A Framework for Stress-testing Islamic Portfolios under IFSB and Basel Capital Frameworks

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Abstract

Purpose — Adverse developments in the global finance industry clearly underlined the importance of stress-testing. One of the key takeaways was the need to strengthen the coverage of the capital framework. Cognisant of this fact, Basel III encapsulates provisions to enhance the financial sector's ability to withstand shocks arising from possible stress events, thereby reducing adverse spillovers into the real economy. Similarly, the IFSB requires Islamic financial institutions to run stress tests as part of capital planning.

Design/methodology/approach — We perform thorough backtests on Islamic and conventional portfolios under widely used risk models, which are characterised by an underlying conditional volatility framework and distribution, to identify the most suitable risk model specification. Associated with an appropriate initial shock and estimation window size, the paper also conducts a model-based stress-test to examine whether the stress losses estimated by the selected models compare favourably to the historical shocks.

Findings — The results suggest that the model-based framework, when combined with an appropriate risk model and distribution, can successfully reproduce past stress periods. The conditional empirical risk model is the most effective one in both long and short portfolio cases — particularly when combined with a long-enough estimation window. The relative performance of normal vs. heavy-tailed distributions and symmetric vs. asymmetric risk models, on the other hand, is highly dependent on whether the portfolio is long or short. Finally, we find that the Islamic portfolio is generally associated with lower historical stress losses as compared to the conventional portfolio.

Originality/value — The model-based framework eliminates some of the key problems associated with traditional scenario-based approaches and is easily adaptable to Islamic finance.

Keywords Stress-testing, Backtesting, Islamic Finance, IFSB, Basel III, Risk Management, Capital adequacy

Paper type Research paper

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1. Introduction

Stress-testing is a key risk management tool within financial institutions. It alerts management to adverse unexpected outcomes related to a variety of risks and, among other things, provides an indication of how much capital might be needed to absorb losses if large shocks occur. While stress-testing by itself cannot address all risk management weaknesses, as part of a comprehensive approach it has a leading role to play in strengthening the resilience not just of individual banks but also of the financial system in its entirety. On the other hand, although the financial crisis evoked a shift towards more comprehensive stress-testing models and methodologies, most of these models and approaches are still exposed to risk model misspecification problems and some of them do not even have an underlying risk model. Furthermore, from Islamic financial institutions perspective, stress-testing remains an underdeveloped area. A stock-taking survey by the Islamic Financial Services Board's (IFSB) Stress-testing Working Group reveals that qualitative methods such as scenario analysis are predominantly of broader use, as compared to quantitative methods such as maximum loss approach and extreme value theory (IFSB, 2012, Para. 1.3.b). More importantly, in the IFSB survey, "availability of models and modelling expertise" is one of the most cited challenges to implementing stress tests (IFSB, 2012, Para. 1.3.f).

In May 2009, in response to the global financial crisis, the Basel Committee on Banking Supervision (BCBS) published its "Principles for Sound Stress Testing Practices and Supervision" (BCBS, 2009). This was followed in August 2010 by the Committee of European Banking Supervisors issuing its "Guidelines on Stress Testing" (CEBS, 2010). And, in June 2011, the Basel Committee published the revised version of Basel III capital rules (originally in December 2010), which attached particular importance to stress-testing in financial institutions (BCBS, 2011). To respond to the special needs of Islamic financial institutions, the constituent central banks of the IFSB, which is the prudential regulatory body for Islamic finance industry, established the Stress Testing Working Group (STWG) in 2009 and tasked it with the preparation of guiding principles on stress-testing for Islamic FIs, which ultimately came out in March 2012 with the intention to complement existing internationally recognised frameworks, some of which were named above (IFSB, 2012).

A recent example, in fact, underlies the on-going popularity as well as importance of stress-testing for the global financial system and its institutions. The European Banking Authority has recently conducted stress-tests on a total of 123 European banks under its supervision. Stress-test results, which were released in late October 2014, left the Authority with a group of 24 failed banks (i.e., one in each five) and a $25 \in$ billion capital hole in the European baking system.

The rest of the study is organised as follows: Section 2 summarises BCBS and IFSB perspectives on stress-testing. Section 3 reviews the selected literature. Section 4 shortly introduces the risk models covered in the present study. Section 5 performs thorough backtesting on these risk models to identify the best-performing ones. Section 6 lays outs the stress-testing methodology and performs stress tests based on the risk models that are found to be outperforming in the preceding section. Then, Section 8 draws some implications for risk capital and concludes.

2. Stress-testing under Basel Committee and IFSB Capital Frameworks

The capital pillar of Basel III recommends the financial institutions produce exposure stress-testing, at least once a month, of principal market risk factors such as interest rates, currencies, equities, credit spreads and commodity prices. The aim is to let financial institutions proactively identify, and when necessary, reduce outsized concentrations to specific directional sensitivities. To make the severity of factor shocks consistent with the purpose of the stress-test, again, the Basel Committee recommends that, when evaluating solvency under stress, factor shocks should be severe enough to capture historical extreme market environments and/or extreme but plausible stressed market conditions. In this regard, the idea of evaluating the impact of such shocks on capital resources, as well as on capital requirements and earnings, is endorsed by the Committee (BCBS, 2011, Stress-testing, para. 115.56). Under Basel III, financial institutions will need to ensure that they have sufficient high-quality liquid resources to survive an acute stress scenario lasting for one month.

It is also articulated under Basel III that stress-test results should be integrated into regular reporting to senior management, which then must take a lead role in the integration of stress-testing into institutional risk management framework and risk culture, and ensure that the results are utilised effectively to manage risks. On the other hand, Basel III casts for the supervisory authorities the role of specifying a number of qualitative criteria that banks would have to meet before they are permitted to use a models-based approach. Then, the extent to which financial institutions meet the qualitative criteria determines the level at which supervisory authorities set a multiplication factor, with those banks that are in full compliance with the qualitative criteria being eligible for a minimum multiplication factor. Again, according to Basel III, the qualitative criteria include, interalia, that the bank must conduct a regular programme of backtesting, and carry out an initial validation as well as an on-going periodic review of its internal model and the risk measures generated by it.

Although it has emerged that Islamic FIs were resistant to the first-round effects of the recent global financial crisis to varying degrees, when the financial crisis turned into an economic crisis, they were exposed to the second-round effects which brought along a general downturn and fall in the value of portfolio assets. With regard to the specificities of Islamic finance, the question remains of how well Islamic financial institutions will be able to absorb stresses and shocks that are more specific to the Islamic financial market. The IFSB, in this regard, in its revised 2012 version of "Guiding Principles on Stresstesting for Institutions Offering Islamic Financial Services" sets out the general principles of stress-testing in Islamic financial institutions (IFSB, 2012). In these principles, there is a particular emphasis on the need for handling "second-round effects" and "fat tails extreme events" when conducting stress tests. Stress tests are required to be based on "exceptional but plausible events" or "low-frequency-high-impact events which may not be reflected in historical data."

Increasingly, individual institutions are taking into account information about plausible worst-case scenarios and, where it is deemed prudent, taking action to avoid the adverse effects of these events. However, a coherent stress-testing framework has yet to be applied, particularly to Islamic FIs. In this regard, this paper aims at introducing a model-based stress-testing methodology which can easily be applied to Islamic financial portfolios,

including market-traded *Sukuk*, *Murabaha* or *Ijarah* products. Moreover, this model can incorporate both volatility clustering and heavy-tails. The ultimate purpose of stress tests in general and the specific model in this paper in particular is to explore the potential impact of plausible price shocks that can occur in financial markets on the portfolios held by a variety of institutions for different purposes, ranging from hedging to speculation.

