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A Benchmark-Suite of real-World constrained multi-objective optimization problems and some baseline results

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ABSTRACT

Generally, Synthetic Benchmark Problems (SBPs) are utilized to assess the performance of metaheuristics. However, these SBPs may include various unrealistic properties. As a consequence, performance assessment may lead to underestimation or overestimation. To address this issue, few benchmark suites containing real-world problems have been proposed for all kinds of metaheuristics except for Constrained Multi-objective Metaheuristics (CMOMs). To fill this gap, we develop a benchmark suite of Real-world Constrained Multi-objective Optimization Problems (RWCMOPs) for performance assessment of CMOMs. This benchmark suite includes 50 problems collected from various streams of research. We also present the baseline results of this benchmark suite by using state-of-the-art algorithms. Besides, for comparative analysis, a ranking scheme is also proposed.

1. Introduction

During the past decades, Constrained Multi-objective Optimization Problems (CMOPs) has gained a lot of attention since the majority of optimization problems of real-world applications contain constraints. Generally, a CMOP has multiple conflicting objectives with one or more constraints that demand to optimize these objectives while satisfying the constraints simultaneously. In CMOPs, Evolutionary Algorithms (EAs) and other metaheuristics have to provide proper tradeoffs among the conflicting objectives while satisfying all constraints, which is a great challenge to them [1,2].

Without losing generality, a CMOP can be defined mathematically:

$$\text{Minimize } f_1(\bar{x}), f_2(\bar{x}), \dots, f_M(\bar{x}), \tag{1}$$

$$\text{Subject to } g_i(\bar{x}) \le 0, i \in \{1, 2, \dots, ng\}$$

$$h_j(x) = 0, j \in \{ng+1, ng+2, \dots, ng+nh\}$$

$$L_k \le x_k \le U_k, k \in \{1, \dots, D\}$$

where f_i represents the i-th objective function, M is the total number of the conflicting objective functions, $\bar{x} = (x_1, x_2, \dots x_D)^T$ is a solution vector of length D, L_k and U_k are the lower and upper bound of the search-space at k-th dimension, ng and nh are the total number of the inequality and equality constraints, respectively. Here, solution $\bar{x}$ can be of two types: feasible and infeasible solution. The feasible solutions satisfy all $(ng + nh)$ constraints of the given problem and blackthe set of all possible feasible solutions within the bound of the search-space creates a subspace in the search-space, called a feasible region. black-However, blackthe solution that does not lie in the feasible region is called an infeasible solution. Similarly, a set of all possible infeasible solutions formed an infeasible subspace in the search-space.

The constraint violation of blackthe solution $\bar{x}_i$ over a j-th constraint can be calculated by the following equation:

$$v_j = \begin{cases} max\left(0, g_j(\bar{x}_i)\right), & j \le ng \\ max\left(0, |h_j(\bar{x}_i)| - \epsilon\right), & ng < j \le (ng+nh) \end{cases}, \tag{2}$$

where v_j is the value of constraint violation for $\bar{x}_i$ on j-th constraint and ϵ is a very small value (10^{-4}) for relaxing the equality constraints. On the basis of this definition, a solution can be called as a feasible solution if that solution has zero constraint violation at each constraint or the

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Nomenclature

SBP	Synthetic Benchmark Problem
CMOM	Constrained Multi-objective Metaheuristic
RWCMOP	Real-world Constrained Multi-objective Optimization Problem
CMOP	Constrained Multi-objective Optimization Problem
EA	Evolutionary Algorithm
CMOO	Constrained Multi-objective Optimization
MOM	Multi-objective Metaheuristic
CHT	Constraint Handling Technique
HV	Hypervolume Indicator
CV	Degree of Constrained Violation

sum of total constraint violations of that solution is zero, i.e.

$$CV(\bar{x}_i) = \sum_{i=1}^{ng+nh} v_i = 0, \tag{3}$$

where $CV(\bar{x}_i)$ is the total constraint violation at solution $\bar{x}_i$. In the case of a nonzero total constraint violation, the solution is termed as an infeasible solution.

Given two solutions $\bar{a}$ and $\bar{b}$ in Constrained Multi-objective Optimization (CMOO), $\bar{a}$ constrained Pareto dominates $\bar{b}$ (can be denoted as $\bar{a} \prec_c \bar{b}$), if and only if

1. $f_i(\bar{b}) \geq f_i(\bar{a}) \; \forall \; i \in \{1, 2, \ldots, M\}$,
2. $f_j(\bar{b}) > f_j(\bar{a}) \; \exists \; i \in \{1, 2, \ldots, M\}$, and
3. $CV(\bar{b}) \geq CV(\bar{a})$.

Here, a feasible solution $\bar{x}^*$ can be said constrained Pareto optimal solution if all possible feasible solutions do not Pareto dominates $\bar{x}^*$. The set of all possible constrained Pareto solutions is termed as Pareto set, and the image formed by this Pareto set on objective space is called Pareto front.

In the majority of CMOPs, some solutions of bound-constrained Pareto front become infeasible and loses its optimility due to some constraints. Therefore, CMOPs cannot be solved by using Multi-objective Metaheuristics (MOMs). We need to incorporate a Constraint Handling Technique (CHT) in the framework of the MOMs to handle the constraints. Several CHTs have been utilized with MOMs in the literature, such as constrained dominance principle [3], self-adaptive penalty function [4], and stochastic ranking [5].

As compared to bound-constrained Pareto front, CMOPs can be divided into four types [6].

1. ***Type I:*** In this case, the constrained Pareto front is the same as the bound-constrained Pareto front, i.e., both Pareto fronts have the same Pareto set.
2. ***Type II:*** In this case, the constrained Pareto set is the subset of the bound-constrained Pareto set.
3. ***Type III:*** In this case, some portions of the constrained Pareto front are the same as the bound-constrained Pareto front, i.e., the intersection of both Pareto sets is not a null set.
4. ***Type IV:*** In this case, the intersection of both Pareto set is a null set, i.e., there is no common region in both Pareto fronts.

