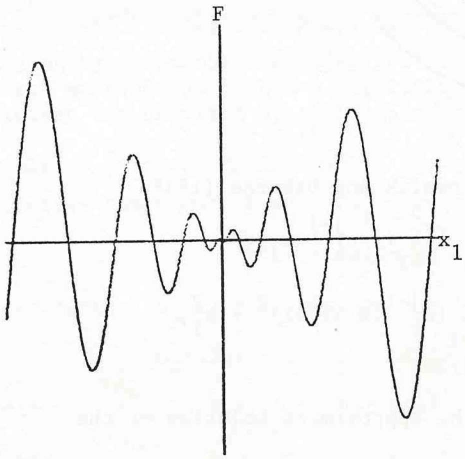
Problem 2.26

Objective function: $F(x) = -x_1 \sin\{\sqrt{(|x_1|)}\}$.

This problem is the same as problem 2.3 except for the modulus. The difference has the effect that the neighbouring minima are further apart here, with a relative maximum separating each of them. The positions of the local minima and maxima are as described under 2.3.

Start: $x_1^{(0)} = 0$; $F(x^{(0)}) = 0$.

Figure A.17 - The objective function $F(x_1)$ of problem 2.26



$-500 \leqq x_1 \leqq 500$

Only the multimembered evolution strategy converged again to the apparent global minimum; all the other methods only converged to the first local minimum.

Problem 2.27 after Zettl (1970)

Objective function: $F(x) = (x_1^2 + x_2^2 - 2x_1)^2 + 0.25x_1$.

Minimum: $x^* \simeq (-0.02990, 0)$; $F(x^*) \simeq -0.003791$.

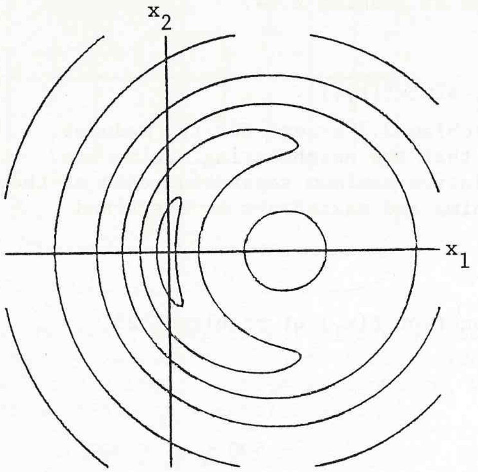
Because of rounding errors this same objective function value is reached for various pairs of values of x_1, x_2.

Maximum: $x' \simeq (1.063,0)$; $F(x') \simeq 1.258$.

Saddle point: $x'' \simeq (1.967,0)$; $F(x'') \simeq 0.4962$.

Start: $x^{(0)} = (2,0)$; $F(x^{(0)}) = 0.5$.

Figure A.18 - Contour diagram for problem 2.27



$-1.5 \leqq x_1 \leqq 2.5$

$-2 \leqq x_2 \leqq 2$

$F(x) = /0.03, 0.3, 1, 3, 10, 30/$

Problem 2.28 of Watson, after Kowalik and Osborne (1968)

Objective function:
$$F(x) = \sum_{i=1}^{30} \{ \sum_{j=1}^{5} (j\, a_i^{j-1}\, x_{j+1}) - (\sum_{j=1}^{6} (a_i^{j-1}\, x_j))^2 - 1\}^2 + x_1^2,$$
where $a_i = (i-1)/29$.

The origin of this problem is the approximate solution of the ordinary differential equation

$$\frac{dz}{dy} - z^2 = 1$$

on the interval $0 \leqq y \leqq 1$ with the boundary condition

$$z(y = 0) = 0.$$

The sought for function $z(y)$ is to be approximated by a polynomial

$$\tilde{z}(y) = \sum_{j=1}^{n} (c_j\, y^{j-1}).$$

In the present case only the first six terms are considered. Suitable values of the polynomial coefficients c_j; j=1(1)6 are to be determined. The deviation from the exact solution of the differential equation is measured in the Gaussian sense as the sum of the squares of the errors at m = 30 argument values y_i, uniformly distributed in the range.

$$F_1 = \sum_{i=1}^{m} \left(\frac{d\tilde{z}}{dy}\Big|_{y_i} - \tilde{z}^2\Big|_{y_i} - 1\right)^2 .$$

The boundary condition is treated as a second simultaneous equation by means of a similarly constructed term:

$$F_2 = \tilde{z}^2\Big|_{y=0} .$$

By inserting the polynomial and redefining the parameters c_i as variables x_i we obtain the objective function $F(x) = F_1 + F_2$, the minimum of which is an approximate solution of the parameterised functional problem.

Minimum: $x^* \simeq (-0.0158, 1.012, -0.2329, 1.260, -1.513, 0.9928)$;

$F(x^*) \simeq 0.002288$.

Start: $x^{(0)} = (0,0,0,0,0,0)$; $F(x^{(0)}) = 30$.

Judging by the number of objective function evaluations all the search methods found this a difficult problem to solve. The best solution was provided by the complex strategy.

Problem 2.29 after Beale (1967)

Objective function: $F(x) = 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3 - 8x_1 - 6x_2 - 4x_3 + 9$.

Constraints: $G_j(x) = x_j \geqq 0$ for $j = 1(1)3$

$G_4(x) = -x_1 - x_2 - 2x_3 + 3 \geqq 0$.

Minimum: $x^* = (4/3, 7/9, 4/9)$; $F(x^*) = 1/9$; G_4 active.

Start: $x^{(0)} = (0.1,0.1,0.1)$; $F(x^{(0)}) = 7.29$.

Problem 2.30

As problem 2.3, but with the constraints:

$G_1(x) = -x_1 + 300 \geqq 0$

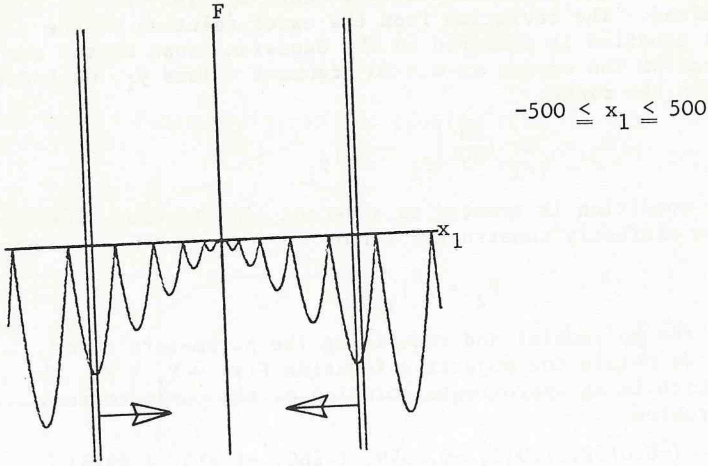
$G_2(x) = x_1 + 300 \geqq 0$.

The introduction of constraints gives rise to two equal valued global minima at the edge of the feasible region:

Minima: $x^* = 300$; $F(x^*) \simeq -299.7$; G_1 active, and
$x^* = -300$; $F(x^*) \simeq -299.7$; G_2 active.

In addition there are five local minima within the feasible region.

Figure A.19 - The function $F(x_1)$ for problem 2.30



Here too the absolute minima were only located by the multimembered evolution strategy.

Problem 2.31

As problem 2.4, but with constraints:

$$G_j(x) = x_j - 1 \geqq 0 \text{ for } j = 1(1)n = 5.$$

Minimum: $x_i^* = 1$ for $i = 1(1)n$; $F(x^*) = 0$; all G_j active.

Start: $x_i^{(0)} = -10$ for $i = 1(1)n$; $F(x^{(0)}) = 61105$ for $n = 5$.

The starting point is in the infeasible region.

Problem 2.32 after Bracken and McCormick (1970)

Objective function: $F(x) = -x_1^2 - x_2^2$.

Constraints: $G_j(x) = x_j \geqq 0$ for $j = 1, 2$

$$G_3(x) = -x_1 + 1 \geqq 0$$

$$G_4(x) = -x_1 - 4x_2 + 5 \geqq 0.$$

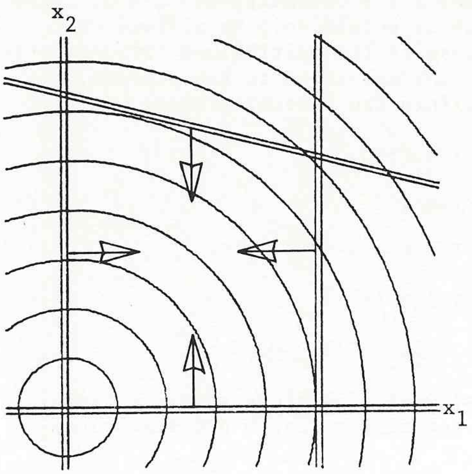
Minimum: $x^* = (1,1)$; $F(x^*) = -2$; G_3 and G_4 active.

Besides this global minimum there is another local one:

$$x' = (0,5/4);\ F(x') = -15/16;\ G_1 \text{ and } G_4 \text{ active}$$

Start: $x^{(0)} = (0,0)$; $F(x^{(0)}) = 0$.

Figure A.20 - Contour diagram for problem 2.32



$-0.25 \leqq x_i \leqq 1.5$ for $i=1,2$

$F(x)=/0.04,0.16,0.36,$
$0.64,1,1.44,$
$1.96,2.56,3.24,$
$4/$

All the search methods converged to the global minimum.

Problem 2.33 after Zettl (1970)

As problems 2.14 and 2.15, but with the constraints:

$$G_j(x) = x_{j+2} - 2 \geqq 0 \text{ for } j = 1,2.$$

Minimum: $x^* \simeq (1.275, 0.6348,2,2);$

$F(x^*) \simeq 189.1$; all G_j active.

Start: $x^{(0)} = (1, 2, 3, 4); \; F(x^{(0)}) = 1512.$

The (1+1) evolution strategy only solved the problem very inaccurately.

Problem 2.34 after Fletcher and Powell (1963)

Objective function: $F(x) = 100\{(x_3 - 10\theta)^2 + (R-1)^2\} + x_3^2,$

where $x_1 = R\cos(2\pi\theta); \; x_2 = R\sin(2\pi\theta).$

and $R = \sqrt{(x_1^2 + x_2^2)}$

$$\theta = \begin{cases} (1/2\pi)\ \arctan\ (x_2/x_1), & \text{if } x_2 \neq 0 \text{ and } x_1 > 0 \\ 1/2 & \text{if } x_2 = 0 \\ (1/2\pi)\{\pi + \arctan\ (x_2/x_1)\}, & \text{if } x_2 \neq 0 \text{ and } x_1 < 0. \end{cases}$$

Constraints: $G_1(x) = -x_3 + 7.5 \geqq 0$

$G_2(x) = x_3 + 2.5 \geqq 0.$

Minimum: $x^* = (1, \simeq 0, 0)$; $F(x^*) = 0$; no constraint is active.

The objective function itself has a discontinuity at $x_2 = 0$, right at the sought for minimum. Thus x_2 should only be allowed to approach closely to zero. Because of the multivalued trigonometric functions there are infinitely many solutions to the problem, of which only one, however, lies within the feasible region.

Start: $x^{(0)} = (-1,0,0)$; $F(x^{(0)}) = 2500$.

Problem 2.35 after Rosenbrock (1960)

Objective function: $F(x) = -x_1 x_2 x_3$.

Constraints: $G_j(x) = x_j \geqq 0$ for $j = 1(1)3$

$$G_4(x) = -x_1 - 2x_2 - 2x_3 + 72 \geqq 0.$$

The underlying problem here was: what dimensions should a parcel of maximum volume have, if the sum of its length and transverse circumference is bounded?

Minimum: $x^* = (24,12,12)$; $F(x^*) = -3456$; G_4 active.

Start: $x^{(0)} = (0,0,0)$; $F(x^{(0)}) = 0$.

All variants of the evolution strategy converged only to within the neighbourhood of the sought for minimum, because in the end only a fraction of all random jumps landed within the feasible region.

Problem 2.36

This is derived from problem 2.35 by treating the constraint G_4, which is active at the minimum, as an equation, and thereby eliminating one of the free variables.

With $x_1' + 2x_2' + 2x_3' = 72$

we obtain

$$F'(x) = -(72 - 2x_2' - 2x_3')x_2 x_3$$

or:

Objective function: $F(x) = -x_1 x_2 (72 - 2x_1 - 2x_2)$.

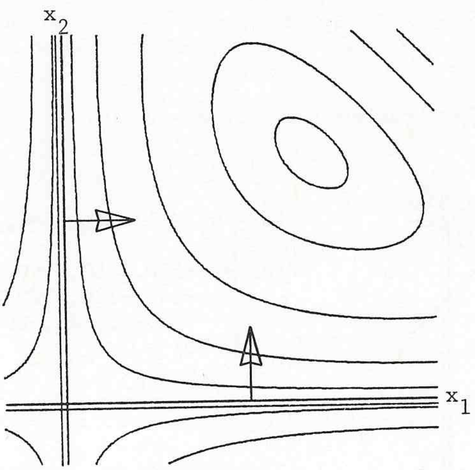
Constraints: $G_j(x) = x_j \geqq 0$ for $j = 1,2$.

Minimum: $x^* = (12,12)$; $F(x^*) = -3456$;

no constraints are active.

Start: $x^{(0)} = (1,1)$; $F(x^{(0)}) = -68$.

Figure A.21 - Contour diagram for problem 2.36



$-3 \leq x_i \leq 18$ for $i=1,2$

$F(x) = / -3400, -3000, -2000, -1000, -300, 300, 1000 /$

Problem 2.37 (corridor model)

Objective function: $F(x) = - \sum_{i=1}^{n} x_i$ for $n = 3$.

Constraints:

$$G_j(x) = \begin{cases} -x_j + 100 \geq 0 & \text{for } j = 1(1)n \\ x_{j-n+1} - \frac{1}{j-n} \sum_{i=1}^{j-n} x_i + \sqrt{\left[\frac{j-n+1}{j-n}\right]} \geq 0 & \text{for } j=n+1(1)2n-1 \\ -x_{j-2n+2} + \frac{1}{j-2n+1} \sum_{i=1}^{j-2n+1} x_i + \sqrt{\left[\frac{j-2n+2}{j-2n+1}\right]} \geq 0 & \text{for } j=2n(1)3n-2 \end{cases}$$

The constraints form a feasible region which could be described as a corridor with a square cross-section (three dimensionally speaking). The axis of the corridor runs along the diagonal in the space:

$$x_1 = x_2 = x_3 = \ldots = x_n .$$

The contours of the linear objective function run perpendicularly to this axis. In order to obtain a finite minimum further constraints were added, whereby a kind of pencil point is placed on the corridor. In the absence of these additional constraints the problem corresponds to the corridor model used by Rechenberg (1973), for which he derived theoretically the rate of progress - a measure of the convergence rate - of the two-membered evolution strategy.

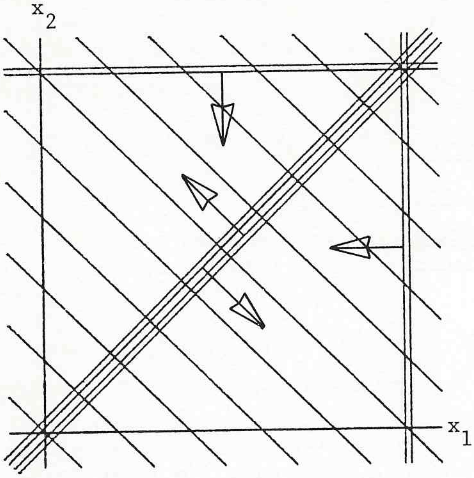
Minimum: $x_i^* = 100$ for $i = 1(1)n$;

$F(x^*) = -300$ for $n = 3$; G_1 to G_n active.

Start: $x_i^{(0)} = 0$ for $i = 1(1)n$;

$F(x^{(0)}) = 0$.

Figure A.22 - Contour diagram for problem 2.37 for n = 2



$-10 \leqq x_i \leqq 110$ for $i=1,2$

$F(x) = /-200,-180,-160,$
$-140,-120,-100,$
$-80,-60,-40,-20,$
$0/$

Problem 2.38

As problem 2.25, but with the additional constraints:

$$G_j(x) = x_j - 1 \geqq 0 \text{ for } j = 1(1)n = 5.$$

Minimum: $x_i^* = 1$ for $i = 1(1)n$;

$F(x^*) = 0$; all G_j active.

Start: $x_i^{(0)} = -10$ for $i = 1(1)n$;

$F(x^{(0)}) = 48884$ for $n = 5$.

The starting point is in the infeasible region.

Problem 2.39 after Rosen and Suzuki (1965)

Objective function: $F(x) = x_1^2+x_2^2+2x_3^2+x_4^2-5x_1-5x_2-21x_3+7x_4$.

Constraints:
$$G_1(x) = -2x_1^2 - x_2^2 - x_3^2 - 2x_1 + x_2 + x_4 + 5 \geqq 0$$
$$G_2(x) = -x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_1 + x_2 - x_3 + x_4 + 8 \geqq 0$$
$$G_3(x) = -x_1^2 - 2x_2^2 - x_3^2 - 2x_4^2 + x_1 + x_4 + 10 \geqq 0.$$

Minimum: $x^* = (0,1,2,-1)$; $F(x^*) = -44$; G_1 active.

Start: $x^{(0)} = (0,0,0,0)$; $F(x^{(0)}) = 0$.

None of the search methods which operate directly with constraints, i.e. without reformulating the objective function, managed to solve the problem to satisfactory accuracy.

Problem 2.40

Objective function: $F(x) = - \sum_{i=1}^{5} x_i$.

Constraints:

$$G_j(x) = \begin{cases} x_j \geqq 0 & \text{for } j = 1(1)5 \\ -\sum_{i=1}^{5} (9+i)x_i + 50000 \geqq 0 & \text{for } j = 6 \end{cases}$$

This is a simple linear programming problem. The solution is in a corner of the allowed region defined by the constraints (simplex).

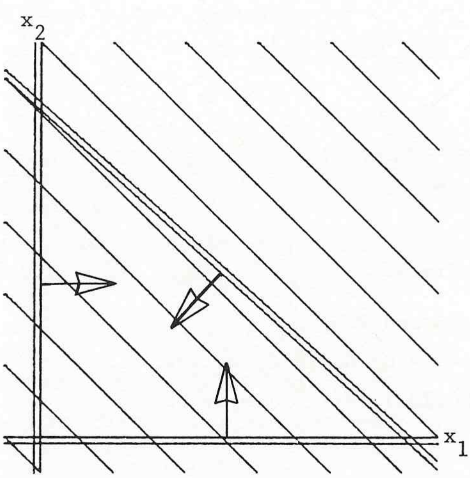
Minimum: $x^* = (5000,0,0,0,0)$

$F(x^*) = -5000$; G_2 to G_6 active.

Start: $x^{(0)} = (250,250,250,250,250)$;

$F(x^{(0)}) = -1250$.

Figure A.23 - Contour diagram for problem 2.40 on the plane $x_3 = x_4 = x_5 = 0$



$-500 \leqq x_i \leqq 5000$ for $i = 1,2$

$F(x)=$/-10500,-9500, -8500,-7500,-6500, -5500,-4500,-3500, -2500,-1500,-500, 500/

In terms of the values of the variables, none of the strategies tested achieved accuracies better than 10^{-2}. The two variants of the (10,100) evolution strategy came closest to the exact solution.

Problem 2.41

Objective function: $F(x) = \sum_{i=1}^{5} (ix_i)$.

Constraints: as for problem 2.40.

Minimum: $x^* = (0,0,0,0,50000/14)$;

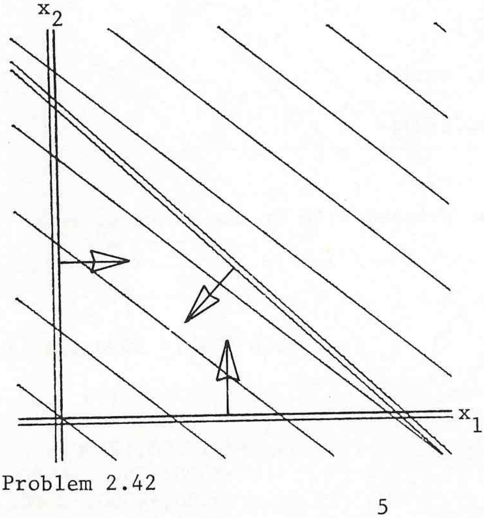
$F(x^*) = -250000/14$; G_j active for $j = 1,2,3,4,6$.

Start: $x^{(0)} = (250,250,250,250,250)$;

$F(x^{(0)}) = -3750$.

This problem differs from the previous one only in the numerical values; regarding the accuracies achieved, the same remarks apply as for problem 2.40.

Figure A.24 - Contour diagram for problem 2.41 on the plane $x_2 = x_3 = x_4 = 0$



$-500 \leq x_i \leq 4500$ for i=1,5

F(x)=/-40000,-35000,-30000,
-25000,-20000,-15000,
-10000,-5000,0/

Problem 2.42

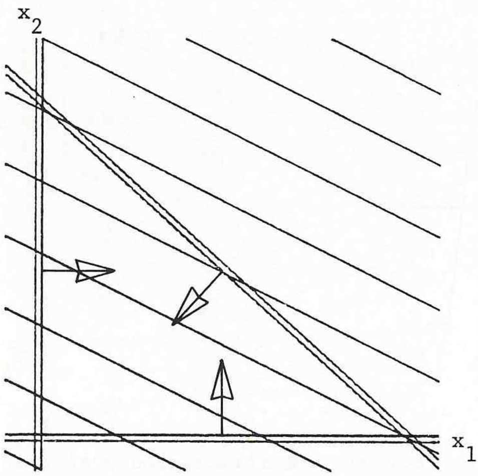
Objective function: $F(x) = \sum_{i=1}^{5} (ix_i)$.

