A two-regime Markov-switching model approach to the U.S. Consumer Sentiment Index via the Gibbs Sampler[1]

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Abstract

The main proposal made in this paper is to use a Markov-switching model in order to determine soaring-confidence and plunging-confidence periods of U.S. economy using U.S. Consumer Sentiment Index data[3]. The differences of index values are used to obtain a stationary series and exploit, if appropriate, autoregressive modeling whereas the transition between different phases of confidence is modeled via a first-order Markov process. A Bayesian approach, namely Gibbs sampling, is adopted to determine the probability of being in a decaying confidence period at any given point in time, which has a number of implications for policy makers beforehand.

[3] Data are published by the Survey Research Center at the Institute for Social Research at the University of Michigan.

1. Preamble

Apart from The Conference Board U.S. Consumer Confidence Index, the index of U.S. Consumer Sentiment published by the University of Michigan is the second most important index for the assessment of current and future economic conditions from consumers' point of view. The index has been produced largely in its current form since 1966. Among 21 questions in overall, a fraction represents the questions for evaluating consumer estimates of personal financial situations, general business environment and economic conditions.

The index level, in time, started to serve as an indicator of current and future economic trends. For the period of 1960-1995 the correlation between expected regime of interest rate policy and actual development is calculated as 0.74 with a 6-month lead on basis. Furthermore, the responses to questions on unemployment have a correlation of 0.80 with future labor market trends with a nine-month lead. It is also worth noting that, particularly in recent years, consumer confidence is strongly affected by stock price trends. The evaluation of the index is represented in Figure-1. What is interesting in the figure is that the index level apparently shows a composition of two dynamics: an increasing-sentiment dynamic in which consumer confidence level does not present big changes, varies randomly around small values and increases gradually, and a decreasing-sentiment dynamic where economic sentiment changes sharply between irregular intervals and plunges significantly.

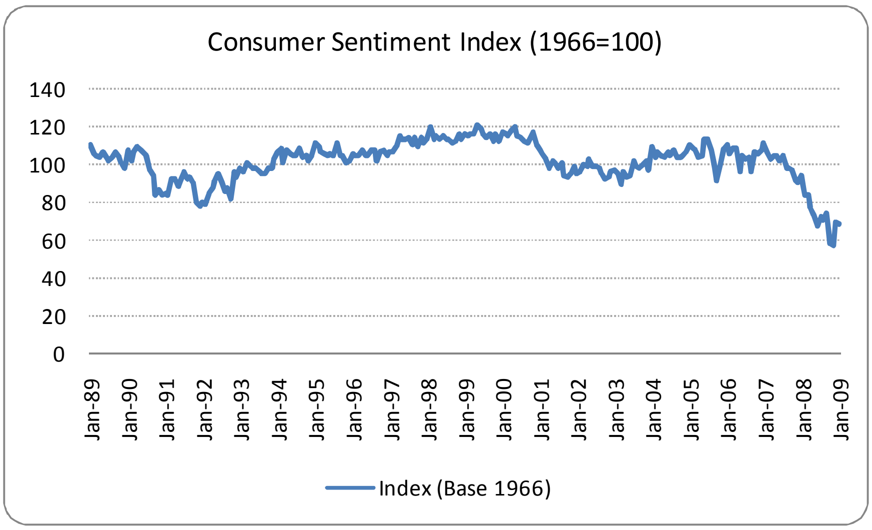

Figure-1 : Evaluation of the U.S. consumer confidence level

For the sake of statistical accuracy, it is useful to use a moving average over, say, 3 months since monthly changes are supposed to have minor significance on the inference of current economic conditions (see Figure-2).

The question of interest here is, as Engel and Hamilton (1990) suggest, how to formalize the concept of long swings in the consumer confidence levels. Another circumstance is that whether long swings (or a directionless drift of a random walk, otherwise) constitute a systematic part in the evaluation of consumer sentiment in time.

