

Topology in Condensed Matter

Summary of Lectures

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1 Useful Notation

iff	if and only if
\Rightarrow	if then
\equiv	defined as
\therefore	therefore
\because	because
\square	end of proof
\mathbb{R}	set of real numbers
\mathbb{C}	set of complex numbers
\mathbb{I}	identity matrix
\forall	the universal quantifier, for all
\exists	the existential quantifier, there exists
\in	is an element of
\subset	is a subset of
\cup	union of sets
\cap	intersection of sets

2 Some Mathematical Background in Topology

- Let X_1 and X_2 be topological spaces. A map $f : X_1 \rightarrow X_2$ is a **homeomorphism** if it is continuous and has an inverse $f^{-1} : X_2 \rightarrow X_1$ which is also continuous. If there exists a homeomorphism between X_1 and X_2 , X_1 is said to be homeomorphic to X_2 and vice versa. If two topological spaces have different topological spaces then they are not homeomorphic to each other..
- Let X be any set and $\mathcal{T} = \{U_i | i \in I\}$ denote a specific collection of subsets of X . The pair (X, \mathcal{T}) is a **topological space** if \mathcal{T} satisfies the following requirements:

- $\emptyset, X \in \mathcal{T}$
- If \mathcal{T} is any subcollection of I the family $\{U_j | j \in J\}$ satisfies $\cup_{k \in K} U_k \in \mathcal{T}$
- If K is any finite subcollection of I the family $\{U_k | k \in K\}$ satisfies $\cap_{k \in K} U_k \in \mathcal{T}$

X itself is often called a **topological space**. The U_i are the open sets and \mathcal{T} gives a topology to X .

- Let X and Y be topological spaces. A map $f : X \rightarrow Y$ is continuous if the inverse image of open set in Y is an open set in X .