A two-regime Markov-switching model approach to the U.S. Consumer Sentiment Index via the Gibbs Sampler

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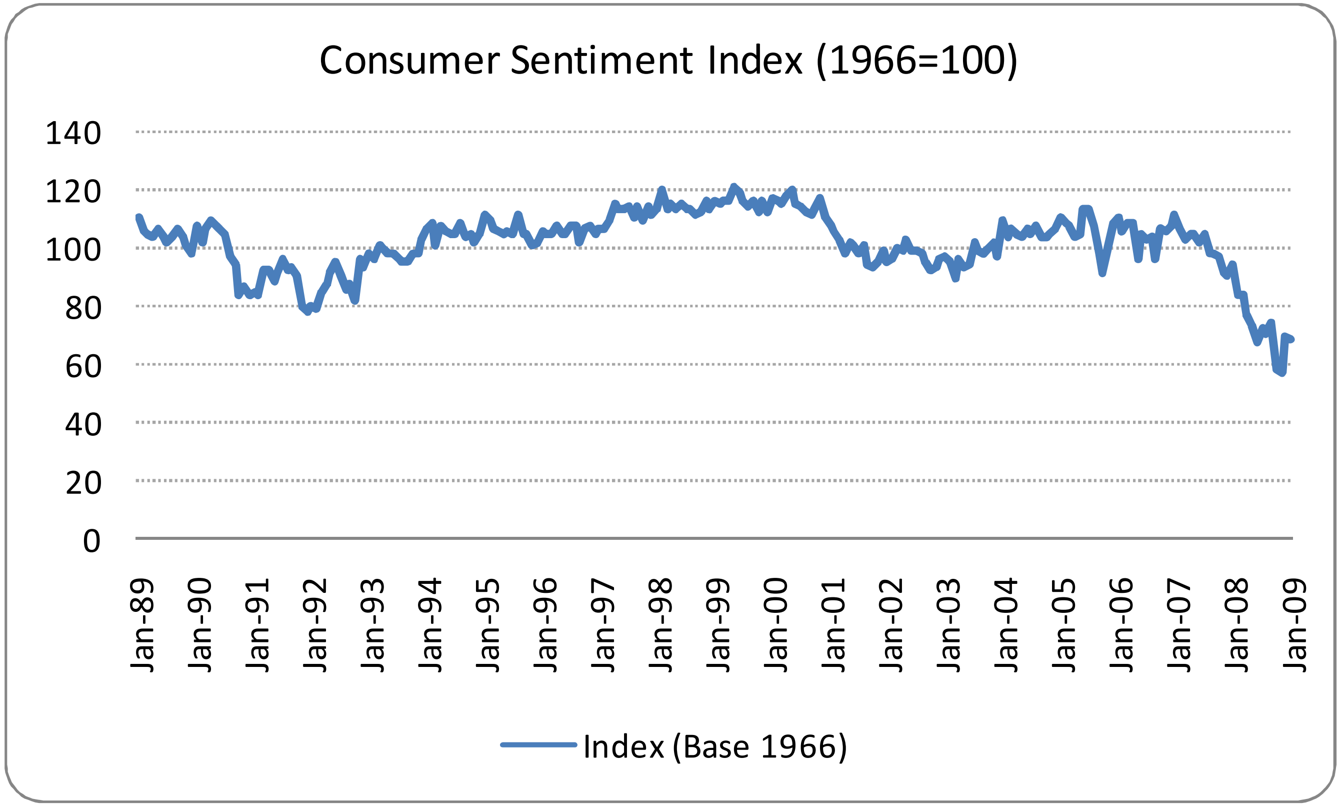
Layout

- Introduction
  - The index (visual inspections)
  - Markov regime-switching models
- Bayesian Gibbs Sampling vs. Maximum Likelihood
- Related Studies
- Model
- Estimation Methodology
- Outcome and its Implications
- In-sample performance