3. Literature on Backtesting of Risk Models and Stresstesting for Islamic FIs

The literature on the comparison of performances of different market risk models has developed quickly since the early works of (Kupiec, 1995), (Christoffersen, 1998) and (Berkowitz, 1999b). (Kupiec, 1995) and (Christoffersen, 1998) proposed general conditional and unconditional efficiency criteria for evaluating VaR forecasts, namely the tests for conditional and unconditional coverage, and applied them on different volatility model settings. (Berkowitz, 1999b), on the other hand, suggested that the information content of forecast distributions combined with ex post loss realisations was enough to construct a powerful test even with small sample sizes and proposed to evaluate the entire forecast distribution, rather than a VaR quantity. More recently, (Predescu and Stancu, 2011) have employed both symmetric and asymmetric conditional risk models along with a simple (unconditional) coverage test to compare the model performances on a number of equity indices, and concluded that symmetric models outperform the asymmetric ones, both for normal and heavy-tailed distributions. Furthermore, (Orhan and Koksal, 2012) compared a broad spectrum of ARCH- and GARCH-type models in terms of their ability to quantify risks under stress times. Employing both conditional and unconditional coverage tests, the authors found that conditional risk models based on a heavy-tailed distribution outperformed those based on normal distribution, and that asymmetric models performed relatively poorly in comparison to symmetric ones. (Gao, Zhang and Zhang, 2012), on the other hand, adopted a Markov Chain Monte Carlo (MCMC) method to compare GARCH models under different distributional assumptions. The authors concluded that a conditional risk model based on generalised error distribution (GED) outperformed those based on Student's t and normal distributions. Finally, (Buberkoku, 1998) found that, although the risk model performance is dependent on the underlying portfolio type, GARCH and the (asymmetric) GJR¹-GARCH models performed better than the (also asymmetric) exponential GARCH (EGARCH) model in most of the cases.

The appearance of stress tests in the mathematical finance literature is rather gradual. The green shoots of foundations of the bridge between stress tests and risk models started with (Kupiec, 1998), who examined cross market effects resulting from a market shock. In a seminal paper, (Berkowitz, 1999a) for the first time came up with the idea of folding stress tests into a risk model, thereby assigning all scenarios certain probabilities. (Aragones, Blanco and Dowd, 2001) criticised traditional stress-testing approaches for being inevitably subjective and difficult to backtest, and for not providing probabilistic outcome to allow sound interpretations about their results. (Dominguez and Alfonso, 2004) developed an empirical stress-testing exercise by using historical scenarios. In a more recent work by (Alexander and Sheedy, 2008), which also significantly influenced the present work, a stress-testing methodology based on a set of most suitable risk

¹Glosten, Jagannathan and Runkle

models on which a rigorous set of backtests are conducted to eliminate model risk is proposed. In (Aydin and Küçüközmen, 2010), this model-based stress-testing methodology was applied to energy derivatives to explore the potential scope of stress losses in energy portfolios. (Kwiatkowski and Rebonato, 2011) presented a methodology to aggregate in a coherent manner conditional stress losses resulting from multiple scenarios provided that the capital charge was levied against the combined stress losses. Finally, (Cuffe and Goldberg, 2012) extended the standard paradigm for portfolio stress-testing by introducing a historical asset covariance matrix which can be modified, through a latent factor, to reflect changing risk climates under possible stress situations.

Yet, the literature on stress-testing of Islamic financial institutions and trading portfolios is relatively scarce. Most notable among them is (Chattha, 2013), who developed a stress-testing methodology under standardised approach covered in IFSB's "Capital Adequacy Standard for Institutions Offering Only Islamic Financial Services" ((IFSB, 2005)) to assess the stability and resilience of Islamic banks. More specifically, this is achieved in the paper through a stress-testing methodology matrix which is used as a benchmark for simulating solvency stress tests for Islamic banks and simulation of pre- and post-stress capital adequacy ratios of a number of Islamic banks.

4. Conditional Risk Models

Before introducing the conditional risk models covered in this study, we briefly present the general spot asset price modelling framework. A well-known approach in the theory of mathematical finance to modelling asset (in particular, equity) prices is to try to recover the underlying asset price behaviour S_t through a geometric Brownian motion of the form $S_t = \exp(X_t)$ where X_t is a standard diffusion process with dynamics

$$dX_t = r_t d_t + \sigma_t dW_t. (1)$$

Here, W_t is a standard Wiener process (Brownian motion) with mean 0 and variance t. The continuous time solution to S_T at any give time T, $t \leq T$, is given by:

$$S_T = S_t \exp\left(\int_t^T \left(r_s - \frac{\sigma_s^2}{2}\right) ds + \int_t^T \sigma_s dW_s\right)$$
 (2)

and, therefore,

$$\ln\left(S_T/S_t\right) = \int_t^T \left(r - \frac{\sigma_s^2}{2}\right) ds + \int_t^T \sigma_s dW_s.$$
 (3)

In the well-known Black-Scholes model, σ_s is simply assumed to be constant, ignoring volatility clustering in financial asset prices. As opposed to this view, GARCH-type conditional risk models have been developed to accommodate the stochastic nature of volatility.

Conditional Symmetric Model

The entire concern of market risk management can perhaps be encapsulated as to model σ and, thereby, the distribution of the asset returns effectively. To model time-dependent market volatility (risk) σ_t , we employ both symmetric and symmetric conditional market risk models. The class of symmetric risk models included in this study consists of GARCH model (Bollerslev, 1986) which presumes a symmetric distribution for innovations (Gaussian or Student's t in our case). Specifically, the conditional variance is assumed to follow a symmetric GARCH process of the form:

$$\sigma_t^2 = \xi_0 + \sum_{i=1}^M \xi_i \sigma_{t-i}^2 + \sum_{i=M+1}^N \xi_i \epsilon_{t-i+M}^2$$
 (4)

which is subject to stationarity and positivity conditions: (i) $\xi_0 > 0$, and (ii) $\xi_i \geq 0$ and $\sum_i \xi_i < 1$ for all $i \geq 1$. In the Gaussian case, we have $\epsilon_{t-i+M} \sim \mathcal{N}(0, \sigma_{t-i+M}^2)$. For GARCH(1,1), Equation 4 simplifies to

$$\sigma_t^2 = \xi_1 + \xi_2 \sigma_{t-1}^2 + \xi_3 \epsilon_{t-1}^2. \tag{5}$$

Using a rolling estimation window method and using estimation windows of size 250, 500 and 1000 trading days (approx. 1, 2 and 4 years, respectively), we calculate for each window one-day-ahead VaR by, first, producing a one-day-ahead volatility forecast based on the corresponding volatility model, then, scaling it by the critical value corresponding to the lower/upper first percentile of the estimated conditional distribution and, finally, calculating the mean equation

$$ln (S_t/S_{t-1}) = \mu + \epsilon_t$$
(6)

where again $\epsilon_t \sim \mathcal{N}(0, \sigma_t^2)$. Note that μ corresponds to the deterministic part in Eq. 3.

