Linear Algebra I
Class Test 1

Dr Nicholas Sedlmayr

Friday January 29th 2016

1. Prove that for two square $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$ that $\operatorname{tr}\mathbf{AB} = \operatorname{tr}\mathbf{BA}$. Hence prove that if $\mathbf{P}$ is a square $n \times n$ invertible matrix and $\mathbf{A}$ is a square $n \times n$ matrix then $\operatorname{tr}\mathbf{P}^{-1}\mathbf{AP} = \operatorname{tr}\mathbf{A}$. [4]

2. Calculate the determinant of the following matrices. Infer whether they have an inverse. [6]

$$\begin{pmatrix} 0 & 0 & 0 & 2 \\ 1 & 4 & 6 & 0 \\ 3 & 0 & 2 & 6 \\ 5 & 0 & 9 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 9 & 10 & 3 \\ 4 & 6 & 10 & 2 \\ 3 & 5 & 8 & 1 \\ 5 & 0 & 5 & 2 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 3 & 2 \\ 1 & 1 & 2 \\ 3 & 0 & 6 \end{pmatrix}.$$

3. Prove that the set of vectors $U = \{(x, y, 0)^T : x, y \in \mathbb{R}\}$ forms a subspace of the vector space $\mathbb{R}^3$. [3]

4. Prove that one can not define a field of order 4 with the integers modulo 4, $\{0, 1, 2, 3\}$. [3]

5. Prove that one can define a field of order 4 from $\{0, 1, a, a+1\}$. [4] *Tip: Start from normal addition and multiplication and determine what needs to be changed to satisfy the definition of a field. Marks will be given if a clear outline of what needs to be done can be given, even without the complete solution.*

Total available marks: 20