Middle East Technical University

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Stress testing of Energy-Related Derivative Instruments based on Conditional Market Risk Models

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- “ 'Stress Test' Leaves Big-Bank Investors Feeling Stressed Out ” (Financial Times, 10.02.2009)

U.S. commercial banks	Assets	Share price performance Feb. 10
J.P. Morgan Chase	$2.25 trillion	-10%
Citigroup	$2.05 trillion	-15%
Bank of America	$1.83 trillion	-19%
Wells Fargo	$622 billion	-14%
State Street	$286 billion	-13%
Bank of New York Mellon	$268 billion	-8%
US Bancorp	$247 billion	-14%
SunTrust Banks	$175 billion	-27%
Capital One Financial	$156 billion	-14%
PNC Financial	$146 billion	-11%
Regions Financial	$144 billion	-30%
BB&T	$137 billion	-13%
Fifth Third	$116 billion	-24%
KeyCorp	$101 billion	-27%

Sources: RBC Capital Markets, SNL Financial, FactSet Research

- Critical decisions
  - Nationalization? Government stake? How much additional capital to raise?...

Origins

Amendment to 1988 Accord [Basel Committee, 1996]

Basel II Framework [Comprehensive version, July 2006]

- Supervisory authorities will not approve the use of internal models unless they're supplemented by stress tests. [Prg.718(Lxxi)]
- A routine and rigorous programme of stress testing, periodical review, reflection of the results in the assessment of capital adequacy as well as the management policies, prompt steps in the case the results reveal particular vulnerability. [Prg.718(Lxxiv) (g)]
- Stress scenarios need to cover a range of factors that can create extraordinary losses or gains in trading portfolios, or make the control of risk in those portfolios very difficult [Prg. 718(Lxxviii)]

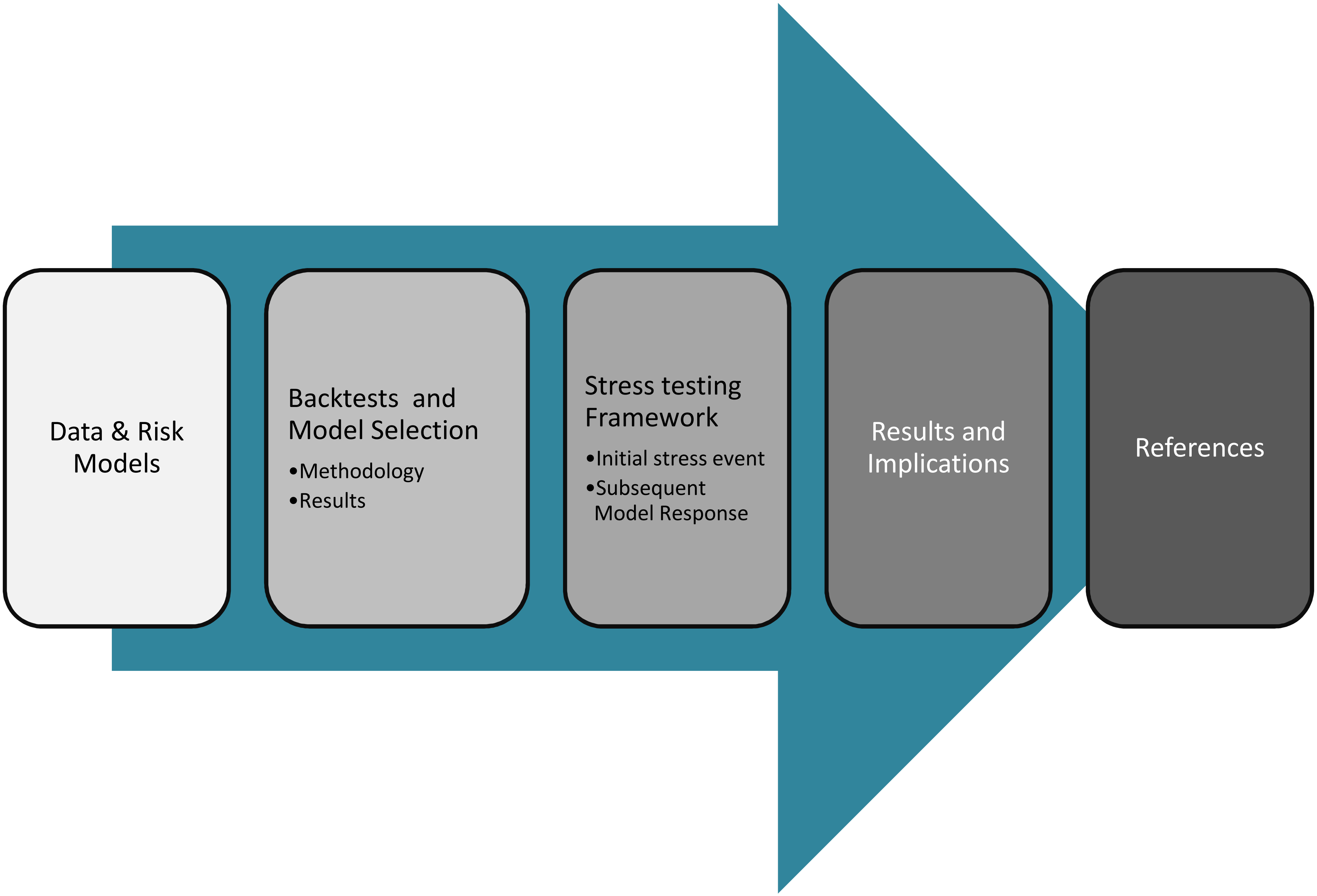
Points of Departure

- Stress testing is fundamental part of the risk management process
- Yet no coherent stress testing framework, particularly for much more volatile energy contracts (i.e. either absence or misspecification of risk models)

Aim of the study is…

- To introduce a methodology for model-based stress testing of energy-related contracts in the context of market risk models that can incorporate both volatility clustering and heavy-tails. [Quantitative criteria]
- To examine the implications of stress test results in terms of Basel II regulatory capital context
- But not to identify steps to be taken to reduce risks and conserve capital.[Qualitative criteria]