The model described above also allows us to incorporate heavy-tailed distributions which have increasingly become more associated with GARCH-type models in the VaR estimation literature. Their recent prominence primarily stems from their conservative nature in estimating the potential losses associated with a certain level of confidence.² For this, assume, for example, that the innovations ϵ are drawn from a Student's t distribution with a degree of freedom v:

$$\epsilon_t (v/(v-2))^{0.5} \sim t_v. \tag{7}$$

Such a distribution is expected to allow for better fitting of the heavy tails of the empirical distribution of the financial asset or portfolio, if any, with the help of the adjustment parameter v. Indeed, financial time series are very much stylised with higher probability of extreme events (see kurtosis figures in Table 1) which are very unlikely to be captured effectively by a normal distribution.

²But, let's keep in mind that our backtesting methodology will be penalising both "too conservative" and "too loose" models as the former will lead to incurring of costs related to holding excessive capital while the latter to insolvency.

Table 1: Descriptive statistics (2004-2014)

	S&P 500 Islamic	S&P 500
Mean	0.04%	0.03%
St.Dev.	1.19%	1.28%
Kurtosis	15.00	14.04
Skewness	-0.12	-0.33
Min	-9.53%	-9.46%
Max	11.58%	10.96%

Conditional Asymmetric Model

A due consideration should also be given to asymmetric models as financial returns largely exhibit skewness. Indeed, the historical data considered in this study exhibit a significant level of skewness, as illustrated, again, in Table 1. Therefore, to accommodate asymmetric models, we employ an exponential GARCH (or, shortly, EGARCH) model proposed in (Nelson, 1991) where innovations are again assumed to match an either Gaussian or Student's t density. In the Gaussian case, the conditional variance equation in a EGARCH (1,1) model can be expressed as:

$$\ln(\sigma_t^2) = \xi_1 + \xi_2 \ln(\sigma_{t-1}^2) + \xi_3 \left[\frac{|\epsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] + \xi_4 \frac{\epsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}, \tag{8}$$

where ξ_4 is called the "leverage parameter" representing the sign effect. The leverage parameter ξ_4 is expected to be negative, thereby allowing large unanticipated downward shocks to further increase the variance. The model also doesn't need any non-negativity constraints on the estimated variance values. Under Student's t innovations assumption, the variance equation 8 would look like:

$$\ln(\sigma_t^2) = \xi_1 + \xi_2 \ln(\sigma_{t-1}^2) + \gamma \frac{\epsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \omega \left[\frac{|\epsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{v-2}{\pi}} \frac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \right].$$

Empirical Model

Using the distribution of past returns directly to forecast future changes in portfolio value is also popular in the financial industry, although, unlike our approach here, this is generally done on a unconditional basis. To make it conditional, here we adopt a slightly different approach which is similar to the one described in (Barone-Adesi, Boutgoin and Giannopoulos, 1998). We make no distributional assumption about the standardised past returns, other than the assumption that there is a mild dependence between them. That is, we first fit either a Gaussian or Student's t GARCH process (see Equation 5) to the historical data and then standardise the sample with the help of the estimated mean and in-sample conditional standard deviation vector. Once we've standardised the sample, the returns are then rescaled to the standard deviation estimate for the day for which a VaR estimation is made to obtain the sample of scaled returns. As a final step, to

calculate VaR, we again simply draw the lower or upper 100α percentile of scaled returns depending on whether the position is long or short.

As for the selection of number of lags associated with past returns and past innovations for the GARCH-type models described above, we consider the findings of earlier studies and employ only the first lags. Most of these studies suggest no evidence about the relative superiority of more sophisticated GARCH models over the simple ones. For example, in (Hansen and Lunde, 2005), 330 different ARCH-type models are compared in terms of their ability to estimate conditional volatility and there is no evidence that more complicated settings can beat the GARCH(1,1). However, they also conclude that the GARCH(1,1) is obviously inferior to the ones that can incorporate leverage effects, i.e., asymmetries — a conclusion which our study does support only under certain circumstances, as we'll be discussing shortly.

In the next section, as part of our backtesting methodology, we'll first examine the one-day-ahead forecasting accuracies of the selected symmetric and asymmetric volatility (risk) models on a Islamic index portfolio under different distributional assumptions. Forecast accuracies will be evaluated on the basis of two well-known methods, namely, the conditional and unconditional coverage tests. We'll then perform a model-based stress test based on the risk models which are found to be relatively more robust in backtesting. We adopt a recursive VaR estimation methodology with the help of a rolling estimation window to obtain more consistent results about the suitability of risk models for stress-testing.

5. Backtesting

This section performs thorough backtests on S&P 500 Islamic index portfolio for the period 2004-2014 to identify the most suitable risk models for stress-testing. S&P 500 Islamic index comprises all Shariah-compliant constituents of S&P 500, which includes U.S. companies with a market cap > US\$ 4,6 billion. All constituents of the S&P 500 Islamic index are screened for Shariah compliance in two categories: sector-based (i.e., whether they are involved in gambling, tobacco or alcohol sectors) and accounting-based (i.e., whether their financial ratios violate Shariah-compliant thresholds). As for the sectoral breakdown of S&P 500 Islamic index, information and communications technology (30.4%), health care (18.6%) and energy (14.3%) sectors collectively account for almost two-thirds of the index weight. We recall Table 1 where descriptive statistics for the S&P 500 Islamic index are depicted. To better reflect the effects of a stress event on portfolio values, generally, higher confidence levels, i.e., 99% and 99.5%, in addition to the traditional 95%, are also included in the backtesting process. The employment of higher confidence levels is argued to help reveal nuanced abilities of heavy-tailed distributions in predicting and capturing extreme tail events. In terms of estimation windows size, taking into account the earlier findings of (Hoppe, 1998) and (Frey and Michaud, 1997), who argued, respectively, that (i) smaller sample sizes could lead to more accurate VaR estimates than larger ones and (ii) that small sample sizes were better in capturing structural changes, and in the light of Basel III stipulation that "backtesting must consider a number of distinct prediction time horizons out to at least one year over a range of various initialisation dates and covering a wide range of market conditions," we include both smaller and larger estimation windows. Structural breaks do apparently exist in our dataset due to the inclusion of the recent financial volatility periods and we expect this

to make a difference in backtesting results.

The most suitable risk models are filtered using two different test statistics, namely, the test for conditional coverage (Christoffersen, 1998) and the test for unconditional coverage (Kupiec, 1995). The former has the null hypothesis that the actual number of model violations is equal to the expected number of violations, whereas the latter hypothesises that model violations are not clustered, and therefore are evenly distributed, over time. In both tests, the smaller the p-value of the test, the more likely will the null hypothesis be rejected (i.e., actual number of model violations is not equal to the expected number of violations / model violations are not evenly distributed over time).³ The test likelihood ratio for unconditional coverage is calculated as

$$\Pi_{uc} = \frac{\alpha^{n_1} (1 - \alpha)^{n_0}}{\hat{\alpha}^{n_1} (1 - \hat{\alpha})^{n_0}}$$

where α and $\hat{\alpha}$ are, respectively, the expected and realised percentage of model violations, and n_0 and n_1 , respectively, the realised number of non-violating and violating returns. For conditional coverage test, on the other hand, Π is defined as

$$\Pi_{cc} = \frac{\alpha^{n_1} (1 - \alpha)^{n_0}}{\hat{\alpha}_{01}^{n_{01}} (\hat{1 - \alpha}_{01})^{n_{00}} \hat{\alpha}_{11}^{n_{11}} (1 - \hat{\alpha}_{11})^{n_{10}}}$$

where n_{00} and n_{01} are, respectively, the number of non-violating and violating returns given that the previous return was non-violating, and n_{10} and n_{11} , respectively, the number of non-violating and violating returns given that the previous return was violating. Accordingly, $\hat{\alpha}_{01}$ and $\hat{\alpha}_{11}$ are, respectively, the percentages of violating returns given that the preceding return was non-violating and violating.

