Linear Algebra I
Summary of Lectures: Matrices

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1. Definition of an $n \times m$ matrix, $\mathbf{A} = (a_{ij})$ with n row and m columns. Addition of matrices $\mathbf{A} + \mathbf{B} = (a_{ij} + b_{ij})$.
   - Associativity, commutativity and existence of a zero for addition.
2. Multiplication of a matrix by a scalar: $\lambda\mathbf{A} = (\lambda a_{ij})$.
3. The matrix multiplication of an $n \times m$ matrix $\mathbf{A}$ and an $m \times k$ matrix $\mathbf{B}$ is an $n \times k$ matrix $\mathbf{C} = \mathbf{AB} = (c_{ij})$ where $c_{ij} = \sum_{r=1}^{n} a_{ir}b_{rj}$.
   - Associativity, existence of a zero matrix ($\mathbf{0}$) and an identity matrix $\mathbb{I}$, distributivity.
   - No commutativity!
4. A matrix $\mathbf{A}$ can have a right inverse $\mathbf{AB} = \mathbb{I}$ and a left inverse $\mathbf{CA} = \mathbb{I}$.
   - Prop. 1.1: If a square matrices has either a left or right inverse then they have a unique inverse from both the left and right.
   - If a non-square matrix has both a left and right inverse then they are the same and the inverse is unique.
   - Prop. 1.2: If $\mathbf{A}$ and $\mathbf{B}$ are invertible square matrices then $\mathbf{AB}$ is also invertible and $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.
5. The transpose of an $n \times m$ matrix is written as $\mathbf{A}^T$, which is an $m \times n$ matrix found by transposing the rows and columns of $\mathbf{A}$.
   - Prop. 1.3: For two $n \times n$ matrices $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$ and $(\mathbf{A}^T)^{-1} = (\mathbf{A}^{-1})^T$.
6. Elementary row operations perform simple operations on the rows of an $n\times m$ matrix $\mathbf{A}$ and can be written as an $n\times n$ matrix $\mathbf{R}$ with the operation preformed by the multiplication $\mathbf{RA}$. ρ_i is used to refer to row i. There are three of them:
   - $\rho_j := \rho_j + \lambda\rho_i$, add λ copies of row i to row j;
   - $\rho_i := \lambda\rho_i$, multiple row i by λ with $\lambda \neq 0$;
   - swap(ρ_i, ρ_j) swap rows i and j.
7. Echelon form: