Elsevier Editorial System(tm) for Applied Soft Computing
Manuscript Draft

Manuscript Number:

Title: Rough decision making approach for solving fuzzy multi- objective fractional programming and its application to fixed-charge transportation problem

Article Type: SI: BAFI 2015

Keywords: Fractional Programming, Fixed-Charge Transportation Problem, Rough Programming, Fuzzy Programming, Robust Ranking Technique, Fuzzy Chance-Constrained Rough Technique.

Corresponding Author: Dr. Sankar Kumar Roy, Ph.D

Corresponding Author's Institution: Vidyasagar University

First Author: Sankar Kumar Roy, Ph.D

Order of Authors: Sankar Kumar Roy, Ph.D; Sudipta Midya, M. Sc; Gerhard Wilhelm Weber, Ph. D ; Nadi Serhan Aydin, M. Sc

Abstract: This paper presents a fuzzy Multi-Objective Fractional Fixed-Charge Transportation Problem (MOFFCTP) in a "rough" decision making environment. The parameters of the formulated model are considered as being of a fuzzy nature. In order to tackle these parameters, we employ the different types of fuzzy measures such as possibility, credibility and necessity measures. Using the fuzzy chance-constrained rough programming technique, we derive the best optimal solution of our proposed MOFFCTP. Then, the obtained result is compared with the results extracted from the Robust ranking technique. We also use rough set theory for extending as well as partitioning the feasible region of the MOFFCTP to accommodate more information by considering two approximations. Using the approximations, we design two models, namely, Lower Approximation Model (LAM) and Upper Approximation Model (UAM) of the proposed MOFFCTP. Finally, by using these models, we obtain the optimal solutions which provide the satisfactory results relevant to our problem. We incorporate a real-life example on the MOFFCTP to show the applicability and performance of our proposed model.

Department of Applied Mathematics with
Oceanology and Computer Programming
Vidyasagar University, Midnapore –721102 West Bengal, India

Date: 01.02.2016

To
Professor Richard Weber
Guest Editor
Special Issue of Applied Soft Computing
Applied Soft Computing for Business Analytics
Universidad de Chile, Chile
Email: rweber@dii.uchile.cl

Dear Sir,

Please find enclosed our manuscript "**Rough decision making approach for solving fuzzy multi-objective fractional programming and its application to fixed-charge transportation problem**" by Roy et al. (Sankar Kumar Roy, Sudipta Midya, Gerhard-Wilhelm Weber and Nadi Serhan Aydin) which we would like to submit for publication as a special issue in "**Applied Soft Computing for Business Analytics**" of the esteemed journal "**Applied Soft Computing**".

To the best of our knowledge, this is a unique way for analyzing the fuzzy multi-objective fractional programming with the application to fixed-charge transportation problem through rough decision making approach. We believe our findings would appeal to the readership of "**Applied Soft Computing**".

The manuscript has not been previously published, is not currently submitted for review to any other journal, and will not be submitted elsewhere before a decision is made by this journal. All authors have approved the manuscript and agree with its submission to "**Applied Soft Computing**".
I look forward from you at your earliest convenience.
Thanking you with regards,

Sincerely yours,

Dr. Sankar Kumar Roy, 01.02.2016
Associate Professor
Department of Applied Mathematics with
Oceanology and Computer Programming
Vidyasagar University, Midnapore-721102
West Bengal, INDIA
Email: sankroy2006@gmail.com

Rough decision making approach for solving fuzzy multi-objective fractional programming and its application to fixed-charge transportation problem

Sankar Kumar Roy[1]*, Sudipta Midya[1], Gerhard Wilhelm Weber[2], and Nadi Serhan Aydin[2]

[1]Department of Applied Mathematics with Oceanology and Computer Programming

Vidyasagar University, Midnapore-721102, West Bengal, India

[2]Institute of Applied Mathematics

Middle East Technical University, 06800, Ankara, Turkey

Email: sankroy2006@gmail.com

Email: sudiptamidya932@gmail.com

Email: gweber@metu.edu.tr

Email: serhan.aydin@metu.edu.tr

Abstract

This paper presents a fuzzy Multi-Objective Fractional Fixed-Charge Transportation Problem (MOFFCTP) in a "rough" decision making environment. The parameters of the formulated model are considered as being of a fuzzy nature. In order to tackle these parameters, we employ the different types of fuzzy measures such as possibility, credibility and necessity measures. Using the fuzzy chance-constrained rough programming technique, we derive the best optimal solution of our proposed MOFFCTP. Then, the obtained result is compared with the results extracted from the Robust ranking technique. We also use rough set theory for extending as well as partitioning the feasible region of the MOFFCTP to accommodate more information by considering two approximations. Using the approximations, we design two models, namely, Lower Approximation Model (LAM) and Upper Approximation Model (UAM) of the proposed MOFFCTP. Finally, by using these models, we obtain the optimal solutions which provide the satisfactory results relevant to

*Corresponding Author

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Keywords: Fractional Programming, Fixed-Charge Transportation Problem, Rough Programming, Fuzzy Programming, Robust Ranking Technique, Fuzzy Chance-Constrained Rough Technique.

1 Introduction

Fractional Programming is a special type of nonlinear programming in which a ratio of two functions is to be maximized or minimized. The study of fractional programming had started long back by Charnes and Cooper [1]. Fractional programming for a single-objective optimization problem was investigated extensively from the view point of its application to several real-life problems which are generally used for modeling of real-life problems, like financial and corporate planning, production planning, university planning, health care and hospital planning, etc. In these areas, fractional programming is frequently faced up with a decision to optimize such type of ratios: profit/cost, actual cost/standard cost, output/employee, nurse/patient ratio, etc.

In a transportation problem, a single product manufactured at different plants (supply points) and transported to a number of different warehouses (demand points). The objective is to satisfy the demand at destinations from the origins so that the transportation cost is minimal. Besides the transportation (shipping) cost, in many real-life applications, a Transportation Problem (TP) is often associated with a fixed cost which is incurred at every origin. That fixed cost may be due to costs of renting a vehicle, permit fees, toll charge, etc. That problem is called Fixed-Charge Transportation Problem (FCTP).

Hirsch and Dantzig [2] first proposed the fixed charge problem. Later on, many researchers, such as Raj and Rajendram [3], Xie and Jia [4], solved FCTP by genetic algorithm. Zavardehi et al. [5] solved fuzzy fixed-charge solid TP by metaheuristics. Kundu et al. [6] solved FCTP with type-2 fuzzy variable. Midya and Roy [7] proposed the model "single-sink fixed-charge multi-objective multi-index stochastic transportation problem" under the light of a stochastic environment and via interval programming.

Since the TP is of the special structure of a linear programming problem, the TP could be defined as a linear fractional programming problem which originates from network models consisting of a finite numbers of nodes and arcs. That type of transportation problem is called a Fractional Transportation Problem (FTP). A linear FTP seeks to optimize the objective function of quotient form with linear functions in numerator and denominator, subject to a set of linear constraints.

The fractional fixed-charge problem was described by Almogy and Levin [8]. After the works Mishra [9] and Toksari [10], Jiao and Liu [11] studied it by fractional programming. In the last 15-20 years, many researchers of scientific community have shown a significant interest in the field of fractional programming and many applications of fractional programming have been found in different areas of TP. Multi-Objective Fractional Fixed-Charge Transportation Problem (MOFFCTP) is an important and extended structure of the classical FTP. In recent years, many researchers have studied FTP, such as Siviri et al. [12], Guzel et al. [13] and Mishra et al. [14]. They have treated the parameters of FTP either of a crisp or of an interval or of a fuzzy in nature.

Fuzzy set theory was first proposed by Zadeh [15], as a powerful mathematical tool for representing uncertainty, inconsistency and imprecise data in real-life situations. Later, Bellman and Zadeh [16] adopted to fuzzy set theory in decision making problem. Possibility theory was also introduced by Zadeh, and after that it was developed by many researchers such as Dubois et al. [17], Ishibuchi and Nii [18].

Rough set theory was introduced by Pawlak [19]; it has often proved to be a strong mathematical tool for analyzing vague description of objects (called action in decision problems). After that, it has been extended by many researchers, e.g., Cheng et al. [20] have discussed positive and converse approximation in interval-valued fuzzy rough set. Tao and Xu [21] have developed rough programming to tackle multi-objective solid transportation problem. They analyzed that the feasible region is not fixed but flexible due to imprecise parameters. Ali et al. [22] have described some properties of generalized rough set. Roy and Mula [23] studied a bi-matrix game by using rough set approach.