While solving the CMOPs, there is a need for the proper balance between minimizing the objective functions and minimizing the constraint violations [7]. Consequently, we can characterize the above-mentioned types of CMOPs according to their required level of balance between minimizing objective functions and minimizing constraint violation [8]. From ***Type I*** to ***Type IV***, the required level is gradually increased. Therefore, in the case of ***Type I*** CMOPs, there is no need of minimizing constraint violation to calculate the constrained Pareto front. While in case of ***Type IV*** CMOPs, more focus is required on minimizing the constraint violations as compared to objective functions.

Generally, theoretical evaluation of the performance of algorithms is difficult due to their stochastic behavior [9]. This is the major reason behind the use of benchmark problems to assess the performance of algorithms empirically. SBPs have been usually used in the performance assessment of the algorithms [10]. The main reasons are that performance evaluation on a real-world application requires domain knowledge of that real-world application and assessment on one problem cannot effectively demonstrate the generality of an algorithm [11].

To cope with this issue, several test-suites having artificial test problems have been designed for CMOPs, see, for example, MFs [6], CFs [12], C-DTLZs [13], SRN [14], TNK [15], OSY [16], and CTPs [17]. There are several advantages to these artificial test suites. They can be easily represented by simple mathematical equations and calculations of objective functions and constraints are computationally cheap and usually fast. Pareto front of these problems is known. Thus, different indicators can be used to represent the experimental results. Most of these problems are scalable to a different number of objectives, the number of decision variables, and the number of constraints. Despite all these advantages, these test problems suffer from serious drawbacks. Usually, they have synthetic properties that may never appear in real-world applications [18,19]. Consequently, the performance of CMOMs can become overrated on some problems and underrated on other problems. For example, most of the problems of these test-suites are ***Type-I*** or ***Type-II*** having a regular Pareto front, which can be easily calculated by some of decomposition-based algorithms [18] (MOEA-D [20] and NSGAIII [13]). Since artificial test problems may contain undesirable characteristics, there is a requirement for a test suite of problems of real-world applications to assess the performance of newly developed algorithms more reliably and effectively. In literature, several benchmark suites have been proposed for assessing the performance of the different class of optimization algorithms, see, for example, [21–26]. However, a benchmark suite of RWCMOPs do not exist, where problems have advantages similar to artificial test problems such as easy to implement, computationally cheap, etc.

To overcome the above-mentioned issues, an easy-to-use test-suite having RWCMOPs is proposed for assessing the performance of CMOPs in this paper. This test suite contains 50 RWCMOPs collected from several areas from mechanical design problems to power system problems. The proposed test-suite provides a diverse set of computationally cheap problems where all problems are implemented by simple mathematical equations. In contrast, the difficulty level of these problems has been maintained at different levels from moderate to high levels. Additionally, these problems do not have unrealistic features as compared to SBPs. However, we do not claim that the proposed test problems will always have better properties than existing synthetic or artificial problems in terms of the performance assessment of CMOMs. We develop this test suite to provide a better tool for conducting the performance assessment of CMOMs over problems of real-world applications in a more realistic way.

The main contributions of this work can be summarized as follows:

1. A test suite of 50 RWCMOPs is proposed where problems are collected from different scientific and engineering fields.
2. In this paper, we have described all RWCMOPs mathematically. Therefore, there is no need to refer to each original article to implement these problems as this paper is self-contained.
3. Moreover, we have implemented this test suite on MATLAB and uploaded it on the official GITHUB page (https://github.com/P-N-Suganthan/2021-RW-MOP). Researchers can easily download this test-suite for examining their CMOMs on RWCMOPs with minimum assistance.
4. The performance of seven state-of-the-art algorithms is assessed on these problems and some baseline results are included in this study.
5. A ranking scheme is also proposed to compare the performance of CMOMs on this test suite.

The remaining parts of this paper are organized as follows. In Section 2, we describe the 50 RWCMOPs mathematically. In Section 3, experimental settings and a ranking scheme are presented for conducting the experiments for the performance assessment of CMOMs on the proposed test suite. In Section 4, the baseline results of this test-suite calculated by seven state-of-the-art algorithms are reported. Finally, Section 5 concludes the works of this paper.

2. Real-World constrained multi-objective optimization test-suite

In this section, the RWCMOPs are described. These problems are classified into five parts according to their domain: mechanical design problems; chemical engineering problems; process design and synthesis problems; power electronics problems; and power system problems.

1) *Mechanical Design Problems:* From mechanical design applications, we have collected 21 RWCMOPs where M, D, and ng vary from 2 to 5, 2 to 10, and 1 to 11, respectively.
2) *Chemical Engineering Problems:* From chemical engineering applications, we have collected 3 RWCMOPs where M, D, ng, and nh vary from 2 to 3, 6 to 9, 0 to 2, and 4 to 6, respectively.
3) *Process, Design, and Synthesis Problems:* From this domain, we have collected bi-objective 5 RWCMOPs where D, ng, and nh vary from 2 to 8, 1 to 9, and 0 to 5, respectively.
4) *Power Electronics Problems:* From this domain, we have collected bi-objective 6 RWCMOPs where D, ng, and nh vary from 2 to 8, 1 to 9, and 0 to 5, respectively.
5) *Power System Optimization Problems:* From this domain, we have collected 15 RWCMOPs where M, D, and nh vary from 2 to 4, 6 to 34, and 1 to 26, respectively.

2.1. Proposed test-suite of RWCMOPs

The above-mentioned 50 problems are combined to create a test-suite for evaluating the performance of CMOMs. The basic details of these problems such as the number of objective functions, number of decision variables, number of equality constraints and inequality constraints are reported in Table 1. As shown in Table 1. the number of objective functions varies from 2 to 5, the number of decision variable varies from 2 to 34, the number of inequality constraints varies from 0 to 29, and the number of equality constraints vary from 0 to 26.