Constraints: as for problems 2.40 and 2.41.

Minimum: $x^* = (0,0,0,0,0)$; $F(x^*) = 0$; G_1 to G_5 active.

Start: $x^{(0)} = (250,250,250,250,250)$; $F(x^{(0)}) = 3750$.

Figure A.25 - Contour diagram for problem 2.42 on the plane $x_3 = x_4 = x_5 = 0$



$-500 \leqq x_i \leqq 5500$ for i=1,2

F(x)=/-1000,1000,3000,5000, 7000,9000,11000,13000, 15000/

The minimum is at the origin of coordinates. The evolution strategies were thus better able to approach the solution by adjusting the individual step lengths. The multimembered strategy with recombination yielded an exact solution with variable values less than 10^{-38}.

Problem 2.43

As problem 2.42, except:

Start: $x^{(0)} = (-250,-250,-250,-250,-250)$;

$F(x^{(0)}) = -3750.$

The starting point is in the infeasible region. The solutions are as in problem 2.42.

Problem 2.44

As problem 2.26, but with additional constraints:

$$G_1(x) = -x_1 + 300 \geqq 0$$

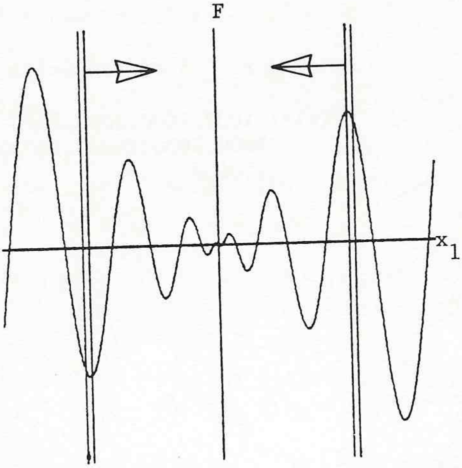
$$G_2(x) = x_1 + 300 \geqq 0.$$

Minimum: $x_1^* = -300$; $F(x^*) \simeq -299.7$; G_2 active.

Besides this global minimum there are five more local (relative) minima within the feasible region.

Start: $x_1^{(0)} = 0$; $F(x^{(0)}) = 0$.

Figure A.26 - The objective function $F(x_1)$ for problem 2.44



$-500 \leqq x_1 \leqq 500$

The global minimum could only be located by multimembered evolution. All the other search strategies converged to the nearest local minimum to the starting point.

Problem 2.45 of Smith and Rudd after Leon (1966a)

Objective function: $F(x) = \sum_{i=1}^{n} \{x_i^i \exp(-x_i)\}$ for n = 5.

Constraints:

$$G_j(x) = \begin{cases} x_j \geqq 0 & \text{for } j = 1(1)n \\ -x_{j-n} + 2 \geqq 0 & \text{for } j = n+1(1)2n. \end{cases}$$

Minimum: $x_i^* = 0$ for $i = 1(1)n$;

$F(x^*) = 0$; all G_1 to G_n active.

Besides this global minimum there is another local one:

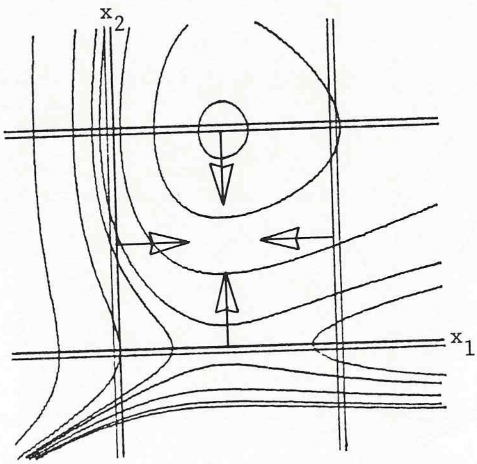
$$x_i' = \begin{cases} 2 \text{ for } i = 1 \\ 0 \text{ for } i = 2(1)n \ ; \end{cases}$$

$F(x') = 2 \exp(-2)$; G_2 to G_{n+1} active.

Start: $x_i^{(0)} = 1$ for $i = 1(1)n$;

$F(x^{(0)}) \simeq 1.84$ for n = 5.

Figure A.27 - Contour diagram for problem 2.45 for n = 2



$-1 \leqq x_i \leqq 3$ for i = 1,2

F(x) = /-1,0,0.3,0.4,0.6,
0.8,0.9/

In the neighbourhood of the sought for minimum, the rate of convergence of a search strategy depends strongly on its ability to make widely different individual adjustments to the step lengths for the changes in the variables. The multimembered evolution solved this problem best when working with recombination. Rosenbrock's method converged to the local minimum, as did the complex and the simple evolution strategies.

Problem 2.46

Objective function: $F(x) = x_1^2 + x_2^2$.

Constraint: $G_1(x) = x_1 + 2x_2 - 2 \geqq 0$.

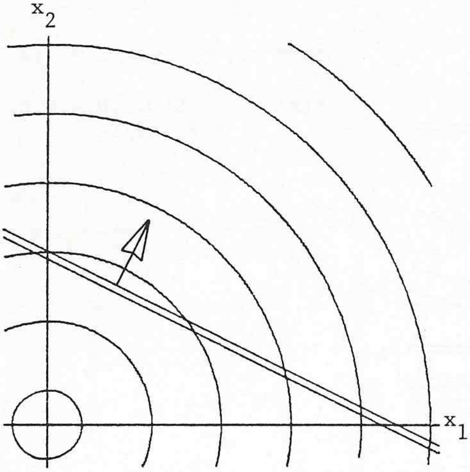
Minimum: $x^* = (0.4, 0.8)$;

$F(x^*) = 0.8$; G_1 active.

Start: $x^{(0)} = (10,10)$;

$F(x^{(0)}) = 200$.

Figure A.28 - Contour diagram for problem 2.46



$-0.25 \leq x_i \leq 2.25$ for $i=1,2$

$F(x)=/0.04,0.36,1,1.96,3.24,4.84,6.76/$

Problem 2.47 after Ueing (1971)

Objective function: $F(x) = -x_1^2 - x_2^2$.

Constraints: $G_j(x) = x_j \geq 0$ for $j = 1,2$

$$G_3(x) = x_1 + x_2 - 17x_1 - 5x_2 + 66 \geq 0$$

$$G_4(x) = x_1 + x_2 - 10x_1 - 10x_2 + 41 \geq 0$$

$$G_5(x) = x_1 + x_2 - 4x_1 - 14x_2 + 45 \geq 0$$

$$G_6(x) = -x_1 + 7 \geq 0$$

$$G_7(x) = -x_2 + 7 \geq 0.$$

Minimum: $x^* = (6,0)$; $F(x^*) = -36$; G_2 and G_3 active.

Besides the global minimum x^* there are three local minima:

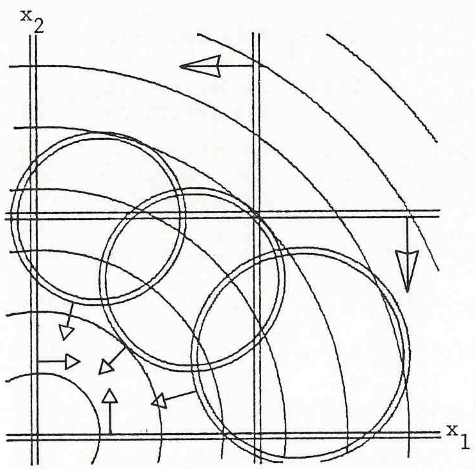
$$x' \simeq (2.116,\ 4.174);\ F(x') \simeq -21.90$$

$$x'' = (0,5);\ F(x'') = -25$$

$$x''' = (5,2);\ F(x''') = -29.$$

Start: $x^{(0)} = (0,0)$; $F(x^{(0)}) = 0$.

Figure A.29 - Contour diagram for problem 2.47



$-1 \leqq x_i \leqq 13$ for $i=1,2$

$-F(x) = /4,16,36,64,100,144,196,256/$

To the original problem have been added the two constraints G_6 and G_7. Without them there are two separate feasible regions and the global minimum is at infinity, in the external, open region. Depending on the initial step lengths, the evolution strategies were sometimes able to go out from the starting point within the inner, closed region into the external region. The multimembered strategies converged to the global minimum, the other search methods all located local minima; which of these was located by the two-membered evolution strategy depended on the sequence of random numbers.

Problem 2.48 after Ueing (1971)

Objective function: $F(x) = -x_1^2 - x_2^2$.

Constraints: $G_j(x) = x_j \geqq 0$ for $j = 1,2$

$G_3(x) = -x_1 + x_2 + 4 \geqq 0$

$G_4(x) = x_1/3 - x_2 + 4 \geqq 0$

$G_5(x) = x_1^2 + x_2^2 - 10x_1 - 10x_2 + 41 \geqq 0.$

Minimum: $x^* = (12,8)$; $F(x^*) = -208$; G_3 and G_4 active.

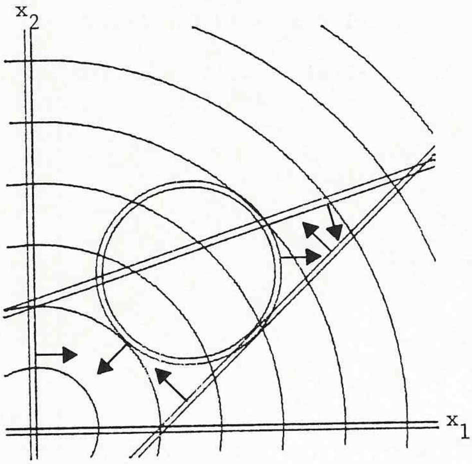
Besides this global minimum there are two local minima:

$x' \simeq (2.018,4.673)$; $F(x') \simeq -25.91$

$x'' \simeq (6.293, 2.293)$; $F(x'') \simeq -44.86$.

Start: $x^{(0)} = (0,0)$; $F(x^{(0)}) = 0$.

Figure A.30 - Contour diagram for problem 2.48



$-1 \leq x_i \leq 13$ for i=1,2

$-F(x) = /4,16,36,64,100,$
$144,196,256/$

There are two allowed regions which are unconnected and closed. The starting point and the global minimum are separated by an infeasible region. Only the (10,100) evolution strategy converged to the global minimum. It sometimes happened with this strategy that one descendant of a generation would jump from one feasible region to the other; however, the group of remaining individuals would converge to one of the local minima. All other strategies converged only to local minima.

Problem 2.49 after Wolfe (1966)

Objective function: $F(x) = (4/3)(x_1^2 + x_2^2 - x_1x_2)^{3/4} + x_3.$

Constraints: $G_j(x) = x_j \geq 0$ for $j = 1(1)3$.

Minimum: $x^* = (0,0,0)$; $F(x^*) = 0$; all G_j active

Start: $x^{(0)} = (10,10,10)$; $F(x^{(0)}) \simeq 52.16$.

Problem 2.50

As problem 2.37, but with some other constraints:

$$G_j(x) = -x_j + 100 \geq 0 \text{ for } j = 1(1)n$$

$$G_{n+1}(x) = -\sum_{i=1}^{n} \{\frac{1}{n} \sum_{j=1}^{n} (x_j) - x_i\}^2 + 1 \geq 0.$$

Minimum: $x_i^* = 100$ for $i = 1(1)n$;

$F(x^*) = -300$ for n=3; G_1 to G_n active.

Start: $x_i^{(0)} = 0$ for $i = 1(1)n$;

$F(x^{(0)}) = 0.$

Instead of the 2n-2 linear constraints of problem 2.37, a non-linear constraint served here to bound the corridor at its sides. From a geometrical point of view, the cross-section of the corridor for n=3 variables is now circular instead of square. For n = 2 variables the two problems become equivalent.

A1.3 Test problems for the third part of the strategy comparison

These are usually n-dimensional extensions of problems from the second set of tests, whose numbers are given in brackets after the new problem number.

Problem 3.1 (analogous to problem 2.4)

Objective function: $F(x) = \sum_{i=1}^{n} \{(x_1 - x_i^2)^2 + (1 - x_i)^2\}.$

Minimum: $x_i^* = 1$ for $i = 1(1)n$; $F(x^*) = 0$.

No noteworthy difficulties arose in the solution of this and the following biquadratic problem with any of the comparison strategies. Away from the minimum, the contour patterns of the objective functions resemble those of the n-dimensional sphere problem (problem 1.1). Nevertheless the slight differences caused most search methods to converge much more slowly (typically by a factor 1/5). The simplex strategy was particularly affected. The computation times it required were about ten to thirty times as long as for the sphere problem with the same number of variables. With n = 100 and greater, the required accuracy was only achieved in problem 3.1 after at least one collapse and subsequent reconstruction of the simplex. The evolution strategies on the other hand were all practically unaffected by the difference with respect to problem 1.1. Also for the complex method the cost was only slightly higher, although with this strategy the computation time increased very rapidly with the number of variables for all problems.

Problem 3.2 (analogous to problem 2.25)

Objective function: $F(x) = \sum_{i=2}^{n} \{(x_1 - x_i^2)^2 + (1 - x_i)^2\}$

Minimum: $x_i^* = 1$ for $i = 1(1)n$; $F(x^*) = 0$.

Problem 3.3 (analogous to problem 2.13)

$$\text{Objective function:} \quad F(x) = \sum_{i=1}^{n} \{ \sum_{j=1}^{n} (a_{ij} \sin \alpha_j + b_{ij} \cos \alpha_j) - \sum_{j=1}^{n} (a_{ij} \sin x_j + b_{ij} \cos x_j)\}^2 ,$$

where a_{ij}, b_{ij} for $i,j = 1(1)n$ are integer random numbers from the range $(-100, 100)$ and α_j $(j = 1(1)n)$ are random numbers from the range $(-\pi, \pi)$.

Minimum: $x_i^* = \alpha_i$ for $i = 1(1)n$; $F(x^*) = 0$.

Besides this desired minimum there are numerous others which have the same value (see problem 2.13). The a_{ij} and b_{ij} require storage space of order $O(n^2)$. For this reason the maximum number of variables for which this problem could be set up had to be limited to $n_{max} = 30$. The computation time per function call also increases as $O(n^2)$. The coordinate strategies ended the search for the minimum before reaching the required accuracy when ten or more variables were involved. The method of Davies, Swann and Campey (DSC) with Gram-Schmidt orthogonalisation and the complex method failed in the same way for thirty variables. For n = 30 the search simplex of the Nelder-Mead strategy also collapsed prematurely; but after a restart the minimum was sufficiently well approximated. Depending on the sequence of random numbers, the two-membered evolution strategy converged either to the desired minimum or to one of the others. This was not seen to occur with the multimembered strategies; however, only one attempt could be made in each case because of the long computation times.

Problem 3.4 (analogous to problem 2.20)

Objective function: $$F(x) = \sum_{i=1}^{n} |x_i| .$$

Minimum: $x_i^* = 0$; $F(x^*) = 0$.

This problem presented no difficulties to those strategies having a line (one-dimensional) search subroutine, since the axis parallel minimisations are always successful. The simplex method on the other hand required several restarts even for just 30 variables, and for n = 100 variables it had to be interrupted as it exceeded the maximum permitted computation time (eight hours) without achieving the required accuracy. The success or failure of the (1+1) evolution strategy and the complex method depended upon the actual random numbers. Therefore in this and the following problems whenever there was any doubt about convergence several (at least three) attempts were made with different sequences of random numbers. It was seen that the two-membered evolution strategy sometimes spent longer near one of the corners formed by the contours of the objective function, where it converged only slowly; however, it finally escaped from this situation. Thus although the computation times were very varied, the search was never terminated prematurely. The success of the multimembered evolution strategy depended on whether or not recombination was implemented. Without recombination the method sometimes failed for just 30 variables, whereas with recombination it converged safely and with no periods of stagnation. In the latter case the computation times taken were actually no longer than for the sphere problem with the same number of variables.

Problem 3.5 (analogous to problem 2.21)

Objective function: $F(x) = \underset{i}{\mathrm{Max}}\{x_i;\ i = 1(1)n\}$.

Minimum: $x_i^* = 0$; $F(x^*) = 0$.

Most of the methods using a one-dimensional search failed here, because the value of the objective function is piecewise constant along the coordinate directions. The methods of Rosenbrock and of Davies, Swann and Campey (whatever the method of orthogonalisation) converged safely since they consider trial steps which do not change the objective function value as successful. If only true improvements are accepted, as in the conjugate gradient, variable metric and coordinate strategies, the search never even leaves the chosen starting point at one of the corners of the contour surface. The simplex and complex strategies failed for n > 30 variables. Even for just ten variables the search simplex of the Nelder-Mead method had to be constructed anew after collapsing 185 times before the desired accuracy could be achieved. For the evolution strategy with only one parent and one descendant, the probability of finding from the starting point a point with a better value of the objective function is

$$w_e = 2^{-n}.$$

For this reason the (1+1) strategy failed for $n \geqq 10$. The multi-membered version without recombination could solve the problem for up to n = 10 variables. With recombination convergence was sometimes still achieved for n = 30 variables, but no longer for n = 100 in the three attempts made.

Problem 3.6 (analogous to problem 2.22)

Objective function: $F(x) = \sum_{i=1}^{n} |x_i| + \prod_{i=1}^{n} |x_i|$.

Minimum: $x_i^* = 0$ for $i = 1(1)n$; $F(x^*) = 0$.

In spite of the even sharper corners on the contour surfaces of the objective function all the strategies behaved in much the same way as they did in the minimum search of problem 3.4. The only notable difference was with the (10,100) evolution strategy without recombination. For n = 30 variables the minimum search always converged; only for n = 100 and above was the search no longer successful.

Problem 3.7 (analogous to problem 2.23)

Objective function: $F(x) = \sum_{i=1}^{n} x_i^{10}$.

Minimum: $x_i^* = 0$ for $i = 1(1)n$;

$F(x^*) = 0$.

The strategy of Powell failed from n = 10 variables. Since all the step lengths were set to zero the search stagnated and the internal termination criterion did not take effect. The optimization had to be interrupted externally. From n = 30, the variable metric method was also ineffective. The quadratic model of the objective function on which it is based led to completely false predictions of suitable search directions. For $n = 10$ the simplex method required 48 restarts, and for $n = 30$ as many as 181 in order to achieve the desired accuracy. None of the evolution strategies had any convergence difficulties in solving the problem. They were not tested further for $n > 300$ simply for reasons of computation time.

Problem 3.8 (similar to problem 2.37) (corridor model)

Objective function: $F(x) = -\sum_{i=1}^{n} x_i$.

Constraints:

$$G_j(x) = \begin{cases} \sqrt{\left[\frac{j+1}{j}\right]} + x_{j+1} - \frac{1}{j}\sum_{i=1}^{j} x_i \geqq 0 & \text{for } j = 1(1)n-1 \\ \sqrt{\left[\frac{j-n+2}{j-n+1}\right]} - x_{j-n+2} + \frac{1}{j-n+1}\sum_{i=1}^{j-n+1} x_i \geqq 0 & \text{for } j=n(1)2n-2 \end{cases}$$

The other constraints of problem 2.37 which bound the corridor in the direction of the sought for minimum were omitted here. The minimum is thus at infinity.

For comparison of the results of this and the following circularly bounded corridor problem with the theoretical rates of progress for this model function, the quantity of interest was the cost not of reaching a given approximation to an objective but of covering a given distance along the corridor axis. For the half-width of the corridor, $b = 1$ was taken. The search was started at the origin and terminated as soon as a distance $s \geqq 10b$ had been covered, or the objective function had reached a value $F \leqq -10\sqrt{n}$.

Start: $x_i^{(0)} = 0$ for $i = 1(1)n$; $F(x^{(0)}) = 0$.

All the tested strategies converged satisfactorily. The number of mutations or generations required by the evolution strategies increased linearly with the number of variables, as expected. Since the number of constraints, as well as the computation time per function call, increased as $O(n)$, the total computation time increased as $O(n^3)$. Because of the maximum of eight hours per search adopted as a limit on the computation time, the two-membered evolution strategy could only be tested to $n = 300$ and the multimembered strategies to $n = 100$. Intermediate results for $n = 300$, however confirm that the expected trend is maintained.

Problem 3.9 (similar to problem 2.50)

Objective function: $F(x) = -\sum_{i=1}^{n} x_i$.

Constraint: $G_1(x) = 1 - \sum_{i=1}^{n} \{\frac{1}{n}\sum_{j=1}^{n} (x_j) - x_i\}^2 \geqq 0$.

Minimum, starting point and convergence criterion as in problem 3.8.

The complex method failed for $n \geq 30$, but the Rosenbrock strategy simply required more objective function evaluations and orthogonalisations compared to the rectangular corridor. The evolution strategies converged safely. They too required more mutations or generations than in the previous problem. However, since only one constraint instead of 2n-2 was to be tested and respected, the time they took only increased as $O(n^2)$. Recombination in the multimembered version was only a very slight advantage for this and the linearly bounded corridor problem.

Problem 3.10 (analogous to problem 2.45)

Objective function: $F(x) = \sum_{i=1}^{n} \{x_i^i \exp(-x_i)\}$.

Constraints:

$$G_j(x) = \begin{cases} x_j \geq 0 & \text{for } j = 1(1)n \\ 2 - x_{j-n} \geq 0 & \text{for } j = n + 1(1)2n. \end{cases}$$

Minimum: $x_i^* = 0$ for $i = 1(1)n$;

$F(x^*) = 0$; all G_1 to G_n active.

Besides this global minimum there is a local one within the feasible region:

$$x_i' = \begin{cases} 2 & \text{for } i = 1 \\ 0 & \text{for } i = 2(1)n \end{cases} ;$$

$F(x') = 2\exp(-2)$.