In our study, we consider the parameters (variable transportation cost, fixed charge, profit, deterioration rate, transporting time, supply, demand) of MOFFCTP to be triangular fuzzy numbers due to various sources of fuzziness such as fluctuation in the financial markets, data collected from multiple sources, linguistic information, etc. Many real-world problems arise where single objective function cannot tackle the situation. So, to deal with these situations, we introduce multi-objective optimization technique in our proposed model. We also incorporate rough set theory to divide the feasible region of MOFFCTP.

Here, we mainly concentrate our attention on the following:

- Solving MOFFCTP by using two types of uncertainty, one is fuzziness and the other one is roughness.

- Fuzzy nonlinear multi-objective fixed-charge transportation problem (i.e., fuzzy-

MOFFCTP) is transformed to fuzzy linear MOFCTP.

- Robust ranking technique is incorporated into fuzzy-MOFCTP to construct the corresponding crisp model.
- Fuzzy chance-constrained rough technique is introduced into fuzzy-MOFCTP to formulate an equivalent crisp model.
- We extend the feasible region of the proposed MOFFCTP by formulating the two approximation models, linear lower approximation model of MOFFCTP (LL-MOFCTP) and linear upper approximation model of MOFFCTP (LU-MOFCTP), using rough set approximation.[1]
- A comparative study is drawn between the solutions extracted from Robust ranking technique and fuzzy chance-constrained rough technique.

Fuzzy MOFFCTP is the extended version of FTP and FCTP. This type of problem arises when the decision maker (DM) cannot determine exact information about data associated to the real-life MOFFCTP. The parameters of MOFFCTP, such as demand, supply, etc., are changeable due to fluctuation of market, road condition, weather condition and other considerable factors. For these phenomena, to the inexact nature of information about the data in real-life MOFFCTP, we adopt fuzzy set theory in our proposed model. Moreover, to flexibly extend and partition the feasible region in the problem, we use rough approximation (lower and upper approximations) technique which produces a better optimal solution compared to other technique. Thus, by using fuzzy set theory and rough set theory on our proposed MOFFCTP, we draw the clear background of a real-life problem, which is our main motivation in the paper.

The remainder of the paper is designed as follows: Some basic knowledge of rough set, fuzzy number, different types of fuzzy measures including the definitions are presented in Section 2. In Section 3, fuzzy-MOFFCTP model and linear fuzzy-MOFCTP model are discussed. Section 4 proposes the deterministic models of fuzzy-MOFCTP. The solution procedure of our proposed MOFFCTP model and bi-objective fractional FCTP are described in Section 5. A real-life example on fuzzy-MOFFCTP is included in Section 6. Section 7 discusses the results of all crisp equivalent models of proposed MOFFCTP. Finally, concluding remarks and an outlook to future study are given in Section 8.

[1]Actually, the two approximation models are connected with the possibility chance constraints and the necessity chance constraints.

2 Preliminaries

Rough Set and Its Approximation

We consider a set of objects U which is called "universe" and an indiscernibility relation, $R \subseteq U \times U$ is also treated, representing our lack of knowledge about the elements of U. For the sake of simplicity, we treat that R by an equivalence relation. Let X be a subset of U. We then want to characterize the set X with respect to R. The fundamental concepts of rough set theory are presented as follows:

• The **lower approximation** of a set X with respect to R is the set of all objects, which can be certainly classified as X with respect to R (are certainly X with respect to R).

• The **upper approximation** of a set X with respect to R is the set of all objects, that can be possibly classified as X with respect to R (are possibly X in view of R).

• The **boundary region** of a set X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R.

Definition 2.1 *The set X is* **crisp** *(exact with respect to R), if the boundary region of X is empty.*

The set X is **rough** *(inexact with respect to R), if the boundary region of X is nonempty.*

The equivalence class of R is determined by the element x and is denoted by $R(x)$. The indiscernibility relation in certain sense describes our lack of knowledge about universe. Equivalence classes of indiscernibility relation, called granules generated by R, representing elementary portion of knowledge, we are able to perceive the data due to R.

Definition 2.2 *[24] The* **lower approximation** *of X with respect to R is denoted by $\underline{R}(X)$ and is defined as follows:*

$$\underline{R}(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}.$$

The **upper approximation** *of X with respect R is denoted by $\overline{R}(X)$ and is described as follows:*

$$\overline{R}(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}.$$

The **boundary region** *of X with respect R is denoted by $BN_R(X)$ and is depicted as follows:*

$$BN_R(X) = \overline{R}(X) \setminus \underline{R}(X).$$

Fuzzy Number:

Definition 2.3 *[25] Let a fuzzy set $\tilde{A}$ be described on the set of real numbers* $\mathbf{R}$ *and is called to be a* **fuzzy number** *if its membership function $\mu_{\tilde{A}} : \mathbf{R} \to [0,1]$ with the following characteristics:*
(1) $\mu_{\tilde{A}}$ *is convex, i.e.,* $\mu_{\tilde{A}}\{\lambda x_1 + (1-\lambda)x_2\} = max\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ *for all* $x_1, x_2 \in \mathbf{R}$,
(2) $\mu_{\tilde{A}}$ *is normal, i.e.,* $\exists$ *in* $x \in \mathbf{R}$ *such that* $\mu_{\tilde{A}}(x) = 1$,
(3) $\mu_{\tilde{A}}$ *is piecewise continuous.*

Triangular Fuzzy Number:

Definition 2.4 *[25] It is a fuzzy number which is described with three points as* $\tilde{A} = (a_1, a_2, a_3)$*, where* a_1*,* a_2 *and* a_3 *are real numbers. It has a membership function and is denoted by* $\mu_{\tilde{A}}(x)$ *and satisfies the following conditions:*
(1) a_1 *to* a_2 *is increasing function,*
(2) a_2 *to* a_3 *is decreasing function,*
(3) $a_1 < a_2 < a_3$.
So, the membership function is depicted as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & if \ x < a_1, \\ \frac{x-a_1}{a_2-a_1}, & if \ a_1 \le x \le a_2, \\ \frac{a_3-x}{a_3-a_2}, & if \ a_2 \le x \le a_3, \\ 0, & if \ x > a_3. \end{cases}$$

Arithmetic Operation in between Two Fuzzy Numbers:
If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then addition and subtraction of the triangular fuzzy numbers can be performed as follows:
$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$,
$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

α-cut:

Definition 2.5 *[25] The α-level set (α-cut) of a fuzzy number $\tilde{A}$ is a* **crisp** *set denoted by $[\tilde{A}]^\alpha$ and is defined as follows:*

$$[\tilde{A}]^\alpha = \{x : \mu_{\tilde{A}}(x) \ge \alpha, \ \forall \ \alpha \in (0,1)\}.$$

If $\tilde{M}$ is a fuzzy number, then $[\tilde{M}]^\gamma$ is a closed interval of $\mathbf{R}$ for all $\gamma \in (0,1)$. Here, we treat an alternative form of $[\tilde{M}]^\gamma$ as follows:

$$[\tilde{M}]^\gamma = [m_1(\gamma), m_2(\gamma)] \subseteq \mathbf{R},$$

where, $m_1(\gamma)$ and $m_2(\gamma)$ are the lower and upper bounds of the interval $\tilde{M}$.

Remark 2.1 *The $\gamma-$cut of a triangular fuzzy number $\tilde{A} = (a, \alpha, \beta)$(where α and β are the left and right spreads of $\tilde{A}$,) is an interval denoted by $[\tilde{A}]^\gamma$ and is represented as follows:*

$$[\tilde{A}]^\gamma = [a - (1-\gamma)\alpha,\ a + (1-\gamma)\beta] \quad (\gamma \in (0,1)).$$

Definition 2.6 *[17] Let φ be a function defined on $\mathbf{R}$, set of real numbers to [0,1]. Then, φ is called the* **reference function** *of a triangular fuzzy variable, if φ satisfies the following conditions:*
(1) $\varphi(x) = \varphi(-x)\ \forall\ x \in \mathbf{R}$,
(2) $\varphi(0) = 1$,
(3) φ *is decreasing on* $[0, +\infty]$.

Definition 2.7 *[17] Let $\tilde{a_1},\ \tilde{a_2}, \ldots, \tilde{a_n}$ be fuzzy variables, and $f : \mathbf{R}^n \to \mathbf{R}$ be a function which is continuous. Then, the* **possibility** *of the fuzzy event characterized by $f(\tilde{a_1},\ \tilde{a_2}, \ldots, \tilde{a_n}) \leq 0$ is defined by*
$Pos\{f(\tilde{a_1},\ \tilde{a_2}, \ldots, \tilde{a_n}) \leq 0\} \ = \ \underset{x_1, x_2 \ldots, x_n}{sup} \Bigg[\underset{1 \leq i \leq n}{min}\ \mu_{\tilde{a_i}}(x_i)\ \ : f(\tilde{x_1},\ \tilde{x_2}, \ldots, \tilde{x_n}) \leq 0 \Bigg].$

Definition 2.8 *[17] Let $(\Theta, P(\Theta), Pos)$ be a possibility space, and A be a set in $P(\Theta)$. Then, the* **necessity measure** *Nec of A is defined as follows:*
$Nec\{A\} \ = \ 1 - Pos\{A^c\}$, *where A^c is the complement of A.*
Thus, the necessity measure is the dual of the possibility measure, i.e.,
$Pos\{A\} + Nec\{A^c\} \ = \ 1$ *for any* $A \in P(\Theta)$.

