

# Bound States and Tomasch Oscillations in TI-SC Heterostructures



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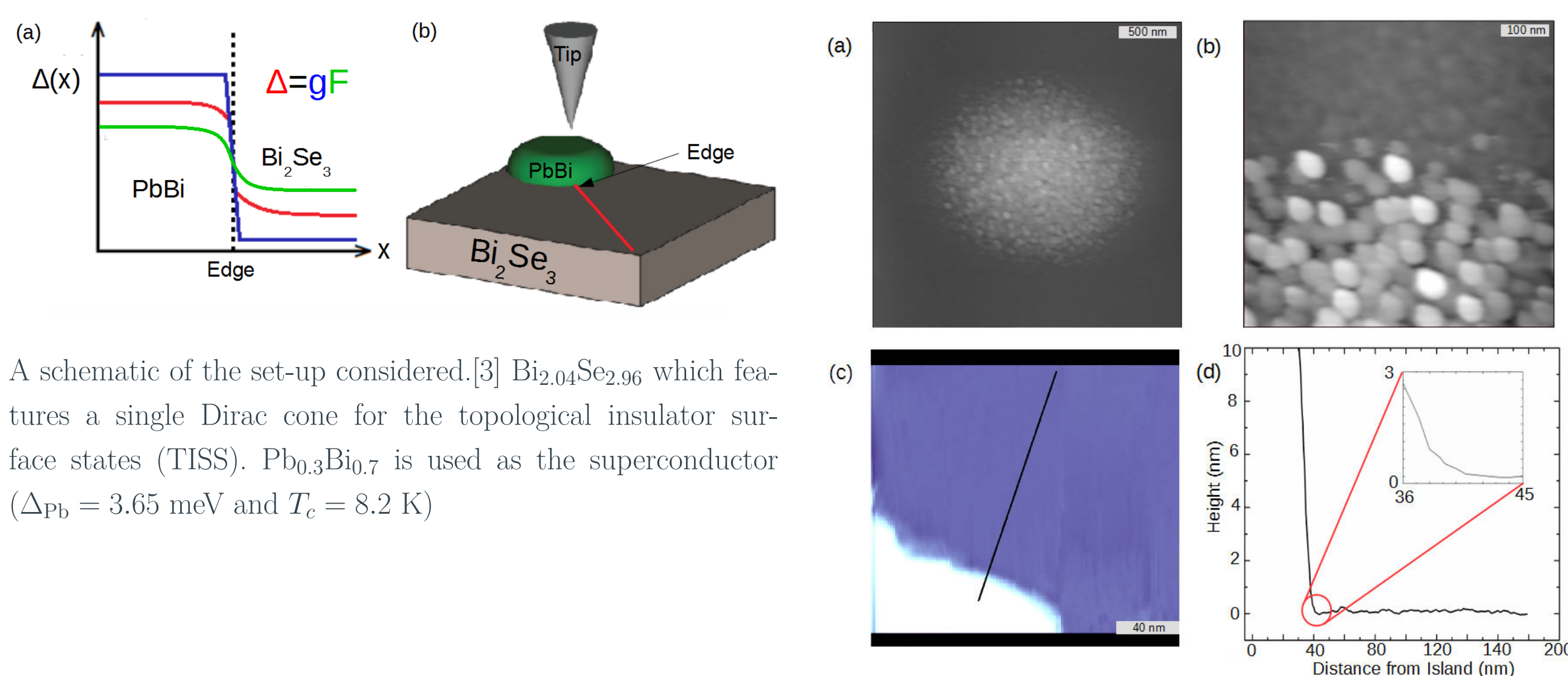
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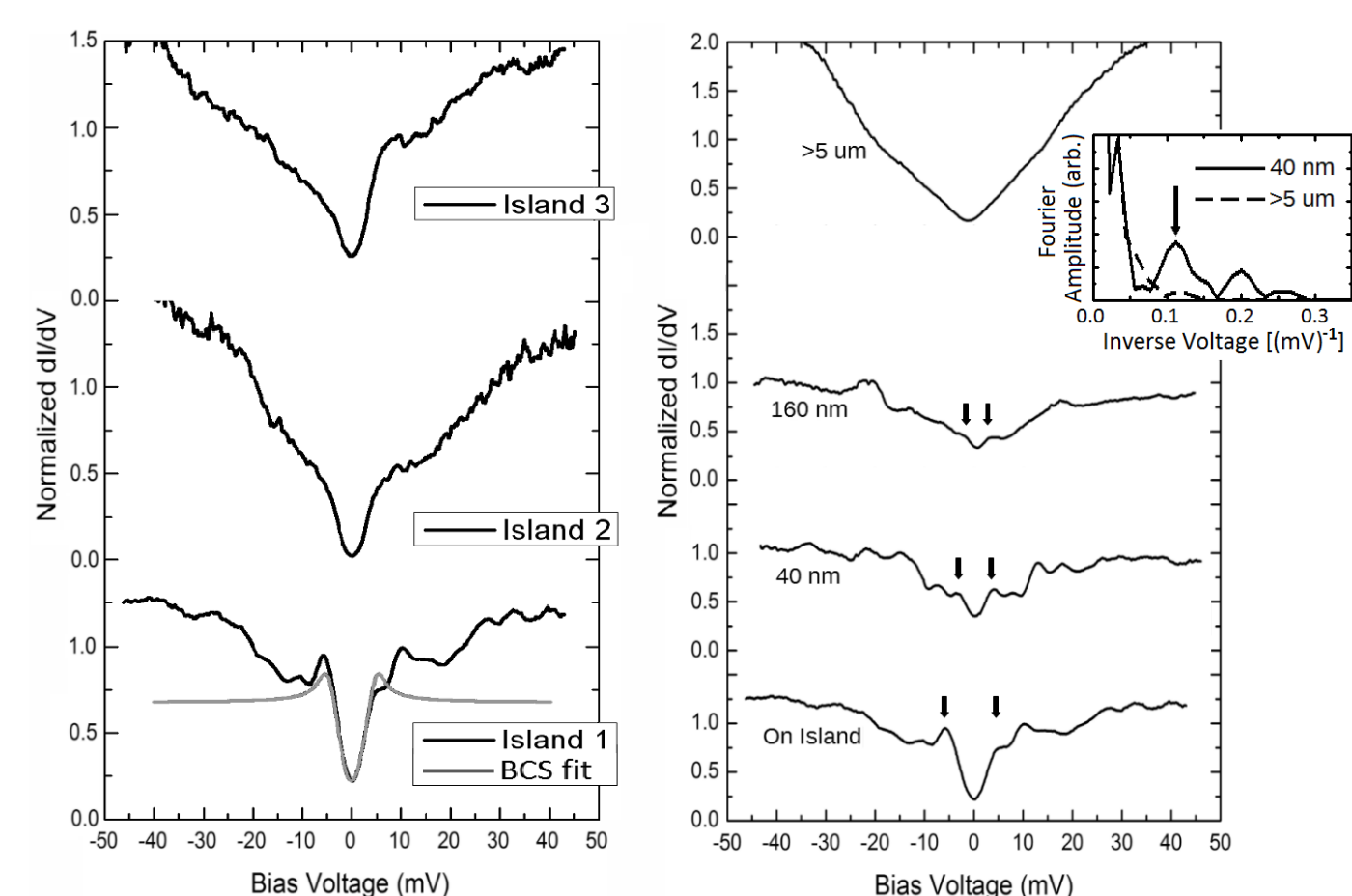
## Abstract

