

1. The dispersion relation for the lattice vibrations of a one dimensional chain of identical atoms of mass M , where each atom is connected to its nearest neighbour by a spring constants K is

$$M\omega^2 = 2K [1 - \cos(ka)] ,$$

with displacements at each atom n given by

$$u_n = Ae^{i(kna - \omega t)} .$$

By sketching $\text{Re } u_n$ show that $k = 2\pi/\lambda$ with λ the wavelength of the wave. Let $M\omega^2 = K$. What is k ? Sketch such a wave for $n = 1, 2, \dots, 5$ at times $t = 0, \pi/(2\omega), \pi/\omega$.

2. Show that the dispersion relation for the lattice vibrations of a one dimensional chain of identical atoms of mass M , where each atom is connected to its nearest and next-nearest neighbour by spring constants K_1 and K_2 is

$$M\omega^2 = 2K_1 [1 - \cos(ka)] + 2K_2 [1 - \cos(2ka)] .$$

Tip: How should equation (1) of the lectures be modified in this case? You can then use the same substitution to solve it.

Show that

- (a) This dispersion relation reduces to the relation for sound waves when the wavelength λ is large enough. What is the velocity of the sound waves?
- (b) The group velocity, $v_g = \partial\omega/\partial k$, vanishes at $k = \pm\pi/a$.
- (c) ω is periodic in k with period $2\pi/a$.

Which of (a-c) would hold if we add longer range connections and why?