Layout


Data & Risk Models
Backtests and Model Selection
•Methodology
•Results
Stress testing Framework
•Initial stress event
•Subsequent Model Response
Results and Implications
References

- NYMEX futures
  - Crude oil: Jan 1983 – Feb 2009
  - Heating oil: Jan 1980 – Feb 2009
  - Natural gas: Jan 1994 – Feb 2009

	Crude Oil	Natural Gas	Heating Oil
Mean	0.004%	0.019%	0.006%
Standard Deviation	2.44%	3.81%	2.34%
Kurtosis	16.82	6.99	20.33
Skewness	-0.94	0.07	-1.53
Min	-40.00%	-37.60%	-39.10%
Max	16.40%	32.40%	14.00%

- Estimation Windows (Backtests)
  - 250, 1000, 2000 days (Rolling Window)

Risk Models

i. Conditional normal model

$$\sigma_t^2 = \xi_1 + \xi_2 \sigma_{t-1}^2 + \xi_3 \varepsilon_{t-1}^2$$

$$\xi_1 \geq 0$$

$$\xi_2, \xi_3 > 0$$

$$\xi_2 + \xi_3 < 1$$

$$\varepsilon_t \sim N(0, \sigma_t^2)$$

ii. Conditional Student's t

$$\varepsilon_t (v/(v-2))^{0.5} \sim t_v$$

Risk Models (cont.)

iii. Conditional empirical model

- Fit a normal GARCH model to historical returns
- Standardize each return
- Finally, scale the standardize returns to current conditional standard deviation estimate
- Use scaled returns in VaR estimations

  [Barone-Adesi et al. (1998)]

Risk Models (cont.)

iv. Asymmetric conditional normal model

$$\ln(\sigma_t^2) = \xi_1 + \xi_2 \ln(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \omega \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

v. Asymmetric conditional Student's t model

$$\ln(\sigma_t^2) = \xi_1 + \xi_2 \ln(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \omega \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{v-2}{\pi}} \frac{\Gamma(\frac{v-1}{2})}{\Gamma(\frac{v}{2})} \right]$$

Test for Unconditional Coverage [Kupiec, 1995]

$$LR_{uc} = \left(\frac{\hat{\alpha}}{\alpha}\right)^{I(\alpha)} \left(\frac{1-\hat{\alpha}}{1-\alpha}\right)^{T-I(\alpha)} \quad , \hat{\alpha} = \frac{1}{T} I(\alpha) \ \text{ and } \ I(\alpha) = \sum_{t=1}^{T} I_t(\alpha)$$

$$2\ln LR_{uc} \sim \chi_1^2$$

Test for Conditional Coverage [Christoffersen, 1998]

$$LR_{cc} = \frac{\hat{\alpha}_{01}^{I_{01}(\alpha)} \hat{\alpha}_{11}^{I_{11}(\alpha)}}{\alpha^{I(\alpha)}} \frac{(1-\hat{\alpha}_{01}^{I_{00}(\alpha)})(1-\hat{\alpha}_{11}^{I_{10}(\alpha)})}{(1-\alpha)^{T-I(\alpha)}}$$

$$2\ln LR_{cc} \sim \chi_2^2$$

Backtests - Results Estimation window: 250 trading days

	100*(1-α)	(1) Conditional normal %	p-tfcc	p-tfuc	(2) Asymmetric Conditional normal %	p-tfcc	p-tfuc	(3) Conditional Student's t %	p-tfcc	p-tfuc	(4) Asymmetric Conditional Student's t %	p-tfcc	p-tfuc	(5) Conditional empirical %	p-tfcc	p-tfuc
long Crude Oil	99	1.91	0.00	0.00	2.20	0.00	0.00	0.85	10.86	22.28	0.96	6.96	76.53	1.48	0.03	0.04
	99.5	1.30	0.00	0.00	1.41	0.00	0.00	0.47	28.89	69.40	0.56	34.29	49.95	0.87	0.02	0.02
	99.8	0.85	0.00	0.00	1.03	0.00	0.00	0.26	60.56	33.73	0.32	13.68	4.97	0.66	0.00	0.00
short Crude Oil	99	1.56	0.01	0.00	1.57	0.01	0.00	0.59	0.11	0.05	0.79	3.73	7.79	1.57	0.01	0.00
	99.5	1.09	0.00	0.00	1.16	0.00	0.00	0.35	4.15	8.22	0.40	46.85	25.15	0.75	2.03	0.82
	99.8	0.77	0.00	0.00	0.69	0.00	0.00	0.16	2.99	46.89	0.22	88.48	66.98	0.48	0.00	0.00
long Natural Gas	99	1.33	8.71	5.75	1.76	0.02	0.00	0.43	0.05	0.01	0.77	15.61	14.70	1.25	20.52	15.18
	99.5	0.88	1.17	0.39	1.19	0.00	0.00	0.26	7.43	2.32	0.31	22.93	8.99	0.62	52.26	31.22
	99.8	0.51	0.24	0.06	0.74	0.00	0.00	0.03	1.66	0.42	0.14	71.36	41.63	0.31	37.35	16.80
short Natural Gas	99	2.13	0.00	0.00	2.22	0.00	0.00	0.74	21.62	10.19	0.82	43.63	27.80	1.16	39.07	33.92
	99.5	1.25	0.00	0.00	1.59	0.00	0.00	0.37	48.96	24.85	0.45	86.17	69.69	0.68	29.75	14.77
	99.8	0.94	0.00	0.00	1.08	0.00	0.00	0.23	92.24	72.36	0.28	56.07	29.42	0.45	1.42	0.38
long Heating Oil	99	1.96	0.00	0.00	2.21	NA	NA	0.89	57.27	36.12	1.01	94.80	95.14	1.46	0.12	0.03
	99.5	1.56	0.00	0.00	1.77	0.00	0.00	0.51	82.46	89.89	0.60	28.31	26.83	0.81	0.27	0.08
	99.8	0.99	0.00	0.00	1.29	0.00	0.00	0.24	72.49	45.37	0.34	5.21	1.65	0.54	0.00	0.00
short Heating Oil	99	1.65	0.00	0.00	1.77	0.00	0.00	0.60	0.06	0.02	0.65	0.45	0.17	1.26	7.79	3.33
	99.5	1.04	0.00	0.00	1.21	0.00	0.00	0.34	2.59	4.38	0.44	24.14	46.42	0.81	0.08	0.08
	99.8	0.65	0.00	0.00	0.77	0.00	0.00	0.07	1.98	0.51	0.20	97.22	97.91	0.48	0.00	0.00