3. Evaluation of the proposed test-suite

In this section, we evaluate the performance of seven state-of-the-art CMOMs on the problems of the proposed test suite. These seven algorithms are ToP [64], TiGE_2 [65], cNSGAIII [13], cMOEA/D [13], CCMO [66], cARMOEA [67], AnD [68]. These algorithms can be treated as state-of-the-art algorithms as these algorithms perform very well on SBPs.

1. cNSGAIII, cMOEA/D, and cARMOEA are the constrained variants of reference-based algorithms NSGAIII, MOEA/D, and ARMOEA, respectively.
2. ToP contains a two-phase optimization strategy. In the first phase, the multi-objective problem is transformed into a single-objective constrained problem and then solved. In the second phase, a popular state-of-the-art algorithm is applied to the original problem [64].
3. TIGE_2 constructs a tri-goal model to provide balance among diversity, convergence, and feasibility [65].
4. CCMO constructs a helper problem derived from the original problem. Two populations are constructed to solve original and helper problems with the same algorithm [66].
5. AnD utilizes vector-angle and shift-based density estimation to solve CMOPs [68].

The source codes of these algorithms are taken from PLATEMO [69], a MATLAB platform for multi- or many-objective optimization.

3.1. Performance indicator

In general, performance indicators are used to assess the quality of the obtained Pareto fronts in the case of CMOPs. Here, we utilize the Hypervolume Indicator (HV) for giving a score to the Pareto fronts obtained by all algorithms as HV has been the only Pareto-compliant indicator available currently in the literature [70]. To calculate HV, feasible Pareto solutions have been used. A larger value of HV of a given Pareto front indicates the better approximation of the original Pareto front of the given problem. Usually, Pareto front of the real-world problem is not known. This is the main reason for not utilizing the other performance indicator which requires a set of reference vectors. As suggested in [18,71], we set the reference vector of length M to $[1.1, 1.1, \ldots, 1.1]^T$ for the calculation of HV in the normalized objective-space. For normalization of objective-space, we use approximated ideal and nadir points of actual objective-space and the normalized i-th objective function value, $f_i(\bar{x})$, for a solution $\bar{x}$ can be obtained by the following equation.

$$\hat{f}_i(\bar{x}) = \begin{cases} \frac{f_i(\bar{x}) - f_i^{ideal}}{f_i^{nadir} - f_i^{ideal}}, & \text{if } f_i^{nadir} \neq f_i^{ideal} \\ f_i(\bar{x}) - f_i^{ideal}, & \text{otherwise} \end{cases} \tag{4}$$

where $\hat{f}_i(\bar{x})$ is the normalized i-th objective function value at solution $\bar{x}$; f_i^{ideal} and f_i^{nadir} are the ideal and nadir points of i-th dimension of the original objective-space. respectively. Here, we use two algorithms, SASS [72] and sCMAgES [73] to calculate the ideal and nadir points of all objectives of all problems of the proposed test-suite as these algorithms are the top-ranked algorithms of *Special Session & Competition on Real-world Constrained Optimization* organised at WCCI 2020 and GECCO 2020 [22]. We adopt the default parameter setting in both algorithms to estimate the ideal and nadir points, except population size and maximum function evaluations. We increase the population size and maximum function evaluations by 10- and 100-times, respectively. Moreover, we run 100 independent trials on each problem. In Table 2, we report the calculated nadir points of each problem. These nadir points are used to calculate the HV value in further experiments.

3.2. Experimental settings

All algorithms are implemented on MATLAB r2017b in a PC with Windows 10 operating system, INTEL Core i7 CPU, and 16 GB RSM. The parameters of all algorithms are set on values suggested in their respective papers. For stopping the optimization process, we apply the same stopping criterion on each algorithm, which is based on the number of objective functions and decision variables. In this stopping criterion, we allot a fixed budget of function evaluations for each problem separately based on population size and the number of iterations. For $M = 2$, $M = 3$, $M = 4$, and $M = 5$, the population-size of each algorithms is set to 80, 105, 143, and 212, respectively [74]. For $D \leq 10$ and $D > 10$, the maximum number of iterations is fixed at 2500 and 10000, respectively. Therefore, the budget of function evaluation, Max_{FEs}, for each problem can be set as follows.

$$Max_{FEs} = \begin{cases} 2 \times 10^4, & \text{if } (M == 2) \;\&\; (D \leq 10) \\ 8 \times 10^4, & \text{elseif } (M == 2) \;\&\; (D > 10) \\ 2.6250 \times 10^4, & \text{elseif } (M == 3) \;\&\; (D \leq 10) \\ 1.05 \times 10^5, & \text{elseif } (M == 3) \;\&\; (D > 10) \\ 3.575 \times 10^4, & \text{elseif } (M == 4) \;\&\; (D \leq 10) \\ 1.43 \times 10^5, & \text{elseif } (M == 4) \;\&\; (D > 10) \\ 5.3 \times 10^4, & \text{elseif } (M == 5) \;\&\; (D \leq 10) \\ 2.12 \times 10^5, & \text{elseif } (M == 5) \;\&\; (D > 10) \end{cases} \tag{5}$$

3.3. Difficulty level evaluation of problems of proposed test-suite

The difficulty level of each problem of the proposed test suite is different from each other. To assess the relative difficulty level of these problems, we adopt the following procedure.

Table 1
Details of the 50 RWCMOPs. M is the total number of objectives, D is the total number of decision variables of the problem, ng is the number of inequality constraints and nh is the number of equality constraints.