Definition 2.9 *[26] Let $(\Theta, P(\Theta), Pos)$ be a possibility space, and A be a set in $P(\Theta)$. Then, the* **credibility measure** *of A is defined as follows:*
$Cr\{A\} \ = \ \frac{1}{2}\Big[Pos\{A\} + Nec\{A\}\Big].$

Thus the credibility measure is *self dual*, i.e., $Cr\{A\} + Cr\{A^c\} \ = \ 1$ for any $A \in P(\Theta)$.

Lemma 2.1 *[17] Let ξ_1 and ξ_2 be two fuzzy variables. Then, we have*
$Pos\{\xi_1 \geq \xi_2\} = \underset{u,v}{sup}\{\mu_{\xi_1}(u) \wedge \mu_{\xi_2}(v) : u \geq v\}$ *and*
$Pos\{\xi_1 \leq \xi_2\} = \underset{u,v}{sup}\{\mu_{\xi_1}(u) \wedge \underset{v}{inf}(1 - \mu_{\xi_2}(v)) : u \leq v\}.$
If the decision maker prefers a pessimistic decision in order to avoid risk, it may be approximated to replace the possibility measure with the necessity measure:
$Nec\{\xi_1 \geq \xi_2\} = \underset{u,v}{inf}\{(1 - \mu_{\xi_1}(u)) \vee \underset{v}{sup}\ \mu_{\xi_2}(v) : u \geq v\}$ *and*
$Nec\{\xi_1 \leq \xi_2\} = \underset{u,v}{inf}\{(1 - \mu_{\xi_1}(u)) \vee (1 - \mu_{\xi_2}(v)) : u \leq v\}.$

Theorem 2.1 *[27] Let* $(\Theta, P(\Theta), Pos)$ *be a possibility space, and* A *be a set in* $P(\Theta)$*. Then,* $Pos\{A\} \geq Cr\{A\} \geq Nec\{A\}$*.*

Proof: First we prove that $Pos\{A\} \geq Nec\{A\}$. If $Pos\{A\} = 1$, then it is obvious that $Pos\{A\} \geq Nec\{A\}$. Otherwise, we must have $Pos\{A^c\} = 1$, which implies that $Nec\{A\} = 1 - Pos\{A^c\} = 0$. Thus, $Pos\{A\} \geq Nec\{A\}$ always holds. Again, it follows from Definition 2.9 that the value of credibility is treated in between possibility and necessity.
Hence, $Pos\{A\} \geq Cr\{A\} \geq Nec\{A\}$. $\square$

3 The Mathematical Model

3.1 Notations and Assumptions:

Notations:

Z_K : the number of objective function for FCTP $(K = 1, 2, \ldots, e)$,
x_{ij} : the amount of transported goods from the i^{th} source to the j^{th} destination,
c_{ij} : shipping cost per unit amount of goods for transporting from the i^{th} source to the j^{th} destination,
f_{ij} : fixed charge associated with the i^{th} source and the j^{th} destination,
y_{ij} : binary variable taking the value of "1" if source i is used, and "0" otherwise,
a_i : capacity of the i^{th} source,
b_j : demand of the j^{th} destination,
m : number of origins (i.e., source points),
n : number of destinations (i.e., demand points),
$\tilde{c_{ij}}$: fuzzy transportation (variable) cost for unit quantity of the product from the i^{th} source to the j^{th} destination,
$\tilde{f_{ij}}$: fuzzy fixed cost associated with the i^{th} source and the j^{th} destination,
$\tilde{p_{ij}}$: fuzzy profit earned for unit quantity from the i^{th} source to the j^{th} destination,
$\tilde{d_{ij}}$: fuzzy deterioration rate for unit quantity of product from the i^{th} source to the j^{th} destination,
$\tilde{t_{ij}}$: fuzzy unit time of transportation of the product from the i^{th} source to the j^{th} destination which is independent of amount of product transported,
$\tilde{a_i}$: fuzzy availability of the product at the i^{th} source point,
$\tilde{b_j}$: fuzzy demand of the product at the j^{th} destination point,
$\tilde{Z_K}$: the number of objective function in fuzzy nature $(K = 1, 2, \ldots, e)$,
$\tilde{z_K}$: the number of objective function in fuzzy nature in the numerator $(K = 1, 2, \ldots, e)$,

$\tilde{g_K}$: the number of objective function in fuzzy nature in the denominator $(K = 1, 2, \ldots, e)$.
Assumptions:
1. $\tilde{a}_i > 0,\ \forall\ i\ ;\ \tilde{b_j} > 0,\ \forall\ j.$
2. $\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\tilde{g_K}(x_{ij})\right] > 0,\ \forall\ K$, for obtaining a feasible solution.
3. Triangular fuzzy numbers whose components are all positive.

3.2 Multi-Objective FCTP

The fixed-charge transportation problem (FCTP) is the extended version of classical transportation problem. FCTP is associated with two types of cost, variable transportation cost (direct cost) and fixed cost (i.e., fixed charge) for transporting the goods from the i^{th} source to the j^{th} destination in such a way that the total transportation cost (direct and fixed cost) are minimized. In a Multi-Objective Fixed-Charge Transportation Problem (MOFCTP), more than one objective functions are to be minimized simultaneously subject to a common set of constraints. The mathematical model of MOFCTP is considered as follows:
Model 1

$$\begin{aligned}
\text{minimize } (Z_K : K = 1, 2, \ldots, e) \Leftrightarrow \text{minimize } \quad & \Bigg[\sum_{i=1}^{m}\sum_{j=1}^{n}\left[Z_1(x_{ij}, y_{ij})\right], \\
& \sum_{i=1}^{m}\sum_{j=1}^{n}\left[Z_K(x_{ij}) : K = 2, 3, \ldots, e\right]\Bigg], \\
\text{subject to } \quad & \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \\
& x_{ij} \geq 0, \quad \forall\ i, j.
\end{aligned}$$

The first objective function is denoted as cost objective function and is defined as follows:

$$Z_1(x_{ij}, y_{ij}) = (c_{ij}x_{ij} + f_{ij}y_{ij}),$$
$$\text{where} \begin{cases} y_{ij} = 0, & \text{if } x_{ij} = 0, \\ y_{ij} = 1, & \text{if } x_{ij} > 0. \end{cases}$$

It is obvious that the model is feasible, if and only if

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j.$$

The first constraint of the **Model 1** implies that the total amount transported from the i^{th} source must not exceed its supply capacity a_i, and the second constraint implies that total amount of transported the goods from all the sources should fulfill the demand of the j^{th} destination.

3.3 Multi-Objective Fractional Programming

In Multi-Objective Fractional Programming Problem (MOFPP), all the objective functions are of rational form and the objective functions are to be optimized with respect to the common set of constraints. The general model of MOFPP can be stated as follows:
Model 2

$$\begin{aligned}
\text{minimize } (Z_K(x): \; K = 1, 2, \ldots, e) \Leftrightarrow \text{minimize} \quad & \left(\frac{f_K(x)}{g_K(x)} : K = 1, 2, \ldots, e\right), \\
\Leftrightarrow \text{minimize} \quad & \left(\frac{f_1(x)}{g_1(x)}, \frac{f_2(x)}{g_2(x)}, \ldots, \frac{f_e(x)}{g_e(x)}\right), \\
\text{subject to} \quad & \upsilon_{l_1}(x) \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} 0 \; (l_1 = 1, 2, \ldots, p), \\
& x \geq 0.
\end{aligned}$$

where x is the decision vector, $Z_K(x)$ and $\upsilon_{l_1}(x)$ $(K = 1, 2, \ldots, e;\; l_1 = 1, 2, \ldots, p)$ are real-valued functions representing the objective functions and the constraints, respectively. The numerator and denominator functions $f_K(x)$ and $g_K(x)$ $(K = 1, 2, \ldots, e)$ are real-valued functions also. The set of feasible solution of the above problem exists only when $g_K(x) > 0$ $(K = 1, 2, \ldots, e)$.