%: proportion of observed model violations
p-tfcc: p-value for the test of conditional coverage
p-tfuc: p-value for the test of unconditional coverage

	100*(1-α)	(1) Conditional normal %	p-tfcc	p-tfuc	(2) Asymmetric Conditional normal %	p-tfcc	p-tfuc	(3) Conditional Student's t %	p-tfcc	p-tfuc	(4) Asymmetric Conditional Student's t %	p-tfcc	p-tfuc	(5) Conditional empirical %	p-tfcc	p-tfuc
long Crude Oil	99	1.57	0.02	0.01	1.70	0.00	0.00	0.51	0.01	0.01	0.64	0.75	0.39	1.02	86.37	87.43
	99.5	1.04	0.00	0.00	0.98	0.00	0.00	0.22	0.39	0.09	0.26	13.73	5.46	0.53	81.92	76.37
	99.8	0.64	0.00	0.00	0.66	0.00	0.00	0.07	5.29	1.54	0.09	12.91	4.33	0.26	65.55	37.93
short Crude Oil	99	1.39	1.77	0.66	1.28	8.44	4.83	0.38	0.00	0.00	0.36	0.00	0.00	1.02	86.37	87.43
	99.5	0.88	0.12	0.04	0.77	2.53	0.96	0.15	0.01	0.00	0.18	0.06	0.01	0.49	87.22	93.65
	99.8	0.53	0.00	0.00	0.47	0.05	0.01	0.07	5.29	1.54	0.11	25.75	10.03	0.29	34.65	15.46
long Natural Gas	99	1.12	58.23	53.76	1.26	26.14	18.11	0.11	0.00	0.00	0.18	0.00	0.00	1.01	75.02	95.59
	99.5	0.69	37.12	18.97	0.76	17.24	7.38	0.04	0.00	0.00	0.07	10.03	9.01	0.54	87.99	76.09
	99.8	0.40	11.86	4.10	0.40	11.86	4.10	0.04	5.88	1.73	0.04	5.88	1.73	0.22	96.90	84.76
short Natural Gas	99	1.66	0.60	0.14	1.62	0.98	0.25	0.40	0.13	0.03	0.43	8.31	10.07	0.90	69.35	59.88
	99.5	1.73	0.00	0.00	1.73	0.00	0.00	0.36	53.08	27.45	0.47	91.54	81.60	0.43	83.28	60.91
	99.8	1.19	0.00	0.00	1.23	0.00	0.00	0.11	49.40	23.61	0.11	49.40	23.61	0.18	96.42	81.47
long Heating Oil	99	1.78	0.00	0.00	0.00	0.00	1.95	0.59	0.14	0.04	0.67	0.67	0.47	1.02	91.33	89.85
	99.5	1.32	0.00	0.00	1.44	0.00	0.00	0.29	3.05	0.87	0.29	3.05	0.87	0.62	34.03	19.62
	99.8	0.89	0.00	0.00	1.02	0.00	0.00	0.17	88.22	64.50	0.17	88.22	64.50	0.25	62.85	35.73
short Heating Oil	99	2.21	0.00	0.00	2.22	0.00	0.00	0.83	7.54	15.13	0.87	2.65	30.11	1.19	19.94	13.98
	99.5	1.57	0.00	0.00	1.59	0.00	0.00	0.44	23.73	52.45	0.52	37.36	78.97	0.60	25.89	26.05
	99.8	1.17	0.00	0.00	1.22	0.00	0.00	0.19	5.51	86.50	0.21	6.55	91.02	0.30	3.43	9.33

%: proportion of observed model violations
p-tfcc: p-value for the test of conditional coverage
p-tfuc: p-value for the test of unconditional coverage

Introducing Stress-testing Framework

- Incorporating the three highlighted characteristics into the stress-testing framework:
  - Heavy-tailed residuals
  - Volatility clustering
  - Leverage effect (Asymmetric volatility response)

- A stress horizon consists of:
  - A stress event
  - Subsequent market response

Stress-testing Framework – Initial Shock Event

- An unanticipated but plausible event that causes a large discontinuity in prices
- That is, for a long position, VaR calculated using a typically low probability region, say α_s , can be used as a stress event.

(i.e. the initial shock at the beginning of the stress period T, (say ε_T^*) can be drawn from the specified distribution.)

e.g.

$$\varepsilon_T^* = t_v^{-1}(\alpha_s)((v-2)/v)^{0.5}\,\bar{\sigma}_T$$

$$\varepsilon_T^* = \Phi^{-1}(\alpha_s)\bar{\sigma}_T$$

- Once the intial shock is imposed on the system at time T, the conditional variance at time T+1 will increase.

  e.g. for the Conditional normal model:

$$\varepsilon_T = \varepsilon_T^* \quad \text{where} \quad \varepsilon_T^* = \Phi^{-1}(\alpha_s)\bar{\sigma}_T$$

$$r_T = \mu + \varepsilon_T$$

$$\hat{\sigma}_{T+1}^2 = \hat{\xi}_1 + \hat{\xi}_2 \bar{\sigma}_T^2 + \hat{\xi}_3 \varepsilon_T^2$$

$$r_{T+1} = \mu + \varepsilon_{T+1} \quad \text{where} \quad \varepsilon_{T+1} \sim N(0, \hat{\sigma}_{T+1})$$

