

Summary of Calculus I

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- **Tangent:** A tangent line is a line that touches a curve, and has the same slope as the curve at the point of contact.
- **Average rate of change:** The average rate of change of a function $f(x)$ between $x = a$ and $x = b$ is

$$\frac{f(b) - f(a)}{b - a}.$$

- **Limit:** Let f be a function defined on some open interval that contains a but not necessarily at a itself. Then we say the limit of $f(x)$ as x approaches a is L and we write

$$\lim_{x \rightarrow a} f(x) = L$$

if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that if

$$0 < |x - a| < \delta \text{ then } |f(x) - L| < \epsilon.$$

- **Left-hand limit:**

$$\lim_{x \rightarrow a^-} f(x) = L,$$

if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that if

$$a - \delta < x < a \text{ then } |f(x) - L| < \epsilon.$$

- **Right-hand limit:**

$$\lim_{x \rightarrow a^+} f(x) = L,$$

if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that if

$$a < x < a + \delta \text{ then } |f(x) - L| < \epsilon.$$

- **Theorem 1:**

$$\lim_{x \rightarrow a} f(x) = L \text{ if and only if } \lim_{x \rightarrow a^-} f(x) = L \text{ and } \lim_{x \rightarrow a^+} f(x) = L.$$

- **Infinit limits:** Let f be a function defined on some open interval that contains a but not necessarily at a itself. Then

$$\lim_{x \rightarrow a} f(x) = \infty$$

means that for every positive number M there is a positive number δ such that if

$$0 < |x - a| < \delta \text{ then } f(x) > M.$$

Let f be a function defined on some open interval that contains a but not necessarily at a itself. Then

$$\lim_{x \rightarrow a} f(x) = -\infty$$

means that for every negative number N there is a positive number δ such that if

$$0 < |x - a| < \delta \text{ then } f(x) < N.$$

- **Vertical asymptote:** The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following is true:

$$\begin{array}{ll} \lim_{x \rightarrow a} f(x) = \infty, & \lim_{x \rightarrow a} f(x) = -\infty, \\ \lim_{x \rightarrow a^-} f(x) = \infty, & \lim_{x \rightarrow a^-} f(x) = -\infty, \\ \lim_{x \rightarrow a^+} f(x) = \infty, \text{ or } & \lim_{x \rightarrow a^+} f(x) = -\infty. \end{array}$$

- **Limit laws:** Suppose that c is a constant and that the limits

$$\lim_{x \rightarrow a} f(x) \text{ and } \lim_{x \rightarrow a} g(x)$$

exist. Then:

1. **Sum law** $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
2. **Difference law** $\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
3. **Constant multiple law** $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$
4. **Product law** $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
5. **Quotient law** $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ if $\lim_{x \rightarrow a} g(x) \neq 0$
6. **Power law** $\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$ where n is a positive integer
7. $\lim_{x \rightarrow a} c = c$
8. $\lim_{x \rightarrow a} x = a$
9. $\lim_{x \rightarrow a} x^n = a^n$ where n is a positive integer
10. $\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer and if n is even then we assume that $a > 0$

11. **Root law** $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$ where n is a positive integer and if n is even then we assume that $\lim_{x \rightarrow a} f(x) > 0$

- **Direct substitution property:** If f is a polynomial or a rational function and a is in the domain of f , then

$$\lim_{x \rightarrow a} f(x) = f(a).$$

Such functions are called continuous at a .

- **Theorem 2:** If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits of both f and g exist as x approaches a then

$$\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x).$$

- **Theorem 3:** The squeeze theorem. If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$$

then

$$\lim_{x \rightarrow a} g(x) = L.$$