

Linear Algebra I

Problem Set 9: Polynomials and Eigenvectors

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Friday March 25th 2016

Due: In class, April 1st 2016

1. (6) Evaluate the polynomials $p(x) = x^3 + 2x^2 + 3x + 1$ and $q(x) = x^2 - x - 1$ at the matrix

$$\mathbf{A} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

2. (6) Consider the infinite sum

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Calculate $f(x)$ at the matrix

$$\mathbf{A} = \frac{\pi}{2} \boldsymbol{\sigma}^x = \frac{\pi}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Express the answer in terms of $\boldsymbol{\sigma}^x$ and the identity matrix. We can take this as the definition of what e^x means for a matrix. *Tip: Note that higher powers of $\boldsymbol{\sigma}^x$ can be written in terms of either $\boldsymbol{\sigma}^x$ or the identity matrix so you can find general expressions for any power of $\boldsymbol{\sigma}^x$.*

3. (8) Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Total available marks: 20