As in the solution of problem 2.45 with 5 variables, the search methods only converged if they could adjust the step lengths individually. The strategy of Rosenbrock failed for only n = 10. The complex method sometimes converged for the same number of variables after about 1000 seconds of computation time, but occasionally not even within the allotted eight hours. For n = 30 variables none of the strategies reached the objective before the time limit expired. The results obtained after eight hours showed clearly that better progress was being made by the two-membered evolution strategy and the multimembered strategy with recombination. The following table gives the best objective function values obtained by each of the strategies compared.

Rosenbrock	10^{-4}*
Complex	10^{-7}
(1+1) evolution	10^{-30}
(10,100) evolution without recombination	10^{-12}
(10,100) evolution with recombination	10^{-26}

*The Rosenbrock strategy ended the search prematurely after about 5 hours. All the other values are intermediate results after 8 hours of computation time when the strategy's own termination criteria were not yet satisfied. The searches could therefore still have come to a successful conclusion.

8

REFERENCES*

Abadie, J. (ed.) (1967), Nonlinear programming, North-Holland, Amsterdam

Abadie, J. (ed.) (1970), Integer and nonlinear programming, North-Holland, Amsterdam

Abadie, J. (1972), Simplex-like methods for non-linear programming, in: Szegö (1972), p.41-60

Abe, K., M. Kimura (1970), Parallel algorithm for solving discrete optimization problems, IFAC Kyoto Symp. Syst. Engng. Appr. Comp. Contr., Aug. 1970, paper 35.1

Adachi, N. (1971), On variable-metric algorithms, JOTA 7, 391-410

Adachi, N. (1973a), On the convergence of variable-metric methods, Computing 11, 111-123

Adachi, N. (1973b), On the uniqueness of search directions in variable-metric algorithms, JOTA 11, 590-604

Adams, R.J., A.Y. Lew (1966), Modified sequential random search using a hybrid computer, Univ. South. California, Electr. Engng. Dept., report, May 1966

Ahrens, J.H., U. Dieter (1972), Computer methods for sampling from the exponential and normal distributions, CACM 15, 873-882 and 1047

Aizerman, M.A., E.M. Braverman, L.I. Rozonoer (1965), The Robbins-Monro process and the method of potential functions, ARC 26, 1882-1885

Akaike, H. (1960), On a successive transformation of probability distribution and its application to the analysis of the optimum gradient method, Ann. Inst. Stat. Math. Tokyo 11, 1-16

Aleksandrov, V.M., V.I. Sysoyev, V.V. Shemeneva (1968), Stochastic optimization, Engng. Cybern. 6, 5, 11-16

Alland, A., jr. (1970), Evolution und menschliches Verhalten, S. Fischer, Frankfurt/Main

Altman, M. (1966), Generalized gradient methods of minimizing a functional, Bull. Acad. Polon. Sci. 14, 313-318

Amann, H. (1968a), Monte-Carlo Methoden und lineare Randwertprobleme, ZAMM 48, 109-116

Amann, H. (1968b), Der Rechenaufwand bei der Monte-Carlo Methode mit Informationsspeicherung, ZAMM 48, 128-131

Anderson, N., Å. Björck (1973), A new high order method of Regula Falsi type for computing a root of an equation, BIT 13, 253-264

Anderson, R.L. (1953), Recent advances in finding best operating conditions, J. Amer. Stat. Assoc. 48, 789-798

Andrews, H.C. (1972), Introduction to mathematical techniques in pattern recognition, Wiley-Interscience, New York

Anscombe, F.J. (1959), Quick analysis methods for random balance screening experiments, Technometrics 1, 195-209

Antonov, G.E., V. Ya. Katkovnik (1972), Method of synthesis of a class of random search algorithms, ARC 32, 990-993

Aoki, M. (1971), Introduction to optimization techniques: fundamentals and applications of nonlinear programming, Macmillan, New York

Apostol, T.M. (1957), Mathematical analysis: a modern approach to advanced calculus, Addison-Wesley, Reading, Mass.

*Glossary of abbreviations on page 291

Arrow, K.J., L. Hurwicz (1956), Reduction of constrained maxima to saddle-point problems, in: Neyman (1956), vol. 5, p.1-20

Arrow, K.J., L. Hurwicz (1957), Gradient methods for constrained maxima, Oper. Res. 5, 258-265

Arrow, K.J., L. Hurwicz, H. Uzawa (eds.) (1958), Studies in linear and non-linear programming, Stanford Univ. Press, Stanford, Calif.

Asai, K., S. Kitajima (1972), Optimizing control using fuzzy automata, Automatica 8, 101-104

Ashby, W.R. (1960), Design for a brain, Wiley, New York, 2nd Ed.

Ashby, W.R. (1965), Constraint analysis of many-dimensional relations, in: Wiener und Schadé (1965), p.10-18

Ashby, W.R. (1968), Some consequences of Bremermann's limit for information-processing systems, in: Oestreicher and Moore (1968), p.69-76

Avriel, M., D.J. Wilde (1966a), Optimality proof for the symmetric Fibonacci search technique, Fibonacci Quart. 4, 265-269

Avriel, M., D.J. Wilde (1966b), Optimal search for a maximum with sequences of simultaneous function evaluations, Mgmt. Sci. 12, 722-731

Avriel, M., D.J. Wilde (1968), Golden block search for the maximum of unimodal functions, Mgmt. Sci. 14, 307-319

Bach, H. (1969), On the downhill method, CACM 12, 675-677

Baer, R.M. (1962), Note on an extremum locating algorithm, Comp. J. 5, 193

Balakrishnan, A.V. (ed.) (1972), Techniques of optimization, Academic Press, New York

Balakrishnan, A.V., L.W. Neustadt (eds.) (1964), Computing methods in optimization problems, Academic Press, New York

Balakrishnan, A.V., L.W. Neustadt (eds.) (1967), Mathematical theory of control, Academic Press, New York

Balakrishnan, A.V., M. Contensou, B.F. DeVeubeke, P. Krée, J.L. Lions, N.N. Moiseev (eds.) (1970), Symposium on optimization, Springer, Berlin

Bandler, J.W. (1969a), Optimization methods for computer-aided design, IEEE Trans. MTT-17, 533-552

Bandler, J.W. (1969b), Computer optimization of inhomogeneous waveguide transformers, IEEE Trans. MTT-17, 563-571

Bandler, J.W., C. Charalambous (1974), Nonlinear programming using minimax techniques, JOTA 13, 607-619

Bandler, J.W., P.A. MacDonald (1969), Optimization of microwave networks by razor search, IEEE Trans. MTT-17, 552-562

Bard, Y. (1968), On a numerical instability of Davidon-like methods, Math. Comp. 22, 665-666

Bard, Y. (1970), Comparison of gradient methods for the solution of nonlinear parameter estimation problems, SIAM J. Numer. Anal. 7, 157-186

Barnes, G.H., R.M. Brown, M. Kato, D.J. Kuck, D.L. Slotnick, R.A. Stokes (1968), The Illiac IV computer, IEEE Trans. C-17, 746-770

Barnes, J.G.P. (1965), An algorithm for solving non-linear equations based on the secant method, Comp. J. 8, 66-72

Barnes, J.L. (1965), Adaptive control as the basis of life and learning systems, Proc. IFAC Tokyo Symp. Syst. Engng. Contr. Syst. Design, Aug. 1965, p.187-191

Barr, D.R., N.L. Slezak (1972), A comparison of multivariate normal generators, CACM 15, 1048-1049

Bass, R. (1972), A rank two algorithm for unconstrained minimization, Math. Comp. 26, 129-143

Bauer, F.L. (1965), Elimination with weighted row combinations for solving linear equations and least squares problems, Numer. Math. 7, 338-352

Bauer, W.F. (1958), The Monte Carlo method, SIAM J. 6 , 438-451

Beale, E.M.L. (1956), On quadratic programming, Nav. Res. Log. Quart. 6, 227-243

Beale, E.M.L. (1958), On an iterative method for finding a local minimum of a function of more than one variable, Princeton Univ., Stat. Techn. Res. Group, techn. report 25, Princeton, N.J., Nov. 1958

Beale, E.M.L. (1967), Numerical methods, in: Abadie (1967), p.133-205

Beale, E.M.L. (1970), Computational methods for least squares, in: Abadie (1970), p.213-228

Beale, E.M.L. (1972), A derivation of conjugate gradients, in: Lootsma (1972a), p.39-43

Beamer, J.H., D.J. Wilde (1969), An upper bound on the number of measurements required by the contour tangent optimization technique, IEEE Trans. SSC-5, 27-30

Beamer, J.H., D.J. Wilde (1970), Minimax optimization of unimodal functions by variable block search, Mgmt. Sci. 16, 529-541

Beamer, J.H., D.J. Wilde (1973), A minimax search plan for constrained optimization problems, JOTA 12, 439-446

Beckman, F.S. (1967), Die Lösung linearer Gleichungssysteme nach der Methode der konjugierten Gradienten, in: Ralston and Wilf (1967), p.106-126

Beckmann, M. (ed.) (1971), Unternehmensforschung heute, Springer, Berlin

Beier, W., K. Glass (1968), Bionik: eine Wissenschaft der Zukunft, Urania, Leipzig

Bekey, G.A., M.H. Gran, A.E. Sabroff, A. Wong (1966), Parameter optimization by random search using hybrid computer techniques, AFIPS Conf. Proc. 29, 191-200

Bekey, G.A., W.J. Karplus (1971), Hybrid-Systeme, Berliner Union und Kohlhammer, Stuttgart

Bekey, G.A., R.B. McGhee (1964), Gradient methods for the optimization of dynamic system parameters by hybrid computation, in: Balakrishnan und Neustadt (1964), p.305-327

Bell, M., M.C. Pike (1966), Remark on algorithm 178 (E4): direct search, CACM 9, 684-685

Bellman, R.E. (1967), Dynamische Programmierung und selbstanpassende Regelprozesse, Oldenbourg, München

Beltrami, E.J., J.P. Indusi (1972), An adaptive random search algorithm for constrained minimization, IEEE Trans. C-21, 1004-1008

Berg, R.L., N.W. Timofejew-Ressowski (1964), Über Wege der Evolution des Genotyps, in: Ljapunov, Kämmerer und Thiele (1964b), p.201-221
Berlin, V.G. (1969), Acceleration of stochastic approximations by a mixed search method, ARC 30, 125-129
Berlin, V.G. (1972), Parallel randomized search strategies, ARC 33, 398-403
Berman, G. (1966), Minimization by successive approximation, SIAM J. Numer. Anal. 3, 123-133
Berman, G. (1969), Lattice approximations to the minima of functions, JACM 16, 286-294
Bernard, J.W., F.J. Sonderquist (1959), Progress report on OPCON: Dow evaluates optimizing control, Contr. Engng. 6, 11, 124-128
Bertram, J.E. (1960), Control by stochastic adjustment, AIEE Trans. II Appl. Ind. 78, 485-491
Beveridge, G.S.G., R.S. Schechter (1970), Optimization: theory and practice, McGraw-Hill, New York
Biggs, M.C. (1971), Minimization algorithms making use of non-quadratic properties of the objective function, JIMA 8, 315-327; errata in 9 (1972)
Biggs, M.C. (1973), A note on minimization algorithms which make use of non-quadratic properties of the objective function, JIMA 12, 337-338
Birkhoff, G., S. MacLane (1965), A survey of modern algebra, Macmillan, New York, 3rd Ed.
Blakemore, J.W., S.H. Davis, jr. (eds.) (1964), Optimization techniques, AIChE Chem. Engng. Progr. Symp. Ser. 60, no.50
Bledsoe, W.W. (1961), A basic limitation on the speed of digital computers, IRE Trans. EC-10, 530
Blum, J.R. (1954a), Approximation methods which converge with probability one, Ann. Math. Stat. 25, 382-386
Blum, J.R. (1954b), Multidimensional stochastic approximation methods, Ann. Math. Stat. 25, 737-744
Boas, A.H. (1962), What optimization is all about, Chem. Engng. 69, 25, 147-152
Boas, A.H. (1963a), How to use Lagrange multipliers, Chem. Engng. 70, 1, 95-98
Boas, A.H. (1963b), How search methods locate optimum in univariable problems, Chem. Engng. 70, 3, 105-108
Boas, A.H. (1963c), Optimizing multivariable functions, Chem. Engng. 70, 5, 97-104
Boas, A.H. (1963d), Optimization via linear and dynamic programming, Chem. Engng. 70, 7, 85-88
Bocharov, I.N., A.A. Feldbaum (1962), An automatic optimizer for the search for the smallest of several minima: a global optimizer, ARC 23, 260-270
Boltjanski, W.G. (1972), Mathematische Methoden der optimalen Steuerung, Hanser, München
Booth, A.D. (1949), An application of the method of steepest descents to the solution of systems of non-linear simultaneous equations, Quart. J. Mech. Appl. Math. 2, 460-468
Booth, A.D. (1955), Numerical methods, Butterworths, London

Booth, R.S. (1967), Location of zeros of derivatives, SIAM J. Appl. Math. 15, 1496-1501

Boothroyd, J. (1965), Certification of algorithm 2: Fibonacci search, Comp. Bull. 9, 105 and 108

Box, G.E.P. (1957), Evolutionary operation: a method for increasing industrial productivity, Appl. Stat. 6, 81-101

Box, G.E.P., D.W. Behnken (1960), Simplex-sum designs: a class of second order rotatable designs derivable from those of first order, Ann. Math. Stat. 31, 838-864

Box, G.E.P., N.R. Draper (1969), Evolutionary operation: a statistical method for process improvement, Wiley, New York

Box, G.E.P., J.S. Hunter (1957), Multi-factor experimental designs for exploring response surfaces, Ann. Math. Stat. 28, 195-241

Box, G.E.P., W.G. Hunter, J.F. MacGregor, J. Erjavec (1973), Some problems associated with the analysis of multiresponse data, Technometrics 15, 33-51

Box, G.E.P., M.E. Muller (1958), A note on the generation of random normal deviates, Ann. Math. Stat. 29, 610-611

Box, G.E.P., K.B. Wilson (1951), On the experimental attainment of optimum conditions, J. Roy. Stat. Soc. B, Methodological, 8, 1-45

Box, M.J. (1965), A new method of constrained optimization and a comparison with other methods, Comp. J. 8, 42-52

Box, M.J. (1966), A comparison of several current optimization methods and the use of transformations in constrained problems, Comp. J. 9, 67-77

Box, M.J., D. Davies, W.H. Swann (1969), Nonlinear optimization techniques, ICI Monograph 5, Oliver and Boyd, Edinburgh

Bracken, J., G.P. McCormick (1970), Ausgewählte Anwendungen nichtlinearer Programmierung, Berliner Union und Kohlhammer, Stuttgart

Brajnes, S.N., V.B. Svečinskij (1971), Probleme der Neurokybernetik und Neurobionik, G. Fischer, Stuttgart, 2nd Ed.

Brandl, V. (1969), Ein wirksames Monte-Carlo-Schätzverfahren zur simultanen Behandlung hinreichend eng verwandter Probleme angewandt auf Fragen der Neutronenphysik, Tagungsbericht der Reaktortagung des Dt. Atomforums, Frankfurt/Main, April 1969, Sektion 1, p.6-7

Branin, F.H., jr., S.K. Hoo (1972), A method for finding multiple extrema of a function of n variables, in: Lootsma (1972a), p.231-237

Bremermann, H.J. (1962), Optimization through evolution and recombination, in: Yovits, Jacobi and Goldstein (1962), p.93-106

Bremermann, H.J. (1963), Limits of genetic control, IEEE Trans. MIL-7, 200-205

Bremermann, H.J. (1967), Quantitative aspects of goal-seeking self-organizing systems, in: Snell (1967), p.59-77

Bremermann, H.J. (1968a), Numerical optimization procedures derived from biological evolution processes, in: Oestreicher and Moore (1968), p.597-616

Bremermann, H.J. (1968b), Principles of natural and artificial intelligence, AGARD report AD-684-952, Sept. 1968, p.6c1-6c2

Bremermann, H.J. (1968c), Pattern recognition, functionals, and entropy, IEEE Trans. BME-15, 201-207
Bremermann, H.J. (1970), A method of unconstrained global optimization, Math. Biosci. 9, 1-15
Bremermann, H.J. (1971), What mathematics can and cannot do for pattern recognition, in: Grüsser and Klinke (1971), p.31-45
Bremermann, H.J. (1973a), On the dynamics and trajectories of evolution processes, in: Locker (1973), p.29-37
Bremermann, H.J. (1973b), Algorithms and complexity of evolution and self-organization, Kybernetik-Kongress der Dt. Ges. für Kybernetik und der Nachrichtentechn. Ges. im VDE, Nürnberg March 1973
Bremermann, H.J., L. Siu-Bik Lam (1970), Analysis of spectra with non-linear superposition, Math. Biosci. 8, 449-460
Bremermann, H.J., M. Rogson, S. Salaff (1965), Search by evolution, in: Maxfield, Callahan and Fogel (1965), p.157-167
Bremermann, H.J., M. Rogson, S. Salaff (1966), Global properties of evolution processes, in: Pattee, Edelsack, Fein and Callahan (1966), p.3-41
Brent, R.P. (1971), An algorithm with guaranteed convergence for finding a zero of a function, Comp. J. 14, 422-425
Brent, R.P. (1973), Algorithms for minimization without derivatives, Prentice-Hall, Englewood Cliffs, N.J.
Bromberg, N.S. (1962), Maximization and minimization of complicated multivariable functions, AIEE Trans. I Comm. Electron. 80, 725-730
Brooks, S.H. (1958), A discussion of random methods for seeking maxima, Oper. Res. 6, 244-251
Brooks, S.H. (1959), A comparison of maximum-seeking methods, Oper. Res. 7, 430-457
Brooks, S.H., M.R. Mickey (1961), Optimum estimation of gradient direction in steepest ascent experiments, Biometrics 17, 48-56
Brown, K.M. (1969), A quadratically convergent Newton-like method based upon Gaussian elimination, SIAM J. Numer. Anal. 6, 560-569
Brown, K.M., J.E. Dennis, jr. (1968), On Newton-like iteration functions: general convergence theorems and a specific algorithm, Numer. Math. 12, 186-191
Brown, K.M., J.E. Dennis, jr. (1972), Derivative free analogues of the Levenberg-Marquardt and Gauss algorithms for non-linear least squares approximation, Numer. Math. 18, 289-297
Brown, R.R. (1959), A generalized computer procedure for the design of optimum systems, AIEE Trans. I Comm. Electron. 78, 285-293
Broyden, C.G. (1965), A class of methods for solving nonlinear simultaneous equations, Math. Comp. 19, 577-593
Broyden, C.G. (1967), Quasi-Newton methods and their application to function minimization, Math. Comp. 21, 368-381
Broyden, C.G. (1969), A new method of solving nonlinear simultaneous equations, Comp. J. 12, 94-99
Broyden, C.G. (1970a), The convergence of single-rank quasi-Newton methods, Math. Comp. 24, 365-382
Broyden, C.G. (1970b), The convergence of a class of double-rank minimization algorithms, part 1: general considerations, JIMA 6, 76-90

Broyden, C.G. (1970c), The convergence of a class of double-rank minimization algorithms, part 2: the new algorithm, JIMA 6,222-231
Broyden, C.G. (1971), The convergence of an algorithm for solving sparse non-linear systems, Math. Comp. 25, 285-294
Broyden, C.G. (1972), Quasi-Newton methods, in: Murray (1972a), p.87-106
Broyden, C.G. (1973), Some condition-number bounds for the Gaussian elimination process, JIMA 12, 273-286
Broyden, C.G., J.E. Dennis, jr., J.J. Moré (1973), On the local and superlinear convergence of quasi-Newton methods, JIMA 12, 223-245
Broyden, C.G., M.P. Johnson (1972), A class of rank-1 optimization algorithms, in: Lootsma (1972a), p.35-38
Bryson, A.E., jr., Y.C.Ho (1969), Applied optimal control, Blaisdell, Waltham, Mass.
Budne, T.A. (1959), The application of random balance designs, Technometrics 1, 139-155
Buehler, R.J., B.V. Shah, O. Kempthorne (1961), Some properties of steepest ascent and related procedures for finding optimum conditions, Iowa State Univ., Stat. Lab., techn. report 1, Ames, Iowa, April 1961
Buehler, R.J., B.V. Shah, O. Kempthorne (1964), Methods of parallel tangents, in: Blakemore and Davis (1964), p.1-7
Burkard, R.E. (1972), Methoden der Ganzzahligen Optimierung, Springer, Vienna
Campbell, D.T. (1960), Blind variation and selective survival as a general strategy in knowledge-processes, in: Yovits and Cameron (1960), p.205-231
Canon, M.D., C.D. Cullum, jr., E. Polak (1970), Theory of optimal control and mathematical programming, McGraw-Hill, New York
Cantrell, J.W. (1969), Relation between the memory gradient method and the Fletcher-Reeves method, JOTA 4, 67-71
Carroll, C.W. (1961), The created response surface technique for optimizing nonlinear restrained systems, Oper. Res. 9, 169-185
Casey, J.K., R.C. Rustay (1966), AID: a general purpose computer program for optimization, in: Lavi and Vogl (1966), p.81-100
Casti, J., M. Richardson, R. Larson (1973), Dynamic programming and parallel computers, JOTA 12, 423-438
Cauchy, A. (1847), Méthode générale pour la résolution des systèmes d'équations simultanées, Compt. Rend. Acad. Sci. URSS, Neue Serie 25, 536-538
Céa, J. (1971), Optimisation: théorie et algorithmes, Dunod, Paris
Chang, S.S.L. (1961), Synthesis of optimum control systems, McGraw-Hill, New York
Chang, S.S.L. (1968), Stochastic peak tracking and the Kalman filter, IEEE Trans. AC-13, 750
Chatterji, B.N., B. Chatterjee (1971), Performance optimization of a self-organising feedback control system in presence of inherent coupling signals, Automatica 7, 599-605
Chazan, D., W.L. Miranker (1970), A nongradient and parallel algorithm for unconstrained minimization, SIAM J. Contr. 8, 207-217
Chichinadze, V.K. (1960), Logical design problems of self-optimizing and learning-optimizing control systems based on random searching, Proc. 1st IFAC Congr., Moscow, June-July 1960, vol. 2, p.653-657

Chichinadze, V.K. (1967), Random search to determine the extremum of the functions of several variables, Engng. Cybern. 5, 1, 115-123

Chichinadze, V.K. (1969), The Psi-transform for solving linear and nonlinear programming problems, Automatica 5, 347-356

Čížek, F., D. Hodáňová (1971), Evolution als Selbstregulation, G. Fischer, Jena

Clayton, D.G. (1971), Algorithm AS-46: Gram-Schmidt orthogonalization, Appl. Stat. 20, 335-338

Clegg, J.C. (1970), Variationsrechnung, Teubner, Stuttgart

Cochran, W.G., G.M. Cox (1950), Experimental designs, Wiley, New York

Cockrell, L.D. (1969), A comparison of several random search techniques for multimodal surfaces, Proc. Nat. Electron. Conf., Chicago, Ill., Dec. 1969, p.18-23

Cockrell, L.D. (1970), On search techniques in adaptive systems, Ph.D. thesis, Purdue Univ., Lafayette, Ind., June 1970

Cohen, A.I. (1972), Rate of convergence of several conjugate gradient algorithms, SIAM, J. Numer. Anal. 9, 248-259

Cohn, D.L. (1954), Optimal systems I: the vascular system, Bull. Math. Biophys. 16, 59-74

Collatz, L., W. Wetterling (1971), Optimierungsaufgaben, Springer, Berlin, 2nd Ed.