We follow a robust backtesting methodology by recursively calculating the one-day-ahead VaR estimations associated with a rolling estimation window which has a fixed length of 250, 1000 or 2000 trading days. Consider, for example, an estimation window of 250 days whereas the first one-day-ahead volatility estimate for a long position occurs on 251st trading day. After each VaR estimation, the sample window is rolled ahead one day and VaR is estimated again. This methodology allows us to obtain almost as many VaR estimations as the overall sample size (less the first window size as burn-in). At each VaR estimation, the actual return $\ln(S_t/S_{t-1})$ on that day is compared to the VaR estimate and any cases where $\ln(S_t/S_{t-1}) < \text{VaR}$ is treated as a violation. The total number of violations as well as their frequencies over time are then used as inputs for tests for conditional and unconditional coverage above. The test results obtained for long and short S&P 500 Islamic index portfolios are presented in Tables 2 and 3 below, respectively, for each risk model / underlying distribution specification.

The use of normally distributed returns with conditional risk models apparently cannot be justified, except for the short portfolio case. The null hypotheses that the actual number of violations is equal to the expected number of violations and that the violations are spread evenly over time are comfortably rejected in most of the cases, as the risk models under normality assumption apparently underestimate the market risk. Results also show that the risk models based on a heavy-tailed distribution, particularly the asymmetric one, perform relatively well in the long portfolio case, but they are also

 $^{^3{\}rm This}$ interpretation will be helpful while reading Tables 2 and 3

Table 2: Backtest results on S&P 500 Shariah Index (Long portfolio)

Cond.Norm.				Cond.t			Asym.Cond.Norm.		
$\alpha = 5.0\%$	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}
250 days	6.97	0.00	0.00	4.72	46.31	53.69	7.01	0.01	0.00
500 days	6.64	0.03	0.12	4.61	9.69	41.67	7.14	0.02	0.00
1000 days	5.80	9.18	16.23	4.15	14.16	11.94	5.93	1.50	10.49
$\alpha = 1.0\%$	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}
250 days	3.13	0.00	0.00	0.75	39.85	21.05	3.31	0.00	0.00
500 days	2.83	0.00	0.00	0.40	0.79	0.19	3.12	0.00	0.00
1000 days	2.24	0.01	0.00	0.33	0.93	0.23	2.04	0.08	0.03
$\alpha = 0.5\%$	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}
250 days	1.90	0.00	0.00	0.35	55.89	29.43	2.12	0.00	0.00
500 days	1.64	0.00	0.00	0.25	20.22	7.52	2.03	0.00	0.00
1000 days	1.19	0.45	0.13	0.07	1.03	0.25	1.05	2.39	0.77
Asym.Cond.t		Cond.Emp.Norm				Cond.Emp.t			
	As	sym.Con	d.t	Con	d.Emp.N	Jorm	$^{\mathrm{C}}$	ond.Emp	o.t
$\alpha = 5.0\%$	$\hat{\alpha}(\%)$	$\frac{\text{sym.Con}}{p_{cc}}$	$\frac{\mathrm{d.t}}{p_{uc}}$	$\frac{\operatorname{Con}}{\hat{\alpha}(\%)}$	$\frac{\text{d.Emp.N}}{p_{cc}}$	$\frac{\text{Norm}}{p_{uc}}$	$\hat{\alpha}(\%)$	ond.Emp p_{cc}	$\frac{\text{o.t}}{p_{uc}}$
$\frac{\alpha = 5.0\%}{250 \text{ days}}$									
	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}
250 days	$\hat{\alpha}(\%)$ 6.00	$\frac{p_{cc}}{9.03}$	p_{uc} 3.41	$\hat{\alpha}(\%)$ 5.65	p_{cc} 23.58	$\frac{p_{uc}}{16.62}$	$\hat{\alpha}(\%)$ 5.56	p_{cc} 42.31	$\frac{p_{uc}}{23.07}$
250 days 500 days	$\hat{\alpha}(\%)$ 6.00 5.45	p_{cc} 9.03 40.69	p_{uc} 3.41 35.66	$\hat{\alpha}(\%)$ 5.65 5.75	p_{cc} 23.58 2.54	p_{uc} 16.62 13.02	$\hat{\alpha}(\%)$ 5.56 5.26	p_{cc} 42.31 15.83	p_{uc} 23.07 60.17