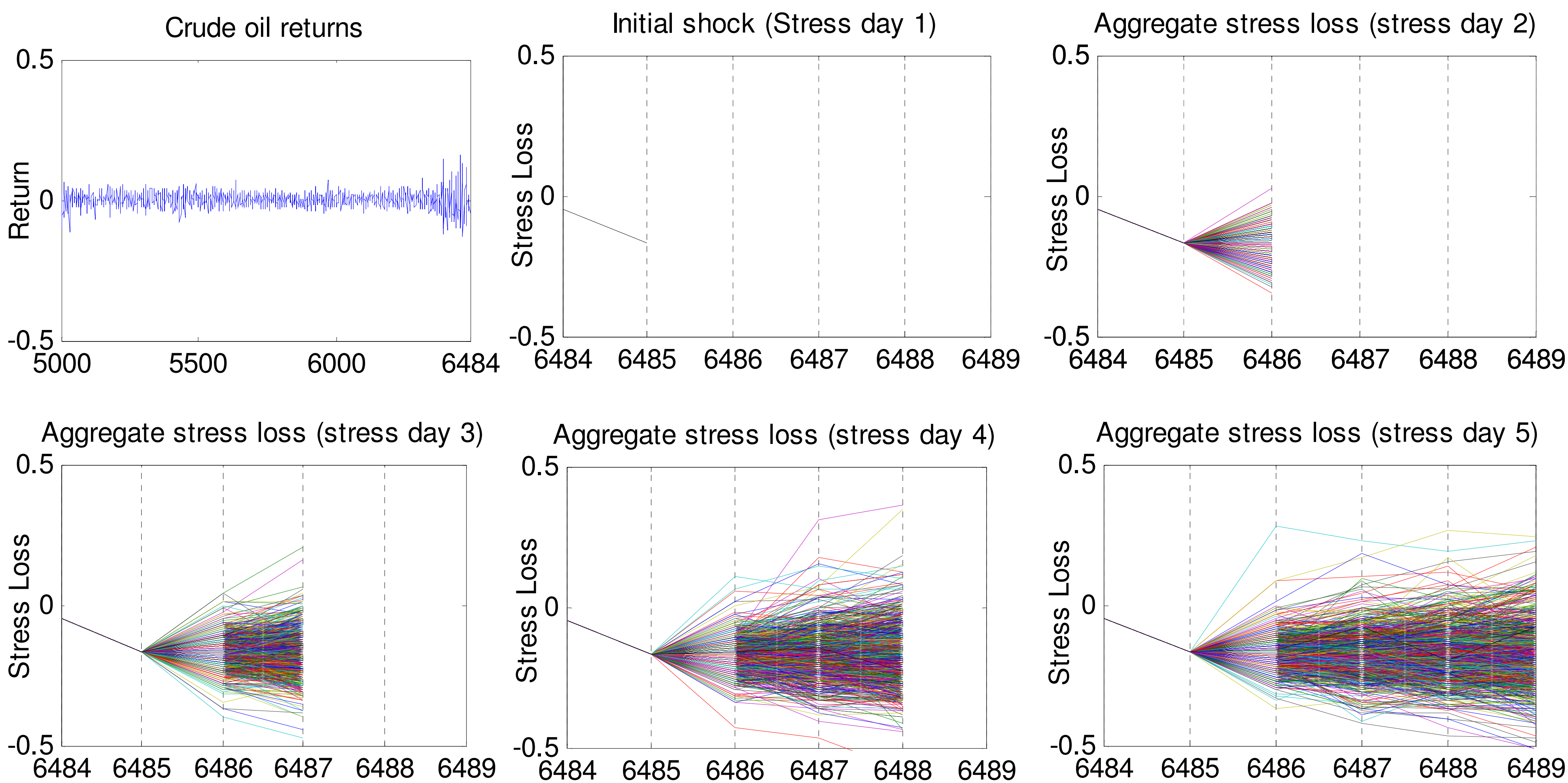
$$\vdots$$

$$\hat{\sigma}_{T+1+i}^2 = \hat{\xi}_1 + \hat{\xi}_2 \hat{\sigma}_{T+i}^2 + \hat{\xi}_3 \varepsilon_{T+i}^2$$

$$r_{T+1+i} = \mu + \varepsilon_{T+1+i} \quad \text{where} \quad \varepsilon_{T+1+i} \sim N(0, \hat{\sigma}_{T+1+i})$$

Stress-testing Framework – Modelling Market Response (Illustration)

Illustration: Simulation of 5-day aggregate stress loss for long crude oil portfolio (Risk Model: Conditional Student's t, Initial shock size=0.0002)



Stress-testing Framework – Parameters

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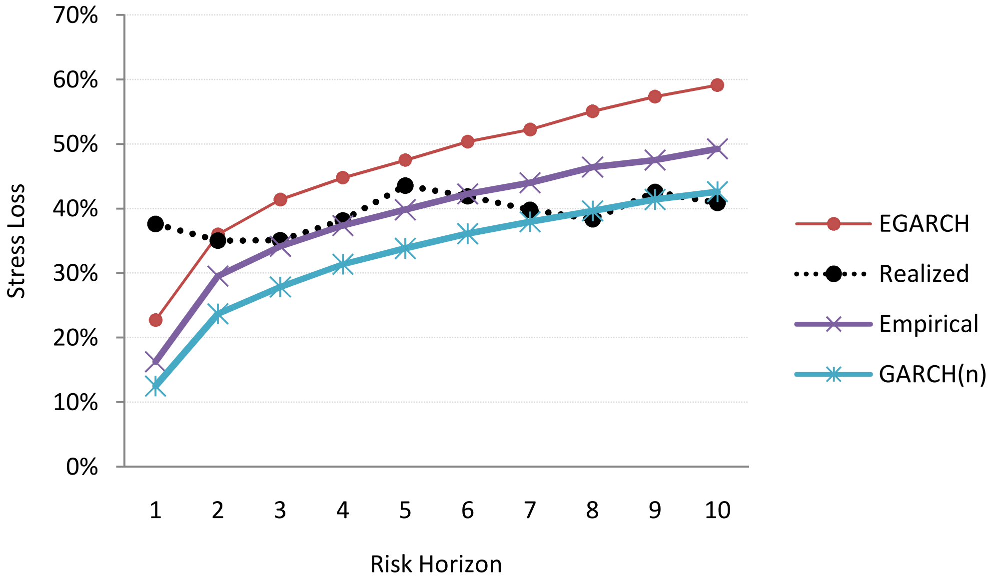
- For an s days of risk horizon, 50.000 paths are simulated based on the initial shock of size α_s and then the worst scenario with a 99% confidence level is called "stress loss".
  - In a stressful market, we're 99% confident that the estimated stress loss will not be exceeded by the actual decrease in the portfolio value in s days.