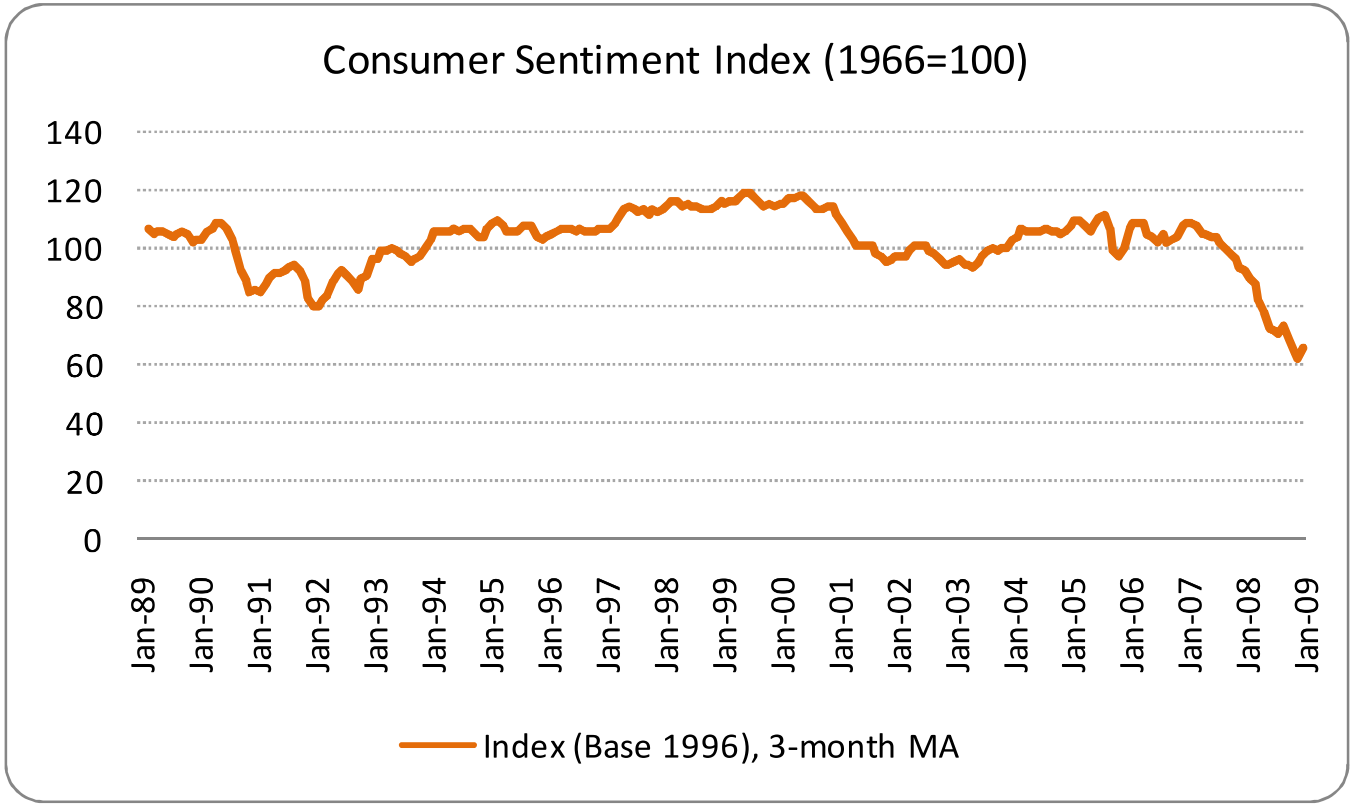
The Index

- In its current form since 1966
- 21 questions overall
- Serve as an indicator of current and future economic conditions
- High accuracy
  - A correlation of 0.74 with interest rate policy (6-month lead on)
  - A correlation of 0.80 with unemployment (9-month lead on)
- Strongly affected by stock price trends (recent years)

The Index


Consumer Sentiment Index (1966=100)
140
120
100
80
60
40
20
0
Jan-89
Jan-90
Jan-91
Jan-92
Jan-93
Jan-94
Jan-95
Jan-96
Jan-97
Jan-98
Jan-99
Jan-00
Jan-01
Jan-02
Jan-03
Jan-04
Jan-05
Jan-06
Jan-07
Jan-08
Jan-09
Index (Base 1966)

