

# Linear Algebra I

## Problem Set 7: Linear Transformations

Dr Nicholas Sedlmayr

Friday March 4th 2016

Due: In class, March 18th 2016

1. Let  $f(\mathbf{u}) = \mathbf{A}\mathbf{u}$  be a linear transformation from  $\mathbb{R}^4$  to  $\mathbb{R}^3$  with  $\mathbf{u} \in \mathbb{R}^4$  and  $\mathbf{A}$  the  $3 \times 4$  matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & -1 & 1 \\ 1 & 3 & -4 & -1 \end{pmatrix}.$$

By converting  $\mathbf{A}$  into echelon form,  $\mathbf{B}$ , calculate the rank and nullity of  $\mathbf{A}$ . If  $\mathbf{R}\mathbf{A} = \mathbf{B}$  what is the matrix  $\mathbf{R}$ ? By constructing a basis which spans  $\text{im}(\mathbf{B})$  find a basis which spans  $\text{im}(\mathbf{A})$ . Note that if  $\mathbf{u}_1, \mathbf{u}_2, \dots$  is a basis for  $\text{im}(\mathbf{A})$  then  $\mathbf{R}^{-1}\mathbf{u}_1, \mathbf{R}^{-1}\mathbf{u}_2, \dots$  is a basis for  $\text{im}(\mathbf{A})$ . (8)

2. For the set of vectors  $S = \{(1, 2, 0)^T, (1, 0, -1)^T\}$  in  $\mathbb{R}^3$  find a linear map  $f : \mathbb{R}^n \rightarrow \mathbb{R}^3$  whose kernel is spanned by  $S$ . (3)
3. Let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation defined by

$$f((x, y, z)^T) = (-x + y - z, x + 2y, -y - 2z)^T.$$

Calculate the images of the vectors (*i.e.*  $f(\mathbf{v}_1)$ , *etc.*)

$$\mathbf{v}_1 = (0, 1, 1)^T, \quad \mathbf{v}_2 = (1, -1, 1)^T, \quad \text{and} \quad \mathbf{v}_3 = (2, 1, 0)^T.$$

Verify that  $f(\mathbf{v}_1) = -\mathbf{v}_1 - 2\mathbf{v}_2 + \mathbf{v}_3$ , and derive similar expressions for  $f(\mathbf{v}_2)$  and  $f(\mathbf{v}_3)$ . Hence write down the matrix of  $f$  with respect to the basis  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$  of  $\mathbb{R}^3$ . (7)

4. Let  $f(u) = \frac{du}{dx}$  be a linear transformation  $f : V \rightarrow V$  where  $V$  is the space of polynomials of order 3 or less over  $\mathbb{R}$ . What is the kernel of  $f$ ? What is the nullity of  $f$ ? (2)

Total available marks: 20