Remark 3.1 *Since the optimization problem (***Model 2***) is of minimization type, we can construct the MOFPP into two single objective linear programming problems as follows:*

$$\begin{aligned}
(i) \; \textit{minimize } (f_K(x): \; K = 1, 2, \ldots, e) \Leftrightarrow \textit{minimize} \quad & \Big(f_1(x), f_2(x), \ldots, f_e(x)\Big), \\
\textit{subject to} \quad & \upsilon_{l_1}(x) \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} 0 \; (l_1 = 1, 2, \ldots, p), \\
& x \geq 0.
\end{aligned}$$

$$(ii)\ maximize\ (g_K(x):\ K=1,2,\ldots,e) \Leftrightarrow maximize \quad \Big(g_1(x), g_2(x), \ldots, g_e(x)\Big),$$

$$subject\ to \quad v_{l_1}(x) \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} 0\ (l_1=1,2,\ldots,p),$$

$$x \geq 0.$$

Using the suitable transformation (shown later), the **Model 2** of MOFPP is transformed to linear multi-objective programming problem. This is designed based on Remark 3.1.

3.4 Fuzzy-MOFFCTP

We consider multi-objective fractional fixed charge transportation problem (MOFFCTP) where the parameters (variable transportation cost, fixed charge, profit, deterioration rate, transporting time, supply, demand) are treated here as fuzzy numbers. As the objective functions are fractional form, so we minimize the objective functions in the numerator, i.e., $\tilde{z}_1(x_{ij}, y_{ij}), \tilde{z}_2(x_{ij}), \ldots, \tilde{z}_e(x_{ij})$, and maximize the objective functions in denominator, i.e., $\tilde{g}_1(x_{ij}), \tilde{g}_2(x_{ij}), \ldots, \tilde{g}_e(x_{ij})$. The mathematical model of fuzzy-MOFFCTP can be formulated as follows:

Model 3

$$\text{minimize } (Z_K : K=1,2,\ldots,e) \Leftrightarrow \text{minimize} \quad \Bigg[\frac{\sum_{i=1}^m \sum_{j=1}^n \big[\tilde{z}_1(x_{ij}, y_{ij})\big]}{\sum_{i=1}^m \sum_{j=1}^n \big[\tilde{g}_1(x_{ij})\big]}, \frac{\sum_{i=1}^m \sum_{j=1}^n \big[\tilde{z_K}(x_{ij}\big]}{\sum_{i=1}^m \sum_{j=1}^n \big[\tilde{g_K}(x_{ij})\big]} : K=2,3,\ldots,e\Bigg],$$

$$\text{subject to} \quad \sum_{j=1}^n x_{ij} \leq \tilde{a}_i \quad (i=1,2,\ldots,m),$$

$$\sum_{i=1}^m x_{ij} \geq \tilde{b}_j \quad (j=1,2,\ldots,n),$$

$$x_{ij} \geq 0\ (i=1,2,\ldots,m\ ; j=1,2,\ldots,n),$$

$$y_{ij} = 0 \text{ if } x_{ij} = 0\ \forall\ i,\ j,$$

$$y_{ij} = 1 \text{ if } x_{ij} > 0\ \forall\ i,\ j.$$

This model is feasible, if and only if

$$\sum_{i=1}^m \tilde{a}_i \ \gtrsim\ \sum_{j=1}^n \tilde{b}_j,$$

where by " $\gtrsim$ " we mean the fuzzy inequality.

3.5 Fuzzy Linear MOFCTP

To solve the **Model 3**, we introduce the transformations [1] $z_{ij} = sx_{ij}$ $(i = 1, 2, \ldots, m\,; j = 1, 2, \ldots, n)$, where $s > 0$ is to be chosen in such a way that Z_K, $K = 1, 2, \ldots, e)$, in **Model 3** is minimal. The equivalent fuzzy linear model is formulated as follows:
Model 4

$$
\begin{aligned}
\text{minimize } (\tilde{Z_K} : K = 1, 2, \ldots, e) \Leftrightarrow \text{minimize} \quad & \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} \big[\tilde{z_1}(z_{ij}/s, y_{ij}) \big], \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} \big[\tilde{z_K}(z_{ij}/s) : \; K = 2, 3, \ldots, e \big] \Bigg], \\
\text{subject to} \quad & \frac{1}{s} \sum_{j=1}^{n} z_{ij} \leq \tilde{a}_i \quad (i = 1, 2, \ldots, m), \\
& \frac{1}{s} \sum_{i=1}^{m} z_{ij} \geq \tilde{b_j} \quad (j = 1, 2, \ldots, n), \\
& s \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{g_1}(z_{ij}/s) \Bigg] \geq 1, \\
& s \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{g_2}(z_{ij}/s) \Bigg] \geq 1, \\
& \vdots \\
& s \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{g_e}(z_{ij}/s) \Bigg] \geq 1, \\
& y_{ij} = 1 \text{ if } z_{ij} > 0, \\
& s > 0, \; z_{ij} \geq 0, \; \forall \; i, \; j.
\end{aligned}
$$

This model is feasible, if and only if

$$
\sum_{i=1}^{m} \tilde{a}_i \;\gtrsim\; \sum_{j=1}^{n} \tilde{b_j},
$$

where by " $\gtrsim$ " we mean the fuzzy inequality.

Remark 3.2 *A real number "a" can be equivalently expressed as triangular fuzzy number (a, a, a), so we can claim that linear fractional programming is a special case of linear fractional programming with triangular fuzzy number as coefficients in the objective function.*

4 Deterministic Model of Fuzzy-MOFCTP

The **Model 4** is a conceptual model rather than mathematical model because
(i) we cannot minimize an uncertain quantity, like fuzzy cost Z_1 and other fuzzy objective functions $Z_2,\ Z_3, \dots, Z_e$,
and
(ii) constraints of above model are not a crisp feasible set.
So, we need to transform the fuzzy-MOFCTP into an approximate deterministic model.
In order to convert the **Model 4** into deterministic form, we adopt two techniques and they are as follows:

- Robust Ranking Technique,
- Fuzzy Chance-Constrained Rough Technique.

4.1 Robust Ranking Technique

Robust ranking technique [28] satisfies the compensation, linearity and additive properties and provides the results which are consistent with human intuition.
Robust ranking for a triangular fuzzy number $\tilde{c}$ is defined as follows:
$R(\tilde{c}) = \int_0^1 (0.5)\big[c_L(\gamma), c_R(\gamma)\big]\, d\gamma,$
where $\big[c_L(\gamma), c_R(\gamma)\big]$ is the γ-level cut of the fuzzy number $\tilde{c}$.
Robust ranking technique for fuzzy objective function gives the crisp value which represents the average value of the fuzzy objective function. In order to convert fuzzy-MOFCTP (**Model 4**) into a crisp model, we introduce Robust ranking technique. The new model can be formulated as follows:

Model 5

$$
\begin{aligned}
\text{minimize} \quad & (R(\tilde{Z_K}) : K = 1, 2, \ldots, e) \\
\Leftrightarrow \text{minimize} \quad & \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} \big[R(\tilde{z}_1)(z_{ij}/s, y_{ij}) \big], \\
& \sum_{i=1}^{m} \sum_{j=1}^{n} \big[R(\tilde{z_K})(z_{ij}/s) : \ K = 2, 3, \ldots, e \big] \Bigg], \\
\text{subject to} \quad & \frac{1}{s} \sum_{j=1}^{n} z_{ij} \leq R(\tilde{a}_i) \quad (i = 1, 2, \ldots, m), \\
& \frac{1}{s} \sum_{i=1}^{m} z_{ij} \geq R(\tilde{b_j}) \quad (j = 1, 2, \ldots, n), \\
& s \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} R(\tilde{g}_1)(z_{ij}/s) \Bigg] \geq 1, \\
& s \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} R(\tilde{g}_2)(z_{ij}/s) \Bigg] \geq 1, \\
& \vdots \\
& s \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} R(\tilde{g}_e)(z_{ij}/s) \Bigg] \geq 1, \\
& y_{ij} = 0 \text{ if } z_{ij} = 0, \\
& y_{ij} = 1 \text{ if } z_{ij} > 0, \\
& s > 0, \ z_{ij} \geq 0, \ \forall \ i, \ j.
\end{aligned}
$$

This model is feasible, if and only if

$$
\sum_{i=1}^{m} R(\tilde{a}_i) \geq \sum_{j=1}^{n} R(\tilde{b_j}).
$$

4.2 Fuzzy Chance-Constrained Programming

Fuzzy chance constrained programming (FCCP) [29] is used to tackle fuzzy parameters involving in the objective functions as well as in constraints. To deal with fuzzy-MOFCTP (**Model 4**), first we formulate FCCP version of **Model 4**. After that, using different fuzzy measures such as Pos, Nec and Cr, hence, we derive the crisp equivalent model of **Model 4**. Therefore, FCCP version of **Model 4** can be formulated as follows (see **Model 6**):