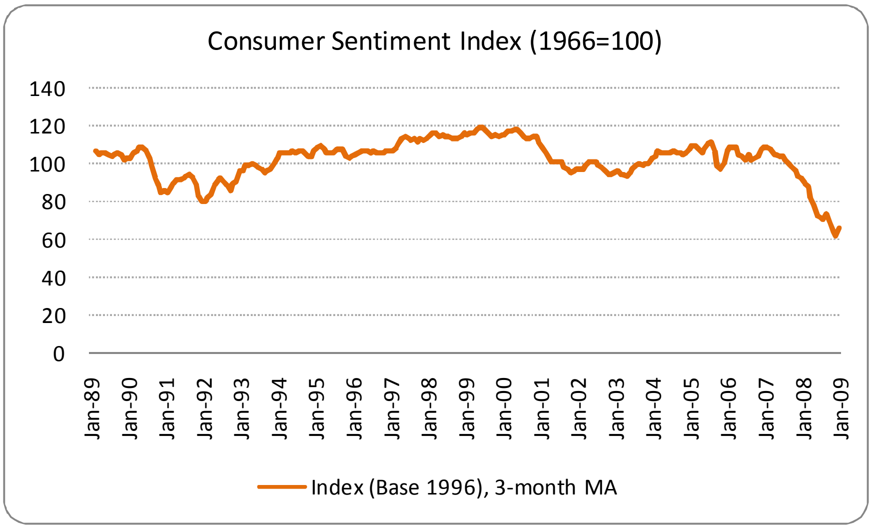

Figure-2 : Evaluation of U.S. confidence level index (3-month average)

In many parts of time series analysis, a Markov switching model and a Markov switching model with time varying transition probability have been used to capture structural changes which cannot be observed. But these models are highly nonlinear and are difficult to be estimated, or is impossible to be done in some cases by using the maximum likelihood estimation. In such cases, other method should be applied. In this paper, I introduce a Bayesian Gibbs sampling method to estimate the model discussed below. Using this method enables us to estimate the highly nonlinear model. However, this paper is not the first attempt to estimate a Markov switching model by Gibbs sampling (see Related Studies).

Using Gibbs sampling method opens a way to estimate highly complicated models. One difference between the maximum likelihood and MCMC is the dealing of latent variables: a latent variable is treated as unobserved variable in the maximum likelihood estimation, while it is treated as parameters in MCMC estimation. This difference in treating latent variables enables us to make the estimated model be easy to be estimated, compared with the maximum likelihood estimation.

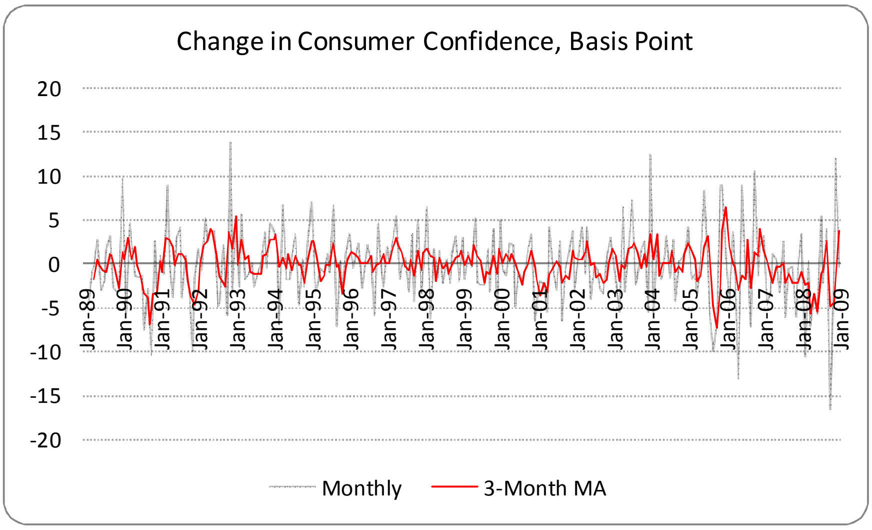

Figure-3 : Change in consumer sentiment (3-month average)

Another advantage of MCMC estimation is the problem of degree of freedom. In the maximum likelihood estimation, if the parameters number is large compared with the observation, the model becomes unstable and the obtained result would not be reliable. MCMC estimation eliminates this problem.

2. Related Studies

After Hamilton (1989) proposed the regime-switching model to examine the persistency of recessions and booms, many papers, including Engel and Hamilton (1990), Bekaert and Hodrick (1993), Engel (1994), Bollen, Gray and Whaley (2000), and Dewachter (2001) applied this model to exchange rate data. Ichiue and Koyama (2007) extended these models to closely investigate the relationship among exchange rate returns, volatilities, and interest rate differentials. Some others used a time-varying dynamic factor model with regime switching to generate regime probabilities from the leading indicators that can be used to signal increases in Turkey's country risk and potential currency crises in real time.

The existence of Gibbs sampling in finance literature cannot be traced back through a long history. Casella and George (1992) introduced basic foundations of Gibbs sampler and analytically established its properties in a sample case followed by more complicated ones. Kim and Nelson (1999) introduced an extensive interface where Bayesian methods were mostly used to estimate switching models. Ichiue and Koyama (2007) employed a regime-switching model to investigate exchange rate volatilities where they once again brought the Gibbs sampling approach under the spotlight. Bauwens and Lubrano (1998) explained how the Gibbs sampler can be used to perform Bayesian inference on GARCH models. Wago (2004), again using Gibbs algorithm, performed a Bayesian estimation of a new kind of asymmetric GARCH in which conditional variance obeys two different regimes with a smooth transition function.

3. Raw Data and Source Description

The raw data is provided by Thomson Datastream. It is also worth noting that the behavior of the consumer sentiment rates cannot be considered as seasonal, since data is seasonally adjusted. Table-1 summarizes the simple characteristics of the data.

Table-1

Summary statistics for difference series of the U.S. consumer sentiment index	
Statistic	
Number of observation	238
Mean	-0.17
Variance	4.19
Skewness	-0.35
Kurtosis	4.00
Minimum	(15.10.2005) -7.40
Maximum	(15.01.2006) 6.4
Jarque-Bera normality test	14.04 [5.99]
Engle's LM test for ARCH effects (up to 5 lags)	31.09 [11.07]

(i) * Denotes significance at 0.01 level
(ii) Critical values in parentheses

The next session discusses Markov-switching models and how Bayesian methods can be useful to estimate model parameters.

4. Hypothesis and the Model

Let ccs_t be the change in 3-month-moving-average consumer sentiment at time t where $t = 1,2,.....,nobs$. The idea behind the Markov-switching models is to associate ccs_t with a random variable z_t, namely unobserved state variable, that indicates which regime the system is in; increasing-confidence (regime 1) or decaying-confidence (regime 2). Since it is unobserved, we cannot know, for any given point in time, which regime the confidence evaluation is in. However, what we assume is that z_t follows a two-state Markov chain of order 1 with transition probabilities:

$$P_{x,y} = P(z_{t+1} = y \mid z_t = x) \text{ where } x, y \in \{1,2\} \text{ and } t = 1,2,.....,nobs$$

This kind of Markov processes (i.e. the ones in which state variables that determine conditional distributions) are called hidden Markov processes.

Our Gibbs sampling estimates corresponds closely to visual impressions of Table-2 and Figure-4. As it can be inferred from the table, decaying-sentiment periods accompanied with relatively greater jumps are persistent during a number of consecutive months, whereas the soaring sentiment periods last longer with relatively smaller jumps. Figure-4 also provides strong evidence how positive-sentiment periods consist of small changes around a slightly positive trend. Thus, taking these into account, conditional distribution of ccs_t is modeled either as a Gaussian or as an order 1 autoregressive process with parameters depend on whether the system is in a decaying-sentiment or a rebounding-sentiment period. That is:

Table-2: Observed subperiodic characteristics

Period:	Average jump size (Monthly, 3-month MA, basis point)	Average change (Monthly, 3-month MA, basis point)
Apr 1989 – Nov 1989	0.9	-0.3
Nov 1989 – Mar 1990	1.9	0.4
Mar 1990 – Dec 1991	2.3	-1.1
Dec 1991 – Jun 1999	1.3	0.4
Jun 1999 – May 2003	1.6	-0.5
May 2003 – Aug 2005	1.3	0.7
Aug 2005 – Jan 2009	2.5	-1.1

$$ccs_t = \begin{cases} \alpha_1 + \sigma_1.\varepsilon_t & if \quad z_t = 1 \\ \alpha_2 + \sigma_2.\varepsilon_t & if \quad z_t = 2 \end{cases}$$

Then, the prior distributions in each regime can be defined as:

$$ccs_t \mid z_t = 1 \sim N(\alpha_1, \sigma_1^2)\,,\ t = 2,3,.....,nobs$$

$$ccs_t \mid z_t = 2 \sim N(\alpha_2, \sigma_2^2)\,,\ t = 2,3,.....,nobs$$

That is, when $z_t = 1$, the observed change in consumer confidence is presumed to have been drawn from $N(\alpha_1, \sigma_1^2)$. Thus, when $z_t = 1$, the trend in the consumer confidence is α_1. Similar inferences about $z_t = 2$ case can follow.

However, since we do not have ccs_{t-1} when $t = 1$, initial conditional distributions is specified as:

$$\begin{pmatrix} ccs_1 \mid z_1 = 1 \sim N(0, \sigma_1^2) \\ ccs_1 \mid z_1 = 2 \sim N(0, \sigma_2^2) \end{pmatrix}$$

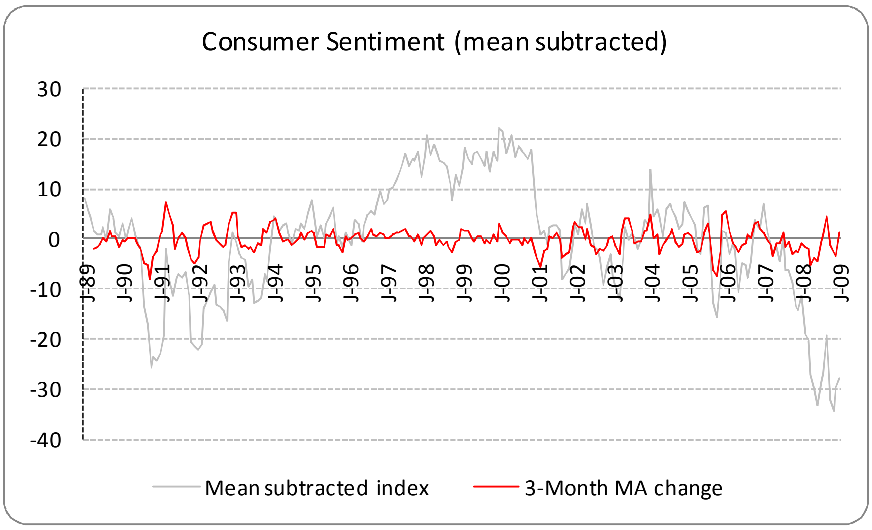

Figure-4 : Change in consumer sentiment (3-month average)

Above subindices of $\sigma_{0,1}^2$ represents the dynamic that the system is in. Once the model is specified, the next step is the estimation of unknown parameters. The set of parameters to be estimated is:

$$\Theta = \{\sigma_1^2, \sigma_2^2, P_{1,1}, P_{2,2}, \alpha_1, \alpha_2\}$$

5. Estimation Methodology

To produce samples from the joint posterior distribution of the parameters given the data, Gibbs sampling algorithm is used to construct a Markov chain with the property that its limiting distribution is the target posterior distribution (Krichene, 2003). For the sake of simplicity, below I give a very clear explanation of how the Gibbs sampling process works:

(i) We have;

a. Starting values for the set of parameters of interest, Θ, and also for the set of nuisance parameters, η (z_t in this case).

b. A series of observed data, ccs_t

(ii) We seek;

a. The pattern in which the nuisance parameters evaluate

b. A sample from posterior distribution of the parameters of interest, $P(\eta, \Theta \mid ccs_t)$

(iii) The procedure to achieve is:

a. Sample η^{i+1} from $P(\eta \mid \Theta^i, ccs_t)$

b. Sample Θ^{i+1} from $P(\Theta \mid \eta^{i+1}, ccs_t)$ and so on.

c. As $i \rightarrow \infty$, the joint distribution of (η , Θ) converges to $P(\eta, \Theta \mid ccs_t)$

Iteration should be run until Monte Carlo error appears to be less than 5% of sample standard deviation.