The Index


Consumer Sentiment Index (1966=100)
140
120
100
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60
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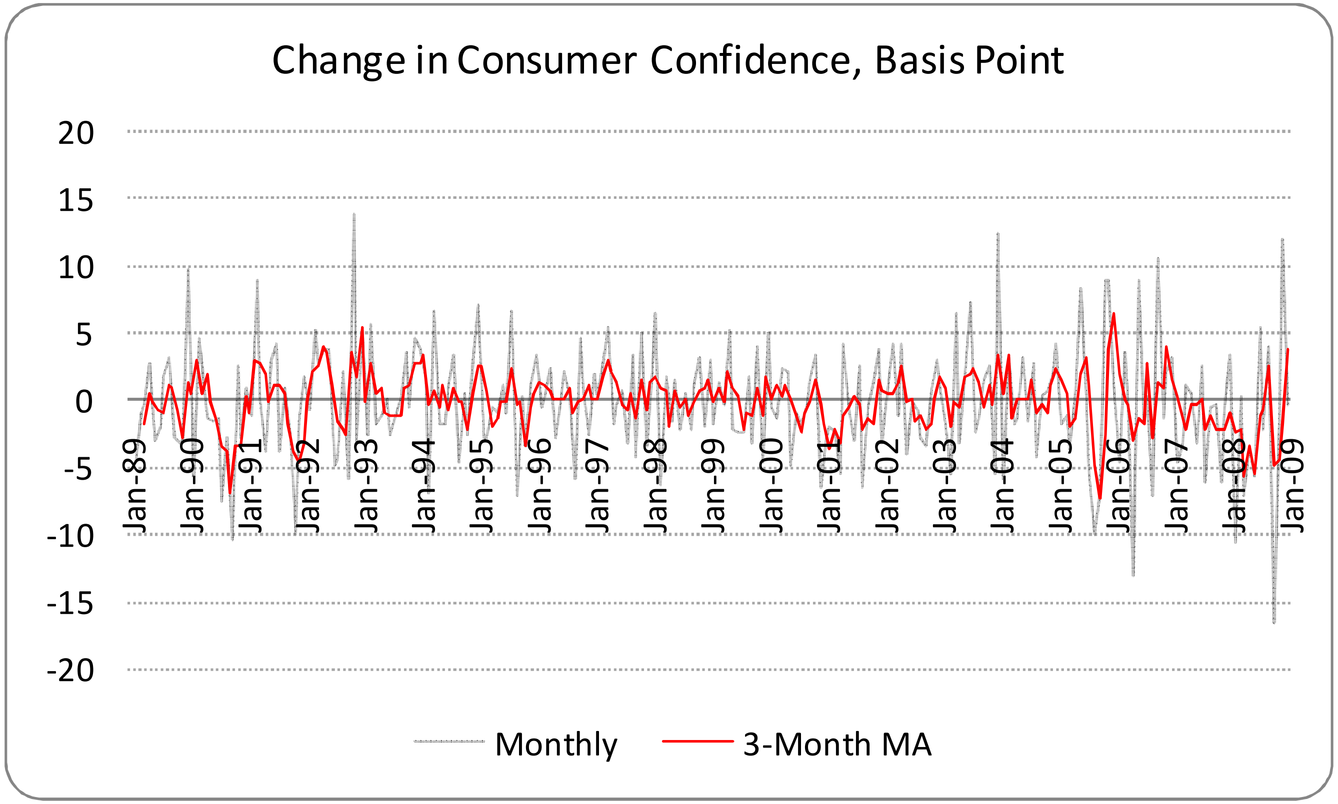