Prob	Name	M	D	ng	nh
	Mechanical Design Problems				
RCM01	Pressure Vessel Design [27]	2	4	2	2
RCM02	Vibrating Platform Design [28]	2	5	5	0
RCM03	Two Bar Truss Design [29]	2	3	3	0
RCM04	Welded Beam Design [30]	2	4	4	0
RCM05	Disc Brake Design [31]	2	4	4	0
RCM06	Speed Reducer Design [32]	2	7	11	0
RCM07	Gear Train Design [33]	2	4	1	0
RCM08	Car Side Impact Design [13]	3	7	9	0
RCM09	Four Bar Plane Truss [34]	2	4	0	0
RCM10	Two Bar Plane Truss	2	2	2	0
RCM11	Water Resources Management	5	3	7	0
RCM12	Simply Supported I-beam Design [35]	2	4	1	0
RCM13	Gear Box Design	3	7	11	0
RCM14	Multiple Disk Clutch Brake Design [36]	2	5	8	0
RCM15	Spring Design [27]	2	3	8	0
RCM16	Cantilever Beam Design [37]	2	2	2	0
RCM17	Bulk Carrier Design [38]	3	6	9	0
RCM18	Front Rail Design [39]	2	3	3	0
RCM19	Multi-product Batch Plant [40]	3	10	10	0
RCM20	Hydro-static Thrust Bearing Design [41]	2	4	7	0
RCM21	Crash Energy Management for High-speed Train [42]	2	6	4	0
	Chemical Engineering Problems				
RCM22	Haverly's Pooling Problem [43]	2	9	2	4
RCM23	Reactor Network Design [44]	2	6	1	4
RCM24	Heat Exchanger Nwteork Design [45]	3	9	0	6
	Process, Design and Synthesis Problems				
RCM25	Process Synthesis Problem [46]	2	2	2	0
RCM26	Process Synthesis and Design Problem [47]	2	3	1	1
RCM27	Process Flow Sheeting Problem [48]	2	3	3	0
RCM28	Two Reactor Problem [46]	2	7	4	4
RCM29	Process Synthesis Problem [46]	2	7	9	0
	Power Electronics Problems				
RCM30	Synchronous Optimal Pulse-width Modulation of 3-level Inverters [49]	2	25	24	0
RCM31	Synchronous Optimal Pulse-width Modulation of 5-level Inverters [50]	2	25	24	0
RCM32	Synchronous Optimal Pulse-width Modulation of 7-level Inverters [51]	2	25	24	0
RCM33	Synchronous Optimal Pulse-width Modulation of 9-level Inverters [52]	2	30	29	0
RCM34	Synchronous Optimal Pulse-width Modulation of 11-level Inverters [53]	2	30	29	0
RCM35	Synchronous Optimal Pulse-width Modulation of 13-level Inverters [53]	2	30	29	0
	Power System Optimization Problems				
RCM36	Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Active Power Loss [54]	2	28	0	24
RCM37	Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Reactive Power Loss [54]	2	28	0	24
RCM38	Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for Minimizing Active and Reactive Power Loss [54]	2	28	0	24
RCM39	Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Active and Reactive Power Loss [54]	3	28	0	24
RCM40	Optimal Power Flow for Minimizing Active and Reactive Power Loss [55]	2	34	0	26
RCM41	Optimal Power Flow for Minimizing Voltage deviation, Active and Reactive Power Loss [56]	3	34	0	26
RCM42	Optimal Power Flow for Minimizing Voltage deviation, and Active Power Loss [57]	2	34	0	26
RCM43	Optimal Power Flow for Minimizing Fuel Cost, and Active Power Loss [58]	2	34	0	26
RCM44	Optimal Power Flow for Minimizing Fuel Cost, Active and Reactive Power Loss [59]	3	34	0	26
RCM45	Optimal Power Flow for Minimizing Fuel Cost, Voltage deviation, and Active Power Loss [55]	3	34	0	26
RCM46	Optimal Power Flow for Minimizing Fuel Cost, Voltage deviation, Active and Reactive Power Loss [55]	4	34	0	26
RCM47	Optimal Droop Setting for Minimizing Active and Reactive Power Loss [60]	2	18	0	12
RCM48	Optimal Droop Setting for Minimizing Voltage Deviation and Active Power Loss [61]	2	18	0	12
RCM49	Optimal Droop Setting for Minimizing Voltage Deviation, Active, and Reactive Power Loss [62]	3	18	0	12
RCM50	Power Distribution System Planning [63]	2	6	0	1

Table 2
Calculated nadir points of each problem.

Nadir Points							
RCM01	3.596489E+05	-7.330383E+03		RCM26	2.926323E+00	-1.579396E+00	
RCM02	-1.274608E-03	3.182549E+02		RCM27	-2.430000E-01	0.000000E+00	
RCM03	1.000000E-01	1.000000E+05		RCM28	1.334321E+02	1.310820E+01	
RCM04	3.667933E+01	1.306667E-02		RCM29	2.999999E+00	1.400000E+01	
RCM05	5.306700E+00	3.028168E+00		RCM30	2.854987E-01	1.024000E-01	
RCM06	5.969822E+03	1.300000E+03		RCM31	7.342985E-01	1.024000E-01	
RCM07	3.465500E+00	4.541780E+01		RCM32	5.888136E-01	1.296000E-01	
RCM08	9.259659E+01	4.000000E+00	1.269973E+01	RCM33	5.217146E-01	1.023999E-01	
RCM09	3.048528E+03	4.000000E-02		RCM34	6.106382E-01	1.110885E-01	
RCM10	1.870486E+02	6.771018E-05		RCM35	1.949969E+00	1.107699E+01	
RCM11	7.345051E+04	1.350000E+03	2.853469E+06	RCM36	6.993850E-01	4.080103E-03	
	6.620032E+06	2.500000E+04		RCM37	6.479325E-01	5.775242E-03	
RCM12	4.379547E+02	6.145910E-02		RCM38	5.022365E-03	3.606036E-03	
RCM13	6.103602E+03	1.300000E+03	1.004676E+03	RCM39	7.018136E-01	4.080331E-03	5.777656E-03
RCM14	1.396752E+00	1.492075E-02		RCM40	6.188716E+00	-2.992047E-01	
RCM15	2.794266E+01	1.879912E+05		RCM41	6.195498E+00	-1.813540E-01	4.060485E+00
RCM16	3.063053E+00	2.040876E-03		RCM42	6.878865E-03	2.969577E-03	
RCM17	-3.151416E+03	8.260630E+03	8.126000E+02	RCM43	5.713952E+00	1.257933E+00	
RCM18	9.336641E-01	1.196596E+00		RCM44	5.720257E+00	1.023138E+00	4.047120E+00
RCM19	2.443518E+05	4.857255E+04	6.000000E+03	RCM45	8.735395E+00	2.286114E+00	4.671025E+00
RCM20	2.672585E+02	-2.767265E-05		RCM46	5.738108E+00	2.357833E+00	8.190989E+00
RCM21	1.315734E+00	2.629774E+01			3.903294E+00		
RCM22	-1.059087E+02	2.000000E+03		RCM47	9.376448E-03	2.473741E-03	
RCM23	-4.019408E-04	4.000000E+00		RCM48	1.429627E-01	8.676186E-04	
RCM24	6.639524E+00	-3.632301E-05	-2.000000E+06	RCM49	1.425345E-01	2.036414E-02	5.830264E-04
RCM25	3.200000E+00	-1.250000E+00		RCM50	6.489562E+04	1.253894E+03	

