

Task 1

As we know from the presentations, in order to find the U value of the double pane window (center region) we should use the following formula:

$$\frac{1}{U_{double\ pane\ window(center\ region)}} \approx \frac{1}{h_i} + \frac{1}{h_{space}} + \frac{1}{h_o}$$

Another fact that we learned from the presentation is that air space coefficient (h_{space}) is equal to air space radiation coefficient (h_{rd}) plus air space convection coefficient (h_{cv}). Since our gap is within optimal range then convection coefficient doesn't exist, therefore $h_{space} = h_{rd}$.

By replacing winter design values of 8.29 and 34.0 $\frac{W}{C^\circ m^2}$ for the inner and outer surface heat transfer coefficients and also value of 5.3 (h_{space} in winter when $\varepsilon_{effective} = 0.72$ for two parallel glass surfaces) we can calculate the benchmark U factor:

$$\frac{1}{U_{double\ pane\ window(center\ region)}} \approx \frac{1}{8.29} + \frac{1}{5.3} + \frac{1}{34} \approx 0.12 + 0.189 + 0.03$$

$$\Rightarrow \frac{1}{U_{double\ pane\ window(center\ region)}} \approx 0.339$$

$$\Rightarrow U_{double\ pane\ window(center\ region)} \approx \boxed{2.95}$$

- In case of using a **low emissivity coating**, due to the lower $\varepsilon_{effective}$ (0.1), our h_{space} is equal to 2.4 $\frac{W}{m^2.k}$ in winter. by replacing this value instead of 5.3 our U factor will be **1.77** $\frac{W}{m^2.k}$. then:

$$\frac{U_{double\ pane\ window\ (center\ region)\text{--with coating}}}{U_{double\ pane\ window\ (center\ region)\text{--without coating}}} \approx \frac{1.77}{2.95} \approx \boxed{60\ \%}$$

- In case of injecting a **low conductive fluid**, due to lower conductivity of the gases, our U-factor is equal to **2.7** (when $\varepsilon_{effective} = 0.84$) and **2.5** (when $\varepsilon_{effective} = 0.1$) for the case of Argon gas and then for Krypton gas the corresponding values are **1.5** (when $\varepsilon_{effective} = 0.84$) and **1.4** (when $\varepsilon_{effective} = 0.1$), then:

$$\frac{U_{double\ pane\ window\ (center\ region)-Argon-\varepsilon=0.84}}{U_{double\ pane\ window\ (center\ region)-without\ coating}} \approx \frac{2.7}{2.95} \approx \boxed{92\ \%}$$

$$\frac{U_{double\ pane\ window\ (center\ region)-Argon-\varepsilon=0.1}}{U_{double\ pane\ window\ (center\ region)-without\ coating}} \approx \frac{2.5}{2.95} \approx \boxed{85\ \%}$$

$$\frac{U_{double\ pane\ window\ (center\ region)-Krypton-\varepsilon=0.84}}{U_{double\ pane\ window\ (center\ region)-without\ coating}} \approx \frac{1.5}{2.95} \approx \boxed{50\ \%}$$

$$\frac{U_{double\ pane\ window\ (center\ region)-Krypton-\varepsilon=0.1}}{U_{double\ pane\ window\ (center\ region)-without\ coating}} \approx \frac{1.4}{2.95} \approx \boxed{48\ \%}$$

- In case of using a **triple glazing**, due to the additional layer of air gap, our U-factor is equal to **2** (when $\varepsilon_{effective} = 0.84$) and **1.3** (when $\varepsilon_{effective} = 0.1$) for the case of air and then it equals to **1.8** (when $\varepsilon_{effective} = 0.84$) and **1** (when $\varepsilon_{effective} = 0.1$) when Argon gas replaces air. In case we replace Argon with Krypton then U-factor will be **1.6** (when $\varepsilon_{effective} = 0.84$) and **0.7** (when $\varepsilon_{effective} = 0.1$), then:

$$\frac{U_{triple\ pane\ window\ (center\ region)-air-\varepsilon=0.84}}{U_{double\ pane\ window\ (center\ region)-without\ coating}} \approx \frac{2}{2.95} \approx \boxed{68\ \%}$$

$$\frac{U_{triple\ pane\ window\ (center\ region)-air-\varepsilon=0.1}}{U_{double\ pane\ window\ (center\ region)-without\ coating}} \approx \frac{1.3}{2.95} \approx \boxed{45\ \%}$$

$$\frac{U_{\text{triple pane window (center region)–Argon–}\varepsilon=0.84}}{U_{\text{double pane window (center region)–without coating}}} \approx \frac{1.8}{2.95} \approx \boxed{61 \%}$$

$$\frac{U_{\text{triple pane window (center region)–Argon–}\varepsilon=0.1}}{U_{\text{double pane window (center region)–without coating}}} \approx \frac{1}{2.95} \approx \boxed{33 \%}$$

$$\frac{U_{\text{triple pane window (center region)–Krypton–}\varepsilon=0.84}}{U_{\text{double pane window (center region)–without coating}}} \approx \frac{1.6}{2.95} \approx \boxed{55 \%}$$

$$\frac{U_{\text{triple pane window (center region)–Krypton–}\varepsilon=0.1}}{U_{\text{double pane window (center region)–without coating}}} \approx \frac{0.7}{2.95} \approx \boxed{24 \%}$$