Prior to the sampling part, we need to specify prior distributions of each parameter involved in the model. This step is where the researcher's former experience about the model parameters comes in. Initially, I consider a number of non-informative prior distributions for $P_{1,1}$ and $P_{2,2}$:

$$P_{1,1} \text{ and } P_{2,2} \sim Beta(0.5, 0.5)$$

Another point at this step is that the variances during decaying-sentiment periods appear to be greater than that of rebounding-sentiment regime. Although we do not specify any inequality with a predefined direction between them, this prior knowledge is expected to be reflected in the results. Including all we know about them, below is expressed prior knowledge about the deviation parameters:

$$\sigma_1 \sim Gamma(\lambda_1, \varphi_1)$$
$$\sigma_2 \sim Gamma(\lambda_2, \varphi_2)$$

Expressing a more informative Gamma allows us to improve estimation results for these parameters accordingly with our level of familiarity with the data and learn about the appropriate range of variation for both regimes. Moreover, inserting such preconditions help MCMC process identify potential separate regimes. Thus, initial bounds to above-mentioned variables directly affect the reliability of the results. Besides, we similarly specify some other appropriate priors for the rest of the parameters of our interest:

$$\alpha_1 \sim N(\mu_1, \tau_1)$$
$$\alpha_2 \sim N(\mu_2, \tau_2)$$

$$z_t \sim Categorical(\pi_{t-1} P)$$

where P is the transition probability matrix that rules the Markov process and π_{t-1} is matrix of the posterior probabilities of being in regime 1 and 2, respectively.

Although the expressions above contain a lot of information about the parameters of interest, they do not yield to a sufficient tool for estimation in analytical terms. That is why we need to exploit the power of numerical methods, Gibbs sampling in this case.

Along with its former versions, WinBugs14 performs all of these steps free of charge. This can be seen as a contribution for further research since other software packages has almost unaffordable costs.

Note the variety of behavior that our model allows; in particular, we do not impose that household sentiment levels are described by long swings. Alternatively, there can be asymmetry in the persistence of the two regimes- upward moves could be short but sharp (i.e. μ_1 large and positive, $P_{1,1}$), whereas downward moves could be gradual and last longer (i.e. μ_2 negative and small in absolute value and $P_{2,2}$ large), or vice versa. Alternatively change in consumer sentiment in any period could be completely independent of the state that appeared last period, as in a random walk, if $P_{1,1} = 1 - P_{2,2} \Leftrightarrow P_{1,1} = P_{2,1}$. A third possibility is the long swings hypothesis, as Engel and Hamilton (1990) suggests, which is represented by the claim that μ_1 and μ_2 are opposite in sign and the values for $P_{1,1}$ and $P_{2,2}$ are both large. Next session is devoted to the results and its potential implications.

6. Outcome and its Implications

Three simultaneous and independent Markov chains were run for every single parameter with at least 5000 simulations for each (i.e. at least 15000 sample draw for each parameter excluding 1000 burn-in). However, we unfortunately did not reach conclusive evidence about the convergence of regime switching autoregressive model. It is worth noting that, before reporting estimation results, all Monte Carlo error values are lower than 5% of sample standard deviation. The parameter estimates are given below:

Table-3: Parameter Estimations

Parameter	Posterior Mean	Standard Deviation
$P_{1,1}$	**0.96**	0.02
$P_{2,2}$	**0.89**	0.10
σ_1^2	**1.43**	0.11
σ_2^2	**2.83**	0.40
α_1	**0.16**	0.14
α_2	**-0.90**	0.55

The points that MCMC appeared to converge (see Appendix-x) associate regime 1 with a consumer confidence increase of 0.16 basis points while regime 2 is characterized by a trend of average 0.90 basis points decrease. A greater variability in regime 2 is apparent, as evidenced by visual impressions and an estimated value of 2.83. Figure-5 plots the probability of being in regime 2 at each point in the sample period, that is $p(z_t = 2 \mid ccs_1,....,ccs_{nobs};\hat{\Theta})$ is plotted as a function of t .

