

# Linear Algebra I

## Problem Set 2: Matrices and Vector Spaces

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Friday January 22nd 2016

Due: In class, January 29nd 2016

1. Are the following matrices invertible? (6)

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 1 & 9 & 6 & 2 \\ 3 & 0 & 0 & 6 \\ 5 & 8 & 9 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 9 & 6 & 2 \\ 3 & 0 & 0 & 6 \\ 5 & 8 & 9 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 1 & 9 & 2 \\ 3 & 0 & 6 \end{pmatrix}.$$

2. Prove proposition 1.8 from the lectures, that the determinant of an upper triangular square matrix is equal to the product of its diagonal entries. (4)
3. Prove that if a square matrix  $\mathbf{A}$  is invertible then  $\det \mathbf{A}^{-1} = (\det \mathbf{A})^{-1}$ . (2)
4. Prove that the set of vectors  $\mathbb{R}^2$  forms a vector space when the addition of two vectors  $\mathbf{u} = (x_1, y_1)^T$  and  $\mathbf{v} = (x_2, y_2)^T$  is defined as  $\mathbf{u} + \mathbf{v} = (x_1 x_2, y_1 y_2)$  and scalar multiplication as  $\lambda \mathbf{u} = (x_1^\lambda, y_1^\lambda)$ . What is the zero vector in this vector space? (5)
5. Let  $V$  be the set of all functions  $f, g, \dots$  from the natural numbers  $\mathbb{N} = \{0, 1, 2, 3, \dots\}$  to  $\mathbb{R}$ , with addition defined by  $(f + g)(n) = f(n) + g(n) \forall n \in \mathbb{N}$  and scalar multiplication  $(\lambda f)(n) = \lambda \cdot f(n) \forall n \in \mathbb{N}$ . (This can also be written as the set of functions  $f : \mathbb{N} \rightarrow \mathbb{R}$ .) Prove that  $V$  is a vector space. What is the zero vector in this vector space? (3)

Total available marks: 20