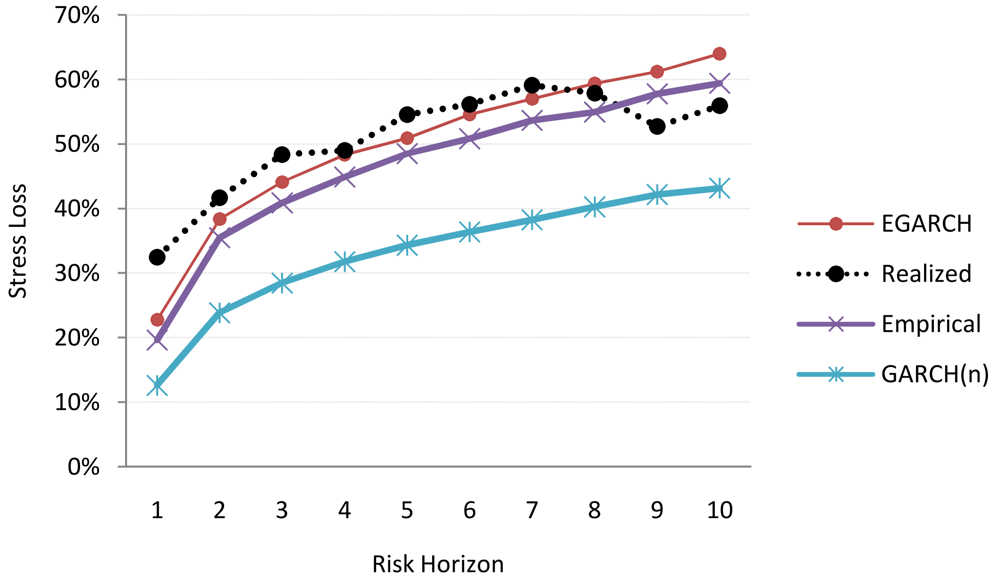
The selection of initial shock size is a part of management strategy. It reflects the risk profile of the company.	Shorter risk horizons may be sufficient for liquid contracts. However, the selection of the risk horizon should reflect the reduced liquidity and the relative size of the portfolio.

Results & Implications

Long natural gas, simulated stress loss over 10 days (alpha=0.0005)



Short natural gas, simulated stress loss over 10 days (alpha=0.0005)



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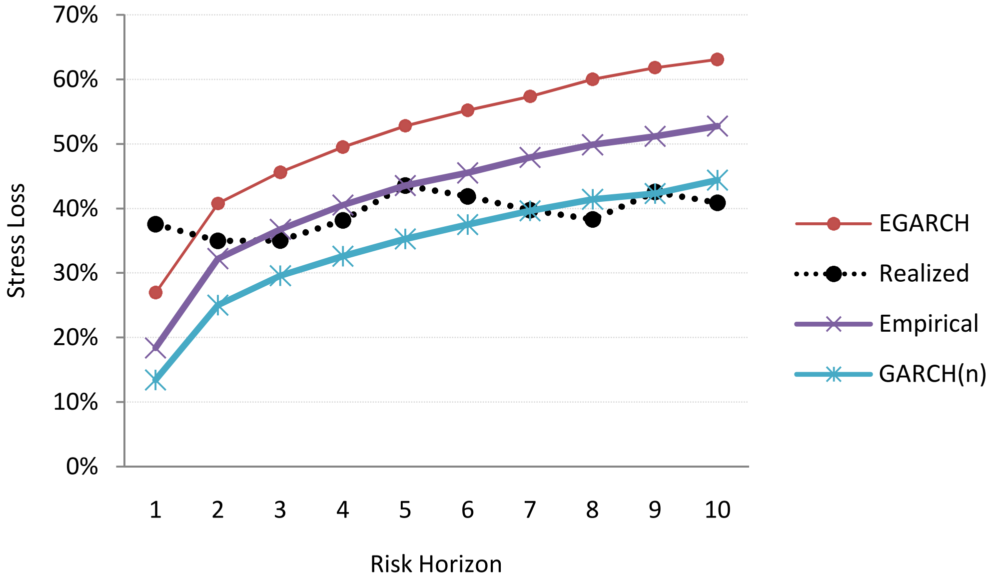
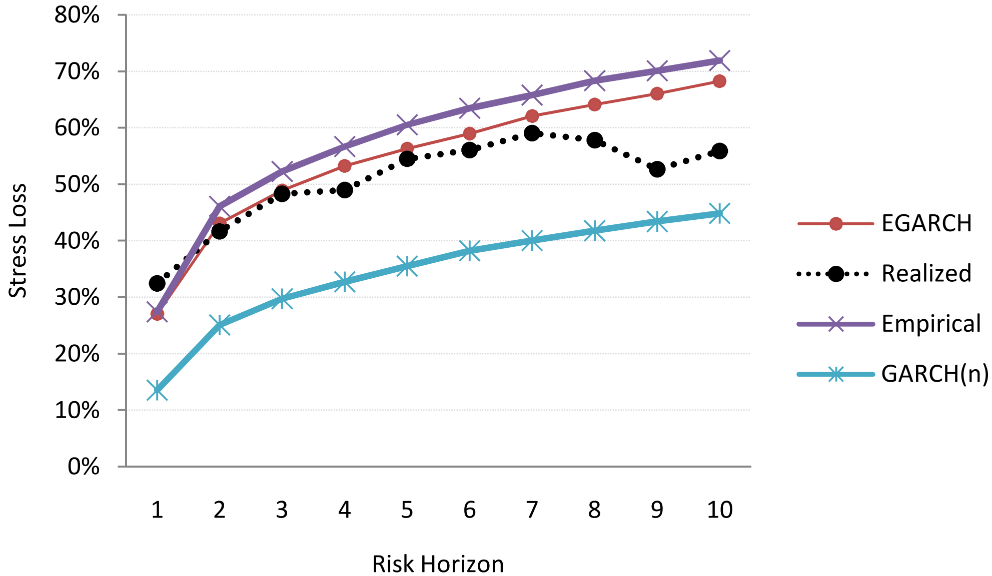
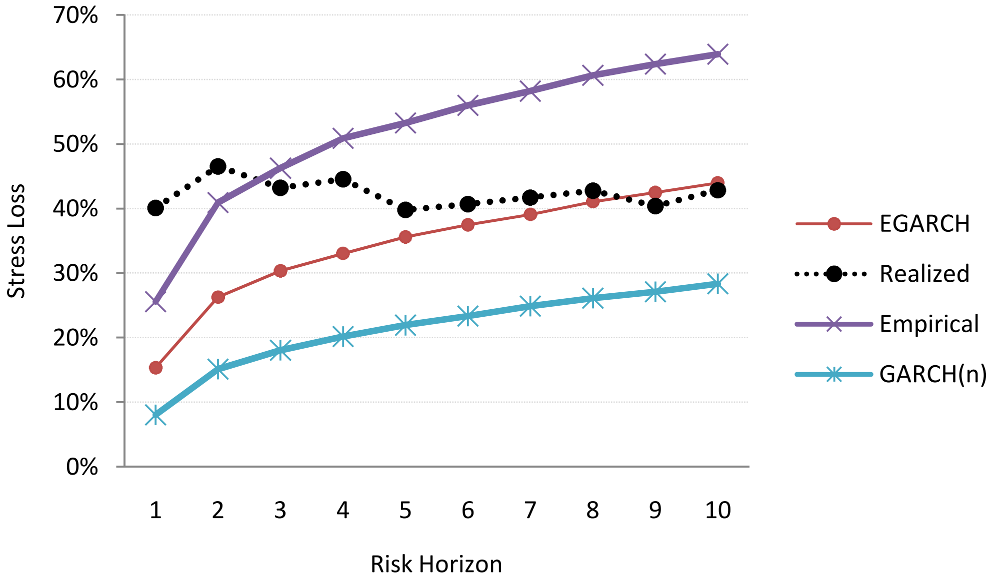


