

# Linear Algebra I

## Problem Set 11: Linear Differential Equations and Diagonalization

Dr Nicholas Sedlmayr

Friday April 8th 2016

Due: In class, April 15th 2016

1. (3) Let  $\mathbf{A}$  be an  $n \times n$  matrix with real entries. If  $\lambda$  is a complex eigenvalue of  $\mathbf{A}$  with eigenvector  $\mathbf{u}$  show that  $\bar{\lambda}$  is also a complex eigenvalue of  $\mathbf{A}$ , where  $\bar{\lambda}$  is the complex conjugate of  $\lambda$ . What is the eigenvector for  $\bar{\lambda}$ ?

2. (9) Solve the set of linear differential equations

$$\begin{aligned}y_1'(x) &= y_1(x) - 2 y_2(x) \text{ and} \\y_2'(x) &= 2 y_1(x) + y_2(x)\end{aligned}$$

subject to the boundary condition  $y_1(x=0) = 0$  and  $y_2(x=0) = 2i$ .

3. (8) Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 2 \\ -4 & 2 & -5 \\ -4 & 0 & -3 \end{pmatrix}.$$

Find the eigenvalues of eigenvectors of  $\mathbf{A}$ . What is the base change matrix  $\mathbf{P}$  from the standard basis to the basis of eigenvectors? What is  $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ ?

Total available marks: 20