# Linear Algebra I
# Class Test 2

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Name: ______________________________

Use both sides of the paper.
If you need more paper please ask.
(And clearly write your name on the top!)

Total available marks: 20

1. (6 marks) Let $V$ be the vector space over $\mathbb{R}$ of all polynomials of degree less than 3. I.e. $V = \{a_0 + a_1 x + a_2 x^2 : a_0, a_1, a_2, x \in \mathbb{R}\}$. We can define an inner product on this space as

$$\langle f|g\rangle = \int_{-1}^{1} \mathrm{d}x f(x) g(x) \, .$$

$1, x, x^2$ is an ordered basis of $V$, find the corresponding matrix of inner products $\mathbf{A}$. Use this matrix to calculate $\langle x+1|x^2\rangle$. *Tip: Don't do the same integral more than once!*

2. (7 marks) Consider $V = \mathbb{R}^3$, with the standard inner product. Starting from the basis $\{(1,1,1)^T, (1,1,0)^T, (1,0,0)^T\}$ use Gram-Schmidt to construct an *orthonormal* basis for $V$.

   You may use without proof the Gram-Schmidt process: Theorem 4.6. If $\{v_1, \ldots v_n\}$ is a basis of a finite dimensional inner product space $V$, then $\{w_1, \ldots w_n\}$ obtained by

$$\begin{aligned} w_1 &= v_1 \\ w_2 &= v_2 - \tfrac{\langle w_1 | v_2 \rangle}{\langle w_1 | w_1 \rangle} w_1 \\ &\vdots \\ w_k &= v_k - \textstyle\sum_{i=1}^{k-1} \tfrac{\langle w_i | v_k \rangle}{\langle w_i | w_i \rangle} w_i \\ &\vdots \end{aligned}$$

   is an orthogonal basis of $V$.

3. (7 marks) Let $f(\mathbf{u}) = \mathbf{A}\mathbf{u}$ be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ with $\mathbf{u} \in \mathbb{R}^3$ and $\mathbf{A}$ the $3 \times 3$ matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 0 & 2 \end{pmatrix} .$$

Calculate the rank and nullity of $\mathbf{A}$. Find a basis for $\ker(\mathbf{A})$ and a basis for $\mathrm{im}(\mathbf{A})$. *Tip: Remember that the columns of a matrix* $\mathbf{A}$ *span* $\mathrm{im}(\mathbf{A})$.