Higher Mathematics in English II
Exercises III:
Complex Analysis

Dr Nicholas Sedlmayr

Summer Semester 2018/2019

1.
   - Reduce
   $$\frac{5}{(1-\mathrm{i})(2-\mathrm{i})(3-\mathrm{i})}$$
   to an imaginary number.
   - Simplify
   $$\frac{2+\mathrm{i}}{3-\mathrm{i}}.$$

2. Sketch the curves in the complex plane given by
   - $\mathrm{Im}(z) = -1$,
   - $|z-1| = |z+\mathrm{i}|$, and
   - $2|z| = |z-2|$.

3. Express $z = -8\mathrm{i}$ in polar form.

4. Find all roots of $z^5 = 4 - 4\mathrm{i}$.

5. Calculate $\cos(\frac{\pi}{3} + \mathrm{i})$.

6. Let C be the upper half circle with $|z| = R$ for some real number $R > 1$. Show that for all z lying on C
   $$\left|\frac{\mathrm{e}^{\mathrm{i}z}}{z^2+z+1}\right| \leq \left|\frac{1}{(R-1)^2}\right|.$$

7. Let
   $$f(z) = \begin{cases} \frac{\bar{z}^2}{|z|} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}.$$
   Show that $f(z)$ is continuous everywhere but analytic nowhere on $\mathbb{C}$.