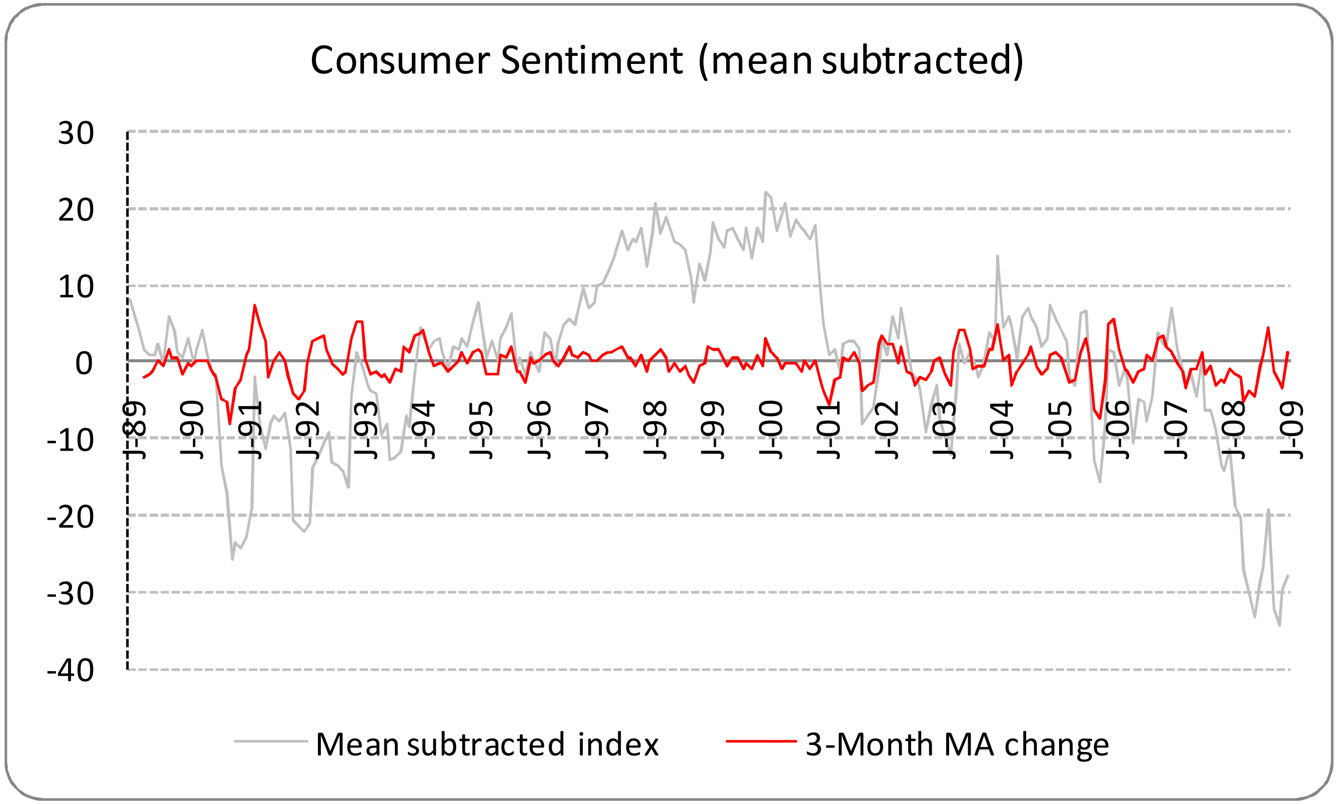
20
0
Jan-89
Jan-90
Jan-91
Jan-92
Jan-93
Jan-94
Jan-95
Jan-96
Jan-97
Jan-98
Jan-99
Jan-00
Jan-01
Jan-02
Jan-03
Jan-04
Jan-05
Jan-06
Jan-07
Jan-08
Jan-09
Index (Base 1996), 3-month MA