Model 6

$$\begin{cases} \text{minimize } (\bar{\tilde{Z}}_K : K = 1, 2, \ldots, e) \\ \Leftrightarrow \text{minimize}\Big[\sum_{i=1}^{m} \sum_{j=1}^{n} \big[\tilde{z}_1(z_{ij}/s, y_{ij})\big],\ \big[\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{z_K}(z_{ij}/s) :\ K = 2, 3, \ldots, e\big]\Big], \\ \text{subject to} \begin{cases} Ch\{Z_K \le \bar{Z_K}\} \ge \delta_K, \\ Ch\{\frac{1}{s} \sum_{j=1}^{n} z_{ij} - \tilde{a}_i \le 0\} \ge \theta_i \ \ (i = 1, 2, \ldots, m), \\ Ch\{\frac{1}{s} \sum_{i=1}^{m} z_{ij} - \tilde{b_j} \ge 0\} \ge \theta_j \ \ (j = 1, 2, \ldots, n), \\ Ch\{s\big[\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{g_1}(z_{ij}/s)\big] - 1 \ge 0\} \ge \theta_1, \\ Ch\{s\big[\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{g_2}(z_{ij}/s)\big] - 1 \ge 0\} \ge \theta_2, \\ \vdots \\ Ch\{s\big[\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{g_e}(z_{ij}/s)\big] - 1 \ge 0\} \ge \theta_e, \\ y_{ij} = 0 \text{ if } z_{ij} = 0, \\ y_{ij} = 1 \text{ if } z_{ij} > 0, \\ s > 0,\ z_{ij} \ge 0,\ \forall\ i,\ j. \end{cases} \end{cases}$$

Here, Ch represents fuzzy measures such as Pos, Nec and Cr; and δ_K $(K = 1, 2, \ldots, e)$, θ_r $(r = 1, 2, \ldots, m, n, e;\ m \ne n \ne e)$ are the predetermined confidence levels. Also, $(\theta_i,\ \theta_j,\ \theta_K) \in \theta_r$. As the problem is of minimization type, we minimize the objective functions $\bar{Z}_1,\ \bar{Z}_2, \ldots,\ \bar{Z}_e$, satisfying the chance-constrained level.

Remark 4.1 *To formulate fuzzy chance-constrained rough model of* **Model 6***, we assume feasible region* $D = \{x : x \in X,\ Ch\{Z_K(z_{ij}/s) \le \bar{Z_K}\} \ge \delta_K\ (K = 1, 2, \ldots, e)$ *and* $Ch\{q_r(z_{ij}/s) \ge 0\} \ge \theta_r\ (r = 1, 2, \ldots, m, n, e;\ m \ne n \ne e)\}$*, where* q_r *represents the set of constraints.* $\bar{Z_K}$ *is the smallest possible value of* Z_K*. We construct two sets* L *and* U *which are defined as follows:*

$$\begin{cases} L = \{x : x \in X,\ Nec\{Z_K(z_{ij}/s) \le \bar{Z_K}\} \ge \delta_K\ (K = 1, 2) \text{ and } Nec\{q_r(z_{ij}/s) \ge 0\} \ge \theta_r\}, \\ U = \{x : x \in X,\ Pos\{Z_K(z_{ij}/s) \le \bar{Z_K}\} \ge \delta_K\ (K = 1, 2) \text{ and } Pos\{q_r(z_{ij}/s) \ge 0\} \ge \theta_r\} \\ \qquad (r = 1, 2, \ldots, m, n, e;\ m \ne n \ne e). \end{cases}$$

Theorem 4.1 *For the feasible region D; we have the relation,* $L \subseteq D \subseteq U$*, where* L *and* U *are already defined as above.*

Proof: For any $z_0 \in X$, if $x_0 \in L$, i.e., $Nec\{Z_K(z_0/s) \le \bar{Z_K}\} \ge \delta_K$ $(K = 1, 2, \ldots, e)$ and $Nec\{q_r(z_0/s)\ \ge \text{ or } \le\ 0\} \ge \theta_r$ $(r = 1, 2, \ldots, m, n, e;\ m \ne n \ne e)$. It follows from

Theorem 2.1 that $Cr\{Z_K(z_0/s) \le \bar{Z_K}\} \ge Nec\{Z_K(z_0/s) \le \bar{Z_K}\} \ge \delta_K, (K=1,2)$ and $Cr\{q_r(z_0/s) \ge \text{or} \le 0\} \ge Nec\{q_r(z_0/s) \ge \text{or} \le 0\} \ge \theta_r, (r = 1, 2, \ldots, m, n, e;\ m \ne n \ne e)$, i.e., $z_0 \in D$. Thus, $L \subseteq D$. In the same way, we can conclude $D \subseteq U$.
Hence, the assertion of the theorem follows. $\square$
Let us consider $L = \underline{D}$ and $U = \overline{D}$; we use $\underline{D}$ and $\overline{D}$ to approximate D. It is obvious that $\underline{D} \subseteq D \subseteq \overline{D}$. Then, **Model 6** can be transformed into **Models 7** and **8**.

4.3 Fuzzy Chance Constrained Rough Technique

Here, we extend the feasible region of **Model 6** using rough approximation technique. Therefore, we transform **Model 6** into two models: the lower approximation model of fuzzy chance-constrained rough MOFCTP (L-MOFCTP) and upper approximation model of fuzzy chance-constrained rough MOFCTP (U-MOFCTP), and these are defined in **Models 7** and **8** as follows:
Model 7 (L-MOFCTP)

$$
\begin{cases}
\text{minimize } (\bar{\bar{Z}}_K : K = 1, 2, \ldots, e), \\
\text{subject to}
\begin{cases}
Nec\{Z_K \le \bar{Z_K}\} \ge \delta_K, \\
Nec\{\frac{1}{s}\sum_{j=1}^{n} z_{ij} - \tilde{a}_i \le 0\} \ge \theta_i \quad (i = 1, 2, \ldots, m), \\
Nec\{\frac{1}{s}\sum_{i=1}^{m} z_{ij} - \tilde{b}_j \ge 0\} \ge \theta_j \quad (j = 1, 2, \ldots, n), \\
Nec\{s\big[\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{g}_1(z_{ij}/s)\big] - 1 \ge 0\} \ge \theta_1, \\
Nec\{s\big[\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{g}_2(z_{ij}/s)\big] - 1 \ge 0\} \ge \theta_2, \\
\vdots \\
Nec\{s\big[\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{g}_e(z_{ij}/s)\big] - 1 \ge 0\} \ge \theta_e, \\
y_{ij} = 0 \text{ if } z_{ij} = 0, \\
y_{ij} = 1 \text{ if } z_{ij} > 0, \\
s > 0,\ z_{ij} \ge 0,\ \forall\ i,\ j.
\end{cases}
\end{cases}
$$

and **Model 8** (U-MOFCTP)

$$\begin{cases} \text{minimize } (\bar{\tilde{Z}}_K : K = 1, 2, \ldots, e), \\ \text{subject to} \begin{cases} Pos\{Z_K \leq \bar{Z_K}\} \geq \delta_K, \\ Pos\{\frac{1}{s}\sum_{j=1}^{n} z_{ij} - \tilde{a}_i \leq 0\} \geq \theta_i \quad (i = 1, 2, \ldots, m), \\ Pos\{\frac{1}{s}\sum_{i=1}^{m} z_{ij} - \tilde{b_j} \geq 0\} \geq \theta_j \quad (j = 1, 2, \ldots, n), \\ Pos\{s\big[\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{g}_1(z_{ij}/s)\big] - 1 \geq 0\} \geq \theta_1, \\ Pos\{s\big[\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{g}_2(z_{ij}/s)\big] - 1 \geq 0\} \geq \theta_2, \\ \vdots \\ Pos\{s\big[\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{g}_e(z_{ij}/s)\big] - 1 \geq 0\} \geq \theta_e, \\ y_{ij} = 0 \text{ if } z_{ij} = 0, \\ y_{ij} = 1 \text{ if } z_{ij} > 0, \\ s > 0, \; z_{ij} \geq 0, \; \forall \; i, \; j. \end{cases} \end{cases}$$

Definition 4.1 *A feasible solution at θ_r-necessity level, z^*_{ij} is said to be a* δ_K-**efficient solution** *to* **Model 7** *if and only if there exists no other feasible solution at θ_r-necessity level z_{ij} such that $Nec\{Z_K(z_{ij}/s)\} \geq \delta_K$ with $Z_K(z_{ij}) \leq \bar{Z_K}(z^*_{ij})$ for all K and $Z_K(z_{ij}) < \bar{Z_K}(z^*_{ij})$ for at least one $K \in \{1, 2, \ldots, e\}$.*

