1. Show that for a quasi-static adiabatic process in a perfect gas, with constant specific heats, that

$$pV^\gamma = \text{const.} \quad \text{where} \quad \gamma = \frac{C_p}{C_V}\,.$$

2. The molar energy of a monatomic gas which obey's van de Waals' equation is given by

$$E = \frac{3}{2}RT - \frac{a}{V}$$

where a is a constant and V is the molar volume at temperature T. From temperature T_1 and volume V_1 the gas is allowed to expand adiabatically into a vacuum os that it occupies a volume V_2. What is the final temperature of the gas?

3. Two vessels contain the same number N of molecules of the same perfect gas. Initially the two vessels are isolated from each other, the gases being at the same temperature T but at different pressures p_1 and p_2. The partition separating the two gases is removed. Find the change of entropy of the system when equilibrium has been re-established, in terms of initial pressures p_1 and p_2. Show that this entropy change is non-negative.

4. By examining variations in E, F, H and G, derive the four different Maxwell relations for the partial derivatives of S, p, T and V.

5. Obtain the partial derivative identity

$$\left.\frac{\partial S}{\partial T}\right|_p = \left.\frac{\partial S}{\partial T}\right|_V + \left.\frac{\partial S}{\partial V}\right|_T \left.\frac{\partial V}{\partial T}\right|_p$$

6. Obtain the partial derivative identity

$$\left.\frac{\partial p}{\partial T}\right|_V \left.\frac{\partial T}{\partial V}\right|_p \left.\frac{\partial V}{\partial p}\right|_T = -1$$

7. Consider a classical ideal gas with equation of state $pV = NkBT$ and constant heat capacity $C_V = Nk_B\alpha$ for some α. Use the results above to show that $C_p = Nk_B(\alpha+1)$, and that the entropy is

$$S = Nk_B \ln(V/N) + NkB\alpha \ln T + \text{const.}$$

Deduce that, for an adiabatic process (with $dS = 0$), VT^α is constant and, equivalently, pV^γ is constant, where $\gamma = C_p/C_V$.