Tomasch oscillations occur in systems where there is a spatial variation in the electron-electron interactions which give rise to the superconducting pairing potential.[1, 2] For example they can occur in states bound within superconducting junctions or islands. Here we report on Tomasch-like oscillations which have been found to occur in Superconducting-Topological Insulator structures.[3] In particular in superconducting islands deposited on top of the topological insulator Bi<sub>2</sub>Se<sub>3</sub>. We go on to consider their existence in junctions formed from Topological Insulators, or topological insulator surface states, and superconductors by calculating the local density of states in these systems.

## Heterostructures



## Experimental Results



We will compare this to two models.

- Check a disorder model using Usadel equation to rule out that the effect is due to trivial surface states.
- In the clean limit we solve a model for the topological surface states.

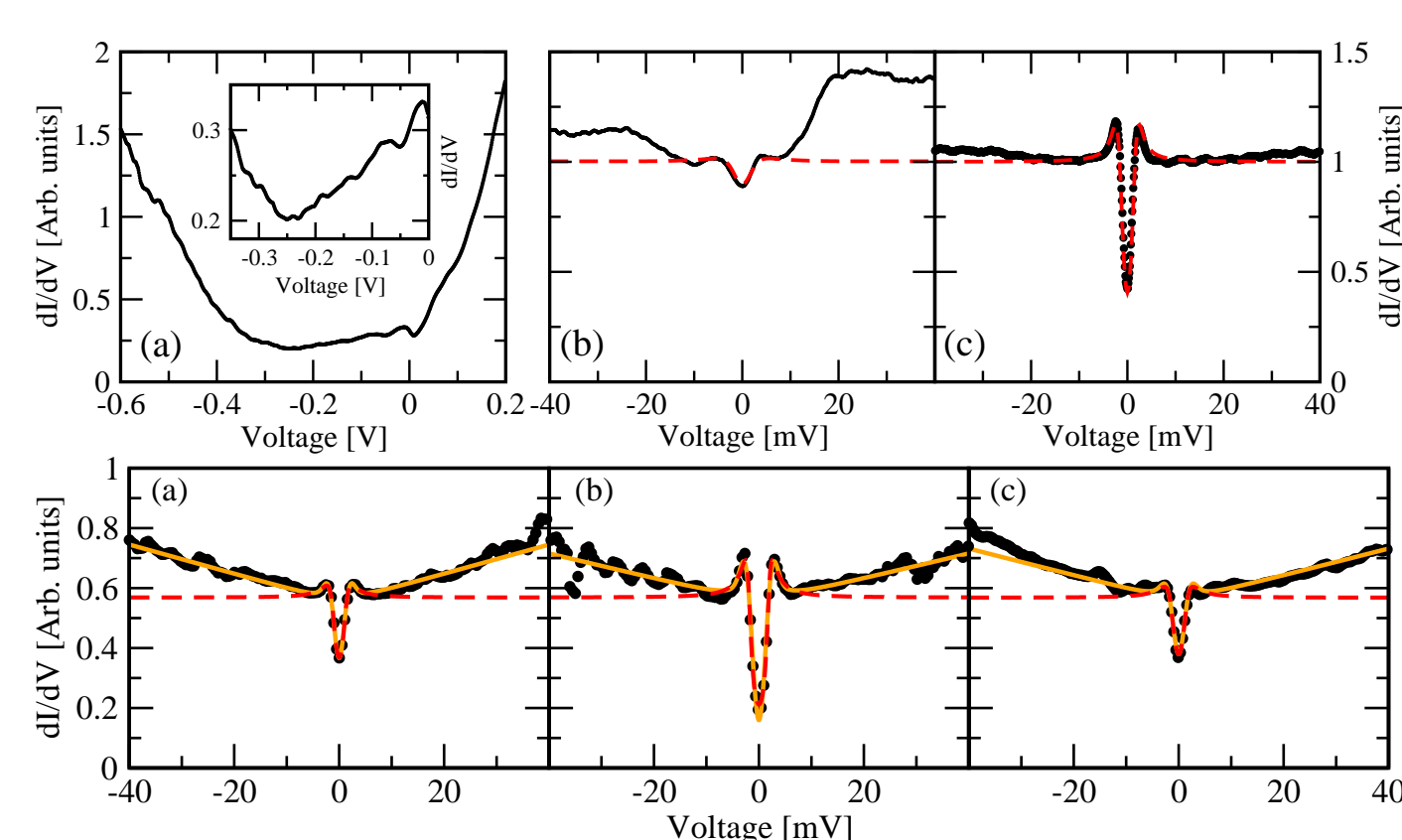
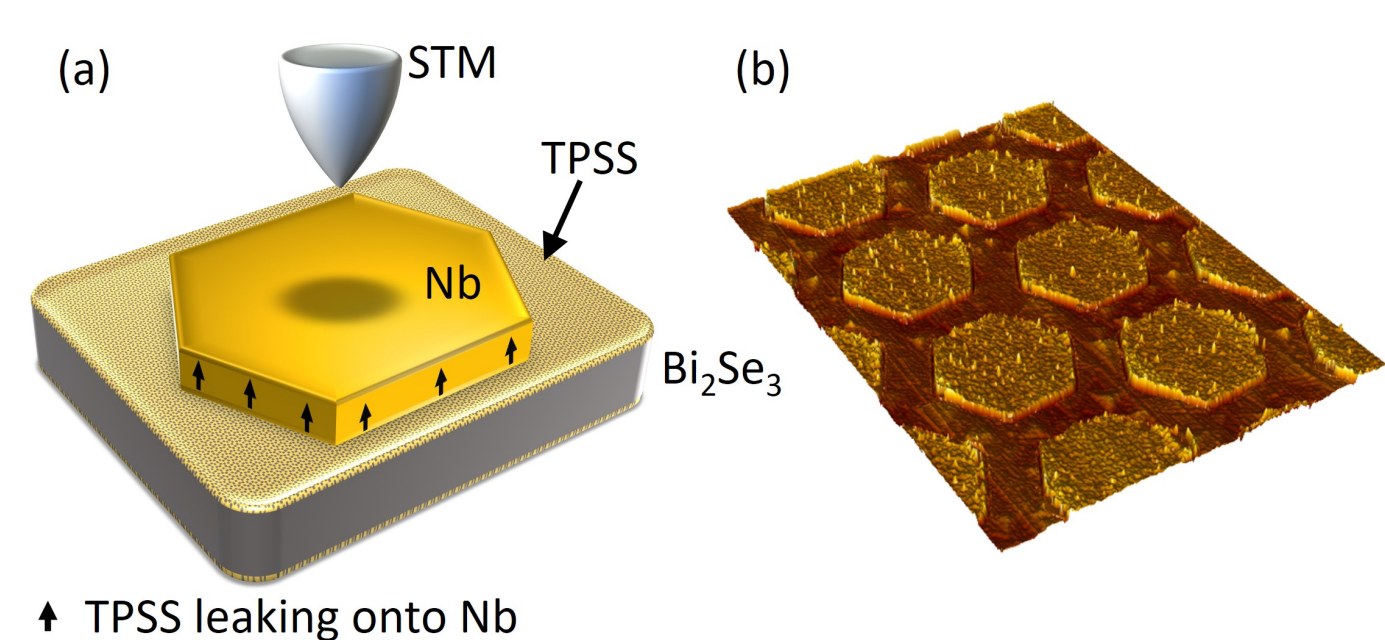
## Inverse Proximity Effect