Short natural gas, simulated stress loss over 10 days (alpha=0.0002)

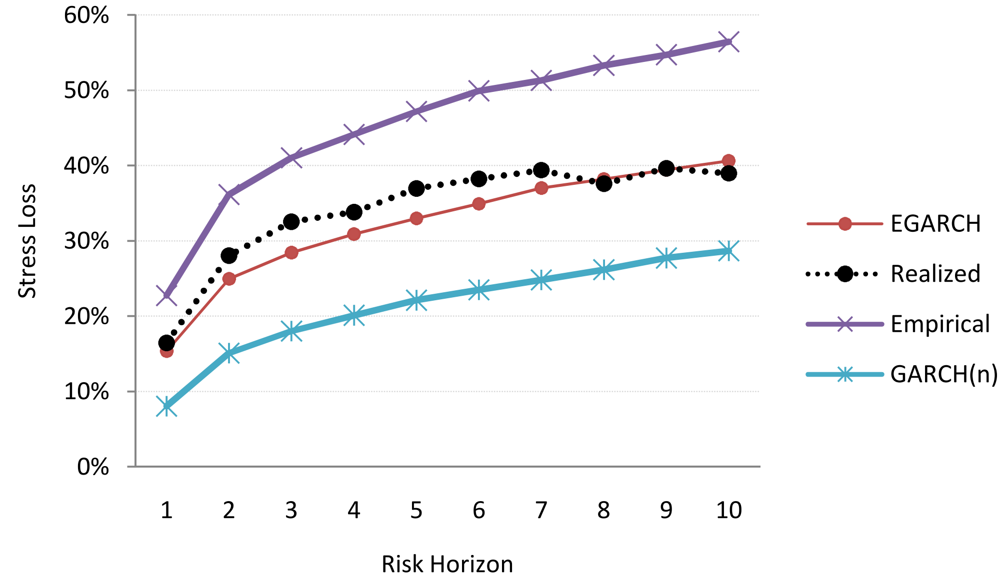


Results & Implications

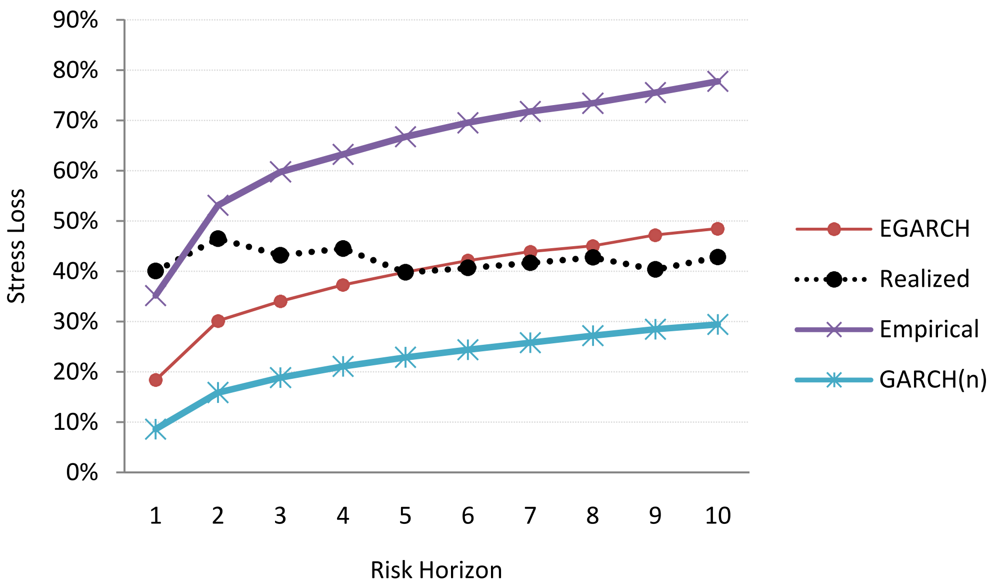
Long crude oil, simulated stress loss over 10 days (alpha=0.0005)



