Thermodynamics and Statistical Physics:
A Summary of Lectures and Some Useful Formulae

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I. FUNDAMENTALS OF STATISTICAL PHYSICS

A. The Microcanonical Ensemble

- Microcanonical ensemble distribution function: $\rho = \delta(E - E')/\Omega(E)$, where $\Omega(E)$ is the number of microstates with energy E.
- Entropy for the Microcanonical ensemble $S = k_B \ln \Omega(E)$.

B. The Canonical Ensemble

- Inverse temperature: $\beta = 1/k_B T$.
- Canonical ensemble: $\rho(E_n) = e^{-\beta E_n}/Z$.
- Partition function for the canonical ensemble: $Z = \sum_n e^{-\beta E_n}$.
- Entropy for the canonical ensemble: $S = -k_B \sum_n \rho(E_n) \ln \rho(E_n)$.
- Heat capacity: $C = \frac{\partial E}{\partial T}$, $\frac{C}{T} = \frac{\partial S}{\partial T}$.
- Pressure: $p = T\frac{\partial S}{\partial V}$
- (Helmholtz) Free energy $F = -k_B T \ln Z$

We can write many useful quantities in terms of the partition function:

- $S = k_B \frac{\partial}{\partial T}(T \ln Z)$.
- $\langle E \rangle = -\frac{\partial}{\partial \beta} \ln Z$.

We can also write many useful quantities in terms of the free energy:

- $C_V = -T\frac{\partial^2 S}{\partial T^2}$.

C. The Grand Canonical Ensemble

- Grand Canonical ensemble: $\rho(E_n) = e^{-\beta E_n + \beta\mu N}/\mathcal{Z}$.
- Partition function for the grand canonical ensemble: $\mathcal{Z} = \sum_n e^{-\beta E_n + \beta\mu N}$.
- Entropy for the canonical ensemble: $S = k_B \frac{\partial}{\partial T}(T \ln \mathcal{Z})$.

- (Helmholtz) Free energy $F = -k_B T \ln Z$

Some useful expressions:

- $\langle N \rangle = \frac{1}{\beta} \frac{\partial}{\partial \mu} \ln \mathcal{Z}$.

D. Free energies/Thermodynamic potentials

- (Helmholtz) Free energy $F = E - TS$.
- H.
- Grand canonical potential $\Phi = F - \mu N$

II. THERMODYNAMIC LAWS