Condensed Matter Physics
Summary of Lectures

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Contents

1 Crystal Structure

- A crystal lattice is the set of all identical points in a crystal: the lattice points.
- Lattice vectors connect nearby sites in the lattice from an arbitrarily picked origin. In a 2D crystal there are 2 ($\mathbf{a}$ and $\mathbf{b}$) and in a 3D crystal we need 3 ($\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$). Typically they will not be orthogonal, and there are some conventional choices for common lattices. The lengths $a = |\mathbf{a}|$, $b = |\mathbf{b}|$, $c = |\mathbf{c}|$, and the angles between these vectors, suffice to define the entire lattice.
- All lattice points can be reached by

$$\mathbf{r} = u\mathbf{a} + v\mathbf{b} + w\mathbf{c} \text{ where } u, v, w, \in \mathbb{Z}\,.$$

- A unit cell is a structure that can be repeated to cover the entire lattice. The unit cell built from the conventional lattice vectors is called the conventional unit cell.
- The smallest possible unit cell is called the primitive unit cell.
- Any 3D lattice of the form $\mathbf{r} = u\mathbf{a} + v\mathbf{b} + w\mathbf{c}$ (or 2D lattice of the form $\mathbf{r} = u\mathbf{a} + v\mathbf{b}$) can be in one of 14 (5) possible types depending on its symmetries. These are called Bravais lattices.
- along with the crystal lattice and the unit cell we require the basis, these three things complete the description of a crystal structure. The basis is the group of atoms associated with each lattice point which is necessary to complete the structure.
- Any crystal can be sorted into one of 32 possible point symmetry groups, and one of 230 possible space symmetry groups.
  - A point symmetry keeps at leats one point fixed during the symmetry transformation. E.g. a rotation or reflection.
  - Space symmetries include translations and combinations of translations with reflections, etc..
- A crystal plane can be identified by Miller indices. A set of crystal planes are parallel and equally spaced, with the same density of lattice points on each plane. The crystal plane which intercepts closest to the origin (but not through the origin) defines the Miller indices. If the intercepts with each lattice vector $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are α, β, γ respectively, then the Miller indices are $(\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma})$.
- Some examples of crystal structures include:

  fcc: face centered cubic

  bcc: body centered cubic