The Index



Change in Consumer Confidence, Basis Point
20
15
10
5
0
-5
-10
-15
-20
Jan-89
Jan-90
Jan-91
Jan-92
Jan-93
Jan-94
Jan-95
Jan-96
Jan-97
Jan-98
Jan-99
Jan-00
Jan-01
Jan-02
Jan-03
Jan-04
Jan-05
Jan-06
Jan-07
Jan-08
Jan-09
Monthly
3-Month MA

The Index



Consumer Sentiment (mean subtracted)
30
20
10
0
-10
-20
-30
-40
J-89
J-90
J-91
J-92
J-93
J-94
J-95
J-96
J-97
J-98
J-99
J-00
J-01
J-02
J-03
J-04
J-05
J-06
J-07
J-08
J-09
Mean subtracted index
3-Month MA change

Regime-switching models

- Is there a dynamic that controls the evaluation of consumer confidence (or it is simply a random walk)?
- Are there unobservable structural changes?

Engel & Hamilton (1990)

Kim & Nelson (1999)

Brooks & Persand (2001)

Hamilton (2005)

[Marcelo Perlin]

Gibbs sampling vs. Maximum Likelihood

- Highly nonlinear and complicated models
- Dealing of latent variables
- Large number of parameters

Casella & George (1992)
Gelfand (2000)
Kim & Nelson (1999)

The Model

Period:	Average jump size (Monthly, 3-month MA, basis point)	Average change (Monthly, 3-month MA, basis point)
Apr 1989 – Nov 1989	0.9	-0.3
Nov 1989 – Mar 1990	1.9	0.4
Mar 1990 – Dec 1991	2.3	-1.1
Dec 1991 – Jun 1999	1.3	0.4
Jun 1999 – May 2003	1.6	-0.5
May 2003 – Aug 2005	1.3	0.7
Aug 2005 – Jan 2009	2.5	-1.1

- Difference series is assumed to be characterized by:

$$ccs_t = \begin{cases} \alpha_1 + \sigma_1.\varepsilon_t & if \quad z_t = 1 \\ \alpha_2 + \sigma_2.\varepsilon_t & if \quad z_t = 2 \end{cases}$$

where, $ccs_t \mid z_t = 1 \sim N(\alpha_1, \sigma_1^2)$, $ccs_t \mid z_t = 2 \sim N(\alpha_2, \sigma_2^2)$, $t = 2,3,\ldots\ldots, nobs$

- Markov chain is controlled by:

$$P_{x,y} = P(z_{t+1} = y \mid z_t = x) \text{ where } x, y \in \{1,2\} \text{ and } t = 1,2,\ldots\ldots, nobs$$

The Model

- Initial draws $\begin{pmatrix} ccs_1 \mid z_1 = 1 \sim N(0, \sigma_1^2) \\ ccs_1 \mid z_1 = 2 \sim N(0, \sigma_2^2) \end{pmatrix}$
- Set of model parameters $\Theta = \{\sigma_1^2, \sigma_2^2, P_{1,1}, P_{2,2}, \alpha_1, \alpha_2\}$
- WinBUGS14

Methodology

- We have:
  - Parameters of interest: Θ and starting values for them
  - Set of latent parameters : η
  - Observed data: ccs_t
- We need:
  - The pattern in which unobserved parameters evaluate
  - A sample from posterior : $P(\eta, \Theta \mid ccs_t)$
- The procedure to achieve this:

Sample η^{i+1} from $P(\eta \mid \Theta^{i}, ccs_t)$

Sample Θ^{i+1} from $P(\Theta \mid \eta^{i+1}, ccs_t)$ and so on.

As $i \to \infty$, the joint distribution of (η, Θ) converges to $P(\eta, \Theta \mid ccs_t)$

Methodology

- Priors

$P_{1,1}$ and $P_{2,2} \sim Beta(0.5, 0.5)$

$\sigma_1 \sim Gamma(\lambda_1, \varphi_1)$

$\sigma_2 \sim Gamma(\lambda_2, \varphi_2)$

$\alpha_1 \sim N(\mu_1, \tau_1)$

$\alpha_2 \sim N(\mu_2, \tau_2)$

$z_t \sim Categorical(\pi_{t-1} P)$

- Models Flexibility
  - A variety of behavior is allowed (Asymmetric persistence, long swings, …. , even a random walk !!!)