Definition 4.2 *A feasible solution at θ_r-possibility level, z^*_{ij} is said to be a* δ_K-**efficient solution** *to* **Model 8** *if and only if there exists no other feasible solution at θ_r-possibility level z_{ij} such that $Pos\{Z_K(z_{ij}/s)\} \geq \delta_K$ with $Z_K(z_{ij}) \leq \bar{Z_K}(z^*_{ij})$ for all K and $Z_K(z_{ij}) < \bar{Z_K}(z^*_{ij})$ for at least one $K \in \{1, 2, \ldots, e\}$.*

Theorem 4.2 *Let $\tilde{c_{ij}}$ be a triangular fuzzy number with membership function $\mu_{\tilde{c_{ij}}}(x)$, is defined as:*

$$\mu_{\tilde{c_{ij}}}(x) = \begin{cases} \frac{x-(c_{ij}-\alpha^c_{ij})}{\alpha^c_{ij}}, & \text{if} \quad (c_{ij} - \alpha^c_{ij}) \leq x \leq c_{ij}, \\ \frac{(c_{ij}+\beta^c_{ij})-x}{\beta^c_{ij}}, & \text{if} \quad c_{ij} \leq x \leq (c_{ij} + \beta^c_{ij}), \\ 0, & \quad\quad \text{otherwise}, \end{cases}$$

where c_{ij} is real number; α^c_{ij} and β^c_{ij} are the left and right spreads of $\tilde{c_{ij}}$ $(i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n)$. If the reference function of $\tilde{c_{ij}}$ is $(1-w), \; w \in (0, 1)$, then, $Pos\{\tilde{c_{ij}}x \leq \bar{Z_K}\} \geq \delta_K$ is equivalent to $\bar{Z_K} \geq c_{ij}x + (1 - \delta_K)\beta^c_{ij}x$ $(K = 1, 2, \ldots, e)$.

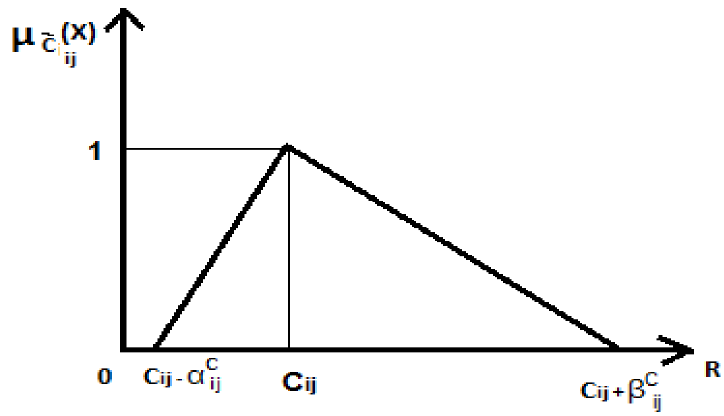

Fig 1: The graph of Triangular Fuzzy Number.

Proof: As $\tilde{c_{ij}}$ is a triangular fuzzy number and its membership function is $\mu_{\tilde{c_{ij}}}$. By extension principle of Zadeh [30], the membership function of fuzzy number $\mu_{\tilde{c_{ij}}}(x)$ is considered as:

$$\mu_{\tilde{c_{ij}}}(x(u)) = \begin{cases} \frac{u-(c_{ij}x-\alpha^c_{ij}x)}{\alpha^c_{ij}x}, & \text{if} \quad (c_{ij}x-\alpha^c_{ij}x) \le u \le c_{ij}x, \\ \frac{(c_{ij}x+\beta^c_{ij}x)-u}{\beta^c_{ij}x}, & \text{if} \quad c_{ij}x \le u \le (c_{ij}x+\beta^c_{ij}x), \\ 0, & \quad \text{otherwise}, \end{cases}$$

for $i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$. For convenience, we denote the triangular fuzzy number $\tilde{c_{ij}} = (c_{ij}, \alpha^c_{ij}, \beta^c_{ij})$, then according to Lemma 2.1, we can write

$$Pos\{\tilde{c_{ij}}x \le \bar{Z_K}\} \ge \delta_K \Leftrightarrow \bar{Z_K} \ge c_{ij}x + (1-\delta_K)\beta^c_{ij}x \quad (K = 1, 2, \ldots, e).$$

Hence, the assertion of our theorem results.

Theorem 4.3 *Let $\tilde{p_{ij}}$ and $\tilde{h_r}$ be the triangular fuzzy numbers with membership functions are defined, as follows, respectively:*

$$\mu_{\tilde{p_{ij}}}(x) = \begin{cases} \frac{x-(p_{ij}-\alpha^p_{ij})}{\alpha^p_{ij}}, & \textit{if} \quad (p_{ij}-\alpha^p_{ij}) \le x \le p_{ij}, \\ \frac{(p_{ij}+\beta^p_{ij})-x}{\beta^p_{ij}}, & \textit{if} \quad p_{ij} \le x \le (p_{ij}+\beta^p_{ij}), \\ 0, & \quad \textit{otherwise}, \end{cases}$$

and

$$\mu_{\tilde{h_r}}(x) = \begin{cases} \frac{x-(h_r-\alpha^h_r)}{\alpha^h_r}, & \textit{if} \quad (h_r-\alpha^h_r) \le x \le h_r, \\ \frac{(h_r+\beta^h_r)-x}{\beta^h_r}, & \textit{if} \quad h_r \le x \le (h_r+\beta^h_r), \\ 0, & \quad \textit{otherwise}, \end{cases}$$

where p_{ij} and h_r are real numbers; α_{ij}^p and β_{ij}^p are the left and right spreads of $\tilde{p_{ij}}$, α_r^h and β_r^h are the left and right spreads of $\tilde{h_r}$ ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $r = 1, 2, \ldots, m, n, e$; $m \neq n \neq e$). If the reference function of $\tilde{p_{ij}}$ and $\tilde{h_r}$ is $(1 - w)$ for some $w \in (0, 1)$ and we suppose that p_{ij} and h_r are independent, then, $Pos\{\tilde{p_{ij}}x \geq \tilde{h_r}\} \geq \theta_r$ is equivalent to

$$h_r + (1 - \theta_r)\beta_r^h \leq p_{ij}x + (1 - \theta_r)\beta_{ij}^p x, \quad r = 1, 2, \ldots, m, n, e; \; m \neq n \neq e.$$

Proof: As $\tilde{p_{ij}}$ is an triangular fuzzy number and its membership function is $\mu_{\tilde{p_{ij}}}$. By extension principle of Zadeh [30], the membership function of fuzzy number $\mu_{\tilde{p_{ij}}}(x)$ is specified as:

$$\mu_{\tilde{p_{ij}}}(x(u)) = \begin{cases} \frac{u - (p_{ij}x - \alpha_{ij}^p x)}{\alpha_{ij}^p x}, & \text{if} \quad (p_{ij}x - \alpha_{ij}^p x) \leq u \leq p_{ij}x, \\ \frac{(p_{ij}x + \beta_{ij}^p x) - u}{\beta_{ij}^p x}, & \text{if} \quad p_{ij}x \leq u \leq (p_{ij}x + \beta_{ij}^p x), \\ 0, & \quad\quad \text{otherwise}, \end{cases}$$

for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; and $\tilde{h_r}$ ($r = 1, 2, \ldots, m, n, e$; $m \neq n \neq e$) is also a triangular fuzzy number with membership function $\mu_{\tilde{h_r}}$. With out any loss of generality, we denote triangular fuzzy number $\tilde{p_{ij}} = (p_{ij}, \alpha_{ij}^p, \beta_{ij}^p)$ and $\tilde{h_r} = (h_r, \alpha_r^h, \beta_r^h)$, then using Lemma 2.1, we obtain
$Pos\{\tilde{p_{ij}}x \geq \tilde{h_r}\} \geq \theta_r \Leftrightarrow h_r - \theta_r\alpha_r^h \leq p_{ij}x + (1 - \theta_r)\beta_{ij}^p x$ ($r = 1, 2, \ldots, m, n, e$; $m \neq n \neq e$).
Hence, the proof of our theorem follows. □

Theorem 4.4 *Assuming that the fuzzy numbers are same as those in Theorem 4.2. Then, $Nec\{\tilde{c_{ij}}x \leq \bar{Z_K}\} \geq \delta_K$ is equivalent to $\bar{Z_K} \geq c_{ij}x - \delta_K\alpha_{ij}^c x$ ($K = 1, 2, \ldots, e$).*

Proof: The proof is similar to the proof of Theorem 4.2.

Theorem 4.5 *Assuming that the fuzzy numbers are same as those in Theorem 4.3. Then, $Nec\{\tilde{p_{ij}}x \geq \tilde{h_r}\} \geq \theta_r$ is equivalent to*

$$h_r + (1 - \theta_r)\beta_r^h \leq p_{ij}x + (1 - \theta_r)\beta_{ij}^p x \quad (r = 1, 2, \ldots, m, n, e; \; m \neq n \neq e).$$

Proof: The proof is similar to the proof of Theorem 4.3.

