Thermal Excitation of Carriers

We want to calculate the number of charge carriers at a temperature T in a simple semiconductor. The probability that a state at energy ε is occupied is

$$f(\varepsilon) = \frac{1}{\mathrm{e}^{(\varepsilon-\mu)/k_B T} + 1}\,.$$

The value of μ is fixed to give the correct number of particles in the system. We will also need the density of states for the conduction and valence band:

$$\text{conduction band} \qquad \mathrm{g}(\varepsilon) = \frac{\mathrm{V}}{2\pi^2\hbar^3}\,(2\mathrm{m_e})^{3/2}\,\sqrt{\varepsilon - \mathrm{E_G}}$$

and

$$\text{valence band} \qquad \mathrm{g}(\varepsilon) = \frac{\mathrm{V}}{2\pi^2\hbar^3}\,(2\mathrm{m_h})^{3/2}\,\sqrt{-\varepsilon}\,.$$

Sketch the density of states and the Fermi function as function of energy on a single plot.

If $\varepsilon - \mu \gg k_B T$ find an approximation for $f(\varepsilon)$. Use this to calculate the number of electrons in the conduction band

$$n = \frac{1}{V}\int_{E_G}^{\infty} f(\varepsilon) g(\varepsilon) d\varepsilon\,,$$

and write the answer in terms of $N_C = 2(2\pi m_e k_B T/h^2)^{3/2}$. Next calculate the number of holes in the valence band

$$p = \frac{1}{V}\int_{-\infty}^{0} [1 - f(\varepsilon)] g(\varepsilon) d\varepsilon\,,$$

and write the answer in terms of $N_V = 2(2\pi m_h k_B T/h^2)^{3/2}$.

Show that np is independent of the value of the chemical potential μ.