To understand the TISS leaking into the superconductor we write a phenomenological model:

$$H = H_{\text{metal}} + H_{\text{BCS}} + H_{\text{TISS}} + H_{\text{coupling}}$$

**Niobium Island**  
**Dirac states leaked into Nb**  
**Local spin independent coupling**

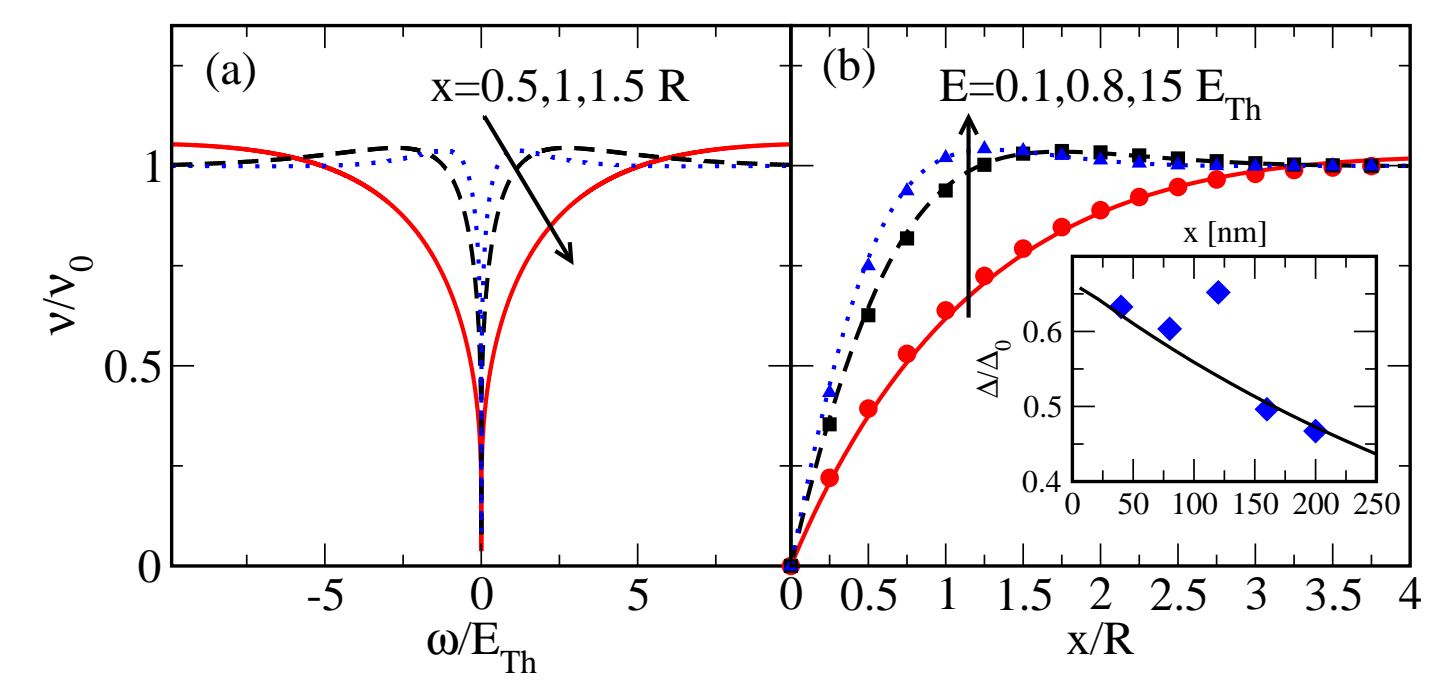
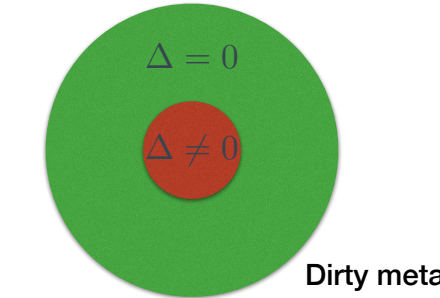
We can compare this to experiments done on superconducting Nb islands:



## Theory for Diffusive Limit

Usadel equation for a circular geometry describing a superconducting island of radius  $R$  surrounded by an infinite normal system. In the normal region:

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x} \frac{\partial \theta}{\partial x} + \frac{i\omega}{E_{\text{Th}}} \sin \theta = 0$$



- $x = r/R$
- Thouless energy:  $E_{\text{Th}} = v_F l / 2R^2$
- $l$  is mean free path
- $v_F$  is Fermi velocity

Superconducting coherence length for disordered surface states is in the range of  $\xi = \sqrt{v_F l / \Delta} \sim 200$  nm. However, in order to fit the inset in (b) a Thouless scale orders of magnitude out is required, and no oscillatory features can be reproduced.

The local density of states is

$$\nu(x, \omega) = \nu_0 \Re \cos[\theta(x, \omega)]$$

$K_0(z)$  is the modified Bessel function

$$\theta_0 = \cos^{-1}(\nu_{\text{BCS}}/\nu_0)$$

Typical values for Bi<sub>2</sub>Se<sub>3</sub> surface states are

- $v_F \simeq 5 \times 10^5$  m/s
- $l \simeq 80$  nm
- $R \simeq 500$

In a linearised regime, for  $x \gg 1$ ,

$$\theta(x, \omega) = \theta_0(\omega) \frac{K_0(x \sqrt{i\omega/E_{\text{Th}}})}{K_0(\sqrt{i\omega/E_{\text{Th}}})}$$

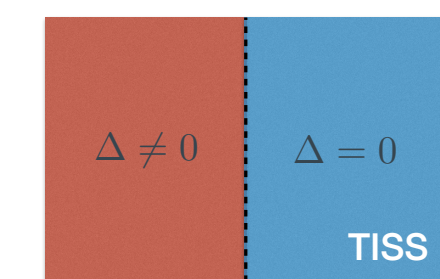
## Theory for TISS Model

Proximity effect relevant for TISS described by Gor'kov's equation:

$$\begin{pmatrix} i\omega_n - H & i\sigma^y \Delta_x \\ -i\sigma^y \Delta_x^\dagger & -i\omega_n - H^* \end{pmatrix} \begin{pmatrix} G_{n,ky}(x, x') \\ F_{n,ky}^{\dagger}(x, x') \end{pmatrix} = \begin{pmatrix} \delta(x - x') \\ 0 \end{pmatrix}$$

with a Hamiltonian

$$H^{(*)} = v_F x \begin{pmatrix} 0 & \pm \hat{k}_x - i k_y \\ \pm \hat{k}_x + i k_y & 0 \end{pmatrix}$$