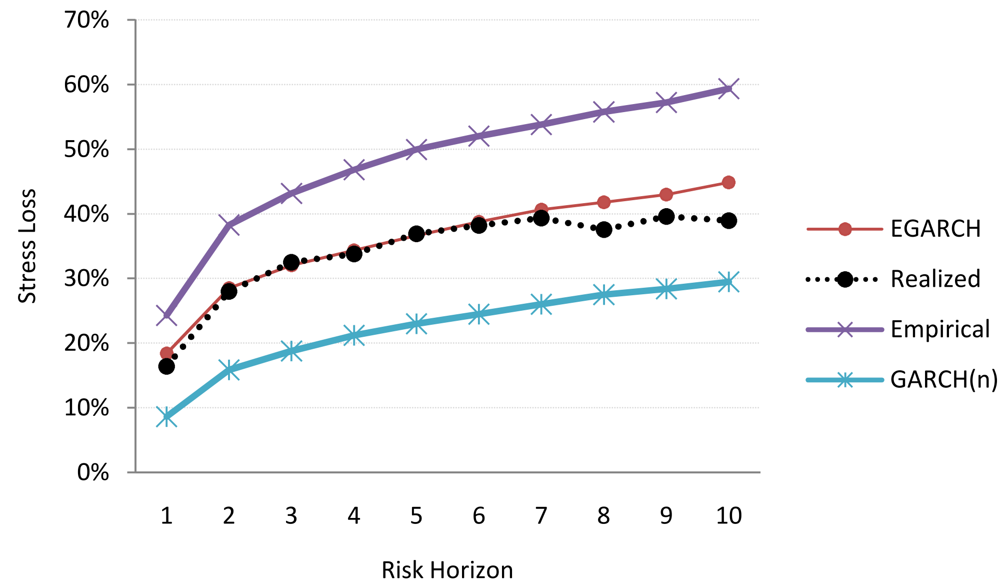
Short crude oil, simulated stress loss over 10 days (alpha=0.0005)



Long crude oil, simulated stress loss over 10 days (alpha=0.0002)

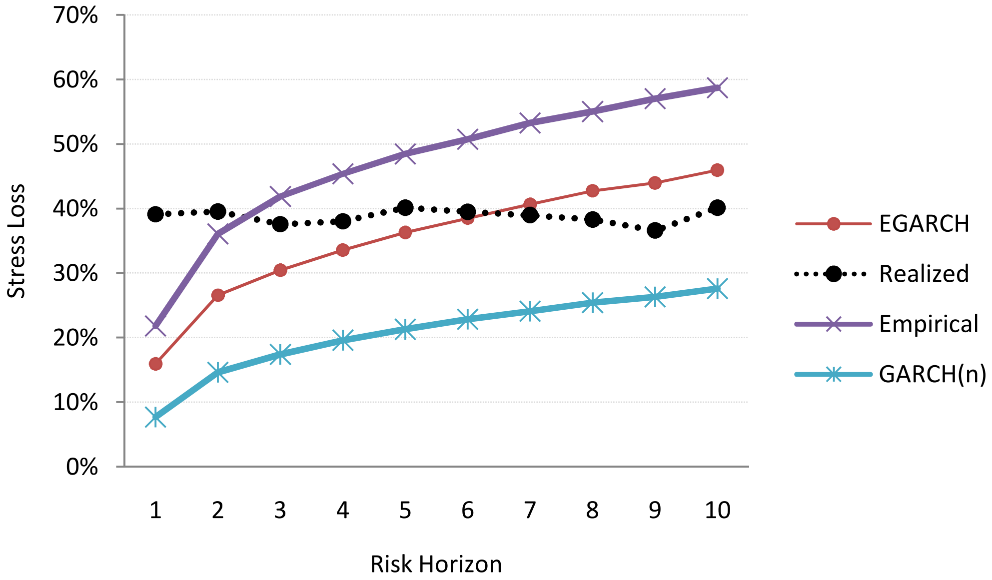


Short crude oil, simulated stress loss over 10 days (alpha=0.0002)