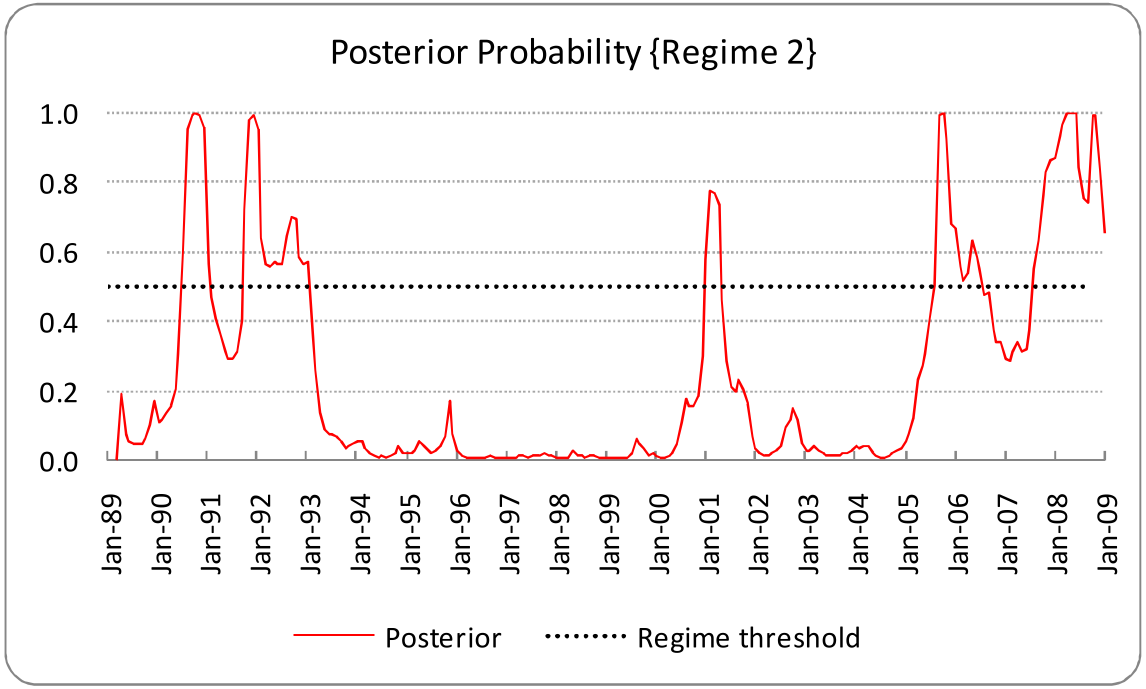
Results and its Implications

- Parameter estimations

$$\Theta = \{\sigma_1^2, \sigma_2^2, P_{1,1}, P_{2,2}, \alpha_1, \alpha_2\} = \{1.43, 2.83, 0.96, 0.89, 0.16, -0.90\}$$

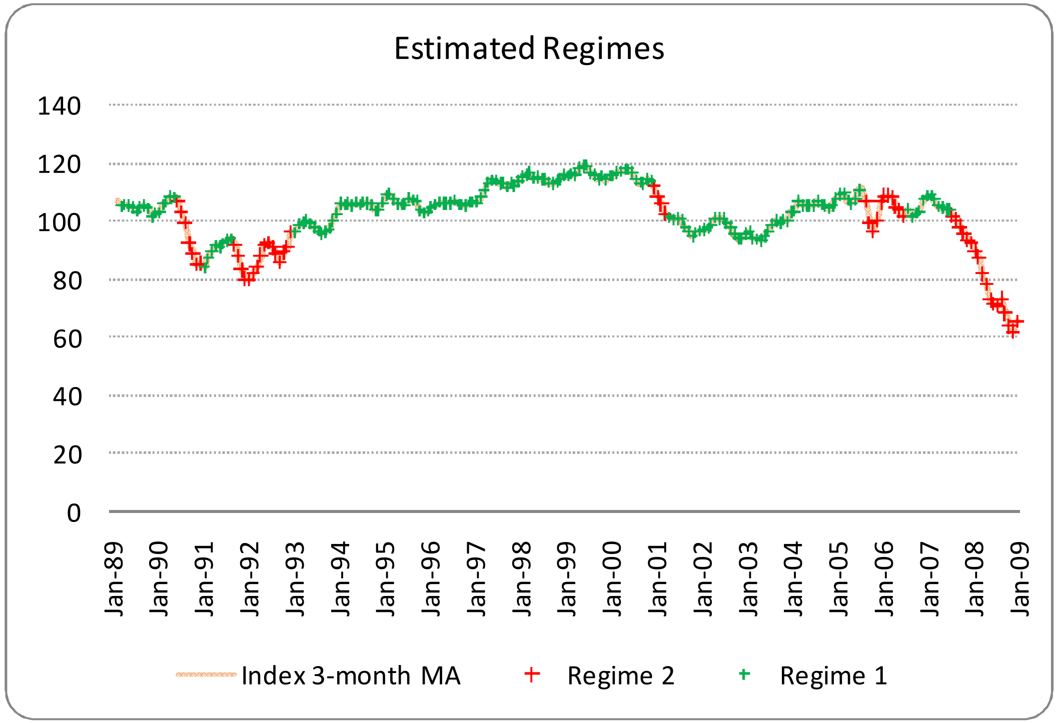
Standard Deviations $[0.11, 0.40, 0.02, 0.10, 0.14, \quad 0.55]$

- $p(z_t = 2 \mid ccs_1, ..., ccs_{nobs}; \hat{\Theta})$ a function of t .



Results and its Implications

- Changes in consumer sentiment are characterized by asymmetric swings
- The system is more likely to be persistent in regime 1.
- Long swings, as in exchange rates, is not apparent.



