

1. Show that \vec{k} , \vec{E} , and \vec{B} are all perpendicular to each other for our solution to the EM wave equation:

$$\vec{E} = \vec{E}_0 \cos \left[\omega t - \vec{k} \cdot \vec{r} + \phi_0 \right] .$$

Sketch the behaviour of \vec{E} and \vec{B} as a function of position for some value of \vec{k} , $\phi_0 = 0$ and $\omega t = 0, \pi/4, \pi/2$.

2. Starting from $\nabla \times \vec{E} = -\partial_t \vec{B}$ and $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \partial_t \vec{E}$ show that

$$\partial_t \left[\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right] = -\frac{1}{\mu_0} \nabla \cdot (\vec{E} \times \vec{B}) - \vec{E} \cdot \vec{J} .$$

By comparing this with the continuity equation $\partial_t \rho = -\nabla \cdot \vec{J} + \sigma$ interpret the above equation. $\vec{S} = \vec{E} \times \vec{H}$ is called the Poynting vector.

3. Consider two infinitely large perfectly conducting parallel plates in the x - z plane. One at $y = 0$ and one at $y = d$.
- What are the boundary conditions for the electric and magnetic fields on the plates?
 - Find a solution to the wave equation for this system for $\vec{k} = (0, 0, k)$.
 - Find a solution to the wave equation for this system which is made from two waves with $\vec{k}_1 = k(0, \sin \theta, \cos \theta)$ and $\vec{k}_2 = k(0, -\sin \theta, \cos \theta)$.
 - Which property of the wave equation, and Maxwells equations more generally ensures that the sum of two solutions is still a solution?
4. Show that the energy of a simple dipole \vec{p} in an electric field \vec{E} is $U = -\vec{p} \cdot \vec{E}$.
5. Starting from the potential for a dipole $\vec{p} = p\hat{z}$ find the electric field produced by a dipole. Find a coordinate free representation.