Summary of Calculus I

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1 Limits

1.1. Tangent: A tangent line is a line that touches a curve, and has the same slope as the curve at the point of contact.

1.2. The average rate of change of a function $f(x)$ between $x = a$ and $x = b$ is

$$\frac{f(b) - f(a)}{b - a}\,.$$

1.3. Limit: Suppose $f(x)$ is defined when x is near the number a. (On an open interval that contains a but not necessarily a itself.) Then we write

$$\lim_{x \to a} f(x) = L\,,$$

"the limit of $f(x)$, as x approaches a, equals L", if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a on either side of a.

1.4. Left-hand limit: We write

$$\lim_{x \to a^-} f(x) = L\,,$$

and say the "the left-hand limit of $f(x)$, as x approaches a, equals L", if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a and $x < a$.

1.5. Right-hand limit: We write

$$\lim_{x \to a^+} f(x) = L\,,$$

and say the "the right-hand limit of $f(x)$, as x approaches a, equals L", if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a and $x > a$.

1.6. We have that the

$$\lim_{x \to a} f(x) = L \text{ if and only if } \lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L\,.$$

1.7. Vertical asymptote: The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following is true:

$$\begin{array}{ll} \lim\limits_{x\to a} f(x) = \infty\,, & \lim\limits_{x\to a} f(x) = -\infty\,, \\ \lim\limits_{x\to a^-} f(x) = \infty\,, & \lim\limits_{x\to a^-} f(x) = -\infty\,, \\ \lim\limits_{x\to a^+} f(x) = \infty\,, \quad \text{or} & \lim\limits_{x\to a^+} f(x) = -\infty\,. \end{array}$$

By $\lim\limits_{x\to a} f(x) = \pm\infty$ we mean that $f(x)$ increases or decreases without bound as x approaches a.