1. For each of the 5 Bravais lattices in 2D:
   (a) Write the properties of a possible pair of lattice vectors.
   (b) Write down the symmetries of the crystal lattice.
   (c) Find a unit cell and a primitive unit cell, and find the number of lattice sites in each.
2. Find the conventional lattice vectors for graphene (see lecture).
3. Show that $c/a = \sqrt{8/3}$ for hexagonal close packing of hard spheres.
4. Sketch a few cubic unit cells and draw the following lattice planes within them: (0 0 1), (1 0 1), (0 1 1), (0 2 1), (2 1 0), (2 1 1), and (1 2 2).
5. Prove that in a lattice of cubic symmetry the direction $[h\,k\,l]$ is perpendicular to the plane $(h\,k\,l)$ with the same indices.
6. Show that the spacing d of the $(h\,k\,l)$ set of lattice planes in a cubic lattice of side a is
$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}\,.$$
7. Consider the pattern
$$\begin{array}{cccccccccccc} q & p & d & b & q & p & d & b & q & p & d & b & \cdots \\ d & b & q & p & d & b & q & p & d & b & q & p & \cdots \\ q & p & d & b & q & p & d & b & q & p & d & b & \cdots \\ \vdots & \vdots & & & & & & & & & & & \end{array}$$
   Indicate:
   (a) a rectangular unit cell;
   (b) a primitive unit cell; and
   (c) the basis of letters associated with each lattice point.
8. Consider the fcc, bcc, hcp, and diamond structures.
   (a) Draw plans of the conventional unit cells of these structures, indicating the height of the atoms as a fraction of the unit cell height.
   (b) What are the coordinates of the atoms in the basis of each structure.
   (c) If the structures were formed out of touching spheres, what would be the volume of space they take up as a fraction of the whole?
9. A crystal has a basis of one atom per lattice point and a set of primitive translation vectors (measured in Å):
$$\mathbf{a} = 3\hat{i}\,, \qquad \mathbf{b} = 3\hat{j}\,, \qquad \mathbf{c} = 1.5\left(\hat{i} + \hat{j} + \hat{k}\right)\,,$$
   where $\hat{i}$, $\hat{j}$, and $\hat{k}$ are the standard unit vectors of a Cartesian coordinate system.

(a) What is the Bravais lattice type of this crystal?

(b) What are the Miller indices of the set of planes most densely populated with atoms?

(c) What are the volumes of the primitive unit cell and the conventional unit cell?

10. For the fcc and bcc structures it is possible to choose a primitive unit cell where the primitive translation vectors **a**, **b**, and **c** are equal in magnitude, as are the angles between them (a rhomb). Sketch a diagram for each case showing **a**, **b**, and **c** and calculate the angles between them.