250 days 500 days 1000 days	$\hat{\alpha}(\%)$ 6.00 5.45 4.15	$ \begin{array}{c} p_{cc} \\ 9.03 \\ 40.69 \\ 14.16 \end{array} $	3.41 35.66 11.94	$\hat{\alpha}(\%)$ 5.65 5.75 4.55	p_{cc} 23.58 2.54 23.49	p_{uc} 16.62 13.02 41.28	$\hat{\alpha}(\%)$ 5.56 5.26 4.48	p_{cc} 42.31 15.83 22.57	p_{uc} 23.07 60.17 34.69
250 days 500 days 1000 days $\alpha = 1.0\%$	$\hat{\alpha}(\%)$ 6.00 5.45 4.15 $\hat{\alpha}(\%)$	p_{cc} 9.03 40.69 14.16 p_{cc}	puc 3.41 35.66 11.94 puc	$\hat{\alpha}(\%)$ 5.65 5.75 4.55 $\hat{\alpha}(\%)$	p_{cc} 23.58 2.54 23.49 p_{cc}	p_{uc} 16.62 13.02 41.28 p_{uc}	$\hat{\alpha}(\%)$ 5.56 5.26 4.48 $\hat{\alpha}(\%)$	p_{cc} 42.31 15.83 22.57 p_{cc}	p_{uc} 23.07 60.17 34.69 p_{uc}
250 days 500 days 1000 days $\alpha = 1.0\%$ 250 days	$\hat{\alpha}(\%)$ 6.00 5.45 4.15 $\hat{\alpha}(\%)$	$\begin{array}{c} p_{cc} \\ 9.03 \\ 40.69 \\ 14.16 \\ \hline p_{cc} \\ 22.26 \end{array}$	p_{uc} 3.41 35.66 11.94 p_{uc} 14.04	$\hat{\alpha}(\%)$ 5.65 5.75 4.55 $\hat{\alpha}(\%)$ 1.59	p_{cc} 23.58 2.54 23.49 p_{cc} 1.91	p_{uc} 16.62 13.02 41.28 p_{uc} 0.95	$\hat{\alpha}(\%)$ 5.56 5.26 4.48 $\hat{\alpha}(\%)$	p_{cc} 42.31 15.83 22.57 p_{cc} 16.03	p_{uc} 23.07 60.17 34.69 p_{uc} 9.58
250 days 500 days 1000 days $\alpha = 1.0\%$ 250 days 500 days	$\hat{\alpha}(\%)$ 6.00 5.45 4.15 $\hat{\alpha}(\%)$ 1.32 0.89	$\begin{array}{c} p_{cc} \\ 9.03 \\ 40.69 \\ 14.16 \\ \hline p_{cc} \\ 22.26 \\ 74.57 \end{array}$	p_{uc} 3.41 35.66 11.94 p_{uc} 14.04 62.09	$\hat{\alpha}(\%)$ 5.65 5.75 4.55 $\hat{\alpha}(\%)$ 1.59 1.44	$\begin{array}{c} p_{cc} \\ 23.58 \\ 2.54 \\ 23.49 \\ \\ p_{cc} \\ 1.91 \\ 11.56 \end{array}$	p_{uc} 16.62 13.02 41.28 p_{uc} 0.95 6.37	$\hat{\alpha}(\%)$ 5.56 5.26 4.48 $\hat{\alpha}(\%)$ 1.37 1.09	p_{cc} 42.31 15.83 22.57 p_{cc} 16.03 71.52	$\begin{array}{c} p_{uc} \\ 23.07 \\ 60.17 \\ 34.69 \\ \hline p_{uc} \\ 9.58 \\ 68.65 \end{array}$
250 days 500 days 1000 days $\alpha = 1.0\%$ 250 days 500 days 1000 days	$\hat{\alpha}(\%)$ 6.00 5.45 4.15 $\hat{\alpha}(\%)$ 1.32 0.89 0.46	$\begin{array}{c} p_{cc} \\ 9.03 \\ 40.69 \\ 14.16 \\ \hline p_{cc} \\ 22.26 \\ 74.57 \\ 5.99 \\ \end{array}$	p_{uc} 3.41 35.66 11.94 p_{uc} 14.04 62.09 1.84	$\hat{\alpha}(\%)$ 5.65 5.75 4.55 $\hat{\alpha}(\%)$ 1.59 1.44 0.92	$\begin{array}{c} p_{cc} \\ 23.58 \\ 2.54 \\ 23.49 \\ \hline p_{cc} \\ 1.91 \\ 11.56 \\ 82.98 \\ \end{array}$	p_{uc} 16.62 13.02 41.28 p_{uc} 0.95 6.37 75.97	$\hat{\alpha}(\%)$ 5.56 5.26 4.48 $\hat{\alpha}(\%)$ 1.37 1.09 0.86	p_{cc} 42.31 15.83 22.57 p_{cc} 16.03 71.52 75.14	$\begin{array}{c} p_{uc} \\ 23.07 \\ 60.17 \\ 34.69 \\ \hline p_{uc} \\ 9.58 \\ 68.65 \\ 56.60 \\ \end{array}$
250 days 500 days 1000 days $\alpha = 1.0\%$ 250 days 500 days 1000 days $\alpha = 0.5\%$	$\hat{\alpha}(\%)$ 6.00 5.45 4.15 $\hat{\alpha}(\%)$ 1.32 0.89 0.46 $\hat{\alpha}(\%)$	$\begin{array}{c} p_{cc} \\ 9.03 \\ 40.69 \\ 14.16 \\ \hline p_{cc} \\ 22.26 \\ 74.57 \\ 5.99 \\ \hline p_{cc} \\ \end{array}$	$\begin{array}{c} p_{uc} \\ 3.41 \\ 35.66 \\ 11.94 \\ \hline p_{uc} \\ 14.04 \\ 62.09 \\ 1.84 \\ \hline p_{uc} \\ \end{array}$	$\hat{\alpha}(\%)$ 5.65 5.75 4.55 $\hat{\alpha}(\%)$ 1.59 1.44 0.92 $\hat{\alpha}(\%)$	$\begin{array}{c} p_{cc} \\ 23.58 \\ 2.54 \\ 23.49 \\ p_{cc} \\ 1.91 \\ 11.56 \\ 82.98 \\ p_{cc} \end{array}$	$\begin{array}{c} p_{uc} \\ 16.62 \\ 13.02 \\ 41.28 \\ \hline p_{uc} \\ 0.95 \\ 6.37 \\ 75.97 \\ \hline p_{uc} \end{array}$	$\hat{\alpha}(\%)$ 5.56 5.26 4.48 $\hat{\alpha}(\%)$ 1.37 1.09 0.86 $\hat{\alpha}(\%)$	p_{cc} 42.31 15.83 22.57 p_{cc} 16.03 71.52 75.14 p_{cc}	$\begin{array}{c} p_{uc} \\ 23.07 \\ 60.17 \\ 34.69 \\ \hline p_{uc} \\ 9.58 \\ 68.65 \\ 56.60 \\ \hline p_{uc} \\ \end{array}$

