Linear Algebra I
Problem Set 10: Eigenvalues and Eigenvectors

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Friday April 1st 2016

Due: In class, April 8th 2016

1. (7) Consider the linear transformation $f : V \to V$ given by

$$f(u) = -\frac{\mathrm{d}^2 u(x)}{\mathrm{d}x^2}\,,$$

where V is the vector space of all continuously differentiable functions on the interval $0 \leq x \leq L$ with L a positive real number. Find the possible eigenvalues λ of $f(u) = \lambda u$ subject to the condition that $u(x = 0) = u(x = L) = 0$. For the smallest two eigenvalues find the corresponding eigenfunction $u(x)$ normalized such that

$$\int_0^L \mathrm{d}x |u(x)|^2 = 1\,.$$

2. (6) Let

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & 9 \\ -1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}\,.$$

Given that $\mathbf{A}$ has eigenvalues 1, 2, and -2 find all eigenvectors of $\mathbf{A}$. Write down a basis of $\mathbb{R}^3$ consisting of eigenvectors of $\mathbf{A}$.

3. (7) Let

$$\mathbf{B} = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & -3 \end{pmatrix}\,.$$

Find all the eigenvalues and eigenvectors of $\mathbf{B}$. Show that there is no basis of $\mathbb{R}^3$ consisting of eigenvectors of $\mathbf{B}$.

Total available marks: 20