Topology in Condensed Matter
Summary of Lectures

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1 Useful Notation

iff	if and only if
$\Rightarrow$	if then
$\equiv$	defined as
$\therefore$	therefore
$\because$	because
$\square$	end of proof
$\mathbb{R}$	set of real numbers
$\mathbb{C}$	set of complex numbers
$\mathbb{I}$	identity matrix
$\forall$	the universal quantifier, for all
$\exists$	the existential quantifier, there exists
$\in$	is an element of
$\subset$	is a subset of
$\cup$	union of sets
$\cap$	intersection of sets

2 Some Mathematical Background in Topology

- Let X_1 and X_2 be topological spaces. A map $f : X_1 \to X_2$ is a homeomorphism if it is continuous and has an inverse $f^{-1} : X_2 \to X_1$ which is also continuous. If there exists a homeomorphism between X_1 and X_2, X_1 is said to be homeomorphic to X_2 and vice versa. If two topological spaces have different topological spaces then they are not homeomorphic to each other..

- Let be any set and $\mathcal{T} = \{U_i | i \in I\}$ denote a specific collection of subsets of X. The pair $(X, \mathcal{T})$ is a topological space if $\mathcal{T}$ satisfies the following requirements:

  (i) $0, X \in \mathcal{T}$

  (ii) If J is any subcollection of I the family $\{U_j | j \in J\}$ satisfies $\cup_{k \in K} U_k \in \mathcal{T}$

  (iii) If K is any finite subcollection of I the family $\{U_k | k \in K\}$ satisfies $\cap_{k \in K} U_k \in \mathcal{T}$

  X itself is often called a topological space. The U_i are the open sets and $\mathcal{T}$ gives a topology to X.

- Let X and Y be topological spaces. A map $f : X \to Y$ is continuous if the inverse image of open set in Y is an open set in X.