Table 3: Backtest results on S&P 500 Shariah Index (Short portfolio)

Cond.Norm.			Cond.t			Asym.Cond.Norm.			
$\alpha = 5.0\%$	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}
250 days	4.19	0.43	6.92	2.65	0.00	0.00	4.85	0.28	74.57
500 days	3.87	4.10	1.52	1.78	0.00	0.00	4.36	0.70	18.00
1000 days	3.76	4.37	2.04	1.85	0.00	0.00	3.49	0.25	0.45
$\alpha = 1.0\%$	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}
250 days	0.88	70.30	56.52	0.26	0.02	0.00	1.01	78.00	94.46
500 days	0.79	54.63	33.30	0.00	-	0.00	0.69	30.96	14.40
1000 days	0.59	21.36	8.49	0.00	-	0.00	0.53	12.05	4.20
$\alpha = 0.5\%$	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}
250 days	0.49	93.84	92.01	0.18	4.12	1.17	0.57	82.03	62.81
500 days	0.35	57.16	30.26	0.00	-	0.00	0.40	76.41	49.46
1000 days	0.20	16.25	5.71	0.00	-	0.01	0.20	16.25	5.71
Asym.Cond.t									
	As	sym.Con	d.t	Con	d.Emp.N	Jorm	\mathbf{C}	ond.Emp	o.t
$\alpha = 5.0\%$	$\hat{\alpha}(\%)$	$\frac{\text{sym.Con}}{p_{cc}}$	$\frac{\mathrm{d.t}}{p_{uc}}$	$\frac{\operatorname{Con}}{\hat{\alpha}(\%)}$	d.Emp.N p_{cc}	$\frac{V_{orm}}{p_{uc}}$	$\hat{\alpha}(\%)$	ond.Emp p_{cc}	p_{uc}
250 days									
	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}	$\hat{\alpha}(\%)$	p_{cc}	p_{uc}
250 days	$\hat{\alpha}(\%)$ 3.44	$\frac{p_{cc}}{0.07}$	p_{uc} 0.03	$\hat{\alpha}(\%)$ 5.34	$\frac{p_{cc}}{0.09}$	p_{uc} 46.57	$\hat{\alpha}(\%)$ 5.16	$\frac{p_{cc}}{0.16}$	$\frac{p_{uc}}{72.64}$
250 days 500 days	$\hat{\alpha}(\%)$ 3.44 2.58	p_{cc} 0.07 0.00	p_{uc} 0.03 0.00	$\hat{\alpha}(\%)$ 5.34 4.86	p_{cc} 0.09 30.00	p_{uc} 46.57 76.99	$\hat{\alpha}(\%)$ 5.16 4.71	p_{cc} 0.16 31.22	p_{uc} 72.64 54.63
250 days 500 days 1000 days	$\hat{\alpha}(\%)$ 3.44 2.58 1.65	$\begin{array}{c} p_{cc} \\ 0.07 \\ 0.00 \\ 0.00 \\ \end{array}$	p_{uc} 0.03 0.00 0.00	$\hat{\alpha}(\%)$ 5.34 4.86 4.88	p_{cc} 0.09 30.00 58.19	p _{uc} 46.57 76.99 82.68	$\hat{\alpha}(\%)$ 5.16 4.71 4.88	p_{cc} 0.16 31.22 58.19	puc 72.64 54.63 82.68
250 days 500 days 1000 days $\alpha = 1.0\%$	$\hat{\alpha}(\%)$ 3.44 2.58 1.65 $\hat{\alpha}(\%)$	$\begin{array}{c} p_{cc} \\ 0.07 \\ 0.00 \\ 0.00 \\ \end{array}$	p_{uc} 0.03 0.00 0.00 p_{uc}	$\hat{\alpha}(\%)$ 5.34 4.86 4.88 $\hat{\alpha}(\%)$	p_{cc} 0.09 30.00 58.19 p_{cc}	puc 46.57 76.99 82.68 puc	$\hat{\alpha}(\%)$ 5.16 4.71 4.88 $\hat{\alpha}(\%)$	p_{cc} 0.16 31.22 58.19 p_{cc}	p_{uc} 72.64 54.63 82.68 p_{uc}
250 days 500 days 1000 days $\alpha = 1.0\%$ 250 days	$\hat{\alpha}(\%)$ 3.44 2.58 1.65 $\hat{\alpha}(\%)$	$\begin{array}{c} p_{cc} \\ 0.07 \\ 0.00 \\ 0.00 \\ \hline p_{cc} \\ 0.05 \end{array}$	p_{uc} 0.03 0.00 0.00 p_{uc} 0.01	$\hat{\alpha}(\%)$ 5.34 4.86 4.88 $\hat{\alpha}(\%)$ 1.37	p_{cc} 0.09 30.00 58.19 p_{cc}	p_{uc} 46.57 76.99 82.68 p_{uc} 9.58	$\hat{\alpha}(\%)$ 5.16 4.71 4.88 $\hat{\alpha}(\%)$ 1.24	p_{cc} 0.16 31.22 58.19 p_{cc}	p_{uc} 72.64 54.63 82.68 p_{uc} 27.78
250 days 500 days 1000 days $\alpha = 1.0\%$ 250 days 500 days	$\hat{\alpha}(\%)$ 3.44 2.58 1.65 $\hat{\alpha}(\%)$ 0.31 0.15	$\begin{array}{c} p_{cc} \\ 0.07 \\ 0.00 \\ 0.00 \\ \\ p_{cc} \\ \hline 0.05 \\ 0.00 \\ \end{array}$	p_{uc} 0.03 0.00 0.00 p_{uc} 0.01 0.00	$\hat{\alpha}(\%)$ 5.34 4.86 4.88 $\hat{\alpha}(\%)$ 1.37 0.99	p_{cc} 0.09 30.00 58.19 p_{cc} 16.03 80.97	p_{uc} 46.57 76.99 82.68 p_{uc} 9.58 96.96	$\hat{\alpha}(\%)$ 5.16 4.71 4.88 $\hat{\alpha}(\%)$ 1.24 0.89	p_{cc} 0.16 31.22 58.19 p_{cc} 38.61 74.57	puc 72.64 54.63 82.68 puc 27.78 62.09
250 days 500 days 1000 days $\alpha = 1.0\%$ 250 days 500 days 1000 days	$\hat{\alpha}(\%)$ 3.44 2.58 1.65 $\hat{\alpha}(\%)$ 0.31 0.15 0.07	$\begin{array}{c} p_{cc} \\ 0.07 \\ 0.00 \\ 0.00 \\ \end{array}$ $\begin{array}{c} p_{cc} \\ 0.05 \\ 0.00 \\ 0.00 \\ \end{array}$	p_{uc} 0.03 0.00 0.00 p_{uc} 0.01 0.00 0.00	$\hat{\alpha}(\%)$ 5.34 4.86 4.88 $\hat{\alpha}(\%)$ 1.37 0.99 0.66	p_{cc} 0.09 30.00 58.19 p_{cc} 16.03 80.97 33.81	p_{uc} 46.57 76.99 82.68 p_{uc} 9.58 96.96 15.49	$\hat{\alpha}(\%)$ 5.16 4.71 4.88 $\hat{\alpha}(\%)$ 1.24 0.89 0.92	p_{cc} 0.16 31.22 58.19 p_{cc} 38.61 74.57 82.98	p_{uc} 72.64 54.63 82.68 p_{uc} 27.78 62.09 75.97
250 days 500 days 1000 days $\alpha = 1.0\%$ 250 days 500 days 1000 days $\alpha = 0.5\%$	$\hat{\alpha}(\%)$ 3.44 2.58 1.65 $\hat{\alpha}(\%)$ 0.31 0.15 0.07 $\hat{\alpha}(\%)$	$\begin{array}{c} p_{cc} \\ 0.07 \\ 0.00 \\ 0.00 \\ \end{array}$ $\begin{array}{c} p_{cc} \\ 0.05 \\ 0.00 \\ 0.00 \\ \end{array}$	p_{uc} 0.03 0.00 0.00 p_{uc} 0.01 0.00 p_{uc}	$\hat{\alpha}(\%)$ 5.34 4.86 4.88 $\hat{\alpha}(\%)$ 1.37 0.99 0.66 $\hat{\alpha}(\%)$	p_{cc} 0.09 30.00 58.19 p_{cc} 16.03 80.97 33.81 p_{cc}	p_{uc} 46.57 76.99 82.68 p_{uc} 9.58 96.96 15.49 p_{uc}	$\hat{\alpha}(\%)$ 5.16 4.71 4.88 $\hat{\alpha}(\%)$ 1.24 0.89 0.92 $\hat{\alpha}(\%)$	p_{cc} 0.16 31.22 58.19 p_{cc} 38.61 74.57 82.98	p_{uc} 72.64 54.63 82.68 p_{uc} 27.78 62.09 75.97 p_{uc}

penalised by the coverage tests for being too conservative in their risk estimations for short portfolios.⁴ The overestimation of short portfolio risk can partly be explained by the negative skewness observed in the underlying series (recall Table 1), which makes large swings on the right-hand-side of the return distribution less likely than those in the left-hand-side. Therefore, while an asymmetric risk model, combined with a heavy-tailed distribution, can effectively mimic negative swings (which are more likely) in the data series, a similar conclusion cannot be made for positive returns. On the other hand, in both long and short portfolios, the conditional empirical risk models which allow for both heavy tails and asymmetry are found to be most consistent with the actual returns. Moreover, this is largely indifferent to the selection of the underlying risk model which is used to estimate the conditional in-sample volatility vector that, in turn, is used to standardise the historical returns – i.e., whether it is associated with a Gaussian or Student's t distribution. With few exceptions, it can also be concluded that the performance of the empirical model is improved with the size of the estimation window, while other models give mixed results. Finally, except for the "long portfolio / Student's t distribution" case, where the asymmetric model outperforms the symmetric one, we don't see much difference in the relative performances of symmetric and asymmetric models.

To conclude this section, we infer from the analysis above that the conditional empirical model can adequately imitate the stochastic behaviour of actual returns and, therefore, can be employed in the next section as a reference model to predict potential stress losses over a pre-determined stress horizon. In addition, the conditional asymmetric model associated with a heavy-tailed distribution can be used to predict stress losses for long portfolios, whereas both the conditional symmetric and asymmetric models associated with a normal distribution can perform fairly well in predicting stress losses for short portfolios. In the next section, where we introduce the model-based stress-testing methodology, the stress test results will be calculated, and compared to past shocks, on the basis of the selected risk models.