4.4 Deterministic Rough Model of Fuzzy-MOFCTP

Using Theorems 4.2, 4.3, 4.4 and 4.5, the equivalent deterministic models of **Models 7** and **8** are stated as linear lower approximation model of fuzzy chance constrained rough MOFCTP (LL-MOFCTP) and linear upper approximation model of fuzzy chance constrained rough MOFCTP (LU-MOFCTP). The deterministic models of LL-MOFCTP (**Model 9**) and LU-MOFCTP (**Model 10**) are defined based on Nec and Pos measures, respectively. Finally, the models are described as follows:

Model 9 (LL-MOFCTP)

$$
\begin{cases}
\text{minimize } (\bar{Z}_K : K = 1, 2, \dots, e), \\
\text{subject to} \begin{cases}
\bar{Z}_1 \geq \Big[\sum_{i=1}^{m} \sum_{j=1}^{n} \big(c_{ij}(z_{ij}/s) - \delta_1 \alpha_{ij}^{1}(z_{ij}/s) \big) + \big(f_{ij}(z_{ij}/s) - \delta_1 \alpha_{ij}^{f}(z_{ij}/s) \big) \Big], \\
\bar{Z}_2 \geq \Big[\sum_{i=1}^{m} \sum_{j=1}^{n} \big(Z_2(z_{ij}/s) - \delta_2 \alpha_{ij}^{2}(z_{ij}/s) \big) \Big], \\
\vdots \\
\bar{Z}_e \geq \Big[\sum_{i=1}^{m} \sum_{j=1}^{n} \big(Z_e(z_{ij}/s) - \delta_e \alpha_{ij}^{e}(z_{ij}/s) \big) \Big], \\
\frac{1}{s} \sum_{j=1}^{n} z_{ij} \leq \big(a_i - \theta_i \alpha_i^{a} \big), \\
\frac{1}{s} \sum_{i=1}^{m} z_{ij} \geq \big(b_j - \theta_j \alpha_j^{b} \big), \\
s \big[\sum_{i=1}^{m} \sum_{j=1}^{n} \Big[g_1(z_{ij}/s) + (1 - \theta_1) \beta_{ij}^{1}(z_{ij}/s) \Big] \geq 1, \\
s \big[\sum_{i=1}^{m} \sum_{j=1}^{n} \Big[g_2(z_{ij}/s) + (1 - \theta_2) \beta_{ij}^{2}(z_{ij}/s) \Big] \geq 1, \\
\vdots \\
s \big[\sum_{i=1}^{m} \sum_{j=1}^{n} \Big[g_e(z_{ij}/s) + (1 - \theta_e) \beta_{ij}^{e}(z_{ij}/s) \Big] \geq 1, \\
y_{ij} = 0 \text{ if } z_{ij} = 0, \\
y_{ij} = 1 \text{ if } z_{ij} > 0, \\
s > 0,\ z_{ij} \geq 0,\ \forall\ i,\ j.
\end{cases}
\end{cases}
$$

Model 10 (LU-MOFCTP)

$$
\begin{cases}
\text{minimize } (\bar{Z}_K : K = 1, 2, \dots, e), \\
\text{subject to}
\begin{cases}
\bar{Z}_1 \le \Big[\sum_{i=1}^{m} \sum_{j=1}^{n} \big(c_{ij}(z_{ij}/s) + (1-\delta_1)\beta_{ij}^{1}(z_{ij}/s) \big) + \big(f_{ij}(z_{ij}/s) + (1-\delta_1)\beta_{ij}^{f}(z_{ij}/s) \big) \Big], \\
\bar{Z}_2 \le \Big[\sum_{i=1}^{m} \sum_{j=1}^{n} \big(Z_2(z_{ij}/s) + (1-\delta_2)\beta_{ij}^{2}(z_{ij}/s) \big) \Big], \\
\vdots \\
\bar{Z}_e \le \Big[\sum_{i=1}^{m} \sum_{j=1}^{n} \big(Z_e(z_{ij}/s) + (1-\delta_e)\beta_{ij}^{e}(z_{ij}/s) \big) \Big], \\
\frac{1}{s} \sum_{j=1}^{n} z_{ij} \le \big(a_i + (1-\theta_i)\beta_i^{a} \big), \\
\frac{1}{s} \sum_{i=1}^{m} z_{ij} \ge \big(b_j + (1-\theta_j)\beta_j^{b} \big), \\
s\big[\sum_{i=1}^{m} \sum_{j=1}^{n} \Big[g_1(z_{ij}/s) - (1-\theta_1)\alpha_{ij}^{1}(z_{ij}/s) \Big] \ge 1, \\
s\big[\sum_{i=1}^{m} \sum_{j=1}^{n} \Big[g_2(z_{ij}/s) - (1-\theta_2)\alpha_{ij}^{2}(z_{ij}/s) \big] \ge 1, \\
\vdots \\
s\big[\sum_{i=1}^{m} \sum_{j=1}^{n} \Big[g_e(z_{ij}/s) - (1-\theta_e)\alpha_{ij}^{e}(z_{ij}/s) \big] \ge 1, \\
y_{ij} = 0 \text{ if } z_{ij} = 0, \\
y_{ij} = 1 \text{ if } z_{ij} > 0, \\
s > 0,\ z_{ij} \ge 0,\ \forall\ i,\ j.
\end{cases}
\end{cases}
$$

5 Solution Procedure

To solve the deterministic models of MOFCTP (**Models 5, 9** and **10**), we apply the fuzzy programming method.

Fuzzy Programming Method

Here, we shortly discuss fuzzy programming method [25], in the following steps:

Step 1: Solve the MOFCTP by considering one objective function at a time and ignoring others. Repeating this process K times for K many different objective functions $(K = 1, 2, \dots, e)$.

Step 2: Using the results of **Step 1**, determine the corresponding value for each objective function and construct a pay-off matrix of size $K \times K$. Then, from the pay-off matrix, we find the lower bound L^K and upper bound U^K for the K^{th} objective function Z_K $(K = 1, 2, \dots, e)$, where L^K are aspiration levels of achievement for K^{th} objective, U^K is the highest acceptable level of achievement for K^{th} objective and $d^K = [U^K - L^K]$ is degradation allowance for K^{th} objective function.

Step 3: From **Step 2**, we obtain, the best (L^K) and worst (U^K) for each objective function. An initial fuzzy model [31] can then be stated as follows.
Find z_{ij} $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$, so as to satisfy $Z_K \leq L^K$ $(K = 1, 2, \ldots, e)$ with the given constraints and non-negative conditions. For the MOFCTP, a membership function $\mu_{Z_K}(Z_K)$ corresponding to K^{th} objective function is defined as follows:

$$\mu_{Z_K}(Z_K) = \begin{cases} 1, & \text{if } Z_K \leq L^K, \\ 1 - (\frac{Z_K - L^K}{U^K - L^K}), & \text{if } L^K < Z_K < U^K (K = 1, 2, \ldots, e), \\ 0, & \text{if } Z_K \geq U^K. \end{cases}$$

Step 4: Converting the fuzzy model of the MOFCTP, obtained in **Step 3**, into the following equivalent crisp model:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & \lambda \leq \frac{U^K - Z_K}{U^K - L^K} \quad (K = 1, 2, \ldots, e), \end{aligned}$$

with the same constraints of **Models 5** or **9** or **10** and $\lambda \geq 0$,

$$\text{where } \lambda \ = \ \min_K [\mu_{Z_K}(Z_K) \ : K = 1, 2, \ldots, e].$$

This linear programming problem can further be simplified as follows:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & Z_K + \lambda(U^K - L^K) \leq U^K \ (K = 1, 2, \ldots, e), \\ & \lambda \geq 0, \end{aligned} \tag{5.1}$$

with the same constraints of **Models 5** or **9** or **10**.

Theorem 5.1 *If x^* is an optimal solution of Equation* (5.1) *then it is also a non-dominated (Pareto-Optimal) solution of* **Models 5, 9** *and* **10**.

5.1 Solution Procedure for Model 5

Step 1: First, we are to find the Robust rank of each triangular fuzzy numbers which is described in Subsection 4.1. So, we obtain the crisp values of all fuzzy parameters of **Model 5**.
Step 2: Then, we solve this crisp MOFCTP (**Model 5**) by using fuzzy programming method which is described in Section 5.
Step 3: Using LINGO software, we derive the optimal solution of each objective function.