1. All algorithms are implemented 25 times independently on each problem to calculate the statistical data for the assessment of performance.
2. This statistical data contains best, mean, worst, and standard deviation of HV values and Degree of Constrained Violation (CV) obtained from 25 times independent implementation are reported in the supplementary document (Tables S1-S8). In addition, we also calculate the Feasibility Rate (FR) of algorithms on each problem.
   - *CV:* CV is the average of the constrained violation of all solutions of the final output population obtained by the algorithm.
   - *FR:* FR is the average fraction of final solutions that are feasible in all independent runs.
3. Finally, we evaluate the difficulty level of problems on the basis of the FR values of all algorithms.

To analyze the performance of each algorithm on the proposed benchmark suite, HV and CV values of all algorithms are depicted in Fig. 1. From this figure, we can summarize the following outcomes. Moreover, we report FR values of all algorithms for each problem in Table 3.

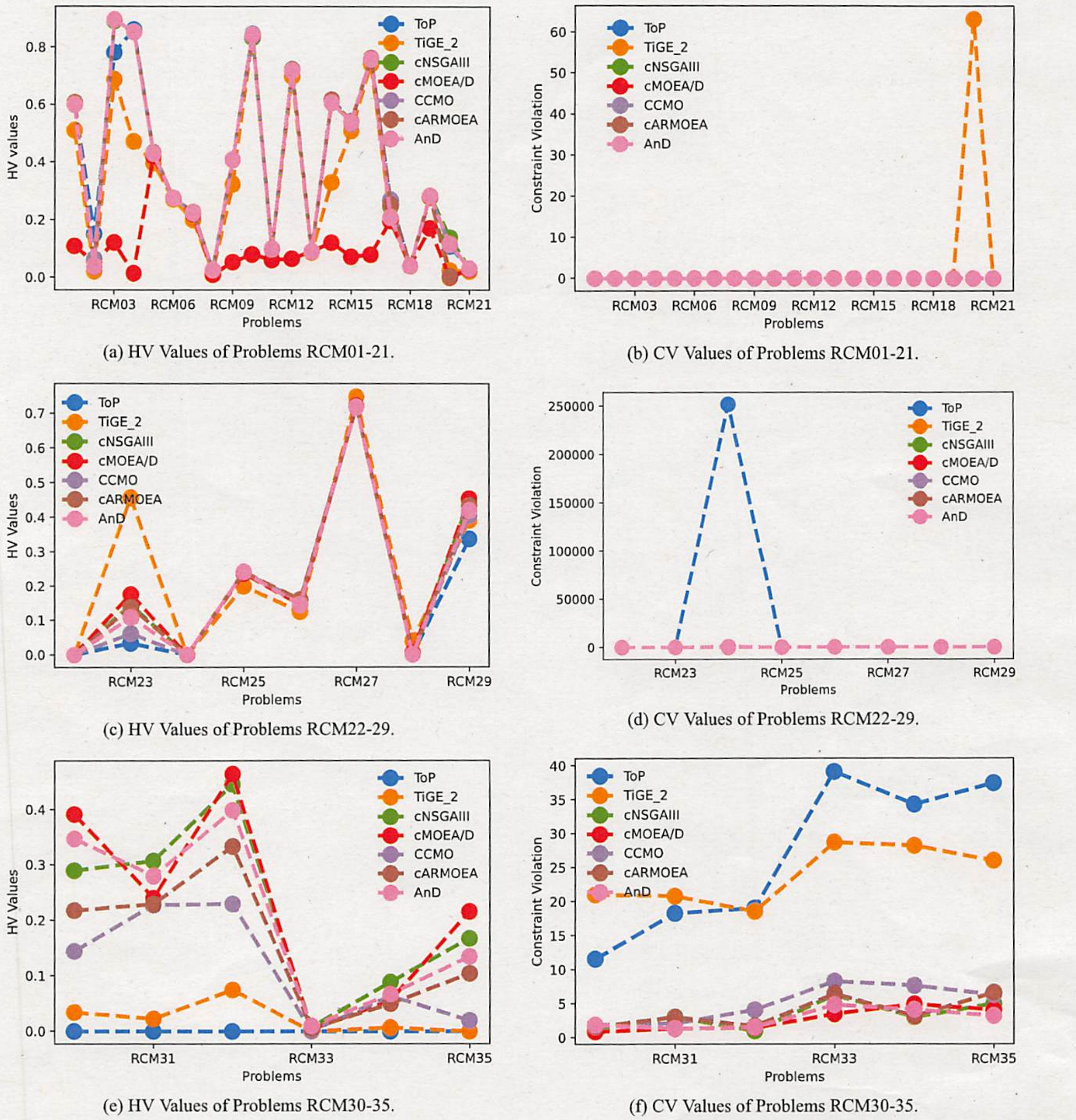
1) *Mechanical Design Problems:* The baseline results of mechanical design problems are shown in Figs. 1a and 1 b and Table 3. By analyzing Table 3, we get that the FR of all algorithms is 1 for most of the mechanical design problems. Therefore, we can conclude that the difficulty level of these problems is relatively low, as state-of-the-art algorithms easily locate the feasible solutions of the constrained Pareto front of these problems.
2) *Chemical Engineering Problems:* It can be seen from the Table 3, FR of two problems out of three is zero. Therefore, it can be concluded that the difficulty level of these problems is relatively high, as state-of-the-art algorithms cannot locate a single feasible solution of two out of three problems. In the case of RCM23, the algorithms locate the feasible solutions in some of the runs, but these feasible solutions are not located on its constrained Pareto front.
3) *Process Design and Synthesis Problems:* In Table 3, Figs. 1c and 1 d, the baseline results of process design and synthesis problems are reported. From Table 3, FR of all problems is 1 on all problems except RCM28 as RCM28 contains four equality constraints. Therefore, it can be concluded that the difficulty level of these problems is relatively low.
4) *Power Electronic Problems:* In Table 3, Figs. 1e and 1 f, the baseline results of power electronics problems are shown. It can be seen from this table that the FR of these problems is relatively low for most of the algorithms. All algorithms cannot locate the feasible solutions in each run. Therefore, the difficulty level of power electronics problems is relatively high.
5) *Power System Problems:* As shown in Table 3, FR of these problems is zero for all algorithms as these problems contain a higher number of equality constraints. Therefore, these problems are relatively more difficult than the problems of other streams as these problems contain a high number of equality constraints. Thus, these problems are hard to solve by current state-of-the-art algorithms. These problems will motivate researchers to design new operators, frameworks, and algorithms to handle the high number of equality constraints.

