1. The dispersion relation for the lattice vibrations of a one dimensional chain of identical atoms of mass M, where each atom is connected to its nearest neighbour by a spring constants K is

$$M\omega^2 = 2K\left[1 - \cos(ka)\right] ,$$

with displacements at each atom n given by

$$u_n = A\mathrm{e}^{\mathrm{i}(kna-\omega t)} .$$

By sketching $\mathrm{Re}\, u_n$ show that $k = 2\pi/\lambda$ with λ the wavelength of the wave. Let $M\omega^2 = K$. What is k? Sketch such a wave for $n = 1, 2, \ldots 5$ at times $t = 0, \pi/(2\omega), \pi/\omega$.

2. Show that the dispersion relation for the lattice vibrations of a one dimensional chain of identical atoms of mass M, where each atom is connected to its nearest and next-nearest neighbour by spring constants K_1 and K_2 is
$$M\omega^2 = 2K_1\left[1-\cos(ka)\right] + 2K_2\left[1-\cos(2ka)\right] .$$
Show that
   (a) This dispersion relation reduces to the relation for sound waves when the wavelength λ is large enough. What is the velocity of the sound waves?
   (b) The group velocity, $v_g = \partial\omega/\partial k$, vanishes at $k = \pm\pi/a$.
   (c) ω is periodic in k with period $2\pi/a$.

   Which of (a-c) would hold if we add longer range connections and why?

3. Obtain expressions for the heat capacity due to longitudinal vibrations of a chain of identical atoms:
   (a) in the Debye approximation;
   (b) using the exact density of states.

   Which gives the higher heat capacity? Show that for low temperatures they give the same value. Why should this be the case?