Colville, A.R. (1968), A comparative study on nonlinear programming codes, IBM New York Sci. Cntr., report 320-2949, June 1968

Colville, A.R. (1970), A comparative study of nonlinear programming codes, in: Kuhn (1970), p.487-501

Converse, A.O. (1970), Optimization, Holt, Rinehart and Winston, New York

Cooper, L., D. Steinberg (1970), Introduction to methods of optimization, Saunders, Philadelphia

Cornick, D.E., A.N. Michel (1972), Numerical optimization of distributed parameter systems by the conjugate gradient method, IEEE Trans. AC-17, 358-362

Courant, R. (1943), Variational methods for the solution of problems of equilibrium and vibrations, Bull. Amer. Math. Soc. 49, 1-23

Courant, R., D. Hilbert (1968a), Methoden der mathematischen Physik, vol. 1, Springer, Berlin, 3rd Ed.

Courant, R., D. Hilbert (1968b), Methoden der mathematischen Physik, vol. 2, Springer, Berlin, 2nd Ed.

Cowdrey, D.R., C.M. Reeves (1963), An application of the Monte Carlo method to the evaluation of some molecular integrals, Comp. J. 6, 277-286

Cox, D.R. (1958), Planning of experiments, Wiley, New York

Cragg, E.E., A.V. Levy (1969), Study on a supermemory gradient method for the minimization of functions, JOTA 4, 191-205

Crippen, G.M., H.A. Scheraga (1971), Minimization of polypeptide energy, X: a global search algorithm, Arch. Biochem. Biophys. 144, 453-461

Crockett, J.B., H. Chernoff (1955), Gradient methods of maximization, Pacif. J. Math. 5, 33-50

Crowder, H., P. Wolfe (1972), Linear convergence to the conjugate gradient method, IBM T.J. Watson Res. Cntr., report RC-3330, Yorktown Heights, N.Y., May 1972

Cryer, C.W. (1971), The solution of a quadratic programming problem using systematic overrelaxation, SIAM J. Contr. 9, 385-392

Cullum, J. (1972), An algorithm for minimizing a differentiable function that uses only function values, in: Balakrishnan (1972), p.117-127

Curry, H.B. (1944), The method of steepest descent for non-linear minimization problems, Quart. Appl. Math. 2, 258-261

Curtis, A.R., J.K. Reid (1974), The choice of step lengths when using differences to approximate Jacobian matrices, JIMA 13, 121-126

Curtiss, J.H. (1956), A theoretical comparison of the efficiencies of two classical methods and a Monte Carlo method for computing one component of the solution of a set of linear algebraic equations, in: Meyer (1956), p.191-233

Dambrauskas, A.P. (1970), The Simplex optimization method with variable step, Engng. Cybern. 8, 28-36

Dambrauskas, A.P. (1972), Investigation of the efficiency of the Simplex method of optimization with variable step in a noise situation, Engng. Cybern. 10, 590-599

Daniel, J.W. (1967a), The conjugate gradient method for linear and nonlinear operator equations, SIAM J. Numer. Anal. 4, 10-26

Daniel, J.W. (1967b), Convergence of the conjugate gradient method with computationally convenient modifications, Numer. Math. 10, 125-131

Daniel, J.W. (1969), On the approximate minimization of functionals, Math. Comp. 23, 573-581

Daniel, J.W. (1970), A correction concerning the convergence rate for the conjugate gradient method, SIAM J. Numer. Anal. 7, 277-280

Daniel, J.W. (1971), The approximate minimization of functionals, Prentice-Hall, Englewood Cliffs, N.J.

Daniel, J.W. (1973), Global convergence for Newton methods in mathematical programming, JOTA 12, 233-241

Dantzig, G.B. (1966), Lineare Programmierung und Erweiterungen, Springer, Berlin

Darwin, C. (1859), Die Entstehung der Arten durch natürliche Zuchtwahl, translation from 'The origin of species by means of natural selection', Reclam, Stuttgart, 1974

Darwin, C. (1874), Die Abstammung des Menschen, translation of the 2nd revised edition of 'The descent of man', Kröner, Stuttgart, 1966

Davidon, W.C. (1959), Variable metric method for minimization, Argonne Nat. Lab., report ANL-5990 rev., Lemont, Ill., Nov. 1959

Davidon, W.C. (1968), Variance algorithm for minimization, Comp. J. 10, 406-410

Davidon, W.C. (1969), Variance algorithm for minimization, in: Fletcher (1969a), p.13-20

Davies, D. (1968), The use of Davidon's method in nonlinear programming, ICI Mgmt. Serv., report MSDH-68-110, Middlesborough, Yorks., Aug. 1968

Davies, D. (1970), Some practical methods of optimization, in: Abadie (1970). p.87-118
Davies, D., W.H. Swann (1969), Review of constrained optimization, in: Fletcher (1969a), p.187-202
Davies, M., I.J. Whitting (1972), A modified form of Levenberg's correction, in: Lootsma (1972a), p.191-201
Davies, O.L. (ed.) (1954), The design and analysis of industrial experiments, Oliver and Boyd, London
Davis, R.H., P.D. Roberts (1968), Method of conjugate gradients applied to self-adaptive digital control systems, IEE Proc. 115, 562-571
DeGraag, D.P. (1970), Parameter optimization techniques for hybrid computers, Proc. VIth Int. Analogue Comp. Meet., Munich, Aug.-Sept. 1970, p.136-139
Dejon, B., P. Henrici (eds.)(1969), Constructive aspects of the fundamental theorem of algebra, Wiley-Interscience, London
Dekker, T.J. (1969), Finding a zero by means of successive linear interpolation, in: Dejon and Henrici (1969), p.37-48
Demyanov, V.F., A.M. Rubinov (1970), Approximate methods in optimization problems, Amer. Elsevier, New York
Denn, M.M. (1969), Optimization by variational methods, McGraw-Hill, New York
Dennis, J.E., jr. (1970), On the convergence of Newton-like methods, in: Rabinowitz (1970), p.163-181
Dennis, J.E., jr. (1971), On the convergence of Broyden's method for nonlinear systems of equations, Math. Comp. 25, 559-567
Dennis, J.E., jr. (1972), On some methods based on Broyden's secant approximation to the Hessian, in: Lootsma (1972a), p.19-34
D'Esopo, D.A. (1956), A convex programming procedure, Nav. Res. Log. Quart. 6, 33-42
DeVogelaere, R. (1968), Remark on algorithm 178(E4): direct search, CACM 11, 498
Dickert, B.F., G. Enyedy, jr., C.E. Huckaba et al. (eds.) (1962), Applied mathematics in chemical engineering, AIChE Chem. Engng. Progr. Symp. Ser. 58, no. 37
Dickinson, A.W. (1964), Nonlinear optimization: some procedures and examples, Proc. XIXth ACM Nat. Conf., Philadelphia, Penns., Aug. 1964, paper E1.2
Dijkhuis,B. (1971), An adaptive algorithm for minimizing a unimodal function of one variable, ZAMM 51, T45-T46
Dinkelbach, W. (1969), Sensitivitätsanalysen und parametrische Programmierung, Springer, Berlin
Dixon, L.C.W. (1972a), Nonlinear optimization, Engl. Univ. Press London
Dixon, L.C.W. (1972b), The choice of step length, a crucial factor in the performance of variable metric algorithms, in: Lootsma (1972a), p.149-170
Dixon, L.C.W. (1972c), Variable metric algorithms: necessary and sufficient conditions for identical behaviour of nonquadratic functions, JOTA 10, 34-40
Dixon, L.C.W. (1973), Conjugate directions without linear searches, JOTA 11, 317-328

Dixon, L.C.W., M.C. Biggs (1972), The advantages of adjoint-control transformations when determining optimal trajectories by Pontryagin's Maximum Principle, Aeronautical J. 76, 169-174
Dobzhansky, T. (1965), Dynamik der menschlichen Evolution: Gene und Umwelt, S. Fischer, Frankfurt/Main
Dowell, M., P. Jarratt (1972), The Pegasus method for computing the root of an equation, BIT 12, 503-508
Drenick, R.F. (1967), Die Optimierung linearer Regelsysteme, Oldenbourg, Munich
Duffin, R.J., E.L. Peterson, C. Zener (1967), Geometric programming: theory and application, Wiley, New York
Dvoretzky, A. (1956), On stochastic approximation, in: Neyman (1956), p.39-56
Edelbaum, T.N. (1962), Theory of maxima and minima, in: Leitmann (1962), p.1-32
Eigen, M. (1971), Selforganization of matter and the evolution of biological macromolecules, Naturwiss. 58, 465-523
Eisenberg, M.A., M.R. McGuire (1972), Further comments on Dijkstra's concurrent programming control problem, CACM 15, 999
Eisenhart, C., M.W. Hastay, W.A. Wallis (eds.) (1947), Selected techniques of statistical analysis for scientific and industrial research and production and management engineering, McGraw-Hill, New York
Elkin, R.M. (1968), Convergence theories for Gauss-Seidel and other minimization algorithms, Univ. Maryland, Comp. Sci. Cntr., techn. report 68-59, College Park, Maryland, Jan. 1968
Elliott, D.F., D.D. Sworder (1969a), A variable metric technique for parameter optimization, Automatica 5, 811-816
Elliott, D.F., D.D. Sworder (1969b), Design of suboptimal adaptive regulator systems via stochastic approximation, Proc. Nat. Electron. Conf., Chicago, Ill., Dec. 1969, p.29-33
Elliott, D.F., D.D. Sworder (1970), Applications of a simplified multidimensional stochastic approximation algorithm, IEEE Trans. AC-15, 101-104
Emery, F.E., M. O'Hagan (1966), Optimal design of matching networks for microwave transistor amplifiers, IEEE Trans. MTT-14, 696-698
Engelhardt, M. (1973), On upper bounds for variances in stochastic approximation, SIAM J. Appl. Math. 24, 145-151
Engeli, M., T. Ginsburg, H. Rutishauser, E. Stiefel (1959), Refined iterative methods for computation of the solution and the eigenvalues of self-adjoint boundary value problems, Mitt. Inst. Angew. Math. ETH Zürich, Birkhäuser, Basel
Erlicki, M.S., J. Appelbaum (1970), Solution of practical optimization problems, IEEE Trans. SSC-6, 49-52
Ermoliev, Yu. (1970), Random optimization and stochastic programming, in: Moiseev (1970), p.104-115
Faber, M.M. (1970), Stochastisches Programmieren, Physica, Würzburg
Fabian, V. (1967), Stochastic approximation of minima with improved asymptotic speed, Ann. Math. Stat. 38, 191-200
Fabian, V. (1968), On the choice of design in stochastic approximation methods, Ann. Math. Stat. 39, 457-465
Faddejew, D.K., W.N. Faddejewa (1973), Numerische Methoden der linearen Algebra, Oldenbourg, Munich, 3rd Ed.

Favreau, R.F., R. Franks (1958), Random optimization by analogue techniques, Proc. IInd Analogue Comp. Meet., Strassbourg, Sept. 1958, p.437-443

Feigenbaum, E.A., J. Feldman (eds.) (1963), Computers and thought, McGraw-Hill, New York

Feldbaum, A.A. (1958), Automatic optimalizer, ARC 19, 718-728

Feldbaum, A.A. (1960), Statistical theory of gradient systems of automatic optimization for objects with quadratic characteristics, ARC 21, 111-118

Feldbaum, A.A. (1962), Rechengeräte in automatischen Systemen, Oldenbourg, Munich

Fend, F.A., C.B. Chandler (1961), Numerical optimization for multi-dimensional problems, Gen. Electr., Gen. Engng. Lab., report 61-GL-78, March 1961

Fiacco, A.V. (1974), Convergence properties of local solutions of sequences of mathematical programming problems in general spaces, JOTA 13, 1-12

Fiacco, A.V., G.P. McCormick (1964), The sequential unconstrained minimization technique for nonlinear programming: a primal-dual method, Mgmt. Sci. 10, 360-366

Fiacco, A.V., G.P. McCormick (1968), Nonlinear programming: sequential unconstrained minimization techniques, Wiley, New York

Fielding, K. (1970), Algorithm 387 (E4): function minimization and linear search, CACM 13, 509-510

Fisher, R.A. (1966), The design of experiments, Oliver and Boyd, Edinburgh, 8th Ed.

Fletcher, R. (1965), Function minimization without evaluating derivatives: a review, Comp. J. 8, 33-41

Fletcher, R. (1966), Certification of algorithm 251(E4): function minimization, CACM 9, 686-687

Fletcher, R. (1968), Generalized inverse methods for the best least squares solution of systems of nonlinear equations, Comp. J. 10, 392-399

Fletcher, R. (ed.) (1969a), Optimization, Academic Press, London

Fletcher, R. (1969b), A review of methods for unconstrained optimization, in: Fletcher (1969a), p.1-12

Fletcher, R. (1970a), A class of methods for nonlinear programming with termination and convergence properties, in: Abadie (1970), p.157-176

Fletcher, R. (1970b), A new approach to variable metric algorithms, Comp. J. 13, 317-322

Fletcher, R. (1971), A modified Marquardt subroutine for non-linear least squares, UKAEA, Res. Group, report AERE-R-6799, Harwell, Berks.

Fletcher, R. (1972a), Conjugate direction methods, in: Murray (1972a), p.73-86

Fletcher, R. (1972b), A survey of algorithms for unconstrained optimization, in: Murray (1972a), p.123-129

Fletcher, R. (1972c), A Fortran subroutine for minimization by the method of conjugate gradients, UKAEA, Res. Group, report AERE-R-7073, Harwell, Berks.

Fletcher, R. (1972d), Fortran subroutines for minimization by quasi-Newton methods, UKAEA, Res. Group, report AERE-R-7125, Harwell, Berks.
Fletcher, R., M.J.D. Powell (1963), A rapidly convergent descent method for minimization, Comp. J. 6, 163-168
Fletcher, R., C.M. Reeves (1964), Function minimization by conjugate gradients, Comp. J. 7, 149-154
Flood, M.M., A. Leon (1964), A generalized direct search code for optimization, Univ. Michigan, Mental Health Res. Inst., preprint 129, Ann Arbor, Mich., June 1964
Flood, M.M., A. Leon (1966), A universal adaptive code for optimization: GROPE, in: Lavi and Vogl (1966), p.101-130
Fogarty, L.E., R.M. Howe (1968), Trajectory optimization by a direct descent process, Simulation 11, 145-155
Fogarty, L.E., R.M. Howe (1970), Hybrid computer solution of some optimization problems, Proc. VIth Int. Analogue Comp. Meet., Munich, Aug.-Sept. 1970, p.127-135
Fogel, L.J., A.J. Owens, M.J. Walsh (1965), Artificial intelligence through a simulation of evolution, in: Maxfield, Callahan and Fogel (1965), p.131-155
Fogel, L.J., A.J. Owens, M.J. Walsh (1966a), Adaption of evolutionary programming to the prediction of solar flares, Gen. Dyn.-Convair, report NASA-CR-417, San Diego, Calif.
Fogel, L.J., A.J. Owens, M.J. Walsh (1966b), Artificial intelligence through simulated evolution, Wiley, New York
Forsythe, G.E. (1968), On the asymptotic directions of the s-dimensional optimum gradient method, Numer. Math. 11, 57-76
Forsythe, G.E. (1969), Remarks on the paper by Dekker, in: Dejon and Henrici, p.49-51
Forsythe, G.E., T.S. Motzkin (1951), Acceleration of the optimum gradient method, Bull. Amer. Math. Soc. 57, 304-305
Fox, R.L. (1971), Optimization methods for engineering design, Addison-Wesley, Reading, Mass.
Frankovič, B., Š. Petráš, J. Skákala, B. Vykouk (1970), Automatisierung und selbsttätige Steuerung, Vlg. Technik, Berlin
Friedberg, R.M. (1958), A learning machine I, IBM J. Res. Dev. 2, 2-13
Friedberg, R.M., B. Dunham, J.H. North (1959), A learning machine II IBM J. Res. Dev. 3, 282-287
Friedmann, M., L.J. Savage (1947), Planning experiments seeking maxima, in: Eisenhart, Hastay and Wallis (1947), p.365-372
Friedrichs, K.O., O.E. Neugebauer, J.J. Stoker (eds.) (1948), Studies and essays, Courant anniversary volume, Interscience, New York
Fu, K.S., L.D. Cockrell (1970), On search techniques for multimodal surfaces, IFAC Kyoto Symp. Syst. Engng. Appr. Comp. Contr., Aug. 1970, paper 17.3
Fu, K.S., Ž.J. Nikolić (1966), On some reinforcement techniques and their relation to the stochastic approximation, IEEE Trans. AC-11, 756-758
Fürst, H., P.H. Müller, V. Nollau (1968), Eine stochastische Methode zur Ermittlung der Maximalstelle einer Funktion von mehreren Veränderlichen mit experimentell ermittelbaren Funktionswerten und ihre Anwendung bei chemischen Prozessen, Chem. Techn. 20, 400-405

Gaidukov, A.L. (1966), Primeneniye sluchainovo poiska pri optimalnom proyektirovanii, Prikladnye zadichi tekhnicheskoi kibernetiki (1966), 420-436

Gal. S. (1971), Sequential minimax search for a maximum when prior information is available, SIAM J. Appl. Math. 21, 590-595

Gal, S. (1972), Multidimensional minimax search for a maximum, SIAM J. Appl. Math. 23, 513-526

Garfinkel, R.S., G.L. Nemhauser (1972), Integer programming, Wiley New York

Garfinkel, R.S., G.L. Nemhauser (1973), A survey of integer programming emphasizing computation and relations among models, in: Hu and Robinson (1973), p.77-155

Gauss, C.F. (1809), Determinatio orbitae observationibus quotcumque quam proxime satisfacientis, Werke, Vol. 7 (Theoria motus corporum coelestium in sectionibus conicis solem ambientium), Liber secundus, Sectio III, p.236-257, Hamburgi sumtibus Frid. Perthes et I.H. Besser, 1809; Teubner, Leipzig, 1906

Gaviano, M., E. Fagiuoli (1972), Remarks on the comparison between random search methods and the gradient method, in: Szegö (1972), p.337-349

Gelfand, I.M., M.L. Tsetlin (1961), The principle of nonlocal search in automatic optimization systems, Soviet Phys. Dokl. 6, 3, 192-194

Geoffrion, A.M. (ed.)(1972), Perspectives on optimization, Addison-Wesley, Reading, Mass.

Gérardin, L. (1968), Natur als Vorbild: die Entdeckung der Bionik, Kindler, Munich

Gersht, A.M., A.I. Kaplinskii (1971), Convergence of the continuous variant of the Robbins-Monro procedure, ARC 32, 71-75

Gessner, P., K. Spremann (1972), Optimierung in Funktionenräumen, Springer, Berlin

Gessner, P., H. Wacker (1972), Dynamische Optimierung: Einführung, Modelle, Computerprogramme, Hanser, Munich

Gilbert, E.G. (1967), A selected bibliography on parameter optimization methods suitable for hybrid computation, Simulation 8, 350-352

Gilbert, P., W.J. Chandler (1972), Interface between communicating parallel processes, CACM 15, 427-437

Gill, P.E., W. Murray (1972), Quasi-Newton methods for unconstrained optimization, JIMA 9, 91-108

Ginsburg, T. (1963), The conjugate gradient method, Numer. Math. 5, 191-200

Girsanov, I.V. (1972), Lectures on mathematical theory of extremum problems, Springer, Berlin

Glass, H., L. Cooper (1965), Sequential search: a method for solving constrained optimization problems, JACM 12, 71-82

Gnedenko, B.W. (1970), Lehrbuch der Wahrscheinlichkeitsrechnung, Akademie Vlg., Berlin, 6th Ed.