From the above analysis, it can be concluded that the proposed test suite contains a variety of problems having different difficulty levels, and it can be utilized to determine the robustness and efficacy of newly proposed algorithms. Due to the higher difficulty level, state-of-the-art algorithms cannot find a single feasible solution in case of majority problems. Although the proposed benchmark suite contains test problems with relatively lower dimensions (maximum 34), most problems are found to be difficult to solve by state-of-the-art algorithms. This phenomenon inspires others to develop more robust CMOMs and CHTs than currently available in the literature.

3.4. Evaluation of performance of algorithms

First of all, we apply Bayesian statistical tests [75] such as the Bayesian signed-rank test and Bayesian Friedman test. Plots of these statistical tests are depicted in Fig. 2. From this figure, we can summarize the following outcomes.

a) In mechanical design problems (RCM01-21), cARMOEA and ToP are the best performers (as shown in Fig. 2b). However, ToP perform better than cARMOEA in one-to-one comparison (as shown



(a) HV Values of Problems RCM01-21.

(b) CV Values of Problems RCM01-21.

(c) HV Values of Problems RCM22-29.

(d) CV Values of Problems RCM22-29.

(e) HV Values of Problems RCM30-35.

(f) CV Values of Problems RCM30-35.

Fig. 1. HV and CV values of Problems RCM01-RCM35.

in Fig. 2a). Moreover, cMOEA/D and TIGE_2 are the worst-ranked algorithms.

b) For other problems except for power system optimization problems, cMOEA/D, and cNSGAiii performs better than others as these algorithms can handle equality constraints better than others.

c) The exciting thing to see here is that ToP and cARMOEA do not perform well on these problems. This outcome suggests that although these algorithms better handle the inequality constraints, their performance degrades when equality constraints introduce in the problems.

Additionally for ranking the CMOMs based on the performance over the proposed benchmark suite, we propose a ranking scheme inspired from [76]. Supposing N algorithms CMOEA_1, CMOEA_2....CMOEA_N participates in the comparative analysis done on P problems. The performance score, S, can be defined as follows:

$$S(CMOEA_i) = \frac{1}{P}\left(\sum_{j=1}^{P} \frac{1}{N-1}\left(\sum_{k=1}^{N} \delta^i_{j,k}\right)\right), \tag{6}$$

where,

$$\delta^i_{j,k} = \begin{cases} 1, & \text{if } CMOEA_j \text{ significantly outperforms } CMOEA_i \text{ on a problem } k \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

Table 3
Baseline results in terms of mean of HV values/FR calculated from 25 independent runs.

