1. A quantum violin string can vibrate at frequencies ω, ω, ω and so on. Each vibration mode can be treated as an independent harmonic oscillator. Ignoring the zero point energy, each mode with frequency $p\omega$ has energy $E = n\hbar p\omega$, $n = 0, 1, 2, \ldots$. Write an expression for the average energy of the string at temperature T. Show that at large temperatures the free energy is given by

$$F = \frac{\pi^2 k_B^2 T^2}{6\hbar\omega}.$$

   (Tip: Riemann zeta function gives $\zeta(2) = \pi2/6$.)

2. Find an expression for the heat capacity at constant volume in terms of the free energy $F = E - TS$.

3. Find the free energy for an ultrarelativistic ideal gas of particles where $E = c|\vec{p}|$. Using the expression from the previous question find the specific heat capacity.