- The consumer confidence seems to be much more variable when it is deteriorating. (Figure)

Results and its Implications

- Posterior probability estimates suggest that the U.S. consumers entered the decaying-confidence stage in the second half of 2007.
- What is interesting here is that posterior probability of being in regime 2 has already got far away from 1 (!!!) and is likely to go below the regime threshold (!!!) unless the consumers perceive another shock looming in very near future.

In-sample goodness-of-fit

Model	Averaged sum of squared errors
AR(1) without reg.	3.2494
ARMA(1,1) without reg.	3.2493
2-Regime Markov-Switching	3.8497
Gaussian white noise	4.1790

Conditional Expectation

Model	Averaged sum of squared errors
AR(1) without reg.	6.5054
ARMA(1,1) without reg.	6.5054
AR(1)+GARCH(1,1) without reg.	6.5576
2-Regime Markov-Switching	7.5101
Gaussian white noise	8.3592
GARCH(1,1) without reg.	8.4638

Simulated Residuals

Questions

?

```
# US consumer sentiment 3-Month MA(regime 1,2)
# Plunges are sharper and shorter in duration, implying a bigger mean in
absolute terms

model{

sentiment.change[1] ~ dnorm(0,precision[1])
precision[1] <- pow(sigma[state[1]],-2)

for (i in 2:nobs) {
sentiment.change[i] ~ dnorm(mu[i],precision[i])
precision[i] <- pow(sigma[state[i]],-2)
mu[i] <- alpha[state[i]]

}

state[1] ~ dcat(P0[])
sigma[1] ~ dgamma(a,b)
sigma[2] ~ dgamma(c,d)
alpha[1] ~ dnorm(e,f)
alpha[2] ~ dnorm(g,h)


for (i in 2:nobs)
{
state[i] ~ dcat(P.mat[state[i-1], ])
}

P0[1]<-0.5
P0[2]<-0.5
P.mat[1,1] ~ dbeta(0.5,0.5)
P.mat[2,2] ~ dbeta(0.5,0.5)
P.mat[1,2]<- 1-P.mat[1,1]
P.mat[2,1]<- 1-P.mat[2,2]
}
```

$$f(x, y) \propto \binom{n}{x} y^{x+\alpha-1}(1-y)^{n-x+\beta-1},$$

$$x = 0, 1, \ldots, n \quad 0 \le y \le 1$$

$f(x \mid y)$ is Binomial (n, y)

$f(y \mid x)$ is Beta $(x + \alpha, n - x + \beta)$

Sample from conditionals

$$f(x) = \binom{n}{x} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(x+\alpha)\Gamma(n-x+\beta)}{\Gamma(\alpha+\beta+n)},$$

$$x = 0, 1, \ldots, n,$$

Draw from marginal

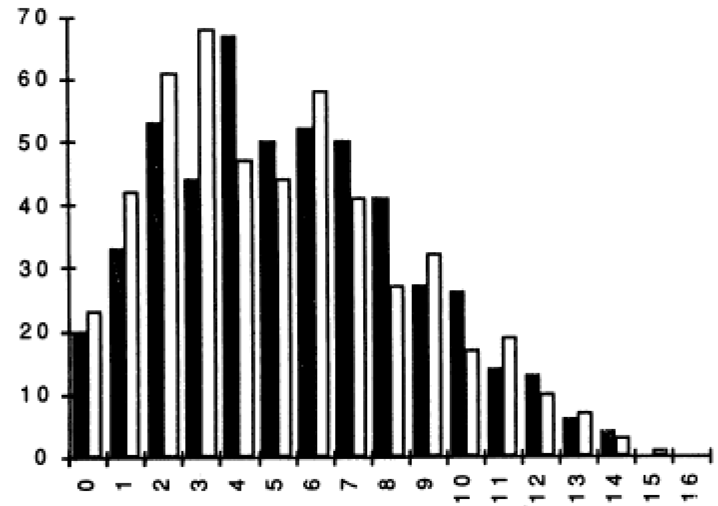


Figure 1. Comparison of Two Histograms of Samples of Size $m = 500$ From the Beta-Binomial Distribution With $n = 16$, $\alpha = 2$, and $\beta = 4$. The black histogram sample was obtained using Gibbs sampling with $k = 10$. The white histogram sample was generated directly from the beta-binomial distribution.