6. Stress-testing

This section defines a stress-testing methodology incorporating the three well-known characteristics of the financial time series highlighted in the previous section: heavy-tails (kurtosis), volatility clustering and asymmetry (skewness). This methodology was initially introduced by (Alexander and Sheedy, 2008) to calculate stress losses for currency positions, and it is two-fold: first an initial shock is fed into the selected conditional risk model (which allows for volatility clustering and asymmetric volatility responses) and the subsequent market response is calculated iteratively throughout the pre-determined stress horizon.

An initial shock (or stress event) can be defined as an abrupt but plausible event that causes a large discontinuity in prices. Therefore, one alternative way to deducing such a stress event from history — relying on management experience — or hypothetically could be to consider extreme outcomes defined by the risk model distribution itself. That is, for a long position, a low probability negative return could be used as an initial shock, ϵ_T^* , which would then evoke consecutive volatility hikes propagating through the conditional

⁴As discussed earlier, although an overestimation of the market risk can prevent the business from incurring unrecoverable losses, it leads to tying up in capital of more resources than necessary.

volatility model, defined in a general sense in Eq. 5, before it subsequently dies away. Under normality assumption, for example, the initial shock ϵ_T^* for a long portfolio could be defined as:

$$\epsilon_T^* = {}^{-1}(\alpha^*)\bar{\sigma}_T$$

where α^* is lower percentile of the distribution associated with the initial stress event and $\bar{\sigma}_T$ is the long-term volatility whose value is to be approximated by historical volatility. Similarly, under heavy-tails assumption and based on relation 7, the stress event could be defined as:

$$\epsilon_T^* = t_v^{-1}(\alpha^*) \left(\frac{v - 2^{0.5}}{v}\right)^{0.5} \bar{\sigma}_T.$$

Under the empirical approach, on the other hand, the initial shock is simply the lower (upper) α^* percentile of the empirical distribution in the case of a long (short) asset position. The typical values for α^* are 0.0002 and 0.0005.

Again, when the initial shock ϵ_T^* is fed into the system at time T, the conditional variance for time T+1 will increase. Taking Eq. 5 as an example, the initial shock can be fed into the system by the following equation:

$$\sigma_{T+1}^{2} = \hat{\xi}_{1} + \hat{\xi}_{2}\bar{\sigma}_{T}^{2} + \hat{\xi}_{3}\left(\epsilon_{T}^{*}\right)^{2}.$$

And, similarly, in EGARCH(1,1) model with Student's t distribution:

$$\ln(\sigma_{T+1}^2) = \hat{\xi}_1 + \hat{\xi}_2 \ln(\bar{\sigma}_T^2) + \hat{\xi}_3 \frac{\epsilon_T^*}{\sqrt{\bar{\sigma}_T^2}} + \hat{\xi}_4 \left[\frac{|\epsilon_T^*|}{\sqrt{\bar{\sigma}_T^2}} - \sqrt{\frac{v-2}{\pi}} \frac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \right].$$

On the next day, stress loss iterations will continue with innovations drawn from appropriate distribution and estimated variance, i.e.,

$$\sigma_{T+2}^2 = \hat{\xi}_1 + \hat{\xi}_2 \sigma_{T+1}^2 + \hat{\xi}_3 \epsilon_{T+1}^2$$

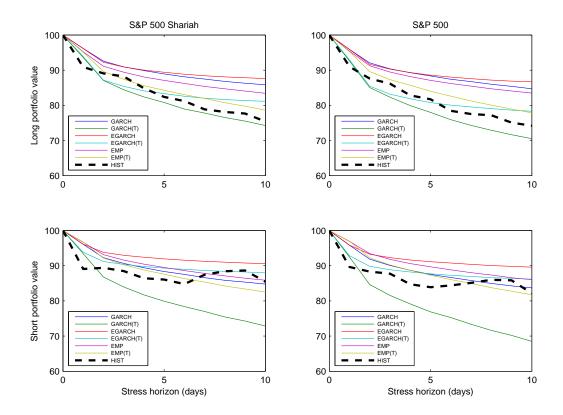
$$\ln(\hat{\sigma}_{T+2}^2) = \hat{\xi}_1 + \hat{\xi}_2 \ln(\hat{\sigma}_{T+1}^2) + \hat{\xi}_3 \frac{\epsilon_{T+1}}{\sqrt{\hat{\sigma}_{T+1}^2}} + \hat{\xi}_4 \left[\frac{|\epsilon_{T+1}|}{\sqrt{\hat{\sigma}_{T+1}^2}} - \sqrt{\frac{v-2}{\pi}} \frac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \right].$$

for GARCH and EGARCH models, respectively, and where $\epsilon_{T+1} \sim l$, $\sigma_{T+\infty}^{\in}$ or $\epsilon_{T+1}(v/(v-2))^{0.5} \sim t_v$. A similar process can be defined for the conditional empirical model where the initial shock and succeeding innovations are drawn from the sample of centred and scaled returns. Both processes above are iterated until we have h single-day returns. Afterwards, these returns are aggregated to obtain a single compound stress loss for each day in the stress horizon. For each day in the stress horizon, we obtain as many as 50.000 Monte Carlo simulations for possible stress losses and then select absolute value of the lower (upper) 1st percentile of the simulated returns for a long (short) portfolio.

The results of a 10-day stress test for the long and short S&P 500 Islamic index portfolio, in terms of the evolution of portfolio values, are presented in Figures 1 and 2 for initial shock sizes $\alpha^* = 0.0005$ and $\alpha^* = 0.0002$. Stress loss results for the S&P 500 index portfolio are also depicted for comparison. Historically observed shocks, i.e., the maximum amount of losses in each possible time period (from 1 to h-day) in the stress horizon, are given in dashed lines in each figure. Stress test results for underperforming "risk model / underlying distribution" specifications are also included for comparison. The results apparently suggest that the risk models which fared relatively well in backtests can successfully imitate past shocks. Yet, in almost all risk models included, estimated stress losses with a relatively small initial shock, as presented in Figure 1, appear to slightly underestimate the historical shocks. In the long portfolio case, the conditional empirical model as well as the conditional symmetric and asymmetric models, all associated with a heavy-tailed distribution, closely replicate the past shocks over the stress horizon. As far as the short portfolio case is concerned, we observe that the asymmetric models can easily adapt to the asymmetric nature of past shocks and give more consistent results, whereas the GARCH model based on Student's t distribution produces overly conservative results. When a more severe initial shock ($\alpha^* = 0.0002$) is fed into and propagated through the system, as presented in Figure 2), risk models react quickly and produce more conservative loss estimations, with most substantial shift taking place in the stress loss estimates of empirical models. In long portfolio case, the conditional asymmetric model with a Student's t distribution and the conditional empirical model with a normal distribution predicts a stress horizon that is apparently closer to historical realisations, whereas the conditional symmetric model and the conditional empirical model – both with a heavytailed distribution – appear to be rather conservative and yield much larger stress losses than those actually observed. Overall, it can be inferred that conditional risk models based on normal distribution are generally inadequate in predicting stress losses effectively, and the conditional symmetric model with Student's t distribution consistently yield conservative results, especially in the short portfolio case and partly because its symmetric nature does not allow it to adapt to the skewed nature of the underlying data. Conditional asymmetric model, in this regard, although it exhibits similar characteristics to symmetric model at the beginning of the stress period, it quickly adapts to the asymetric nature of volatility clustering in long and short portfolios. The stress test results also mark lower bounds for maximum potential losses in an Islamic equity index portfolio as compared to conventional equity index portfolio, as well as to those observed in most liquid energy contracts and major exchange rates, as reported in (Aydin and Küçüközmen, 2010) and (Alexander and Sheedy, 2008), respectively.