Goldfarb, D. (1969), Sufficient conditions for the convergence of a variable metric algorithm, in: Fletcher (1969a), p.273-282

Goldfarb, D. (1970), A family of variable-metric methods derived by variational means, Math. Comp. 24, 23-26

Goldfeld, S.M., R.E. Quandt, H.F. Trotter (1966), Maximization by quadratic hill-climbing, Econometrica 34, 541-551
Goldfeld, S.M., R.E. Quandt, H.F. Trotter (1968), Maximization by improved quadratic hill-climbing, Princeton Univ., Econ. Res. Progr., res. memo. RM-95, Princeton, N.J., April 1968
Goldstein, A.A. (1962), Cauchy's method of minimization, Numer. Math. 4, 146-150
Goldstein, A.A. (1965), On Newton's method, Numer. Math. 7, 391-393
Goldstein, A.A., J.F. Price (1967), An effective algorithm for minimization, Numer. Math. 10, 184-189
Goldstein, A.A., J.F. Price (1971), On descent from local minima, Math. Comp. 25, 569-574
Goliński, J., Z.K. Leśniak (1966), Optimales Entwerfen von Konstruktionen mit Hilfe der Monte-Carlo-Methode, Bautechnik 43, 307-311
Goll, R. (1972), Der Evolutionismus: Analyse eines Grundbegriffs neuzeitlichen Denkens, Beck, Munich
Golub, G.H. (1965), Numerical methods for solving linear least squares problems, Numer. Math. 7, 206-216
Golub, G.H., M.A. Saunders (1970), Linear least squares and quadratic programming, in: Abadie (1970), p.229-256
Gonzalez, R.S. (1970), An optimization study on a hybrid computer, Ann. Assoc. Int. Calcul Analog. 12, 138-148
Gorvits, G.G., O.I. Larichev (1971), Comparison of search methods for the solution of nonlinear identification problems, ARC 32, 272-280
Gottfried, B.S., J. Weisman (1973), Introduction to optimization theory, Prentice-Hall, Englewood Cliffs, N.J.
Gran, R. (1973), On the convergence of random search algorithms in continuous time with applications to adaptive control, IEEE Trans. SMC-3, 62-66
Grassé, P.P. (1973), Allgemeine Biologie, Vol. 5: Evolution, G. Fischer, Stuttgart
Grassmann, P. (1967) Verfahrenstechnik und Biologie, Chem. Ing. Techn. 39, 1217-1226
Grassmann, P. (1968), Verfahrenstechnik und Medizin, Chem. Ing. Techn. 40, 1094-1100
Graves, R.L., P. Wolfe (eds.)(1963), Recent advances in mathematical programming, McGraw-Hill, New York
Greenberg, H. (1971), Integer programming, Academic Press, New York
Greenstadt, J. (1967a), On the relative efficiencies of gradient methods, Math. Comp. 21, 360-367
Greenstadt, J. (1967b), Bestimmung der Eigenwerte einer Matrix nach der Jacobi-Methode, in: Ralston and Wilf (1967), p.152-168
Greenstadt, J. (1970), Variations on variable-metric methods, Math.Comp. 24, 1-22
Greenstadt, J. (1972), A quasi-Newton method with no derivatives, Math. Comp. 26, 145-166
Grüsser, O.J., R. Klinke (eds.) (1971), Zeichenerkennung durch biologische und technische Systeme, Springer, Berlin
Guilfoyle, G., I. Johnson, P. Wheatley (1967), One-dimensional search combining golden section and cubic fit techniques, Anal. Mech. Assoc. Inc., quart. report 67-1, Westbury, Long Island, N.Y., Jan. 1967

Guin, J.A. (1968), Modification of the Complex method of constrained optimization, Comp. J. 10, 416-417
Gurin, L.S. (1966), Random search in the presence of noise, Engng. Cybern. 4, 3, 252-260
Gurin, L.S., V.P. Lobac (1963), Combination of the Monte Carlo method with the method of steepest descents for the solution of certain extremal problems, AIAA J. 1, 2708-2710
Gurin, L.S., L.A. Rastrigin (1965), Convergence of the random search method in the presence of noise, ARC 26, 1505-1511
Hadamard, J. (1908), Mémoire sur le problème d'analyse relatif à l'équilibre des plaques élastiques encastrées, Mém.prés. Acad. Sci. Inst. France, 2nd Series, Vol. 33 (savants étrangers) no. 4, p. 1-128
Hadley, G. (1962), Linear programming, Addison-Wesley, Reading, Mass.
Hadley, G. (1969), Nichtlineare und dynamische Programmierung, Physica, Würzburg
Hague, D.S., C.R. Glatt (1968), An introduction to multivariable search techniques for parameter optimization and program AESOP, Boeing, Space Div., report NASA-CR-73200, Seattle, Wash., March 1968
Hamilton, P.A., J. Boothroyd (1969), Remark on algorithm 251(E4): function minimization, CACM 12, 512-513
Hammersley, J.M., D.C. Handscomb (1964), Monte Carlo methods, Methuen, London
Hancock, H. (1960), Theory of maxima and minima, Dover Publ., New York
Hansen, P.B. (1972), Structured multiprogramming, CACM 15, 574-578
Harkins, A. (1964), The use of parallel tangents in optimization, in: Blakemore and Davis (1964), p.35-40
Hartmann, D. (1974), Optimierung balkenartiger Zylinderschalen aus Stahlbeton mit elastischem und plastischem Werkstoffverhalten, Dr.-Ing. Diss.,Univ. Dortmund, July 1974
Haubrich, J.G.A. (1963), Algorithm 205(E4): ative, CACM 6, 519
Heinhold, J., K.W. Gaede (1972), Ingenieur-Statistik, Oldenbourg, Munich, 3rd Ed.
Henn, R., H.P.Künzi (1968), Einführung in die Unternehmensforschung I and II, Springer, Berlin
Herschel, R. (1961), Automatische Optimisatoren, Elektron. Rechenanlg. 3, 30-36
Hertel, H. (1963), Biologie und Technik, Vol. 1: Struktur, Form, Bewegung, Krausskopf, Mainz
Hesse, R. (1973), A heuristic search procedure for estimating a global solution of nonconvex programming problems, Oper. Res. 21, 1267-1280
Hestenes, M.R. (1956), The conjugate-gradient method for solving linear systems, Proc. Symp. Appl. Math. 6, 83-102
Hestenes, M.R. (1966), Calculus of variations and optimal control theory, Wiley, New York
Hestenes, M.R. (1969), Multiplier and gradient methods, in: Zadeh, Neustadt and Balakrishnan (1969a), p.143-163
Hestenes, M.R. (1973), Iterative methods for solving linear equations, JOTA 11, 323-334 (Original: 1951)

Hestenes, M.R., M.L. Stein (1973), The solution of linear equations by minimization, JOTA 11, 335-359 (Original: 1951)
Hestenes, M.R., E. Stiefel (1952), Methods of conjugate gradients for solving linear systems, NBS J. Res. 49, 409-436
Heydt, G.T. (1970), Directed random search, Ph.D. thesis, Purdue Univ., Lafayette, Ind., Aug. 1970
Heynert, H. (1972), Einführung in die allgemeine Bionik, Dt. Vlg. Wissensch., Berlin
Hildebrand, F.B. (1956), Introduction to numerical analysis, McGraw-Hill, New York
Hill, J.C. (1964), A hillclimbing technique using piecewise cubic approximation, Ph.D. thesis, Purdue Univ., Lafayette, Ind., June 1964
Hill, J.C., J.E. Gibson (1965), Hillclimbing on hills with many minima, Proc. IInd IFAC Symp. Theory Self Adapt. Contr. Syst., Teddington, Engl., Sept. 1965, p.322-334
Hill, J.D. (1969), A search technique for multimodal surfaces, IEEE Trans. SSC-5, 2-8
Hill, J.D., K.S.Fu (1965), A learning control system using stochastic approximation for hill-climbing, VIth Joint Autom. Contr. Conf., Troy, N.Y., June 1965, session 14, paper 2
Hill, J.D., G.J. McMurtry, K.S. Fu (1964), A computer simulated on-line experiment in learning control systems, AFIPS Conf. Proc. 25, 315-325
Himmelblau, D.M. (1972a), A uniform evaluation of unconstrained optimization techniques, in: Lootsma (1972b), p.69-97
Himmelblau, D.M. (1972b), Applied nonlinear programming, McGraw-Hill, New York
Himsworth, F.R. (1962), Empirical methods of optimization, Trans. Inst. Chem. Engrs. 40, 345-349
Höfler, A., U. Leyssner, J. Wiedemann (1973), Optimization of the layout of trusses combining strategies based on Michell's theorem and on the biological principles of evolution, AGARD Conf. Proc. 123, IInd Symp. Struct. Optim., Mailand, April 1973, app. A
Hoffmann, U., H. Hofmann (1970), Einführung in die Optimierung mit Anwendungsbeispielen aus dem Chemie-Ingenieur-Wesen, Vlg. Chemie, Weinheim
Hooke, R. (1957), Control by automatic experimentation, Chem. Engng. 64, 6, 284-286
Hooke, R., T.A. Jeeves (1958), Comments on Brooks' discussion of random methods, Oper. Res. 6, 881-882
Hooke, R., T.A. Jeeves (1961), Direct search solution of numerical and statistical problems, JACM 8, 212-229
Hooke, R., R.I. VanNice (1959), Optimizing control by automatic experimentation, ISA J. 6, 7, 74-79
Hopper, M.J. (ed.) (1971), Harwell subroutine library: a catalogue of subroutines, UKAEA, Res. Group, report AERE-R-6912, Harwell, Berks.
Horwitz, L.B., P.E. Sarachik (1968), Davidon's method in Hilbert space, SIAM J. Appl. Math. 16, 676-695
Hoshino, S. (1971), On Davies, Swann and Campey minimisation process, Comp. J. 14, 426-427

Hoshino, S. (1972), A formulation of variable metric methods, JIMA 10, 394-403
Hotelling, H. (1941), Experimental determination of the maximum of a function, Ann. Math. Stat. 12, 20-45
House, F.R. (1971), Remark on algorithm 251(E4): function minimisation, CACM 14, 358
Householder, A.S. (1953), Principles of numerical analysis, McGraw-Hill, New York
Householder, A.S. (1970), The numerical treatment of a single nonlinear equation, McGraw-Hill, New York
Houston, B.F., R.A. Huffman (1971), A technique which combines modified pattern search methods with composite designs and polynomial constraints to solve constrained optimization problems, Nav. Res. Log. Quart. 18, 91-98
Hu, T.C. (1972), Ganzzahlige Programmierung und Netzwerkflüsse, Oldenbourg, Munich
Hu, T.C., S.M. Robinson (eds.) (1973), Mathematical programming, Academic Press, New York
Huang, H.Y. (1970), Unified approach to quadratically convergent algorithms for function minimization, JOTA 5, 405-423
Huang, H.Y. (1974), Method of dual matrices for function minimization, JOTA 13, 519-537
Huang, H.Y., J.P. Chambliss (1973), Quadratically convergent algorithms and one-dimensional search schemes, JOTA 11, 175-188
Huang, H.Y., J.P. Chambliss (1974), Numerical experiments on dual matrix algorithms for function minimization, JOTA 13, 620-634
Huang, H.Y., A.V. Levy (1970), Numerical experiments on quadratically convergent algorithms for function minimization, JOTA 6, 269-282
Huelsman, L.P. (1968), GOSPEL: a general optimization software package for electrical network design, Univ. Arizona, Dept. Electr. Engng., report, Tucson, Arizona, Sept. 1968
Hull, T.E. (1967), Random-number generation and Monte-Carlo methods, in: Klerer and Korn (1967), p.63-78
Humphrey, W.E., B.J. Cottrell (1962/66), A general minimizing routine, Univ. California, Lawrence Rad. Lab. int. memo. P-6, Livermore, Calif., July 1962, rev. March 1966
Hupfer, P. (1970), Optimierung von Baukonstruktionen, Teubner, Stuttgart
Hyslop, J. (1972), A note on the accuracy of optimisation techniques, Comp. J. 15, 140
Idelsohn, J.M. (1964), Ten ways to find the optimum, Contr. Engng. 11, 6, 97-102
Imamura, H., K. Uosaki, M. Tasaka, T. Suzuki (1970), Optimization methods in the multimodal case and their application to automatic lens design, IFAC Kyoto Symp. Syst. Engng. Appr. Comp. Contr., Aug. 1970, paper 7.4
Inomata, S., M. Kumada (1961), On the golf method, Denki shikensho iho Bull. Electrotechn. Lab., Tokyo, 25, 495-512
Ivakhnenko, A.G. (1970), Heuristic self-organization in problems of engineering cybernetics, Automatica 6, 207-219
Jacobson, D.H., D.Q.Mayne (1970), Differential dynamic programming, Amer. Elsevier, New York

Jacoby, S.L.S., J.S. Kowalik, J.T. Pizzo (1972), Iterative methods for nonlinear optimization problems, Prentice-Hall, Englewood Cliffs, N.J.
Jahnke-Emde-Lösch (1966), Tafeln höherer Funktionen, Teubner, Stuttgart, 7th Ed.
Janáč, K. (1971), Adaptive stochastic approximations, Simulation 16, 51-58
Jarratt, P. (1967), An iterative method for locating turning points, Comp. J. 10, 82-84
Jarratt, P. (1968), A numerical method for determining points of inflexion, BIT 8, 31-35
Jarratt, P. (1970), A review of methods for solving nonlinear algebraic equations in one variable, in: Rabinowitz (1970), p.1-26
Jarvis, R.A. (1968), Hybrid computer simulation of adaptive strategies, Ph.D. thesis, Univ. Western Australia, March 1968
Jarvis, R.A. (1970), Adaptive global search in a time-variant environment using a probabilistic automaton with pattern recognition supervision, IEEE Trans. SSC-6, 209-217
Jeeves, T.A. (1958), Secant modification of Newton's method, CACM 1, 9-10
Jöhnk, M.D. (1969), Erzeugen und Testen von Zufallszahlen, Physica, Würzburg
Johannsen, G. (1970), Entwicklung und Optimierung eines vielparametrigen nichtlinearen Modells für den Menschen als Regler in der Fahrzeugführung, Dr.-Ing. Diss., Tech. Univ. Berlin, Oct. 1970
Johannsen, G. (1973), Optimierung vielparametriger Bezugsmodelle mit Hilfe von Zufallssuchverfahren, Regelungstechn. Prozess-Datenverarb. 21, 234-239
John, F. (1948), Extremum problems with inequalities as subsidiary conditions, in: Friedrichs, Neugebauer and Stoker (1948), p.187-204
John, P.W.M. (1971), Statistical design and analysis of experiments, Macmillan, New York
Johnson, S.M. (1956), Best exploration for maximum is Fibonaccian, RAND Corp., report P-856, Santa Monica, Calif.
Jones, A. (1970), Spiral: a new algorithm for nonlinear parameter estimation using least squares, Comp. J. 13, 301-308
Jones, D.S. (1973), The variable metric algorithm for non-definite quadratic functions, JIMA 12, 63-71
Jordan, P. (1970), Schöpfung und Geheimnis, Stalling, Oldenburg
Kamiya, A., T. Togawa (1972), Optimal branching structure of the vascular tree, Bull. Math. Biophys. 34, 431-438
Kammerer, W.J., M.Z. Nashed (1972), On the convergence of the conjugate gradient method for singular linear operator equations, SIAM J. Numer. Anal. 9, 165-181
Kantorovich, L.V. (1940), A new method of solving of some classes of extremal problems, Compt. Rend. Acad. Sci. URSS, Neue Serie 28, 211-214
Kantorovich, L.V. (1945), On an effective method of solving extremal problems for quadratic functionals, Compt. Rend. Acad. Sci. URSS, Neue Serie 48, 455-460

Kantorovich, L.V. (1952), Functional analysis and applied mathematics, NBS report 1509, March 1952
Kaplinskii, A.I., A.I. Propoi (1970), Stochastic approach to non-linear programming problems, ARC 31, 448-459
Kappler, H. (1967), Gradientenverfahren der nichtlinearen Programmierung, O. Schwartz, Göttingen
Karnopp, D.C. (1961), Search theory applied to parameter scan optimization problems, Ph.D. thesis, MIT, Cambridge, Mass., June 1961
Karnopp, D.C. (1963), Random search techniques for optimization problems, Automatica 1, 111-121
Karnopp, D.C. (1966), Ein direktes Rechenverfahren für implizite Variationsprobleme bei optimalen Prozessen, Regelungstechn. 14, 366-368
Karp, R.M., W.L. Miranker (1968), Parallel minimax search for a maximum, J. Comb. Theory 4, 19-35
Karreman, H.F. (ed.)(1968), Stochastic optimization and control, Wiley, New York
Karumidze, G.V. (1969), A method of random search for the solution of global extremum problems, Engng. Cybern. 7, 6, 27-31
Katkovnik, V. Ya., O. Yu. Kulchitskii (1972), Convergence of a class of random search algorithms, ARC 33, 1321-1326
Katkovnik, V.Ya., L.I. Shimelevich (1972), A class of heuristic methods for solution of partially-integer programming problems, Engng. Cybern. 10, 390-394
Kaupe, A.F. (1963), Algorithm 178(E4): direct search, CACM 6, 313-314
Kaupe, A.F. (1964), On optimal search techniques, CACM 7, 38
Kavanaugh, W.P., E.C. Stewart, D.H. Brocker (1968), Optimal control of satellite attitude acquisition by a random search algorithm on a hybrid computer, AFIPS Conf. Proc. 32, 443-452
Kawamura, K., R.A. Volz (1973), On the rate of convergence of the conjugate gradient reset method with inaccurate linear minimizations, IEEE Trans. AC-18, 360-366
Kelley, H.J. (1962), Methods of gradients, in: Leitmann (1962), p.205-254
Kelley, H.J., G.E. Myers (1971), Conjugate direction methods for parameter optimization, Astron. Acta 16, 45-51
Kelley, H.J., J.L. Speyer (1970), Accelerated gradient projection, in: Balakrishnan, Contensou, DeVeubeke et al. (1970), p.151-158
Kempthorne, O. (1952), The design and analysis of experiments, Wiley, New York
Kenworthy, I.C. (1967), Some examples of simplex evolutionary operation in the paper industry, Appl. Stat. 16, 211-224
Kesten, H. (1958), Accelerated stochastic approximation, Ann. Math. Stat. 29, 41-59
Khovanov, N.V. (1967), Stochastic optimization of parameters by the method of variation of the search region, Engng. Cybern. 5, 4, 34-39
Kiefer, J. (1953), Sequential minimax search for a maximum, Proc. Amer. Math. Soc. 4, 502-506
Kiefer, J. (1957), Optimum sequential search and approximation methods under minimum regularity assumptions, SIAM J. 5, 105-136

Kiefer, J., J. Wolfowitz (1952), Stochastic estimation of the maximum of a regression function, Ann. Math. Stat. 23, 462-466
King, R.F. (1973), An improved Pegasus method for root finding, BIT 13, 423-427
Kivelidi, V.Kh., Ya. I. Khurgin (1970), Construction of probabilistic search, ARC 31, 1892-1894
Kjellström, G. (1965), Network optimization by random variation of component values, Ericsson Techn. 25, 133-151
Klerer, M., G.A. Korn (eds.) (1967), Digital computer user's handbook, McGraw-Hill, New York
Klessig, R., E. Polak (1972), Efficient implementations of the Polak-Ribière conjugate gradient algorithm, SIAM J. Contr. 10, 524-549
Klessig, R., E. Polak (1973), An adaptive precision gradient method for optimal control, SIAM J. Contr. 11, 80-93
Klingman, W.R., D.M. Himmelblau (1964), Nonlinear programming with the aid of a multiple-gradient summation technique, JACM 11, 400-415
Klötzler, R. (1970), Mehrdimensionale Variationsrechnung, Birkhäuser, Basel
Koch, H.W. (1973), Der Sozialdarwinismus: seine Genese und sein Einfluss auf das imperialistische Denken, Beck, Munich
Kopp, R.E. (1967), Computational algorithms in optimal control, IEEE Int. Conv. Record 15, part 3 (Automatic Control), 5-14
Korbut, A.A., J.J. Finkelstein (1971), Diskrete Optimierung, Akademie Vlg., Berlin
Korn, G.A. (1966), Random process simulation and measurement, McGraw-Hill, New York
Korn, G.A. (1968), Hybrid computer Monte Carlo techniques, in: McLeod (1968), p.223-234
Korn, G.A., T.M. Korn (1961), Mathematical handbook for scientists and engineers, McGraw-Hill, New York
Korn, G.A., T.M. Korn (1964), Electronic analog and hybrid computers, McGraw-Hill, New York
Korn, G.A., H. Kosako (1970), A proposed hybrid-computer method for functional optimization, IEEE Trans. C-19, 149-153
Kovács, Z., S.A. Lill (1971), Note on algorithm 46: a modified Davidon method for finding the minimum of a function, using difference approximation for derivatives, Comp. J. 14, 214-215
Kowalik, J. (1967), A note on nonlinear regression analysis, Austral. Comp. J. 1, 51-53
Kowalik, J., J.F. Morrison (1968), Analysis of kinetic data for allosteric enzyme reactions as a nonlinear regression problem, Math. Biosci. 2, 57-66
Kowalik, J., M.R. Osborne (1968), Methods for unconstrained optimization problems, Amer. Elsevier, New York
Krasnushkin, E.V. (1970), Multichannel automatic optimizer having a variable sign for the feedback, ARC 31, 2057-2061
Krasovskii, A.A. (1962), Optimal methods of search in continuous and pulsed extremum control systems, Proc. 1st IFAC Symp. Optim. Adapt. Contr., Rome, April 1962, p.19-33
Krasovskii, A.A. (1963), Problems of continuous systems theory of extremal control of industrial processes, Proc. IInd IFAC Congr., Basel, Aug.-Sept. 1963, vol. 1, p.519-526

Krasulina, T.P. (1972), Robbins-Monro process in the case of several roots, ARC 33, 580-585

Kregting, J., R.C. White, jr. (1971), Adaptive random search, Eindhoven Univ. of Technology, Dept. Electr. Engng., Group Measurement and Control, report TH-71-E-24, Eindhoven, Netherlands, Oct. 1971

Krelle, W., H.P. Künzi (1958), Lineare Programmierung, Vlg. Industr. Organ., Zurich

Krolak, P.D. (1968), Further extensions of Fibonaccian search to nonlinear programming problems, SIAM J. Contr. 6, 258-265

Krolak, P.D., L. Cooper (1963), An extension of Fibonaccian search to several variables, CACM 6, 639-641

Künzi, H.P. (1967), Mathematische Optimierung grosser Systeme, Ablauf- und Planungsforschung 8, 395-407

Künzi, H.P., W. Krelle (1969), Einführung in die mathematische Optimierung, Vlg. Industr. Organ., Zurich

Künzi, H.P., W. Krelle, W. Oettli (1962), Nichtlineare Programmierung, Springer, Berlin

Künzi, H.P., W. Oettli (1969), Nichtlineare Optimierung: neuere Verfahren - Bibliographie, Springer, Berlin

Künzi, H.P., S.T. Tan (1966), Lineare Optimierung grosser Systeme, Springer, Berlin

Künzi, H.P., H.G. Tzschach, C.A. Zehnder (1966), Numerische Methoden der mathematischen Optimierung, Teubner, Stuttgart

Künzi, H.P., H.G. Tzschach, C.A. Zehnder (1970), Numerische Methoden der mathematischen Optimierung mit Algol- und Fortran-Programmen: Gebrauchsversion der Computerprogramme, Teubner, Stuttgart

Kuester, J.L., J.H. Mize (1973), Optimization techniques with Fortran, McGraw-Hill, New York

Kuhn, H.W. (ed.) (1970), Proc. Princeton Symposium Math. Programm., Aug. 1967, Princeton Univ. Press, Princeton, N.J.