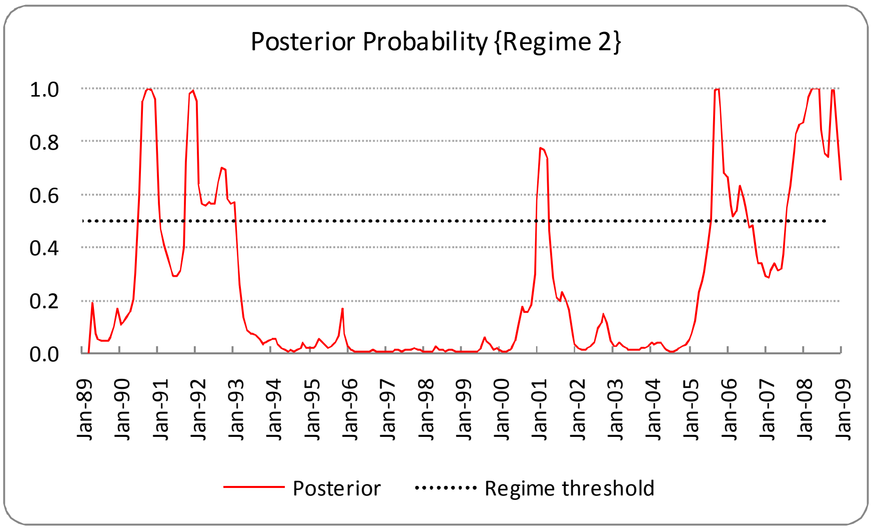

Figure-5 : Posterior probabilities of being in Regime 2

The horizontal dashed line marks the threshold value at which the researchers typically conclude that the process has switched between regimes. Either the parameter estimates in

Table-3 or posterior probabilities given in Figure-5 show that movements in consumer sentiment in the U.S. are characterized by long but asymmetric swings. The Gibbs sampling estimates of 0.96 and 0.89 for $P_{1,1}$ and $P_{2,2}$ respectively indicate that the system is more likely to be persistent in regime 1 than it would be in regime 2. This result does not only suggest that the dynamics that control the process of change in consumer confidence do not show very similar properties with symmetrically long swings hypothesis that Engel and Hamilton (1990) found strong evidence for exchange rates, but also support the claim that the behavior of consumers' state of mood regarding the economic conditions have some common attributes, as also implied by high correlations, with stock markets that are also characterized by long-lasting consistent climbs followed by steeper and short-lived declines. The U.S. consumer sentiment index was in a state of rise till the beginning of 1990, and then a number of large fluctuations followed until it starts recovering from the beginning of 1993. The years 1999 and 2000, which mark the burst of dot-com bubble and shake of faith in markets, gave an end to this upward trend, as the model successfully captures. The 2001-2003 period, during which mostly the side-effects of the panic created by 9/11 attacks are echoed, was recorded as a new bottom line of the level at which the U.S. people feel confident about the current economic situation. From 2003 on, an euphoria, again marked by our model, was evident in the consumers' feelings, supported by high liquidity and loose lending owing to rising oil prices. (September 2005?). The symptoms of global financial crisis were started to be felt in the market with tightening credit conditions at the first half of the year 2007. Thus, our model of asymmetric swings tends to match closely what one might be led to believe from casual inspection of Figure-4.

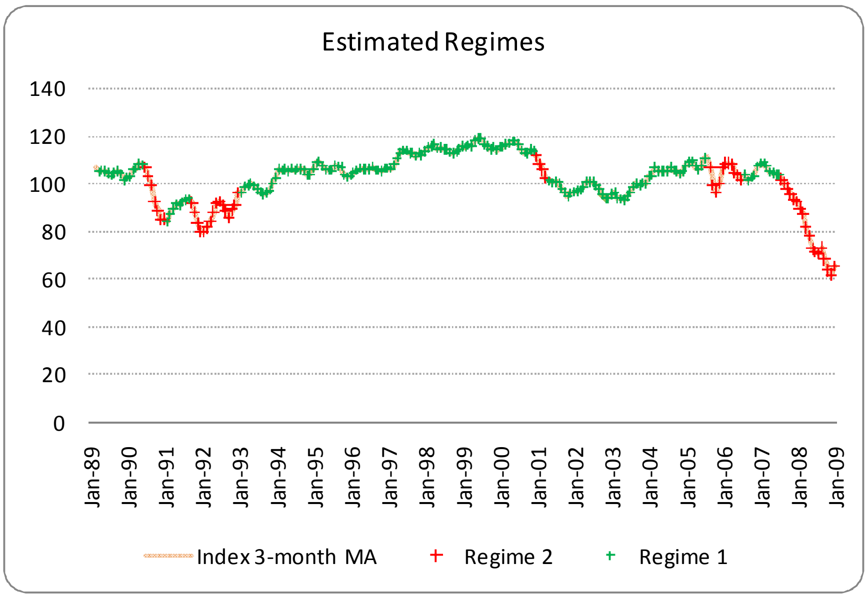

Figure-6 : Change in consumer sentiment (3-month average)

Regimes 1 and 2 can be distinguished not only by their means but also by the variances of the conditional distributions. The consumer confidence seems to be much more variable when it is deteriorating. Our estimates suggest that the U.S. consumers entered the decaying-confidence stage in the second half of 2007. Another interesting point is that posterior probability of being in regime 2 has already got far away from 1 and is likely to go below the regime threshold unless the consumers perceive another shock looming in very near future.