Problem	ToP [64]	TiGE_2 [65]	cNSGAIII [13]	cMOEA/D [13]	CCMO [66]	cARMOEA [67]	AnD [68]
Mechanical Design Problems							
RCM01	6.06E-01/1.00	5.11E-01/1.00	6.06E-01/1.00	1.09E-01/1.00	6.04E-01/1.00	**6.07E-01/1.00**	5.99E-01/1.00
RCM02	**1.50E-01/1.00**	2.16E-02/1.00	5.35E-02/1.00	5.22E-02/1.00	6.37E-02/1.00	3.60E-02/1.00	3.84E-02/1.00
RCM03	7.81E-01/1.00	6.88E-01/1.00	8.92E-01/1.00	1.21E-01/1.00	8.97E-01/1.00	**8.98E-01/1.00**	8.97E-01/1.00
RCM04	**8.61E-01/1.00**	4.72E-01/1.00	8.54E-01/1.00	1.44E-02/1.00	8.53E-01/1.00	8.53E-01/1.00	8.53E-01/1.00
RCM05	**4.34E-01/1.00**	3.97E-01/1.00	4.33E-01/1.00	4.21E-01/1.00	4.33E-01/1.00	4.33E-01/1.00	4.31E-01/1.00
RCM06	2.74E-01/1.00	2.72E-01/1.00	2.77E-01/1.00	2.77E-01/1.00	2.77E-01/1.00	**2.77E-01/1.00**	2.77E-01/1.00
RCM07	2.27E-01/1.00	2.00E-01/1.00	2.26E-01/1.00	2.21E-01/1.00	**2.27E-01/1.00**	2.26E-01/1.00	2.24E-01/1.00
RCM08	2.56E-02/1.00	2.04E-02/1.00	2.54E-02/1.00	9.37E-03/1.00	2.58E-02/1.00	**2.59E-02/1.00**	2.58E-02/1.00
RCM09	4.09E-01/1.00	3.23E-01/1.00	4.09E-01/1.00	5.31E-02/1.00	4.09E-01/1.00	**4.10E-01/1.00**	4.07E-01/1.00
RCM10	**8.47E-01/1.00**	8.41E-01/1.00	8.33E-01/1.00	7.95E-02/1.00	8.39E-01/1.00	8.41E-01/1.00	8.45E-01/1.00
RCM11	9.73E-02/1.00	9.79E-02/1.00	**9.97E-02/1.00**	6.04E-02/1.00	9.92E-02/1.00	9.71E-02/1.00	9.89E-02/1.00
RCM12	**7.23E-01/1.00**	6.98E-01/1.00	7.22E-01/1.00	6.45E-02/1.00	7.20E-01/1.00	7.22E-01/1.00	7.18E-01/1.00
RCM13	8.92E-02/1.00	8.67E-02/1.00	9.01E-02/1.00	9.02E-02/1.00	8.88E-02/1.00	**9.03E-02/1.00**	9.03E-02/1.00
RCM14	**6.17E-01/1.00**	3.30E-01/1.00	6.16E-01/1.00	1.21E-01/1.00	6.14E-01/1.00	6.17E-01/1.00	6.06E-01/1.00
RCM15	**5.43E-01/1.00**	5.09E-01/1.00	5.41E-01/1.00	7.20E-02/1.00	5.35E-01/1.00	5.41E-01/1.00	5.39E-01/1.00
RCM16	**7.63E-01/1.00**	7.42E-01/1.00	7.62E-01/1.00	7.91E-02/1.00	7.62E-01/1.00	7.62E-01/1.00	7.59E-01/1.00
RCM17	2.65E-01/1.00	2.04E-01/1.00	2.47E-01/1.00	1.97E-01/1.00	**2.71E-01/1.00**	2.53E-01/1.00	2.09E-01/1.00
RCM18	4.05E-02/1.00	3.93E-02/1.00	4.05E-02/1.00	4.03E-02/1.00	4.05E-02/1.00	**4.05E-02/1.00**	4.04E-02/1.00
RCM19	2.85E-01/1.00	2.78E-01/1.00	**2.85E-01/1.00**	1.71E-01/1.00	2.81E-01/1.00	2.80E-01/1.00	2.84E-01/1.00
RCM20	1.09E-01/0.97	2.44E-02/0.97	**1.39E-01/0.97**	0.00E+00/1.00	1.14E-01/1.00	3.07E-03/1.00	1.16E-01/1.00
RCM21	**3.18E-02/1.00**	2.12E-02/1.00	3.17E-02/1.00	2.93E-02/1.00	3.17E-02/1.00	3.17E-02/1.00	3.17E-02/1.00
Chemical Engineering Problems							
RCM22	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM23	3.33E-02/0.03	**4.57E-01/0.90**	1.44E-01/0.57	1.76E-01/0.73	6.11E-02/0.27	1.38E-01/0.50	1.08E-01/0.43
RCM24	0.00E+00/0.00	0.00E+00/0.10	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
Process, Design and Synthesis Problems							
RCM25	2.41E-01/1.00	1.99E-01/1.00	2.41E-01/1.00	2.37E-01/1.00	**2.41E-01/1.00**	2.41E-01/1.00	2.41E-01/1.00
RCM26	1.56E-01/1.00	1.24E-01/1.00	1.53E-01/1.00	1.45E-01/1.00	1.55E-01/1.00	**1.59E-01/1.00**	1.45E-01/1.00
RCM27	7.18E-01/1.00	**7.48E-01/1.00**	7.19E-01/1.00	7.22E-01/1.00	7.20E-01/1.00	7.19E-01/1.00	7.17E-01/1.00
RCM28	0.00E+00/0.00	**3.95E-02/0.97**	0.00E+00/0.00	4.97E-03/0.10	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM29	3.36E-01/1.00	3.91E-01/1.00	4.48E-01/1.00	**4.53E-01/1.00**	4.04E-01/1.00	4.32E-01/1.00	4.18E-01/1.00
Power Electronics Problems							
RCM30	0.00E+00/0.10	3.46E-02/0.07	2.89E-01/0.50	**3.92E-01/0.60**	1.44E-01/0.27	2.18E-01/0.40	3.48E-01/0.57
RCM31	0.00E+00/0.00	2.29E-02/0.07	**3.07E-01/0.47**	2.40E-01/0.47	2.28E-01/0.40	2.29E-01/0.37	2.80E-01/0.53
RCM32	0.00E+00/0.00	7.49E-02/0.10	4.46E-01/0.60	**4.64E-01/0.63**	2.30E-01/0.33	3.33E-01/0.47	3.99E-01/0.53
RCM33	0.00E+00/0.00	0.00E+00/0.03	7.99E-03/0.20	3.47E-03/0.30	8.51E-04/0.13	5.53E-03/0.13	**9.72E-03/0.17**
RCM34	0.00E+00/0.00	6.82E-03/0.07	**8.88E-02/0.23**	5.55E-02/0.37	6.35E-02/0.27	4.95E-02/0.23	6.68E-02/0.27
RCM35	0.00E+00/0.00	0.00E+00/0.00	1.67E-01/0.30	**2.16E-01/0.40**	1.97E-02/0.03	1.04E-01/0.17	1.35E-01/0.23
Power System Optimization Problems							
RCM36	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM37	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM38	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM39	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM40	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM41	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM42	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM43	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM44	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM45	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM46	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM47	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM48	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM49	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00
RCM50	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00	0.00E+00/0.00

Here, we use the Wilcoxon rank-sum test at a 0.05 significance level to determine the significant difference between the performance of two algorithms on a problem. The lower value of S of an algorithm suggests that the algorithm performs better on the proposed test-suite.

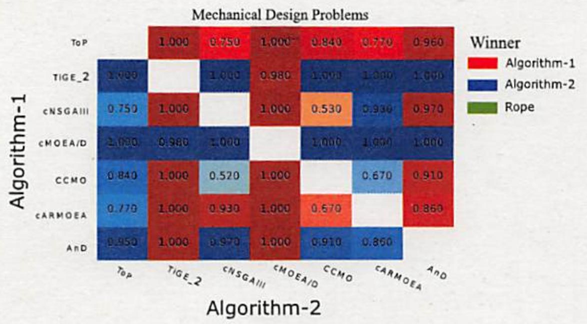
The performance score of all algorithms on the proposed benchmark suite is shown in Table (4). As shown in Table (4), cNSGAIII and cARMOEA provide the lowest performance score, i.e., performs better than other algorithms.