Kuhn, H.W., A.W. Tucker (1951), Nonlinear programming, in: Neyman (1951), p.481-492

Kulchitskiy, O. Yu. (1972), A non-gradient random search method for an extremum in a Hilbert space, Engng. Cybern. 10, 773-780

Kuo, F.F., J.F. Kaiser (eds.) (1966), System analysis by digital computer, Wiley, New York

Kushner, H.J. (1963), Hill climbing methods for the optimization of multiparameter noise disturbed systems, Trans. ASME D, J. Basic Engng. (1963), 157-164

Kushner, H.J. (1972), Stochastic approximation algorithms for the local optimization of functions with nonunique stationary points, IEEE Trans. AC-17, 646-654

Kussul, E., A. Luk (1971), Evolution als Optimierungsprozess, Ideen des exakten Wissens (1971), 821-826

Kwakernaak, H. (1965), On-line iterative optimization of stochastic control systems, Automatica 2, 195-208

Kwakernaak, H. (1966), On-line dynamic optimization of stochastic control systems, Proc. IIIrd IFAC Congr., London, June 1966, paper 29-D

Kwatny, H.G. (1972), A note on stochastic approximation algorithms in system identification, IEE Trans. AC-17, 571-572

Land, A.H., S. Powell (1973), Fortran codes for mathematical programming: linear, quadratic and discrete, Wiley, London
Lange-Nielsen, T., G.M. Lance (1972), A pattern search algorithm for feedback-control system parameter optimization, IEEE Trans. C-21, 1222-1227
Langguth, V. (1972), Ein Identifikationsverfahren für lineare Systeme mit Hilfe von stochastischen Suchverfahren und unter Anwendung der Sequentialanalyse für stochastische Fehlersignale, messen-steuern-regeln 15, 293-296
Lapidus, L., E. Shapiro, S. Shapiro, R.E. Stillman (1961), Optimization of process performance, AIChE J. 7, 2, 288-294
Larichev, O.I., G.G. Gorvits (1974), New approach to comparison of search methods used in nonlinear programming problems, JOTA 13, 635-659
Larson, R.E., E. Tse (1973), Parallel processing algorithms for the optimal control of nonlinear dynamic systems, IEEE Trans. C-22, 777-786
Lasdon, L.S. (1970), Conjugate direction methods for optimal control, IEEE Trans. AC-15, 267-268
Lavi, A., T.P. Vogl (eds.)(1966), Recent advances in optimization techniques, Wiley, New York
Lawrence, J.P., III, F.P. Emad (1973), An analytic comparison of random searching and gradient searching of a known objective function, IEEE Trans. AC-18, 669-671
Lawrence, J.P., III, K. Steiglitz (1972), Randomized pattern search, IEEE Trans. C-21, 382-385
LeCam, L.M., J. Neyman (eds.)(1967), Proc. Vth Berkeley Symp. Math. Stat. Probab., 1965/66, Univ. Calif. Press, Berkeley, Calif.
LeCam, L.M., J. Neyman, E.L. Scott (eds.)(1972), Proc. VIth Berkeley Symp. Math. Stat. Probab., 1970/71, Univ. Calif. Press, Berkeley, Calif.
Lee, R.C.K. (1964), Optimal estimation, identification, and control, MIT Press, Cambridge, Mass.
Leibniz, G.W. (1710), Theodicee, 4th rev. Ed.: Förster, Hannover, 1744
Leitmann, G. (ed.)(1962), Optimization techniques with applications to aerospace systems, Academic Press, New York
Leitmann, G. (1964), Einführung in die Theorie optimaler Steuerung und der Differentialspiele: eine geometrische Darstellung, Oldenbourg, Munich
Leitmann, G. (ed.)(1967), Topics in optimization, Academic Press, New York
Leon, A. (1966a), A comparison among eight known optimizing procedures, in: Lavi and Vogl (1966), p.23-46
Leon, A. (1966b), A classified bibliography on optimization, in: Lavi and Vogl (1966), p.599-649
Lerner, A. Ja., E.A. Rosenman (1973), Optimale Steuerungen, Vlg. Technik, Berlin
Leśniak, Z.K. (1970), Methoden der Optimierung von Konstruktionen unter Benutzung von Rechenautomaten, W. Ernst, Berlin
Levenberg, K. (1944), A method for the solution of certain nonlinear problems in least squares, Quart. Appl. Math. 2, 164-168

Levine, L. (1964), Methods for solving engineering problems using analog computers, McGraw-Hill, New York
Levine, M.D., T. Vilis (1973), On-line learning optimal control using successive approximation techniques, IEEE Trans. AC-18, 279-284
Lew, H.S. (1972), An arithmetical approach to the mechanics of blood flow in small caliber blood vessels, J. Biomech. 5, 49-69
Leyssner, U. (1974), Über den Einsatz Linearer Programmierung beim Entwurf optimaler Leichtbaustabwerke, Dr.-Ing. Diss., Techn. Univ. Berlin, June 1974
Lill, S.A. (1970), Algorithm 46: a modified Davidon method for finding the minimum of a function, using difference approximation for derivatives, Comp. J. 13, 111-113
Lill, S.A. (1971), Note on algorithm 46: a modified Davidon method, Comp. J. 14, 106
Little, W.D. (1966), Hybrid computer solutions of partial differential equations by Monte Carlo methods, AFIPS Conf. Proc. 29, 181-190
Ljapunov, A.A. (ed.), W. Kämmerer, H. Thiele (eds.)(1964a), Probleme der Kybernetik, vol. 4, Akademie Vlg., Berlin
Ljapunov, A.A. (ed.), W. Kämmerer, H. Thiele (eds.)(1964b), Probleme der Kybernetik, vol. 5, Akademie Vlg., Berlin
Locker, A. (ed.) (1973), Biogenesis, Evolution, Homeostasis, Springer, Berlin
Loginov, N.V. (1966), Methods of stochastic approximation, ARC 27, 706-728
Lootsma, F.A. (ed.) (1972a), Numerical methods for non-linear optimization, Academic Press, London
Lootsma, F.A. (1972b), A survey of methods for solving constrained minimization problems via unconstrained minimization, in: Lootsma (1972a), p.313-347
Lowe, C.W. (1964), Some techniques of evolutionary operation, Trans. Inst. Chem. Engrs. 42, T334-T344
Lucas, E. (1876), Note sur l'application des séries récurrentes à la recherche de la loi de distribution de nombres premiers, Compt. Rend. Hebdomad. Séances Acad. Sci. Paris, 82, 165-167
Luenberger, D.G. (1972), Mathematical programming and control theory: trends of interplay, in:Geoffrion (1972), p.102-133
Luenberger, D.G. (1973), Introduction to linear and nonlinear programming, Addison-Wesley, Reading, Mass.
Machura, M., A. Mulawa (1973), Algorithm 450(E4): Rosenbrock function minimization, CACM 16, 482-483
Madsen, K. (1973), A root-finding algorithm based on Newton's method, BIT 13, 71-75
Mamen, R., D.Q. Mayne (1972), A pseudo Newton-Raphson method for function minimization, JOTA 10, 263-277
Mangasarian, O.L. (1969), Nonlinear programming, McGraw-Hill, New York
Marfeld, A.F. (1970), Kybernetik des Gehirns: ein Kompendium der Grundlagenforschung, Safari Vlg., Berlin
Marquardt, D.W. (1963), An algorithm for least-squares estimation of nonlinear parameters, SIAM J. 11, 431-441

Masters, C.O., H. Drucker (1971), Observations on direct search procedures, IEEE Trans. SMC-1, 182-184
Matthews, A., D. Davies (1971), A comparison of modified Newton methods for unconstrained optimisation, Comp. J. 14, 293-294
Matyas, J. (1965), Random optimization, ARC 26, 244-251
Matyas, J. (1967), Das zufällige Optimierungsverfahren und seine Konvergenz, Proc. Vth Int. Analogue Comp. Meet., Lausanne, Aug.-Sept. 1967, vol. 1, p.540-544
Maxfield, M., A. Callahan, L.J. Fogel (eds.) (1965), Biophysics and cybernetic systems, Spartan, Washington, D.C.
Maybach, R.L. (1966), Solution of optimal control problems on a high-speed hybrid computer, Simulation 9, 238-245
McArthur, D.S. (1961), Strategy in research: alternative methods for design of experiments, IRE Trans. EM-8, 34-40
McCormick, G.P. (1969), Anti-zig-zagging by bending, Mgmt. Sci. 15, 315-320
McCormick, G.P. (1972), Attempts to calculate global solutions of problems that may have local minima, in: Lootsma (1972a), p.209-221
McCormick, G.P., J.D. Pearson (1969), Variable metric methods and unconstrained optimization, in: Fletcher (1969a), p.307-326
McCormick, G.P., K. Ritter (1972), Projection method for unconstrained optimization, JOTA 10, 57-66
McCormick, G.P., K. Ritter (1974), Alternative proofs of the convergence properties of the conjugate-gradient method, JOTA 13, 497-518
McGhee, R.B. (1967), Some parameter-optimization techniques, in: Klerer and Korn (1967), p.234-255
McGhee, R.B., R.B.Walford (1968), A Monte Carlo approach to the evaluation of conditional expectation parameter estimates for nonlinear dynamic systems, IEEE Trans. AC-13, 29-37
McGrew, D.R., Y.Y. Haimes (1974), Parametric solution to the joint system identification and optimization problems, JOTA 13, 582-605
McLeod, J. (ed.) (1968), Simulation, McGraw-Hill, New York
McMillan, C., jr. (1970), Mathematical programming: an introduction to the design and application of optimal decision machines, Wiley, New York
McMurtry, G.J. (1965), A study of stochastic automata as models of adaptive and learning controllers, Ph.D. thesis, Purdue Univ., Lafayette, Ind., June 1965
McMurtry, G.J., K.S. Fu (1966), A variable structure automaton used as a multimodal searching technique, IEEE Trans. AC-11, 379-387
Medvedev, G.A. (1963), Behaviour of automatic search systems under the influence of random noise, Engng. Cybern. 1, 3, 53-60
Medvedev, G.A. (1968), Stepping automatic search systems with random step length, ARC 29, 264-269
Medvedev, G.A., A.I. Ruban (1967), Automatic search system using step signals, Engng. Cybern. 5, 2, 146-158
Meerkov, S.M. (1972), Deceleration in the search for the global extremum, ARC 33, 2029-2038
Meissinger, H.F. (1964), Parameter optimization by an automatic open-loop computing method, Proc. IVth Int. Analogue Comp. Meet., Brighton, Sept. 1964, p.268-274

Meissinger, H.F., G.A. Bekey (1966), An analysis of continuous parameter identification methods, Simulation 6, 94-102

Merzenich, W. (1972), Ein einfaches mathematisches Evolutionsmodell, GMD, Mitt. 21, Bonn

Meyer, H.A. (ed.) (1956), Symposium on Monte Carlo methods, Wiley, New York

Michie, D. (1971), Heuristic search, Comp. J. 14, 96-102

Miele, A. (1969), Variational approach to the gradient method: theory and numerical experiments, in: Zadeh, Neustadt and Balakrishnan (1969b), p.143-157

Miele, A., J.W. Cantrell (1969), Study on a memory gradient method for the minimization of functions, JOTA 3, 459-470

Miele, A., J.W. Cantrell (1970), Memory gradient method for the minimization of functions, in: Balakrishnan, Contensou, DeVeubeke et al. (1970), p.252-263

Miele, A., J.N. Damoulakis, J.R. Cloutier, J.L. Tietze (1974), Sequential gradient-restoration algorithm for optimal control problems with nondifferential constraints, JOTA 13, 218-255

Miele, A., H.Y. Huang, J.C. Heidemann (1969), Sequential gradient-restoration algorithm for the minimization of constrained functions: ordinary and conjugate gradient versions, JOTA 4, 213-243

Miele, A., A.V. Levy, E.E. Cragg (1971), Modifications and extensions of the conjugate gradient-restoration algorithm for mathematical programming problems, JOTA 7, 450-472

Miele, A., J.L. Tietze, A.V. Levy (1972), Summary and comparison of gradient-restoration algorithms for optimal control problems, JOTA 10, 381-403

Miller, R.E. (1973), A comparison of some theoretical models of parallel computation, IEEE Trans. C-22 710-717

Millstein, R.E. (1973), Control structures in Illiac IV Fortran, CACM 16, 621-627

Minot, O.N. (1969), Artificial intelligence and new simulations, Simulation 13, 214-215

Minsky, M. (1961), Steps toward artificial intelligence, IRE Proc. 49, 8-30

Miranker, W.L. (1969), Parallel methods for approximating the root of a function, IBM J. Res. Dev. 13, 297-301

Miranker, W.L. (1971), A survey of parallelism in numerical analysis, SIAM Review 13, 524-547

Mitchell, B.A., jr., (1964), A hybrid analog-digital parameter optimizer for Astrac II, AFIPS Conf. Proc. 25, 271-285

Mitchell, R.A., J.L. Kaplan (1968), Nonlinear constrained optimization by a non-random Complex method, NBS J. Res. C, Engng. Instr., 72, 249-258

Mlynski, D. (1964a), Der Wirkungsgrad experimenteller Optimierungsstrategien, Dr.-Ing. Diss., Techn. Hochsch. Aachen, Dec. 1964

Mlynski, D. (1964b), Maximalisierung durch logische Suchprozesse, in: Steinbuch and Wagner (1964), p.82-94

Mlynski, D. (1966a), Ein Beitrag zur statistischen Theorie der Optimierungsstrategien I and II, Regelungstechnik 14, 209-215 and 325-330

Mlynski, D. (1966b), Efficiency of experimental strategies for optimizing feedback control of disturbed processes, Proc. IIIrd IFAC Congr., London, June 1966, paper 29-G
Mockus, J.B. see under Motskus, I.B.
Moisseev, N.N. (ed.)(1970), Colloquium on methods of optimization, Springer, Berlin
Moran, P.A.P. (1967), Unsolved problems in evolutionary theory, in: LeCam and Neyman (1967), vol. 4, p.457-480
Morrison, D.D. (1968), Optimization by least squares, SIAM J. Numer. Anal. 5, 83-88
Motskus, I.B. (1965), Some experiments related to the capabilities of man in solving multiextremal problems heuristically, Engng. Cybern. 3, 3, 40-44
Motskus, I.B. (1967), Mnogoekstremalnye sadachi v projektirovanii, Nauka, Moscow
Mockus, J.B. (1971), On the optimization of power distribution systems, in: Schwarz (1971), techn. papers vol. 3, p.6.3.2-1 to 6.3.2-14
Motskus, I.B., A.A. Feldbaum (1963), Symposium on multiextremal problems, Trakay, June 1963, Engng. Cybern. 1, 5, 154-155
Movshovich, S.M. (1966), Random search and the gradient method in optimization problems, Engng. Cybern. 4, 6, 39-48
Müller-Merbach, H. (1971), Operations Research: Methoden und Modelle der Optimalplanung, F. Vahlen, Berlin, 2nd Ed.
Mufti, I.H. (1970), Computational methods in optimal control problems, Springer, Berlin
Mugele, R.A. (1961), A nonlinear digital optimizing program for process control systems, AFIPS Conf. Proc. 19, 15-32
Mugele, R.A. (1962), A program for optimal control of nonlinear processes, IBM Syst. J. 1, 2-17
Mugele, R.A. (1966), The probe and edge theorems for non-linear optimization, in: Lavi and Vogl (1966), p.131-144
Munson, J.K., A.I. Rubin (1959), Optimization by random search on the analogue computer, IRE Trans. EC-8, 200-203
Murata, T. (1963), The use of adaptive constrained descent in systems design, Univ. Illinois, Coord. Sci. Lab., report R-189, Urbana, Ill., Dec. 1963
Murray, W. (ed.)(1972a), Numerical methods for unconstrained optimization, Academic Press, London
Murray, W. (1972b), Second derivative methods, in: Murray (1972a), p.57-71
Murray, W. (1972c), Failure, the causes and cures, in: Murray (1972a), p.107-122
Murtagh, B.A. (1970), A short description of the variable-metric method, in: Abadie (1970), p.525-528
Murtagh, B.A., R.W.H. Sargent (1970), Computational experience with quadratically convergent minimization methods, Comp. J. 13, 185-194
Mutseniyeks, V.A., L.A. Rastrigin (1964), Extremal control of continuous multiparameter systems by the method of random search, Engng. Cybern. 2, 1, 82-90
Myers, G.E. (1968), Properties of the conjugate-gradient and Davidon methods, JOTA 2, 209-219

Nachtigall, W. (1971), Biotechnik: statische Konstruktionen in der Natur, Quelle and Meyer, Heidelberg
Nake, F. (1966), Zertifikat zu Algorithmus 2: Orthonormierung von Vektoren nach E. Schmidt, Computing 1, 281
Neave, H.R. (1973), On using the Box-Müller transformation with multiplicative congruential pseudo-random number generators, Appl. Stat. 22, 92-97
Nelder, J.A., R. Mead (1965), A Simplex method for function minimization, Comp. J. 7, 308-313
Nenonen, L.K., B. Pagurek (1969), Conjugate gradient optimization applied to a copper converter model, Automatica 5, 801-810
Newman, D.J. (1965), Location of the maximum on unimodal surfaces, JACM 12, 395-398
Neyman, J. (ed.)(1951), Proc. IInd Berkeley Symp. Math. Stat. Probab., 1950, Univ. California Press, Berkeley, Calif.
Neyman, J. (ed.)(1956), Proc. IIIrd Berkeley Symp. Math. Stat. Probab., 1954/55, Univ. California Press, Berkeley, Calif.
Neyman, J. (ed.)(1961), Proc. IVth Berkeley Symp. Math. Stat. Probab., 1960, Univ. California Press, Berkeley, Calif.
Nickel, K. (1967), Allgemeine Forderungen an einen numerischen Algorithmus, ZAMM 47, Sonderheft, T67-T68
Nickel, K., K. Ritter (1972), Termination criterion and numerical convergence, SIAM J. Numer. Anal. 9, 277-283
Niemann, H. (1974), Methoden der Mustererkennung, Akadem. Verlagsges., Frankfurt/Main
Nikolić, Ž.J., K.S. Fu (1966), An algorithm for learning without external supervision and its application to learning control systems, IEEE Trans. AC-11, 414-442
Norkin, K.B. (1961), On one method for automatic search for the extremum of a function of many variables, ARC 22, 534-538
Odell, P.L. (1961), An empirical study of three stochastic approximation techniques applicable to sensitivity testing, report NAVWEPS-7837
Oestreicher, H.L., D.R. Moore (eds.)(1968), Cybernetic problems in bionics, Gordon and Breach, New York
Oi, K., H. Sayama, T.Takamatsu (1973), Computational schemes of the Davidon-Fletcher-Powell method in infinite-dimensional space, JOTA 12, 447-458
Oldenburger, R. (ed.) (1966), Optimal and self optimizing control, MIT Press, Cambridge, Mass.
Oliver, L.T., D.J. Wilde (1964), Symmetric sequential minimax search for a maximum, Fibonacci Quart. 2, 169-175
O'Neill, R. (1971), Algorithm AS 47: function minimization using a Simplex procedure, Appl. Stat. 20, 338-345
Opačić, J. (1973), A heuristic method for finding most extrema of a nonlinear functional, IEEE Trans. SMC-3, 102-107
Oren, S.S. (1973), Self-scaling variable metric algorithms without line search for unconstrained minimization, Math. Comp. 27, 873-885
Ortega, J.M., W.C. Rheinboldt (1967), Monotone iterations for nonlinear equations with application to Gauss-Seidel methods, SIAM J. Numer. Anal. 4, 171-190

Ortega, J.M., W.C. Rheinboldt (1970), Iterative solution of nonlinear equations in several variables, Academic Press, New York

Ortega, J.M., W.C. Rheinboldt (1972), A general convergence result for unconstrained minimization methods, SIAM J. Numer. Anal. 9, 40-43

Ortega, J.M., M.L. Rockoff (1966), Nonlinear difference equations and Gauss-Seidel type iterative methods, SIAM J. Numer. Anal. 3, 497-513

Osborne, M.R. (1972), Some aspects of nonlinear least squares calculations, in: Lootsma (1972a), p.171-189

Osche, G. (1972), Evolution: Grundlagen, Erkenntnisse, Entwicklungen der Abstammungslehre, Herder, Freiburg

Ostrowski, A.M. (1966), Solution of equations and systems of equations, Academic Press, New York, 2nd Ed.