7. In-sample goodness-of-fit performance

Below provided in-sample performances of a variety of models. The numbers indicate the averaged sum of squared residuals. To illustrate, for Markov-Switching model, this number corresponds to the difference between the actual change in the sentiment index, ccs_{t+1}, and estimated change, e_{t+1}, where:

$$e_{t+1} = \begin{cases} N(\alpha_1, \sigma_1) & with \quad probability \quad \pi_{1,t+1} \\ N(\alpha_2, \sigma_2) & with \quad probability \quad \pi_{2,t+1} \end{cases}$$

and,

$$\begin{bmatrix} \pi_{1,t+1} & \pi_{2,t+1} \end{bmatrix} = \begin{bmatrix} \pi_{1,t+1} & \pi_{2,t+1} \end{bmatrix} * \begin{bmatrix} P_{1,1} & P_{1,2} \\ P_{2,1} & P_{2,2} \end{bmatrix}$$

or more generally,

$$\Pi_{t+s} = \Pi_t * \mathrm{P}^s$$

Table-4: Average in-sample errors using conditional expectations

Model	Averaged sum of squared errors
AR(1) without reg.	3.2494
ARMA(1,1) without reg.	3.2493
2-Regime Markov-Switching	3.8497
Gaussian white noise	4.1790

As the given values in Table-4 suggest, autoregressive models provides a better fit for the monthly changes in the U.S. Consumer Sentiment Index partially since an infinite amount of state is embedded in them at each state. However, this circumstance does not cast shadow over the evidence of regime-switching behavior of the corresponding data. We still have a better estimate than a pure white noise process. In Table-5 provided a similar measure of the goodness-of-fit where, in this case, innovations are simulated via Monte Carlo method using the model parameters. Similar inferences can follow.

Table-5: Average in-sample errors using conditional expectations with simulated innovations

Model	Averaged sum of squared errors
AR(1) without reg.	6.5054
ARMA(1,1) without reg.	6.5054
AR(1)+GARCH(1,1) without reg.	6.5576
2-Regime Markov-Switching	7.5101
Gaussian white noise	8.3592
GARCH(1,1) without reg.	8.4638

Our interest in this paper has been to describe a two-regime Markov-switching model to detect the different structural dynamics of the U.S. Consumer Sentiment Index as well as to define the unobservable state variable which evaluates according to a Markov chain. A possible extension to this study could be to explore if these different regimes and the pattern in which they evaluate could be used by policymakers to develop proactive policy responses beforehand.

8. Appendices

Appendix : Some of the questions included in the calculation process of U.S. Consumer Sentiment Index

- **Personal Finance:** We are interested in how people are getting along financially these days. Would you say that you (and your family living there) are better off or worse off financially than you were one year ago? What about next one-year-term from now?
- **General Business Conditions:** Would you say that at present time business conditions are better or worse than they were a year ago? And what do you think thay will be during the next 12 months?
- **Interest Rates:** No one can say for sure, but what do you think will happen to interest rates for borrowing money during the next 12 months – will they go up, stay the same or go down? (Go up, stay the same, go down, don't know/not applicable)
- **Economic Policy:** As to the economic policy of government – I mean steps taken to fight inflation or unemployment – would you say the government is doing a good job, only fair, or a poor job? (Good job, only fair, poor job, don't know/not applicable)
- **General Economic Conditions:** Looking ahead, which would you say is more likely – that in the country as a whole we will have continuous good times during the next five years

or so, or that we will have periods of widespread unemployment or depression, or what?

- **Consumption:** About the big things people buy for their homes – such as furniture, a refrigerator, stove, television, and things like that. Generally speaking, do you think now is a good or a bad time for people to buy major household items? (Good time to buy, uncertain; depends, bad time to buy)

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