Linear Algebra I
Problem Set 8: Revision

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Friday March 18th 2016

Due: In class, March 25th 2016

1. (6) In which of these is $V = U \oplus W$?
   (a) $V = \mathbb{C}^3$, $U = \{(a, b, c)^T : a + b = 0\}$, $W = \{(a, b, c)^T : a = b = 2c\}$.
   (b) $V = \mathbb{R}_4[x]$, polynomials of degree less than 4,
   $U = \{$polynomials of even degree, and degree less than 4$\}$, and
   $W = \{$polynomials of odd degree, and degree less than 4$\}$.

2. (6) Use Gram-Schmidt orthogonalization to find an orthonormal basis for $V = \mathbb{R}_3[x]$, the polynomials of degree less than 3, starting from the basis $\{1, x, x^2\}$ and with the inner product

$$\langle f|g\rangle = \int_0^2 \mathrm{d}x f(x)g(x)\,.$$

3. (8) Which of the following are bases of $\mathbb{R}^4$? Say why! Which are bases of the subspaces they span?
   (a) $\{(1, 1, 0, 3)^T, (2, 1, 1, 1)^T\}$.
   (b) $\{(1, 1, 0, 3)^T, (2, 1, 1, 1)^T, (1, 0, 1, -2)^T, (0, 0, 0, 1)\}$.
   (c) $\{(1, 1, 0, 3)^T, (2, 1, 1, 1)^T, (1, 0, 0, -2)^T, (0, 0, 0, 1)^T, (0, 1, 0, 0)^T\}$.
   (d) $\{(1, 1, 0, 3)^T, (2, 1, 1, 1)^T, (1, 0, 0, -2)^T, (0, 0, 0, 1)^T\}$.

Total available marks: 20