Ostrowski, A.M. (1967), Contributions to the theory of the method of steepest descent, Arch. Ration. Mech. Anal. 26, 257-280

Overholt, K.J. (1965), An instability in the Fibonacci and golden section search methods, BIT 5, 284-286

Overholt, K.J. (1967a), Note on algorithm 2: Fibonacci search, and algorithm 7: Minx, and the golden section search, Comp. J. 9, 414

Overholt, K.J. (1967b), Algorithm 16: Gold, Comp. J. 9, 415

Overholt, K.J. (1967c), Algorithm 17: Goldsec, Comp. J. 9, 415

Overholt, K.J. (1973), Efficiency of the Fibonacci search method, BIT 13, 92-96

Pagurek, B., C.M. Woodside (1968), The conjugate gradient method for optimal control problems with bounded control variables, Automatica 4, 337-349

Palmer, J.R. (1969), An improved procedure for orthogonalising the search vectors in Rosenbrock's and Swann's direct search optimisation methods, Comp. J. 12, 69-71

Papentin, F. (1972), A Darwinian evolutionary system, Dr. rer. nat. Diss., Univ. Tübingen

Parkinson, J.M., D. Hutchinson (1972a), A consideration of non-gradient algorithms for the unconstrained optimization of functions of high dimensionality, in: Lootsma (1972a), p.99-113

Parkinson, J.M., D. Hutchinson (1972b), An investigation into the efficiency of variants on the Simplex method, in: Lootsma (1972a), p.115-135

Pask, G. (1962), Physical and linguistic evolution in self-organising systems, Proc. Ist IFAC Symp. Optim. Adapt. Contr., Rome, April 1962, p.199-227

Pask, G. (1971), A cybernetic experimental method and its underlying philosophy, Int. J. Man-Machine Stud. 3, 279-337

Patrick, M.L. (1972), A highly parallel algorithm for approximating all zeros of a polynomial with only real zeros, CACM 15, 952-955

Pattee, H.H., E.A. Edelsack, L. Fein, A.B. Callahan (eds.) (1966), Natural automata and useful simulations, Spartan, Washington, D.C.

Paviani, D.A., D.M. Himmelblau (1969), Constrained nonlinear optimization by heuristic programming, Oper. Res. 17, 872-882

Pearson, J.D. (1969), Variable metric methods of minimization, Comp. J. 12, 171-178

Peckham, G. (1970), A new method for minimising a sum of squares without calculating gradients, Comp. J. 13, 418-420
Pierre, D.A. (1969), Optimization theory with applications, Wiley, New York
Pierson, B.L., S.G. Rajtora (1970), Computational experience with the Davidon method applied to optimal control problems, IEEE Trans. SSC-6, 240-242
Pike, M.C., I.D. Hill, F.D. James (1967), Note on algorithm 2: Fibonacci search, and on algorithm 7: Minx, and algorithm 2 modified: Fibonacci search, Comp. J. 9, 416-417
Pike, M.C., J. Pixner (1965), Algorithm 2: Fibonacci search, Comp. Bull. 8, 147
Pincus, M. (1970), A Monte Carlo method for the approximate solution of certain types of constrained optimization problems, Oper. Res. 18, 1225-1228
Pinkham, R.S. (1964), Random root location, SIAM J. 12, 855-864
Pinsker, I. Sh., B.M. Tseitlin (1962), A nonlinear optimization problem, ARC 23, 1510-1518
Plane, D.R., C. McMillan, jr. (1971), Discrete optimization: integer programming and network analysis for management decisions, Prentice-Hall, Englewood Cliffs, N.J.
Plaschko, P., K. Wagner (1973), Evolutions-Linearisierungs-Programm zur Darstellung von numerischen Daten durch beliebige Funktionen, DFVLR Inst. Turbulenzforsch., Berlin, Forschungsbericht DLR-FB-73-55
Pluznikov, L.N., V.O. Andreyev, E.S. Klimenko (1971), Use of random search method in industrial planning, Engng. Cybern. 9, 229-235
Polak, E. (1971), Computational methods in optimization: a unified approach, Academic Press, New York
Polak, E. (1972), A survey of methods of feasible directions for the solution of optimal control problems, IEEE Trans. AC-17, 591-596
Polak, E. (1973), An historical survey of computational methods in optimal control, SIAM Review 15, 553-584
Polak, E., G. Ribière (1969), Note sur la convergence de méthodes de directions conjugées, Rev. Franc. Inf. Rech. Opér. 3, 16-R1 (Jan.-Feb.), 35-43
Polyak, B.T. (1969), The conjugate gradient method in extremal problems, USSR Comp. Math. and Math. Phys. 9, 4, 94-112
Ponstein, J. (1967), Seven kinds of convexity, SIAM Review 9, 115-119
Pontrjagin, L.S., V.G. Boltjanskij, R.V. Gamkrelidze, E.F. Misčenko (1967), Mathematische Theorie optimaler Prozesse, Oldenbourg, Munich, 2nd Ed.
Powell, D.R., J.R. MacDonald (1972), A rapidly convergent iterative method for the solution of the generalised nonlinear least squares problem, Comp. J. 15, 148-155
Powell, M.J.D. (1962), An iterative method for finding stationary values of a function of several variables, Comp. J. 5, 147-151
Powell, M.J.D. (1964), An efficient method for finding the minimum of a function of several variables without calculating derivatives, Comp. J. 7, 155-162
Powell, M.J.D. (1965), A method for minimizing a sum of squares of nonlinear functions without calculating derivatives, Comp. J. 7, 303-307

Powell, M.J.D. (1966), Minimization of functions of several variables, in: Walsh (1966), p.143-158
Powell, M.J.D. (1968a), On the calculation of orthogonal vectors, Comp. J. 11, 302-304
Powell, M.J.D. (1968b), A Fortran subroutine for solving systems of nonlinear algebraic equations, UKAEA, Res. Group, report AERE-R-5947, Harwell, Oxon
Powell, M.J.D. (1969), A theorem on rank one modifications to a matrix and its inverse, Comp. J. 12, 288-290
Powell, M.J.D. (1970a), Rank one methods for unconstrained optimization, in: Abadie (1970), p.139-156
Powell, M.J.D. (1970b), A survey of numerical methods for unconstrained optimization, SIAM Review 12, 79-97
Powell, M.J.D. (1970c), A Fortran subroutine for unconstrained minimization, requiring first derivatives of the objective function, UKAEA, Res. Group. report AERE-R-6469, Harwell, Oxon
Powell, M.J.D. (1970d), A Fortran subroutine for solving systems of nonlinear algebraic equations, in: Rabinowitz (1970), p. 115-161
Powell, M.J.D. (1970e), A hybrid method for nonlinear equations, in: Rabinowitz (1970), p.87-114
Powell, M.J.D. (1970f), Subroutine VA04A (Fortran), updated 20.5.1970, in: Hopper (1970), p.72
Powell, M.J.D. (1970g), Recent advances in unconstrained optimization, UKAEA, Res. Group., techn. paper AERE-TP-430, Harwell, Oxon, Nov. 1970
Powell, M.J.D. (1971), On the convergence of the variable metric algorithm, JIMA 7, 21-36
Powell, M.J.D. (1972a), Some properties of the variable metric algorithm, in: Lootsma (1972a), p.1-17
Powell, M.J.D. (1972b), Quadratic termination properties of minimization algorithms, I: statement and discussion of results, JIMA 10, 333-342
Powell, M.J.D. (1972c), Quadratic termination properties of minimization algorithms, II: proofs and theorems, JIMA 10, 343-357
Powell, M.J.D. (1972d), A survey of numerical methods for unconstrained optimization, in: Geoffrion (1972), p.3-21
Powell, M.J.D. (1972e), Problems related to unconstrained optimization, in: Murray (1972a), p.29-55
Powell, M.J.D. (1972f), Unconstrained minimization algorithms without computation of derivatives, UKAEA, Res. Group. techn. paper AERE-TP-483, Harwell, Oxon, April 1972
Poznyak, A.S. (1972), Use of learning automata for the control of random search, ARC 33, 1992-2000
Pugachev, V.N. (1970), Determination of the characteristics of complex systems using statistical trials and analytical investigation, Engng. Cybern. 8, 1109-1117
Pugh, E.L. (1966), A gradient technique of adaptive Monte Carlo, SIAM Review 8, 346-355
Pun, L. (1969), Introduction to optimization practice, Wiley, New York
Rabinowitz, P. (ed.) (1970), Numerical methods for nonlinear algebraic equations, Gordon and Breach, London

Ralston, A., H.S. Wilf (eds.) (1967), Mathematische Methoden für Digitalrechner, Oldenbourg, Munich
Ralston, A., H.S. Wilf (eds.) (1969), Mathematische Methoden für Digitalrechner II, Oldenbourg, Munich
Rastrigin, L.A. (1960), Extremal control by the method of random scanning, ARC 21, 891-896
Rastrigin, L.A. (1963), The convergence of the random search method in the extremal control of a many-parameter system, ARC 24, 1337-1342
Rastrigin, L.A. (1965a), Sluchainyi poisk v zadachakh optimisatsii mnogoparametricheskikh sistem, Zinatne, Riga, Translation (August 1967):
Rastrigin, L.A. (1965b), Random search in optimization problems for multiparameter systems, Air Force Syst. Comd., Foreign Techn. Div., FTD-HT-67-363
Rastrigin, L.A. (1966), Stochastic methods of complicated multi-parameter system optimization, Proc. IIIrd IFAC Congr., London, June 1966, paper 3-F
Rastrigin, L.A. (1967), Raboty po teorii i primeneniyu statisticheskikh metodov optimisatsii v institute elektroniki i vychislitelnoi tekhniki akademii nauk Latviiskoi SSR, Avtomatika i Vychislitelnaya Tekhnika (1967), 5, 31-40
Rastrigin, L.A. (1968), Statisticheskiye metody poiska, Nauka, Moscow
Rastrigin, L.A. (1969), Teorija i primenenije slučainogo poiska, Zinatne, Riga
Rastrigin, L.A. (1972), Adaptivnye sistemy, vol. 1, Zinatne, Riga
Rauch, S.W. (1973), A convergence theory for a class of nonlinear programming problems, SIAM J. Numer. Anal. 10, 207-228
Rechenberg, I. (1964), Cybernetic solution path of an experimental problem, Roy. Aircr. Establ., libr. transl. 1122, Farnborough, Hants., Aug. 1965, Translation of the unpublished written summary of the lecture 'Kybernetische Lösungsansteuerung einer experimentellen Forschungsaufgabe', delivered at the joint annual meeting of the WGLR and DGRR, Berlin, 1964
Rechenberg, I. (1973), Evolutionsstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution, Frommann-Holzboog, Stuttgart
Rein, H., M. Schneider (1971), Einführung in die Physiologie des Menschen, Springer, Berlin
Rhead, D.G. (1971), Some numerical experiments on Zangwill's method for unconstrained minimization, Univ. London, Inst. Comp. Sci., working paper ICSI-319
Ribière, G. (1970), Sur la méthode de Davidon-Fletcher-Powell pour la minimisation des fonctions, Mgmt. Sci. 16, 572-592
Rice, J.R. (1966), Experiments on Gram-Schmidt orthogonalization, Math. Comp. 20, 325-328
Richardson, J.A., J.L. Kuester (1973), Algorithm 454 (E4): the Complex method for constrained optimization, CACM 16, 487-489
Robbins, H., S. Monro (1951), A stochastic approximation method, Ann. Math. Stat. 22, 400-407
Roberts, P.D., R.H. Davis (1969), Conjugate gradients, Control 13, 206-210

Roberts, S.M., H.I. Lyvers (1961), The gradient method in process control, Ind. Engng. Chem. 53, 877-882

Rosen, J.B. (1960), The gradient projection method for nonlinear programming I: linear constraints, SIAM J. 8, 181-217

Rosen, J.B. (1961), The gradient projection method for nonlinear programming II: nonlinear constraints, SIAM J. 9, 514-532

Rosen, J.B. (1966), Iterative solution of nonlinear optimal control problems, SIAM J. Contr. 4, 223-244

Rosen, J.B., O.L. Mangasarian, K. Ritter (eds.) (1970), Nonlinear programming, Academic Press, New York

Rosen, J.B., S. Suzuki (1965), Construction of nonlinear programming test problems, CACM 8, 113

Rosen, R. (1967), Optimality principles in biology, Butterworths, London

Rosenblatt, F. (1958), The perceptron: a probabilistic model for information storage and organization in the brain, Psychol. Rev. 65, 386-408

Rosenbrock, H.H. (1960), An automatic method for finding the greatest or least value of a function, Comp. J. 3, 175-184

Rosenbrock, H.H., C. Storey (1966), Computational techniques for chemical engineers, Pergamon Press, Oxford

Ross, G.J.S. (1971), The efficient use of function minimization in non-linear maximum-likelihood estimation, Appl. Stat. 19, 205-221

Rothe, R. (1959), Höhere Mathematik für Mathematiker, Physiker und Ingenieure, I: Differentialrechnung und Grundformeln der Integralrechnung nebst Anwendungen, Teubner, Leipzig, 18th Ed.

Rutishauser, H. (1966), Algorithmus 2: Orthonormierung von Vektoren nach E. Schmidt, Computing 1, 159-161

Rybashov, M.V. (1965a), The gradient method of solving convex programming problems on electronic analog computers, ARC 26, 1886-1898

Rybashov, M.V. (1965b), Gradient method of solving linear and quadratic programming problems on electronic analog computers, ARC 26, 2079-2089

Rybashov, M.V. (1969), Insensitivity of gradient systems in the solution of linear problems on analog computers, ARC 30, 1679-1687

Ryshik, I.M., I.S. Gradstein (1963), Summen-, Produkt- und Integraltafeln, Dt. Vlg. Wissensch., Berlin, 2nd Ed.

Saaty, T.L. (1955), The number of vertices of a polyhedron, Amer. Math. Monthly 62, 326-331

Saaty, T.L. (1963), A conjecture concerning the smallest bound on the iterations in linear programming, Oper. Res. 11, 151-153

Saaty, T.L. (1970), Optimization in integers and related extremal problems, McGraw-Hill, New York

Saaty, T.L., J. Bram (1964), Nonlinear mathematics, McGraw-Hill, New York

Sacks, J. (1958), Asymptotic distribution of stochastic approximation procedures, Ann. Math. Stat. 29, 373-405

Sameh, A.H. (1971), On Jacobi and Jacobi-like algorithms for a parallel computer, Math. Comp. 25, 579-590

Samuel, A.L. (1963), Some studies in machine learning using the game of checkers, in: Feigenbaum and Feldman (1963), p.71-105

Sargent, R.W.H., D.J. Sebastian (1972), Numerical experience with algorithms for unconstrained minimization, in: Lootsma (1972a), p.45-68

Sargent, R.W.H., D.J. Sebastian (1973), On the convergence of sequential minimization algorithms, JOTA 12, 567-575

Saridis, G.N. (1968), Learning applied to successive approximation algorithms, Proc. 1968 Joint Autom. Contr. Conf., Ann Arbor, Mich., p.1007-1013

Saridis, G.N. (1970), Learning applied to successive approximation algorithms, IEEE Trans. SSC-6, 97-103

Saridis, G.N., H.D. Gilbert (1970), Self-organizing approach to the stochastic fuel regulator problem, IEEE Trans. SSC-6, 186-191

Satterthwaite, F.E. (1959a), REVOP or random evolutionary operation, Merrimack College, report 10-10-59, North Andover, Mass.

Satterthwaite, F.E. (1959b), Random balance experimentation, Technometrics 1, 111-137

Satterthwaite, F.E., D. Shainin (1959), Pinpoint important process variable with polyvariable experimentation, J. Soc. Plast. Engrs. 15, 225-230

Savage, J.M. (1966), Evolution, Bayer. Landwirtsch. Vlg., Munich

Sawaragi, Y., T. Takamatsu, K. Fukunaga, E. Nakanishi, H. Tamura (1971), Dynamic version of steady state optimizing control of a distillation column by trial method, Automatica 7, 509-516

Schechter, S. (1962), Iteration methods for nonlinear problems, Trans. Amer. Math. Soc. 104, 179-189

Schechter, S. (1968), Relaxation methods for convex problems, SIAM J. Numer. Anal. 5, 601-612

Schechter, S. (1970), Minimization of a convex function by relaxation, in: Abadie (1970), p.177-190

Scheeffer, L. (1886), Über die Bedeutung der Begriffe 'Maximum und Minimum' in der Variationsrechnung, Mathem. Annalen 26, 197-208

Scheuer, E.M., D.S. Stoller (1962), On the generation of normal random deviates, Technometrics 4, 278-281

Schinzinger, R. (1966), Optimization in electromagnetic system design, in: Lavi and Vogl (1966), p.163-214

Schley, C.H., jr. (1968), Conjugate gradient methods for optimization, Gen. Electr., Res. Dev. Cntr., report 68-C-008, Schenectady, N.Y., Jan. 1968

Schmalhausen, I.I. (1964), Grundlagen des Evolutionsprozesses vom kybernetischen Standpunkt, in: Ljapunov, Kämmerer and Thiele (1964a), p.151-188

Schmetterer, L. (1961), Stochastic approximation, in: Neyman (1961), vol. 1, p.587-609

Schmidt, J.W., H. Schwetlick (1968), Ableitungsfreie Verfahren mit höherer Konvergenzgeschwindigkeit, Computing 3, 215-226

Schmidt, J.W., H.F. Trinkaus (1966), Extremwertermittlung mit Funktionswerten bei Funktionen von mehreren Veränderlichen, Computing 1, 224-232

Schmidt, J.W., K. Vetters (1970), Ableitungsfreie Verfahren für nichtlineare Optimierungsprobleme, Numer. Math. 15, 263-282

Schmitt, E. (1969), Adaptive computer algorithms for optimization and root-finding, NTZ-Report 6, VDE Vlg., Berlin

Schrack, G., N. Borowski (1972), An experimental comparison of three random searches, in: Lootsma (1972a), p.137-147
Schumer, M.A. (1967), Optimization by adaptive random search, Ph. D. thesis, Princeton Univ., Princeton, N.J., Nov. 1967
Schumer, M.A. (1969), Hill climbing on a sample function of a Gaussian Markov process, JOTA 4, 413-418
Schumer, M.A., K. Steiglitz (1968), Adaptive step size random search, IEEE Trans. AC-13, 270-276
Schuster, P. (1972), Vom Makromolekül zur primitiven Zelle: die Entstehung biologischer Funktion, Chemie in unserer Zeit 6, 1, 1-16
Schwarz, H. (ed.) (1971), Multivariable technical control systems, North-Holland, Amsterdam
Schwarz, H.R., H. Rutishauser, E. Stiefel (1968), Numerik symmetrischer Matrizen, Teubner, Stuttgart
Schwetlick, H. (1970), Algorithmus 12: Ein ableitungsfreies Verfahren zur Lösung endlich-dimensionaler Gleichungssysteme, Computing 5, 82-88 and 393
Sergiyevskiy, G.M., A.P. Ter-Saakov (1970), Factor experiments in many-dimensional stochastic approximation of an extremum, Engng. Cybern. 8, 949-954
Shah, B.V., R.J. Buehler, O. Kempthorne (1964), Some algorithms for minimizing a function of several variables, SIAM J. 12, 74-92
Shanno, D.F. (1970a), Parameter selection for modified Newton methods for function minimization, SIAM J. Numer. Anal. 7, 366 - 372
Shanno, D.F. (1970b), Conditioning of quasi-Newton methods for function minimization, Math. Comp. 24, 647-656
Shanno, D.F., P.C. Kettler (1970), Optimal conditioning of quasi-Newton methods, Math. Comp. 24, 657-664
Shapiro, I.J., K.S. Narendra (1969), Use of stochastic automata for parameter self-optimization with multimodal performance criteria, IEEE Trans. SSC-5, 352-360
Shedler, G.S. (1967), Parallel numerical methods for the solution of equations, CACM 10, 286-291
Shimizu, T. (1969), A stochastic approximation method for optimization problems, JACM 16, 511-516
Shubert, B.O. (1972), A sequential method seeking the global maximum of a function, SIAM J. Numer. Anal. 9, 379-388
Silverman, G. (1969), Remark on algorithm 315(E4): the damped Taylor's series method for minimizing a sum of squares and for solving systems of non-linear equations, CACM 12, 513
Singer, E. (1962), Simulation and optimization of oil refinery design, in: Dickert, Enyedy, Huckaba et al. (1962), p.62-74
Sirisena, H.R. (1973), Computation of optimal controls using a piecewise polynomial parameterization, IEEE Trans. AC-18, 409-411
Slagle, J.R. (1972), Einführung in die heuristische Programmierung: künstliche Intelligenz und intelligente Maschinen, Vlg. Moderne Industrie, Munich
Smith, C.S. (1962), The automatic computation of maximum likelihood estimates, Nat. Coal Board, Sci. Dept., report SC-846-MR-40, June 1962

Smith, D.E. (1973), An empirical investigation of optimum-seeking in the computer simulation situation, Oper. Res. 21, 475-497
Smith, F.B., jr., D.F. Shanno,(1971), An improved Marquardt procedure for non-linear regressions, Technometrics 13, 63-74
Smith, L.B. (1969), Remark on algorithm 178 (E4): direct search, CACM 12, 638
Smith, N.H., D.F. Rudd (1964), The feasibility of directed random search, Univ. Wisconsin, Dept. Chem. Engng., report
Snell, F.M. (ed.) (1967), Progress in theoretical biology, vol. 1, Academic Press, New York
Sorenson, H.W. (1969), Comparison of some conjugate direction procedures for function optimization, J.Franklin Inst. 288, 421-441
Southwell, R.V. (1940), Relaxation methods in engineering science: a treatise on approximate computation, Oxford Univ. Press, Oxford
Southwell, R.V. (1946), Relaxation methods in theoretical physics, Clarendon Press, Oxford
Späth, H. (1967), Algorithm 315(E4,C5): the damped Taylor's series method for minimizing a sum of squares and for solving systems of nonlinear equations, CACM 10, 726-728
Spang, H.A., III (1962), A review of minimization techniques for nonlinear functions, SIAM Review 4, 343-365
Spedicato, E. (1973), Stability of Huang's update for the conjugate gradient method, JOTA 11, 469-479
Spendley, W. (1969), Nonlinear least squares fitting using a modified Simplex minimization method, in: Fletcher (1969a), p.259-270
Spendley, W., G.R. Hext, F.R. Himsworth (1962), Sequential application of Simplex designs in optimization and evolutionary operation, Technometrics 4, 441-461
Speyer, J.L., H.J. Kelley, N. Levine, W.F. Denham (1971), Accelerated gradient projection technique with application to rocket trajectory optimization, Automatica 7, 37-43
Stanton, E.L. (1969), A discrete element analysis of elastoplastic plates by energy minimization, Ph.D. thesis, Case Western Reserve Univ., Jan. 1969
Stark, R.M., R.L. Nicholls (1972), Mathematical foundations for design: civil engineering systems, McGraw-Hill, New York
Stebbins, G.L. (1968), Evolutionsprozesse, G. Fischer, Stuttgart
Stein, M.L. (1952), Gradient methods in the solution of systems of linear equations, NBS J. Res. 48, 407-413
Steinbuch, K. (1971), Automat und Mensch, Springer, Berlin, 4th Ed.
Steinbuch, K., S.W. Wagner (eds.) (1964), Neuere Ergebnisse der Kybernetik, Oldenbourg, Munich
Stewart, E.C., W.P. Kavanaugh, D.H. Brocker (1967), Study of a global search algorithm for optimal control, Proc. Vth Int. Analogue Comp. Meet., Lausanne, Aug.-Sept. 1967, p.207-230
Stewart, G.W. (1967), A modification of Davidon's minimization method to accept difference approximations of derivatives, JACM 14, 72-83
Stewart, G.W. (1973), Conjugate direction methods for solving systems of linear equations, Numer. Math. 21, 285-297

Stiefel, E. (1952), Über einige Methoden der Relaxationsrechnung, ZAMP 3, 1-33

Stiefel, E. (1965), Einführung in die numerische Mathematik, Teubner, Stuttgart, 4th Ed.