Assuming a perfect interface the local density of states is

$$\nu(x, \omega) = -\frac{1}{\pi} \Im \int_{-\infty}^{+\infty} \frac{dk_y}{2\pi} \text{Tr} \left[ G_{n,ky}^{T,S}(x, x) \right]_{i\omega_n - \gamma \omega + i\delta}$$

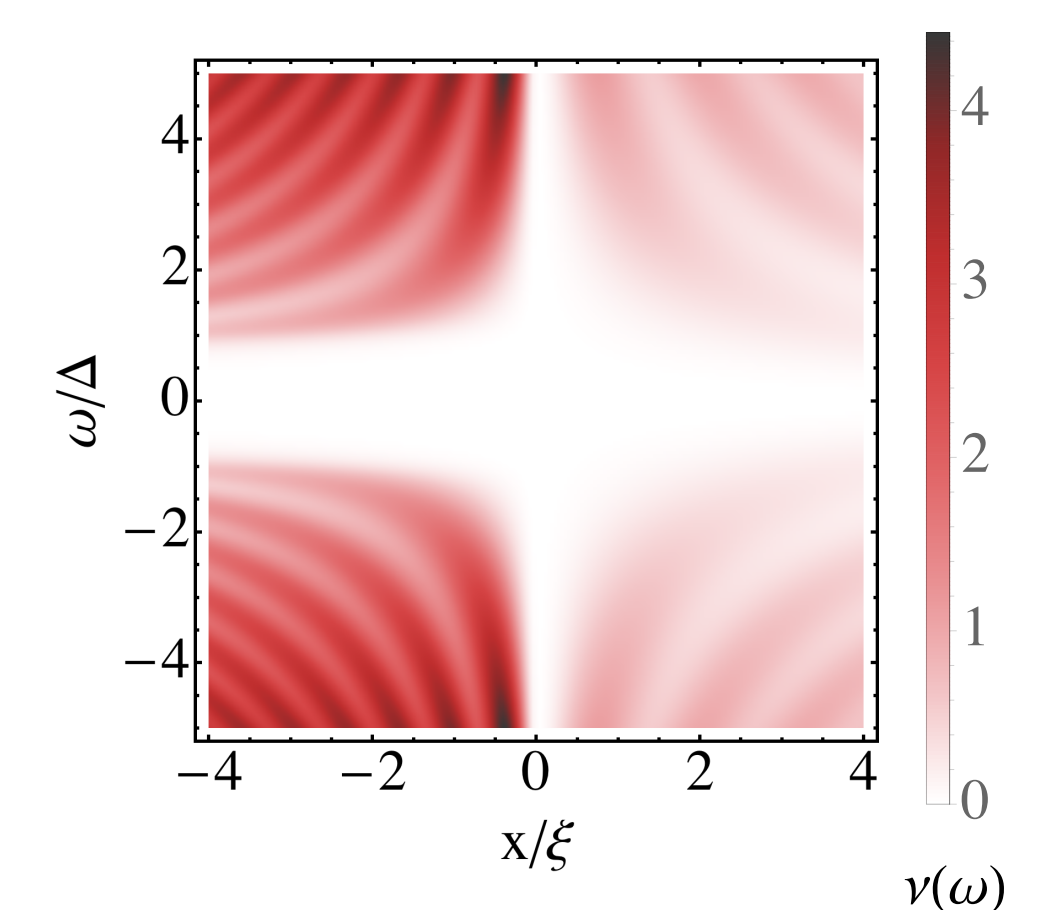
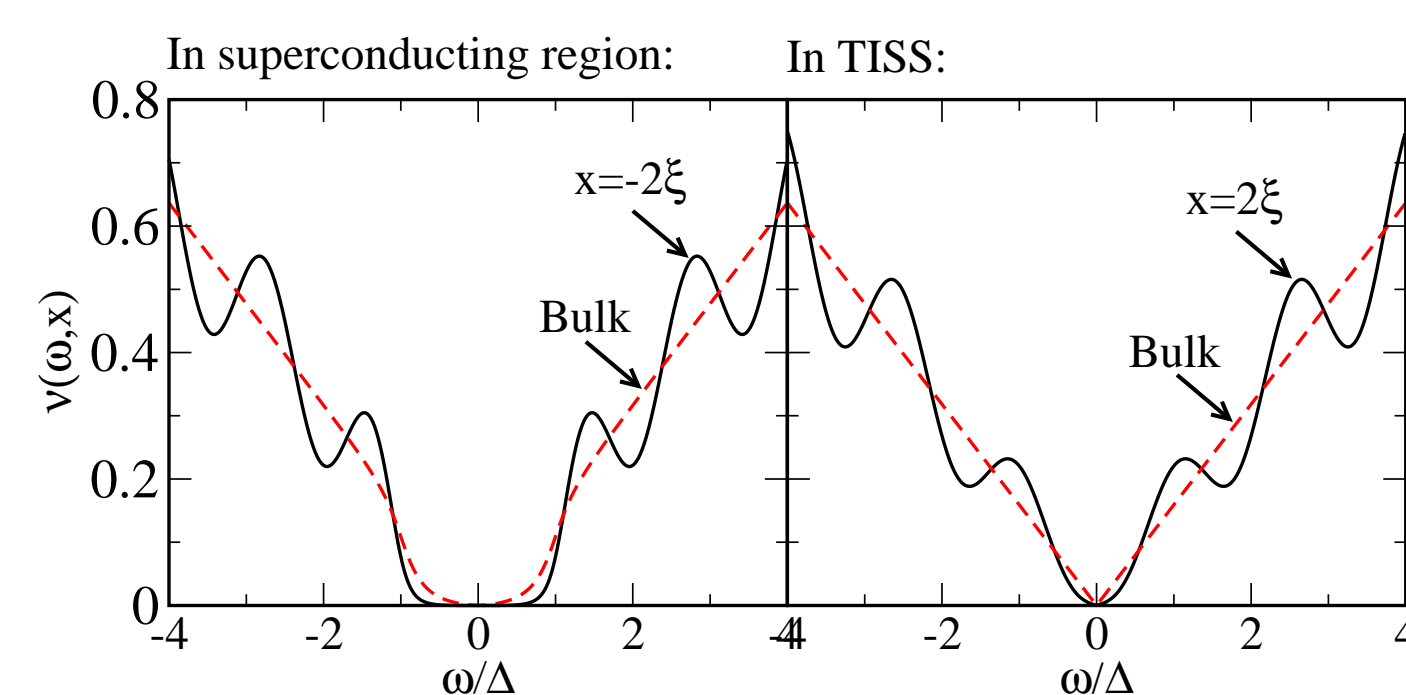
resulting in

