Curie Law

Assuming that the permanent moments in a paramagnet are independent we can write the occupation of an energy level at a temperature T as a Boltzmann factor

$$\mathrm{e}^{-E_p/k_B T} = \mathrm{e}^{-g\mu_B B J_z/k_B T}\,.$$

The partition function is then defined as

$$Z = \sum_{J_z=-J}^{J} \mathrm{e}^{-g\mu_B B J_z/k_B T}\,.$$

By noting that Z is a geometric series calculate the sum.

The magnetization can be found from

$$M = -\frac{N k_B T^2}{B}\left(\frac{\partial \ln Z}{\partial T}\right)_B$$

where the subscript B means the partial derivative is performed for constant B. Show that, with $x = g\mu_B BJ/k_B T$,

$$M = N g \mu_B J B_J(x)\,,$$

where

$$B_J(x) = \frac{2J+1}{2J}\coth\left[\frac{2J+1}{2J}x\right] - \frac{1}{2J}\coth\left[\frac{x}{2J}\right]\,.$$

Find the leading order term in an approximation for $B_J(x)$ for small x.

Show that the susceptibility is

$$\chi = \frac{M}{H} \approx \mu_0 \frac{M}{B} = \frac{N p^2 \mu_B^2 \mu_0}{3 k_B T} \equiv \frac{C}{T}$$

where $p = g\sqrt{J(J+1)}$. C is the Curie constant.