5.2 Solution Procedure for Models 7 and 8

Step 1: First, we set the confidence levels (choice of decision maker) of δ_K ($K = 1, 2, \ldots, e$) for objective functions and confidence levels of θ_i ($i = 1, 2, \ldots, m$), θ_j ($j = 1, 2, \ldots, n$) and θ_K ($K = 1, 2, \ldots, e$) for constraints. So, we obtain the simplest form of **Models 9** and **10**. Note that, for convenience, we consider three types of confidence level θ_i, θ_j, θ_K for constraints which lie in θ_r.
Step 2: We solve the crisp MOFCTPs (**Models 9** and **10**) by using fuzzy programming method described in Section 5.
Step 3: Using LINGO software, we obtain the optimal solution of each objective function.

5.3 Application of proposed MOFFCTP

To show the applicability of our proposed fuzzy-MOFFCTP model **(Model 3)**, we consider a real-life fuzzy Bi-Objective Fractional Fixed Charge Transportation Problem (fuzzy-BOFFCTP). When we set $K = 1, 2$, in **Model 3**, then fuzzy-MOFFCTP model is converted to fuzzy-BOFFCTP model. The parameters of fuzzy-BOFFCTP (variable transportation cost, fixed charge, profit, deterioration rate, transporting time, supply, demand) are taken as fuzzy in nature. The objective functions Z_1 and Z_2 can be described as:
(I) minimizing the total transporting cost (variable and fixed transportation cost) with respect to profit (i.e., we find the minimum cost and maximum profit,) and
(II) minimizing the deterioration rate of goods with respect to time (i.e., we calculate the minimum deterioration rate of goods when the transporting time is maximal).
Fuzzy-BOFFCTP can be described as follows:

$\tilde{z_1}(x_{ij}, y_{ij}) = \sum_{i=1}^{m} \sum_{j=1}^{n} (\tilde{c_{ij}} x_{ij} + \tilde{f_{ij}} y_{ij})$ and $\tilde{z_2}(x_{ij}) = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij}$,

$\tilde{g_1}(x_{ij}) = \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} x_{ij}$ and $\tilde{g_2}(x_{ij}) = \max\{\, \tilde{t_{ij}} : x_{ij} > 0,\ \forall i,\ j\}$.

6 Numerical Example

A natural gas production company in India processes liquefied natural gas. The company has three production centers and four depots situated in different places in India. The company transports liquefied natural gas from production center to depots via tankers over the highways and railways. The Decision Maker (DM) desires that minimizing the total transporting cost (variable cost per unit and fixed cost) in which profit is maximum and minimizing the deterioration rate of goods in respect to maximum transporting time (from production center to depots). Because of that it is more economical to cool down (-160^oC) the gas such that it becomes liquid and transport it via ship. The transportation cost, fixed charge and profit in million dollars per ton, deterioration rate in litre, time in hour are considered. The DM is also interested to find the amount of natural gas products in ton to be transported from the i^{th} production center (warehouse) to the j^{th} depot (demand point) so as to satisfy the total requirement.

The data shown in Tables 1, 2 and 3 are considered to describe the whole problem.

Table 1: The coefficients of transportation cost ($\tilde{c_{ij}}$) and fixed charge ($\tilde{f_{ij}}$) in terms of triangular fuzzy numbers.

	D_1	D_2	D_3	Supply
S_1	(3,4,5), (6,7,8)	(6,7,8), (8,9,10)	(5,7,9), (9,10,11)	(24,26,28)
S_2	(8,9,10), (10,11,12)	(10,11,12), (12,13,14)	(6,8,10), (11,12,13)	(30,32,34)
S_3	(6,10,14), (9,12,15)	(11,12,13), (13,14,15)	(10,12,14), (13,15,17)	(32,34,36)
Demand	(18,20,22)	(25,26,27)	(31,32,33)	

Table 2: The coefficients of earned profit ($\tilde{p_{ij}}$) in terms of triangular fuzzy numbers.

	D_1	D_2	D_3	Supply
S_1	(8,9,10)	(14,15,16)	(12,13,14)	(24,26,28)
S_2	(16,18,20)	(18,19,20)	(15,17,19)	(30,32,34)
S_3	(20,21,22)	(20,22,24)	(17,18,19)	(32,34,36
Demand	(18,20,22)	(25,26,27)	(31,32,33)	

Table 3: The coefficients of deterioration rate ($\tilde{d_{ij}}$) and transporting time ($\tilde{t_{ij}}$) in terms of triangular fuzzy numbers.

	D_1	D_2	D_3	Supply
S_1	(1,2,3), (12,13,14)	(3,4,5), (14,15,16)	(2,3,4), (17,18,19)	(24,26,28)
S_2	(4,5,6), (13,15,17)	(7,8,9), (16,18,20)	(6,7,8), (17,19,21)	(30,32,34)
S_3	(5,7,9), (19,20,21)	(2,4,6), (20,21,22)	(5,6,7), (18,20,22)	(32,34,36)
Demand	(18,20,22)	(25,26,27)	(31,32,33)	

7 Result and Discussion

In this section, we discuss the optimal solutions of the equivalent crisp **Models 5, 9** and **10** for $K = 1, 2$ which are also the optimal solutions of **Model 4** and as well as **Model 3** for $K = 1, 2$.

(i) Using the data from **Tables 1, 2** and **3** in the formulated **Model 5** for $K = 1, 2$; applying the procedure described in Subsection 5.1, we obtain the results which are shown in **Table 4**.

Table 4: The optimal solution of proposed MOFFCTP for $K = 1, 2$.

Robust Ranking Technique	Optimal Values of the Objective Functions $[Z_1, Z_2]$	Values of λ and s
Model 3	0.54, 0.49	0.75, 0.0007

(ii) Using the data from **Tables 1, 2** and **3** in the prescribed **Models 9** and **10** for $K = 1, 2$, and applying the procedure described in Subsection 5.2. To solve these models, we set confidence levels of $\delta_1 = \delta_2 = 0.85$ (choice of the DM) and confidence levels of $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 0.90$. So we obtain the simplest form of **Models 9** and **10** and the obtained the results which are shown in **Table 5**.

Table 5: The optimal solution of proposed MOFFCTP for $K = 1, 2$.

Fuzzy-Chance Constrained Rough Technique	Optimal Values of the Objective Functions $[Z_1, Z_2]$	Values of λ and s
Model 7(LL-MOFCTP)	0.45, 0.20	0.70, 0.0008
Model 8(LU-MOFCTP)	0.52, 0.29	0.61, 0.0007

From **Tables 4** and **5**, we can conclude that the optimal value of the proposed MOFFCTP (i.e., **Model 3**) for $K = 1, 2$, is $(0.45,\ 0.20)$, more efficient than $(0.52,\ 0.29)$ and $(0.54,\ 0.49)$, respectively. Also, we observe that the optimal solution derived from fuzzy chance constrained rough technique is better than the optimal solution calculated from Robust ranking technique of proposed MOFFCTP. Here, we extend the feasible region (for optimal solution) of the proposed MOFFCTP which lies on lower approximation, i.e., $(0.45,\ 0.20)$; and on the upper approximation, i.e., $(0.52,\ 0.29)$. But a better optimal solution occurred when we consider the case LL-MOFCTP (i.e., **Model 9**) for $K = 1, 2$.

8 Conclusion and Outlook

In this paper, for the first time, multi-objective fractional fixed charge transportation problem is formulated by using two types of uncertainty and to extend the feasible region for optimal solutions. To convert the proposed MOFFCTP into a deterministic form,

we have proposed two techniques: one is Robust ranking technique and the other one is fuzzy chance-constrained rough technique. We have applied fuzzy chance-constrained rough technique to divide crisp form of MOFCTP into two parts: LL-MOFCTP and LU-MOFCTP. From lower and upper approximation models of MOFCTP, we have shown that the feasible region of MOFCTP (also, MOFFCTP) is extended as well as partitioned to produce better optimal solutions. To show the effectiveness of our proposed MOFFCTP, we have compared the solutions between the solutions extracted from Robust ranking technique and fuzzy chance-constrained rough technique.
The proposed models may be extended to various types of TP, fractional optimization problem and allocation problem. In future study, one can analyze the multi-objective fractional fixed-charge solid transportation problem with all parameters may be considered a fuzzy-rough variables.

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*Highlights (for review)

Highlights for Review

- Studying the Multi-Objective Fractional Fixed-Charge Transportation Problem (MOFFCTP) in a rough decision making approach under a new view.
- The parameters of the proposed model are considered as fuzzy numbers; and different types of fuzzy measures such as Possibility, Credibility and Necessity are employed to deal the fuzziness of the proposed model.
- We use rough set theory for extending as well as partitioning the feasible region of the MOFFCTP to accommodate more information by considering two approximations.
- Robust Ranking technique and Fuzzy Chance-Constrained Rough technique are utilized to derive the best optimal solution of the proposed model and the results are compared.
- A real-life example is included to illustrate the applicability of the proposed model.

*Graphical abstract (for review)