Stoer, J., C. Witzgall (1970), Convexity and optimization in finite dimensions I, Springer, Berlin

Stolz, O. (1893), Grundzüge der Differential- und Integralrechnung, erster Theil: reelle Veränderliche und Functionen, Abschnitt V: die grössten und kleinsten Werthe der Functionen, p.199-258, Teubner, Leipzig

Stone, H.S. (1973a), Parallel computation: an introduction, IEEE Trans. C-22, 709-710

Stone, H.S. (1973b), An efficient parallel algorithm for the solution of a tri-diagonal linear system of equations, JACM 20, 27-38

Storey, C. (1962), Applications of a hill climbing method of optimization, Chem. Engng. Sci. 17, 1, 45-52

Storey, C., H.H. Rosenbrock (1964), On the computation of the optimal temperature profile in a tubular reaction vessel, in: Balakrishnan and Neustadt (1964), p.23-64

Stratonovich,R.L.(1968), Does there exist a theory of synthesis of optimal adaptive, self-learning and self-adaptive systems? ARC 29,83-92

Stratonovich, R.L. (1970), Optimal algorithms of the stochastic approximation type, Engng. Cybern. 8, 20-27

Strongin, R.G. (1970), Multi-extremal minimization, ARC 31, 1085-1088

Strongin, R.G. (1971), Minimization of many-extremal functions of several variables, Engng. Cybern. 9, 1004-1010

Suchowitzki, S.I., L.I. Awdejewa (1969), Lineare und konvexe Programmierung, Oldenbourg, Munich

Sugie, N. (1964), An extension of Fibonaccian searching to multidimensional cases, IEEE Trans. AC-9, 105

Sutti, C., L. Trabattoni, P. Brughiera (1972), A method for minimization of a one-dimensional nonunimodal function, in: Szegö (1972), p.181-192

Svechinskii, V.B. (1971), Random search in probabilistic iterative algorithms, ARC 32, 76-80

Swann, W.H. (1964), Report on the development of a new direct searching method of optimization, ICI, Centr. Instr. Lab., research note 64-3, Middlesborough, Yorks., June 1964

Swann, W.H. (1969), A survey of non-linear optimization techniques, FEBS-Letters 2, March, Suppl., pp.S39-S55

Swann, W.H. (1972), Direct search methods, in: Murray (1972a), p.13-28

Sweschnikow, A.A. (ed.) (1970), Wahrscheinlichkeitsrechnung und mathematische Statistik in Aufgaben, Teubner, Leipzig

Sydow, A. (1968), Eine Methode zur exakten Realisierung des Gradientenverfahrens auf dem iterativ-arbeitenden Analogrechner, messen-steuern-regeln 11, 462-465

Synge, J.L. (1944), A geometrical interpretation of the relaxation method, Quart. Appl. Math. 2, 87-89

Szegö, G.P. (ed.) (1972), Minimization algorithms, mathematical theories, and computer results, Academic Press, New York

Szegö, G.P., G. Treccani (1972), Axiomatization of minimization algorithms and a new conjugate gradient method, in: Szegö (1972), p.193-216

Tabak, D. (1969), Comparative study of various minimization techniques used in mathematical programming, IEEE Trans. AC-14, 572

Tabak, D. (1970), Applications of mathematical programming techniques in optimal control: a survey, IEEE Trans. AC-15, 688-690

Talkin, A.I. (1964), The negative gradient method extended to the computer programming of simultaneous systems of differential and finite equations, AFIPS Conf. Proc. 26, 539-543

Tapley, B.D., J.M. Lewallen (1967), Comparison of several numerical optimization methods, JOTA 1, 1-32

Taran, V.A. (1968a), A discrete adaptive system with random search for the optimum, Engng. Cybern. 6, 4, 142-150

Taran, V.A. (1968b), Adaptive systems with random extremum search, ARC 29, 1447-1455

Tazaki, E., A. Shindo, T. Umeda (1970), Decentralized optimization of a chemical process by a feasible method, IFAC Kyoto Symp. Syst. Engng. Appr. Comp. Contr., Aug. 1970, paper 25.1

Thom, R. (1969), Topological models in biology, Topology 8, 313-336

Thomas, M.E., D.J. Wilde (1964), Feed-forward control of over-determined systems by stochastic relaxation, in: Blakemore and Davis (1964), p.16-22

Titterington, D.M. (1973), A method of extremum adaptation, JIMA 11, 297-315

Todd, J. (1949), The condition of certain matrices I, Quart. J. Mech. Appl. Math. 2, 469-472

Tokumaru, H., N. Adachi, K. Goto (1970), Davidon's method for minimization problems in Hilbert space with an application to control problems, SIAM J. Contr. 8, 163-178

Tolle, H. (1971), Optimierungsverfahren für Variationsaufgaben mit gewöhnlichen Differentialgleichungen als Nebenbedingungen, Springer, Berlin

Tomlin, F.K., L.B. Smith (1969), Remark on algorithm 178(E4): direct search, CACM 12, 637-638

Tovstucha, T.I. (1960), The effect of random noise on the steady-state operation of a step-type extremal system for an object with a parabolic characteristic, ARC 21, 398-404

Traub, J.F. (1964), Iterative methods for the solution of equations, Prentice-Hall, Englewood Cliffs, N.J.

Treccani, G., L. Trabattoni, G.P. Szegö (1972), A numerical method for the isolation of minima, in: Szegö (1972), p.239-289

Zypkin, Ja. S. (1966), Adaption und Lernen in automatischen Systemen, Oldenbourg, Munich

Zypkin, Ja. S. (1967), Probleme der Adaption in automatischen Systemen, messen-steuern-regeln 10, 362-365

Tsypkin, Ya. Z. (1968a), All the same, does a theory of synthesis of optimal adaptive systems exist? ARC 29, 93-98

Tsypkin, Ya.Z. (1968b), Optimal hybrid adaptation and learning algorithms, ARC 29, 1271-1276

Tsypkin, Ya.Z. (1968c), Self-learning, what is it? IEEE Trans. AC-13, 608-612

Zypkin, Ja.S. (1970a), Adaption und Lernen in kybernetischen Systemen, Oldenbourg, Munich
Tsypkin, Ya.Z. (1970b), On learning systems, IFAC Kyoto Symp. Syst. Engng. Appr. Comp. Contr., Aug. 1970, paper 34.1
Tsypkin, Ya.Z. (1970c), Generalized learning algorithms, ARC 31, 86-92
Tsypkin, Ya.Z. (1971), Smoothed randomized functionals and algorithms in adaptation and learning theory, ARC 32, 1190, 1209
Tsypkin, Ya. Z., A.I. Kaplinskiy, K.A. Larionov (1970), Adaptation and learning algorithms under nonstationary conditions, Engng. Cybern. 8, 829-840
Tsypkin, Ya. Z., A.S. Poznyak (1972), Finite learning automata, Engng. Cybern. 10, 478-490
Ueing, U. (1971), Zwei Lösungsmethoden für nichtkonvexe Programmierungsprobleme, Springer, Berlin
Ueing, U. (1972), A combinatorial method to compute a global solution of certain non-convex optimization problems, in: Lootsma (1972a), p.223-230
Unbehauen, H. (1971), On the parameter optimization of multivariable control systems, in: Schwarz (1971), techn.papers vol. 2, p.2.2.10-1 to 2.2.10-11
Vagin, V.N., L. Ye. Rudelson (1968), An example of a self-organizing system, Engng. Cybern. 6, 6, 33-40
Vajda, S. (1961), Mathematical programming, Addison-Wesley, Reading, Mass.
Vajda, S. (1967), The mathematics of experimental design, Griffin, London
VanNorton, R. (1967), Lösung linearer Gleichungssysteme nach dem Verfahren von Gauss-Seidel, in: Ralston and Wilf (1967), p.92-105
Varah, J.M. (1965), Certification of algorithm 203 (E4): steep 1, CACM 8, 171
Varga, J. (1974), Praktische Optimierung: Verfahren und Anwendungen der linearen und nichtlinearen Optimierung, Oldenbourg, Munich
Varga, R.S. (1962), Matrix iterative analysis, Prentice-Hall, Englewood Cliffs, N.J.
Vaysbord, E.M.(1967), Asymptotic estimates of the rate of convergence of random search, Engng. Cybern. 5, 4, 22-32
Vaysbord, E.M. (1968), Convergence of a method of random search, Engng. Cybern. 6, 3, 44-48
Vaysbord, E.M. (1969), Convergence of a certain method of random search for a global extremum of a random function, Engng. Cybern. 7, 1, 46-50
Vaysbord, E.M., D.B. Yudin (1968), Multiextremal stochastic approximation, Engng. Cybern. 6, 5, 1-11
Venter, J.H. (1967), An extension of the Robbins-Monro procedure, Ann. Math. Stat. 38, 181-190
Viswanathan, R., K.S. Narendra (1972), A note on the linear reinforcement scheme for variable-structure stochastic automata, IEEE Trans. SMC-2, 292-294
Vitale, P., G. Taylor (1968), A note on the application of Davidon's method to nonlinear regression problems, Technometrics 10, 843-849
Vogelsang, R. (1963), Die mathematische Theorie der Spiele, Dümmler, Bonn

Voltaire, F.M. Arouet de (1759), Candide oder der Optimismus, Insel, Frankfurt/Main, 1973
Volz, R.A. (1965), The minimization of a function by weighted gradients, IEEE Proc. 53, 646-647
Volz, R.A. (1973), Example of function optimization via hybrid computation, Simulation 21, 43-48
von Neumann, J. (1960), Die Rechenmaschine und das Gehirn, Oldenbourg, Munich
von Neumann, J., O. Morgenstern (1961), Spieltheorie und wirtschaftliches Verhalten, Physica, Würzburg
Waddington, C.H. (ed.) (1968), Towards a theoretical biology I: prolegomena, Edinburgh Univ. Press, Edinburgh
Wald, A. (1966), Sequential analysis, Wiley, New York, 8th Ed.
Wallack, P. (1964), Certification of algorithm 203(E4): steep 1, CACM 7, 585
Walsh, J. (ed.) (1966), Numerical analysis: an introduction, Academic Press, London
Ward, L., A. Nag, L.C.W. Dixon (1969), Hill-climbing techniques as a method of calculating the optical constants and thickness of a thin metallic film, Brit. J. Appl. Phys. (J. Phys. D), Ser. 2, 2, 301-304
Wasan, M.T. (1969), Stochastic approximation, Cambridge Univ. Press, Cambridge
Wasscher, E.J. (1963a), Algorithm 203(E4): steep 1, CACM 6, 517-519
Wasscher, E.J. (1963b), Algorithm 204(E4): steep 2, CACM 6, 519
Wasscher, E.J. (1963c), Remark on algorithm 129(E4): minifun, CACM 6, 521
Wasscher, E.J. (1965), Remark on algorithm 205(E4): ative, CACM 8, 171
Weber, H.H. (1972), Einführung in Operations Research, Akad. Verlagsges., Frankfurt/Main
Wegge, L. (1966), On a discrete version of the Newton-Raphson method, SIAM J. Numer. Anal. 3, 134-142
Weinberg, F. (ed.) (1968), Einführung in die Methode Branch and Bound, Springer, Berlin
Weinberg, F., C.A. Zehnder (eds.)(1969), Heuristische Planungsmethoden, Springer, Berlin
Weisman, J., C.F. Wood (1966), The use of optimal search for engineering design, in: Lavi and Vogl (1966), p.219-228
Weisman, J., C.F. Wood, L. Rivlin (1965), Optimal design of chemical process systems, AIChE Chem. Engng. Progr. Symp. Ser. 61, no.55, p.50-63
Weiss, E.A., D.H. Archer, D.A. Burt (1961), Computer sets tower for best run, Petrol Refiner 40, 10, 169-174
Wells, M. (1965), Algorithm 251(E4)): function minimization (Flepomin), CACM 8, 169-170
Werner, J. (1974), Über die Konvergenz des Davidon-Fletcher-Powell-Verfahrens für streng konvexe Minimierungsaufgaben im Hilbert-raum, Computing 12, 167-176
Wheeling, R.F. (1960), Optimizers: their structure, CACM 3, 632-638
White, L.J., R.G. Day (1971), An evaluation of adaptive step-size random search, IEEE Trans. AC-16, 475-478

White, R.C., jr. (1970), Hybrid-computer optimization of systems with random parameters, Ph.D. thesis, Univ. Arizona, Tucson, Ariz., June 1970
White, R.C., jr. (1971), A survey of random methods for parameter optimization, Simulation 17, 197-205
Whitley, V.W. (1962), Algorithm 129(E4): minifun, CACM 5, 550-551
Whittle, P. (1971), Optimization under constraints: theory and applications of nonlinear programming, Wiley-Interscience, London
Wiener, N. (1963), Kybernetik: Regelung und Nachrichtenübertragung in Lebewesen und Maschine, Econ, Düsseldorf
Wiener, N., J.P. Schadé (eds.) (1965), Progress in biocybernetics, vol. 2, Elsevier, Amsterdam
Wilde, D.J. (1963), Optimization by the method of contour tangents, AIChE J. 9, 2, 186-190
Wilde, D.J. (1964), Optimum seeking methods, Prentice-Hall, Englewood Cliffs, N.J.
Wilde, D.J. (1965), A multivariable dichotomous optimum-seeking method, IEEE Trans. AC-10, 85-87
Wilde, D.J. (1966), Objective function indistinguishability in unimodal optimization, in: Lavi and Vogl (1966), p.341-350
Wilde, D.J., C.S. Beightler (1967), Foundations of optimization, Prentice-Hall, Englewood Cliffs, N.J.
Wilkinson, J.H. (1965), The algebraic eigenvalue problem, Oxford Univ. Press, London
Wilkinson, J.H., C. Reinsch (1971), Handbook for automatic computation, vol 2: linear algebra, Springer, Berlin
Wilson, E.O., W.H. Bossert (1973), Einführung in die Populationsbiologie, Springer, Berlin
Witte, B.F.W., W.R. Holst (1964), Two new direct minimum search procedures for functions of several variables, AFIPS Conf. Proc. 25, 195-209
Witten, I.H. (1972), Comments on 'use of stochastic automata for parameter self-optimization with multimodal performance criteria', IEEE Trans. SMC-2, 289-292
Wolf, F., A. Schmitt (1966), Modelle lernender Automaten, Vieweg, Braunschweig
Wolfe, P. (1959a), The simplex method for quadratic programming, Econometrica 27, 382-398
Wolfe, P. (1959b), The secant method for simultaneous nonlinear equations, CACM 2, 12-13
Wolfe, P. (1966), On the convergence of gradient methods under constraints, IBM Zurich, Res. Lab., report RZ-204, March 1966
Wolfe, P. (1967), Another variable metric method, IBM working paper
Wolfe, P. (1969), Convergence conditions for ascent methods, SIAM Review 11, 226-235
Wolfe, P. (1970), Convergence theory in nonlinear programming, in: Abadie (1970), p.1-36
Wolfe, P. (1971), Convergence conditions for ascent methods II: some corrections, SIAM Review 13, 185-188
Wood, C.F. (1960), Application of direct search to the solution of engineering problems, Westinghouse, Res. Lab., sci. paper 6-41210-1-P1, Pittsburgh, Pa., Oct. 1960

Wood, C.F. (1962), Recent developments in direct search techniques, Westinghouse, Res. Lab., res. paper 62-159-522-R1, Pittsburgh, Pa.
Wood, C.F. (1965), Review of design optimization techniques, IEEE Trans. SSC-1, 14-20
Yates, F. (1967), A fresh look at the basic principles of the design and analysis of experiments, in: LeCam and Neyman (1967), vol. 4, p.777-790
Youden, W.J., O. Kempthorne, J.W. Tukey, G.E.P. Box, J.S. Hunter, F.E. Satterthwaite, T.A. Budne (1959), Discussion of the papers of Messrs. Satterthwaite and Budne, Technometrics 1, 157-193
Yovits, M.C., S. Cameron (eds.) (1960), Self-organizing systems, Pergamon Press, Oxford
Yovits, M.C., G.T. Jacobi, D.G. Goldstein (eds.) (1962), Self-organizing systems, Spartan, Washington, D.C.
Yudin, D.B. (1965), Quantitative analysis of complex systems I, Engng. Cybern. 3, 1, 1-9
Yudin, D.B. (1966), Quantitative analysis of complex systems II, Engng. Cybern. 4, 1, 1-13
Yudin, D.B. (1972), New approaches to formalizing the choice of decisions in complex situations, ARC 33, 747-756
Yvon, J.P., (1972), On some random search methods, in: Szegö (1972), p.313-335
Zach, F. (1974), Technisches Optimieren, Springer, Vienna
Zadeh, L.A., L.W. Neustadt, A.V. Balakrishnan (eds.) (1969a), Computing methods in optimization problems 2, Academic Press, London
Zadeh, L.A., L.W. Neustadt, A.V. Balakrishnan (eds.) (1969b), Computing methods in optimization problems, Springer, Berlin
Zadeh, N. (1970), A note on the cyclic coordinate ascent method, Mgmt. Sci. 16, 642-644
Zahradnik, R.L. (1971), Theory and techniques of optimization for practicing engineers, Barnes and Noble, New York
Zakharov, V.V. (1969), A random search method, Engng. Cybern. 7, 2, 26-30
Zakharov, V.V. (1970), The method of integral smoothing in many-extremal and stochastic problems, Engng. Cybern. 8, 637-642
Zangwill, W.I. (1967), Minimizing a function without calculating derivatives, Comp. J. 10, 293-296
Zangwill, W.I. (1969), Nonlinear programming: a unified approach, Prentice-Hall, Englewood Cliffs, N.J.
Zeleznik, F.J. (1968), Quasi-Newton methods for nonlinear equations, JACM 15, 265-271
Zellnik, H.E., N.E. Sondak, R.S. Davis (1962), Gradient search optimization, Chem. Engng. Progr. 58, 8, 35-41
Zettl, G. (1970), Ein Verfahren zum Minimieren einer Funktion bei eingeschränktem Variationsbereich der Parameter, Numer. Math. 15, 415-432
Zigangirov, K.S. (1965), Optimal search in the presence of noise, Engng. Cybern. 3, 4, 112-116
Zoutendijk, G. (1960), Methods of feasible directions: a study in linear and nonlinear programming, Elsevier, Amsterdam
Zoutendijk, G. (1970), Nonlinear programming: computational methods, in: Abadie (1970), p.37-86

Zurmühl, R. (1965), Praktische Mathematik für Ingenieure und Physiker, Springer, Berlin, 5th Ed.
Zwart, P.B. (1970), Nonlinear programming: a quadratic analysis of ridge paralysis, JOTA 6, 331-339
Zwart, P.B. (1973), Nonlinear programming: counterexample to two global optimization algorithms, Oper. Res. 21, 1260-1266
Zypkin, Ja. S. see under Tsypkin, Ya. Z.