Table 4
Ranking of all algorithms on the proposed benchmark suite.

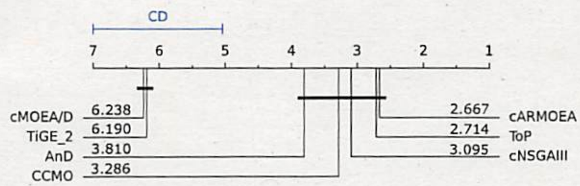
Algorithm	Performance Score	Rank
ToP [64]	0.6567	6
TiGE_2 [65]	0.7300	7
cNSGAIII [13]	0.3433	1.5
cMOEA/D [13]	0.4900	4
CCMO [66]	0.5033	5
cARMOEA [67]	0.3433	1.5
AnD [68]	0.4333	3

4. Conclusion

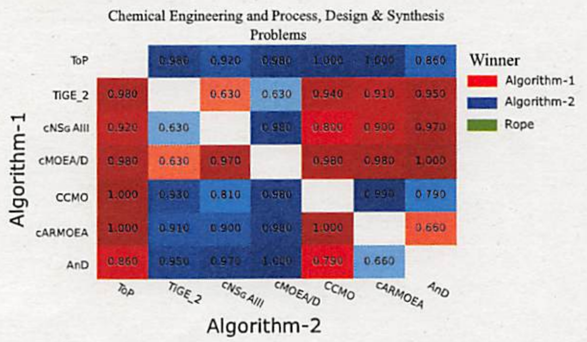
While evaluation on RWCMOPs is an important aspect of performance assessment of newly developed CMOMs, it is a difficult task to establish due to domain knowledge requirements and other obstacles. To resolve this issue, we develop a test-suite containing RWCMOPs selected from various engineering streams. This test suite contains 50 RWCMOPs of different difficulty levels from low to high. To evaluate the difficulty



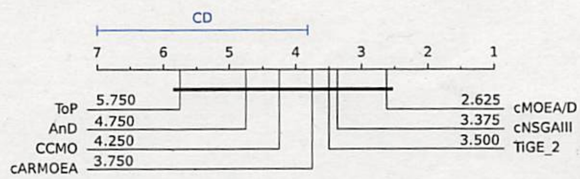
(a) Bayesian Signed-Rank Test for Problems RCM01-21.



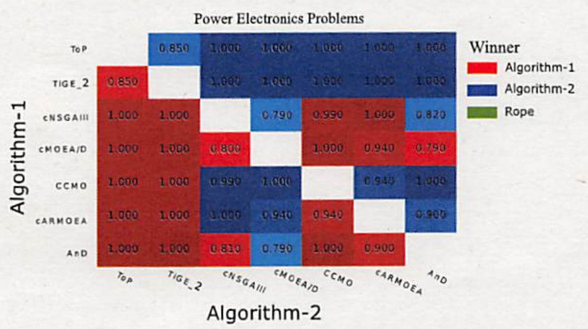
(b) Critical Diagram for Problems RCM01-21.



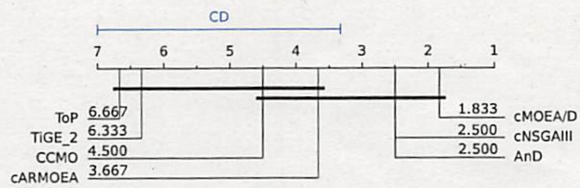
(c) Bayesian Signed-Rank Test for Problems RCM22-29.



(d) Critical Diagram for Problems RCM22-29.



(e) Bayesian Signed-Rank Test for Problems RCM30-35.



(f) Critical Diagram for Problems RCM30-35.

Fig. 2. Plots of Statistical Tests.

level of these problems, we select seven state-of-the-art algorithms for calculating the baseline results of these problems. The baseline results obtained from the experiments suggest that some of the problems are not solved by all algorithms, i.e., hard to solve by currently available algorithms. We also present the performance comparison of the selected algorithms on the proposed benchmark suite. The main findings of this work are as follows.

- A benchmark suite of 50 RWCMOPs is proposed, where these problems have been collected from diverse real-world application problems.
- The performance of seven state-of-the-art algorithms is assessed on these problems.
- ToP performs well on mechanical design problems. However, its performance degrades on RWCMOPs with smaller feasible regions compared to other RWCMOPs.
- TiGE_2 shows relatively better performance on chemical engineering problems and process, design, and synthesis problems compared to other RWCMOPs.
- cMOEA/D provides better performance on Power Electronic problems than other algorithms.
- Overall performance of cARMOEA and cNSGAIII is better than other algorithms on the proposed benchmark suite.
- All seven algorithms cannot find a single feasible solution for Power System Optimization problems.

Performance analysis of other state-of-the-art algorithms is proposed for a further research topic. Moreover, further analysis of each problem, such as the shape of the feasible region and Pareto fronts, is also a future work pathway. This benchmark suite may motivate researchers to develop efficient and robust algorithms for solving problems of real-world applications.

Declaration of Competing Interest

The authors whose names are listed immediately below certify that the work reported in the paper is solely ours and has not submitted elsewhere. In addition, we declare that there is NO conflict of interest for any of the authors.

CRediT authorship contribution statement

Abhishek Kumar: Data curation, Formal analysis, Writing – original draft. **Guohua Wu:** Data curation, Formal analysis, Writing – original draft. **Mostafa Z. Ali:** Data curation, Formal analysis, Writing – original draft. **Qizhang Luo:** Data curation, Formal analysis, Writing – original draft. **Rammohan Mallipeddi:** Data curation, Formal analysis, Writing – original draft. **Ponnuthurai Nagaratnam Suganthan:** Formal analysis. **Swagatam Das:** Formal analysis.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.swevo.2021.100961

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