$$\nu(x, \omega) = \frac{|\omega| \Theta(|\omega| - \Delta_x)}{2\pi v_F^2} \left[ 1 - J_0 \left( \frac{2x \sqrt{\omega^2 - \Delta_x^2}}{v_F x} \right) \right]$$

$J_0$  is the Bessel function of the first kind

Displays Friedel-like oscillations induced in the LDoS in the normal side of the junction. The energy scale of these oscillations,  $v_F T / 2x$ , are of the same order of magnitude as those we see experimentally

On the superconducting side it implies oscillatory LDoS with Tomasch-like functional dependence on energy and position, which physically originates from quasiparticle scattering as induced by a nonuniform superconducting order parameter



The local density of states, in arbitrary units, as a function of energy for the topological insulator system with  $\Delta = 1$  and  $v_{FT} = 2v_{FS} = 1$ . A phenomenological damping of magnitude  $\Gamma = 0.25\Delta$  is included. The position is measured in units of the superconducting coherence length  $\xi = v_{FS}/\Delta$ . [3]

## References

- [1] T. Wolfram and G. W. Lehman, "Theory of the tomasch effect," Physics Letters A, vol. 24, no. 2, pp. 101-102, 1967.
- [2] T. Wolfram, "Tomasch oscillations in the density of states of superconducting films," Physical Review, vol. 170, pp. 481-490, jun 1968.
- [3] I. M. Dayton, N. Sedlmayr, V. Ramirez, T. C. Chassapis, R. Loloee, M. G. Kanatzidis, A. Levchenko, and S. H. Tessmer, "Scanning tunneling microscopy of superconducting topological surface states in Bi<sub>2</sub>Se<sub>3</sub>," Physical Review B, vol. 93, p. 220506, jun 2016.
- [4] N. Sedlmayr, E. W. Goodwin, M. Gottschalk, I. M. Dayton, C. Zhang, E. Huemiller, R. Loloee, T. C. Chassapis, M. Sdeh, N. Koirala, G. Kanatzidis, S. Oh, D. J. V. Harlingen, A. Levchenko, and S. H. Tessmer, "Dirac surface states in superconductors: a dual topological proximity effect," arXiv preprint, p. 1805.12330, 2018.