This result is particularly important for institutions that are highly engaged in Islamic finance equity portfolios. Here we refer to Basel III and IFSB capital rules which recommend a more direct link between stress tests and risk capital. Considering the possibilities of (i) a substantially reduced market liquidity in a prolonged period of severe market conditions, (ii) large portfolio size, and (iii) delays in managerial reactions, we use a 10-day risk horizon to draw some conclusions for the minimum capital requirements. The 10-day stress test results imply that up to 30% of the Islamic equity portfolio value could be required to be set aside as capital to absorb potential stress losses during a 10-day stress period, which is quite significant particularly if one considers the fact that IFSB requires Islamic financial institutions to hold a minimum regulatory capital of only 8% of their total risk-weighted assets at all times.

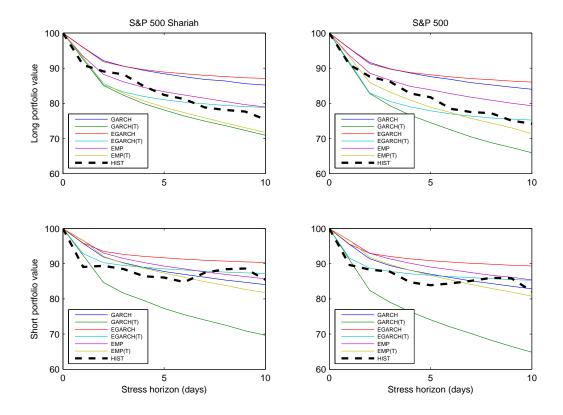
Figure 1: Islamic and conventional portfolio values under a 10-day stress event associated with an initial shock $\alpha^* = 0.0005$



7. Second-Round Effects

Financial institutions and vendors rarely tend to develop stress-tests that do account for second-round effects (BCBS, 2013). The BCBS particularly lack any emphasis on (in fact, do not even mention) the need to model second-round effects ((BCBS, 2009); (BCBS, 2011)). This type of effects can actually have a more substantial effect than firstround effects (i.e., risks can be highly non-linear). (van den End, 2010) categorises secondround effects into two possible channels, namely, (i) idiosyncratic risk, which generally stems from reputational risk, and (ii) systemic risk, which results from collective acts of financial institutions. The IFSB, on the other hand, gives particular attention to the second-round effects on capital and grant Islamic financial institutions with the flexibility to determine how such effects are relevant to their businesses and how they can incorporate them into their stress tests (IFSB, 2012). One suggested way of incorporating secondround effects into stress-testing in the IFSB Guiding Principles on Stress-testing is to consider the role of select macroeconomic factors which, while unscathed by the first shock, can act as a transmission mechanism for a second wave of shock should international market conditions further deteriorate. In terms of reputational risk, the IFSB Guidelines put particular emphasis on Shariah non-compliance and fiduciary risks, which might have systemic implications for the Islamic financial system as a whole. Considering all these, second-round effects can be incorporated into the conditional volatility models mentioned above through introducing exogenous variables into the variance equation (e.g., a real

Figure 2: Islamic and conventional portfolio values under a 10-day stress event associated with an initial shock $\alpha^* = 0.0002$



estate sector performance index) which will take account of possible sectoral channels through which external shock can be transmitted to Islamic financial system, and the use of an appropriate multivariate GARCH model whereby cross-market volatility spillovers between conventional and Islamic markets can be modelled in a more dynamical manner (e.g., using GARCH-BEKK model proposed first in (Baba, Engle, Kraft and Kroner, 1991). The endogeneous dynamics of the second-round effects, on the other hand, can be modelled by introducing a latent variable into the variance equation as a threshold value for first-round losses of financial institutions which, when exceeded, will trigger a counteraction to restore capital buffer. These counter-actions, in turn, will lead to reputational risk and systemic risk, the latter being commensurate to the size and number of reacting financial institutions as well as similarity of their reactions ((van den End, 2010); (van den End, 2012)). We leave the discussion of second-round effects here as a future research item as they are beyond the scope of the present study.

8. Conclusion

Model-based stress loss estimations reveals the fact that the risk models that performed better in backtests are able to generate consistent price paths for artificial stressful periods (triggered by different initial tail events), which mostly match the historical price discontinuities observed in previous financial crises. Taking into account a minimum of

10-day risk horizon, the selected models predict significant lower bounds for necessary risk capital to absorb potential large losses in both conventional and Islamic finance portfolios. Given the current volatile nature of financial markets, Islamic financial institutions must indispensably perform regular stress tests on their asset positions and hold adequate capital against unexpected shocks. Indeed, under IFSB and Basel III frameworks, financial institutions are required by regulators to establish a stress test framework as one of the main components of daily risk management activities. The paper, in this regard, must appeal to both finical institutions and supervisory authorities.

Our backtest results clearly revealed the fact that Gaussian distribution is not appropriate for predicting stress losses in an Islamic equity portfolio, as in many conventional asset portfolios. Another interesting finding is that the superiority of conditional volatility models which incorporate asymmetric volatility response over symmetric ones in predicting stress losses is largely determined by the selection of the underlying distribution. Under normality assumption, we find almost no difference between symmetric and asymmetric models. However, as far as an heavy-tailed distribution is concerned, the asymmetric model apparently does a better job. Finally, it is also found that the size of the estimation window has diverse effects on backtest performances of different risk models: although working with a larger estimation window generally leads to an increase in the performance of the conditional empirical model, it has a diminishing effect on that of the GARCH model with Student's t innovations causing it to produce overly conservative results.

Our study also reveals a number of superiorities that the model-based stress-testing framework has over traditional methods. In traditional methods where hypothetical stress scenarios are employed, risk managers may subjectively overestimate, underestimate or even ignore the potential for some scenarios. The model based approach presented here eliminates this subjectivity problem and is apparently much more objective. Another problem with the traditional stress tests is that the results are difficult to interpret because they give no idea about the probabilities of the scenarios concerned. The current model avoids this complexity by allowing the management to fix a probability for an extreme shock accordingly with the company's risk attitude and by providing the risk manager with the clear understanding of the potential risks surrounding the firm. Furthermore, the modelbased framework can easily be integrated into risk management process. This is not quite easy in traditional models because market risk measurement is a probabilistic approach whereas the stress scenarios in the traditional models are discrete events without certain probabilities and each of them is likely to generate different outcomes. Yet, after all, the model-based approach is still vulnerable to risk model specification and structural breaks in the market behaviour.

To conclude, and as an outlook, it would be appealing to extend the discussion to modelling of tighter liquidity conditions in a more stressful market, which could be helpful in the selection of the minimum risk horizon to hedge positions. Also, the potential implications of a stress loss for capital adequacy further encourage the exploration of multi-asset portfolio cases where the correlations between assets turn out to be important. Finally, the calibrating the size of the initial shock parameter α^* to the market data by minimising the distances between the past stress losses and model-estimated stress losses could be another interesting area for future research. Reliability of coverage test as candidate unbiased transmitters of model performance should also be challenged, at least, by broadening the spectrum of backtesting models included in this study.

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