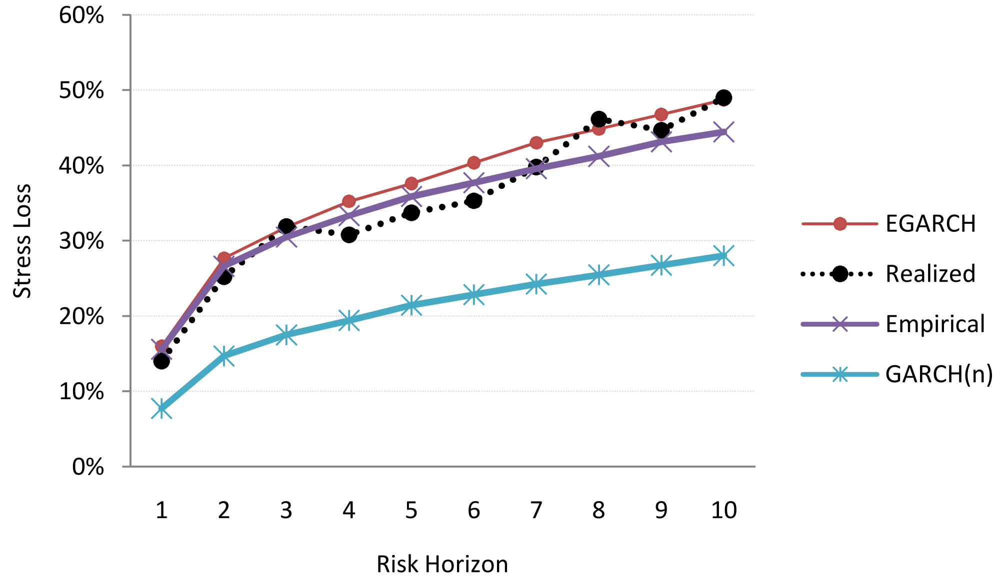


Results & Implications

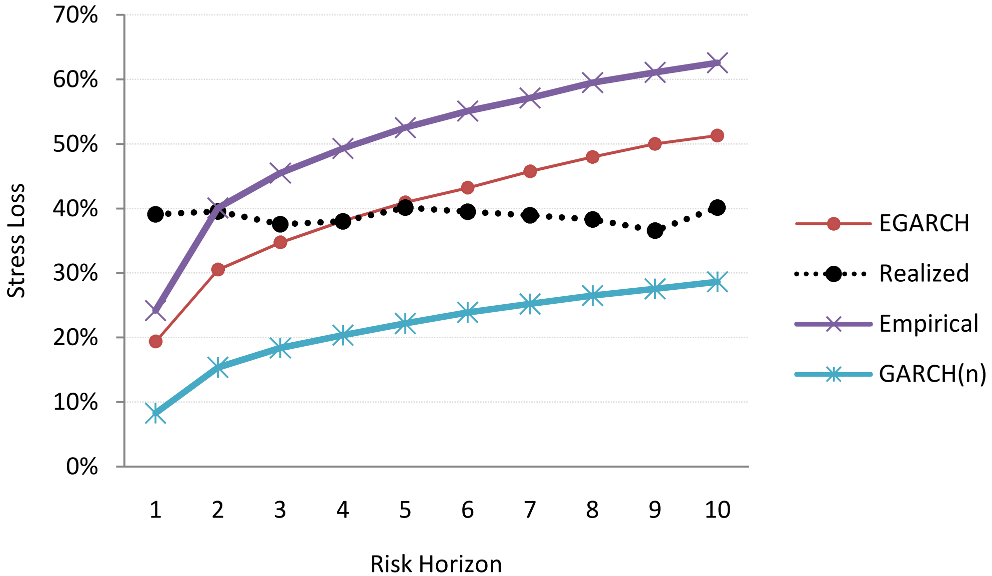
Long heating oil, simulated stress loss over 10 days (alpha=0.0005)



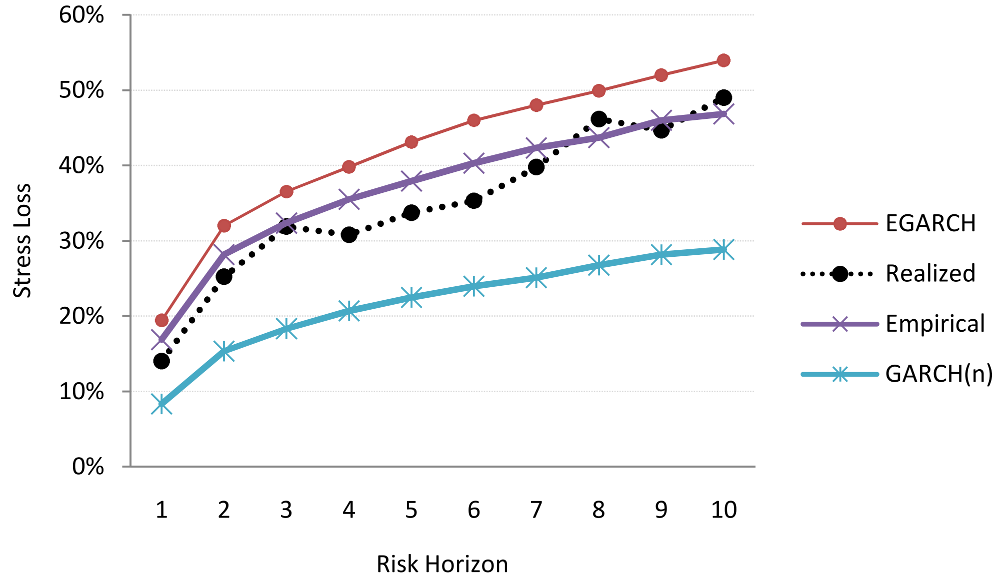
Short heating oil, simulated stress loss over 10 days (alpha=0.0005)



Long heating oil, simulated stress loss over 10 days (alpha=0.0002)



Short heating oil, simulated stress loss over 10 days (alpha=0.0002)



Results and Implications for risk capital

- Backtest results strongly indicate that the risk model with volatility adjusted innovations is the most suitable model for stress testing purposes.
- Volatile structure of the energy markets is successfully reflected in the stress test results.
- Significant stress losses predicted by asymmetric conditional models mostly match the historical shocks when 99.98% initial shock is considered.
  - Model-based stress test is able to simulate potential price paths for energy derivatives which are consistent with historical realizations.
- Basel II rules imply a lower bound for capital that must be set aside for single asset energy portfolios ranging between 45%–78% of the portfolio value.

Results and Implications for risk capital

- "...an instantaneous price shock equivalent to 10 day movement in prices is to be used, i.e. minimum holding period will be ten trading days...". [Prg.718(Lxxi)]

Implications for risk capital		Empirical	EGARCH
	Maximum historical loss over 10 days=42.82%		
long Crude Oil	*Stress loss (99% confidence)*		
	s=10,alpha=0.0005	63.87%	43.97%
	s=10,alpha=0.0002	77.77%	48.47%
	Maximum historical loss over 10 days=38.94%		
short Crude Oil	*Stress loss (99% confidence)*		
	s=10,alpha=0.0005	40.61%	56.41%
	s=10,alpha=0.0002	44.85%	59.33%
	Maximum historical loss over 10 days=40.86%		
long Natural Gas	*Stress loss (99% confidence)*		
	s=10,alpha=0.0005	49.23%	59.11%
	s=10,alpha=0.0002	52.73%	63.10%
	Maximum historical loss over 10 days=55.90%		
short Natural Gas	*Stress loss (99% confidence)*		
	s=10,alpha=0.0005	63.95%	59.38%
	s=10,alpha=0.0002	68.24%	71.91%
	Maximum historical loss over 10 days=40.13%		
long Heating Oil	*Stress loss (99% confidence)*		
	s=10,alpha=0.0005	58.70%	45.92%
	s=10,alpha=0.0002	62.56%	51.31%
	Maximum historical loss over 10 days=48.99%		
short Heating Oil	*Stress loss (99% confidence)*		
	s=10,alpha=0.0005	48.73%	44.46%
	s=10,alpha=0.0002	53.96%	46.82%

Advantages & Disadvantages

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The model-based stress test

- avoids subjective assumptions of the management,
- provides a coherent risk management methodology,
- can easily be linked to statistical risk models,
- incorporates two key components in modeling extreme risk: heavy tails and volatility clustering,
- is still vulnerable to risk model misspecification and significant structural changes in the market

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Thanks...