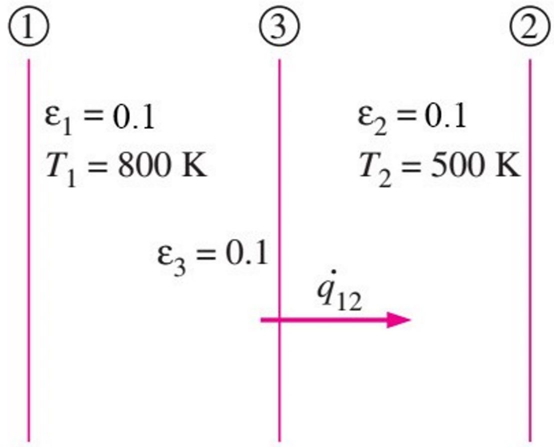
Week6_Li, Junkai

2019年11月6日 20:58

Define the radiative heat transfer rate between two parellel plates shown in the picture:



$$\dot{q}_{net_{1-2}} = \frac{\dot{Q}_{net_{1-2}}}{A} = \frac{A\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \div A$$

$$= \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{\left(5.67\ \times 10^{-8} \frac{\mathrm{W}}{m^2\,K^4}\right) \times (800^4 - 500^4) K^4}{\frac{1}{0.1} + \frac{1}{0.1} - 1}$$

$$\approx 1035.82\ \frac{W}{m^2}$$

The new heat transfer rate should be 1% of the $\dot{q}_{net_{1-2}}$,

$i.e., \dot{q}'_{net_{1-2}} = \dot{q}_{net_{1-2,\, n\, shiels}} = \frac{1}{100} \times \dot{q}_{net_{1-2}},$

$$\dot{q}_{net_{1-2,\, n\, shiels}} = \frac{\dot{Q}_{net_{1-2,\, n\, shields}}}{A}$$

$$= \frac{A\sigma(T_2^4 - T_1^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_{3,1}} + \frac{1}{\epsilon_{3,2}} - 1\right) + \cdots \left(\frac{1}{\epsilon_{n,1}} + \frac{1}{\epsilon_{n,2}} - 1\right)} \div \mathrm{A}$$

$$= \frac{\sigma(T_2^4 - T_1^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_{3,1}} + \frac{1}{\epsilon_{3,2}} - 1\right) + \cdots \left(\frac{1}{\epsilon_{n,1}} + \frac{1}{\epsilon_{n,2}} - 1\right)}$$

Autem, $\epsilon_1 = \epsilon_2 = \epsilon_3 = \cdots = \epsilon_n = 0.1$

Substitute $\epsilon = 0.1$ for $\epsilon_1, \epsilon_2, \epsilon_3, \ldots, \epsilon_n, and\ introduce\ to\ the\ equation$:

$$\dot{q}_{net_{1-2,\, n\, shiels}} = \frac{\sigma(T_2^4 - T_1^4)}{(n+1)(\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1)} = \frac{1}{\mathrm{n}+1} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1}$$

$$Since\ \dot{q}'_{net_{1-2}} = \dot{q}_{net_{1-2,\, n\, shiels}} = \frac{1}{100} \times \dot{q}_{net_{1-2}} = \frac{1}{100} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{1}{100} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1}$$

$$i.e., \quad \frac{1}{\mathrm{n}+1} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1} = \frac{1}{100} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1}$$

$n = 99$

To have the new heat transfer rate be 1% of the previous rate without any shields, we need 99 shields which $\epsilon = 0.1$

Q.E.D.

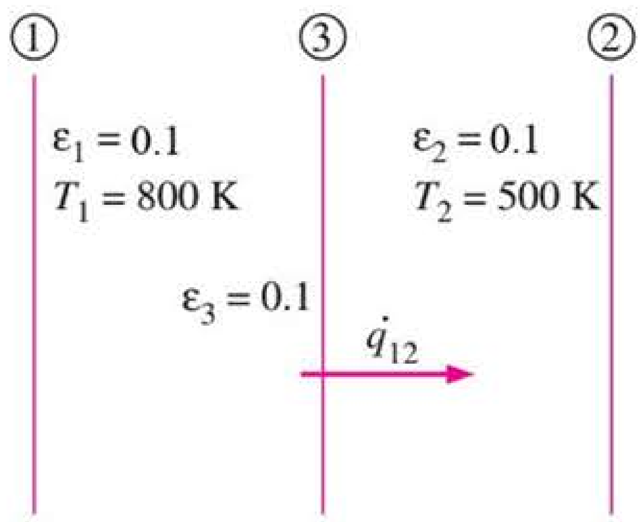
Week6_Li, Junkai

2019年11月6日 20:58

Define the radiative heat transfer rate between two parellel plates shown in the picture:

$$\boxed{\dot{q}_{net_{1-2}} = \frac{\dot{Q}_{net_{1-2}}}{A} = \frac{A\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \div A}$$

Calculating the heat trander rate between two parellel plates.



$$= \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{\left(5.67 \times 10^{-8} \frac{W}{m^2 K^4}\right) \times (800^4 - 500^4) K^4}{\frac{1}{0.1} + \frac{1}{0.1} - 1}$$

$$\approx 1035.82 \ \frac{W}{m^2}$$

The new heat transfer rate should be 1% of the $\dot{q}_{net_{1-2}}$,

$$i.e., \dot{q}'_{net_{1-2}} = \dot{q}_{net_{1-2,\, n\, shiels}} = \frac{1}{100} \times \dot{q}_{net_{1-2}},$$

$$\boxed{\begin{aligned} \dot{q}_{net_{1-2,\, n\, shiels}} &= \frac{\dot{Q}_{net_{1-2,\, n\, shields}}}{A} \\ &= \frac{A\sigma(T_2^4 - T_1^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_{3,1}} + \frac{1}{\epsilon_{3,2}} - 1\right) + \cdots \left(\frac{1}{\epsilon_{n,1}} + \frac{1}{\epsilon_{n,2}} - 1\right)} \div A \\ &= \frac{\sigma(T_2^4 - T_1^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_{3,1}} + \frac{1}{\epsilon_{3,2}} - 1\right) + \cdots \left(\frac{1}{\epsilon_{n,1}} + \frac{1}{\epsilon_{n,2}} - 1\right)} \end{aligned}}$$

Calculating the heat trander rate between two parellel plates with shields*n.

Autem, $\epsilon_1 = \epsilon_2 = \epsilon_3 = \cdots = \epsilon_n = 0.1$

Substitute $\epsilon = 0.1$ for $\epsilon_1, \epsilon_2, \epsilon_3, \ldots, \epsilon_n$, *and introduce to the equation:*

$$\boxed{\dot{q}_{net_{1-2,\, n\, shiels}} = \frac{\sigma(T_2^4 - T_1^4)}{(n+1)(\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1)} = \frac{1}{n+1} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1}}$$

Calculating the heat trander rate between two parellel plates with shields*n when each plate and shield has the same the value of emmisivity.

$$Since \ \dot{q}'_{net_{1-2}} = \dot{q}_{net_{1-2,\, n\, shiels}} = \frac{1}{100} \times \dot{q}_{net_{1-2}} = \frac{1}{100} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{1}{100} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1}$$

$$i.e., \quad \frac{1}{n+1} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1} = \frac{1}{100} \times \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1}$$

$n = 99$

To have the new heat transfer rate be 1% of the previous rate without any shields, we need 99 shields which $\epsilon = 0.1$

Q.E.D.