Linear Algebra I
Problem Set 12: Hermitian Matrices

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Friday April 15th 2016

Due: In class, April 22nd 2016

1. (10) Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix} .$$

   As we have seen in the lectures these eigenvectors form an orthogonal basis with respect to the standard inner product on $\mathbb{C}^3$. By considering a basis transformation to an orthonormal basis of eigenvectors find a diagonalizing matrix $\mathbf{P}$, and hence $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ where $\mathbf{B}$ is diagonal. *(Hint: $\mathbf{P}^{-1} = \bar{\mathbf{P}}^T$ for an orthogonal transformation from one orthonormal basis to another.)*

2. (5) Without explicitly calculating the diagonalizing matrix $\mathbf{P}$, write down a possible diagonal matrix $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ similar to the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 1 \end{pmatrix} .$$

3. (5) Prove theorem 6.23 from the lectures, i.e. that if f is a self-adjoint transformation of an inner product space V, and λ is an eigenvalue of f, then λ is real. *(Hint: Consider the properties of $\langle v|f(v)\rangle$ for a self-adjoint transformation f where v is an eigenvector of f.)*

Total available marks: 20