Task 2

We know from the last session that in order to calculate the heating and cooling load we need to use following formulas:

- $Q_{\text{heating}} = HF \times A$ where HF is the heating load factor and A is the fenestration area including frame. We can calculate the heating load factor by multiplying Window's U-factor by the heating temperature difference.

- $Q_{\text{cooling}} = CF \times A$ where CF is the surface cooling factor and A is the fenestration area including frame. CF can be calculated as following:

$$CF = U (\Delta T_{\text{cooling}} - 0.46DR) + PXI \times SHGC \times IAC \times FF_3$$

Where DR is **cooling daily range**, PXI is **peak irradiance** and is equal to sum of E_D (**diffuse irradiance**) and E_d (**direct irradiance**) in case of no exterior shading (as we imagined), SHGC is solar heat gain coefficient, IAC is **interior shading attenuation coefficient** which equals to 1 in our case since we have no interior shading (as we imagined) and finally FF_3 is **fenestration solar load factor**.

For the new windows the following values can be plotted from corresponding tables:

Window_West_Fixed	
U-factor	2.84 (Fixed Double Glazed Window with Wooden Frame)
PXI	$E_D + E_d = 559 + 188 = 747$ (Based on Latitude of Piacenza)
SHGC	0.54 (Fixed Double Glazed Window with Wooden Frame)
IAC	1 (No Interior Shading)
FF_3	0.56 (Single Family Detached House)

Window_South_Fixed	
U-factor	2.84 (Fixed Double Glazed Window with Wooden Frame)
PXI	$E_D + E_d = 348 + 209 = 557$ (Based on Latitude of Piacenza)
SHGC	0.54 (Fixed Double Glazed Window with Wooden Frame)
IAC	1 (No Interior Shading)
FF_3	0.47 (Single Family Detached House)

Window_South_Operable	
U-factor	2.87 (Fixed Double Glazed Window with Wooden Frame)
PXI	$E_D + E_d = 348 + 209 = 557$ (Based on Latitude of Piacenza)
SHGC	0.46 (Fixed Double Glazed Window with Wooden Frame)
IAC	1 (No Interior Shading)
FF_3	0.47 (Single Family Detached House)

We also know from the presentation that $\Delta T_{heating} = 24.8^{\circ}C$, $\Delta T_{cooling} = 7.9^{\circ}C$ and $DR = 11.9^{\circ}C$, then:

$$Q_{heating_window_{west}} = HF_{window_{west}} \times A_{window_{west}}$$

$$HF_{window_{west}} = U_{window_{west}} \times \Delta T_{heating} = 2.84 \times 24.8 \approx 70.44 \frac{W}{m^2}$$

$$\rightarrow Q_{heating_window_{west}} = 70.44 \times 14.4 \approx \boxed{1014.34 \text{ W}}$$

$$Q_{cooling_window_{west}} = CF_{window_{west}} \times A_{window_{west}}$$

$$CF_{window_{west}} = U_{window_{west}} (\Delta T_{cooling} - 0.46DR) + P_{XI_{window_{west}}} \times SHGC_{window_{west}} \times IAC_{window_{west}} \times FF_{3 \text{ window}_{west}}$$

$$\rightarrow = 2.84 (7.9 - 0.46 \times 11.9) + 747 \times 0.54 \times 1 \times 0.56 \approx 226 \frac{W}{m^2}$$

$$\rightarrow Q_{cooling} = 226 \times 14.4 \approx \boxed{3255 \text{ W}}$$

$$Q_{heating_window_{south-f}} = HF_{window_{south-f}} \times A_{window_{south-f}}$$

$$HF_{window_{south-f}} = U_{window_{south-f}} \times \Delta T_{heating}$$

$$\rightarrow = 2.84 \times 24.8 \approx 70.44 \frac{W}{m^2}$$

$$\rightarrow Q_{heating_window_{south-f}} = 70.44 \times 3.6 \approx \boxed{254 \text{ W}}$$

$$Q_{cooling_window_{south-f}} = CF_{window_{south-f}} \times A_{window_{south-f}}$$

$$CF_{window_{south-f}} = U_{window_{south-f}} (\Delta T_{cooling} - 0.46DR) + PXI_{window_{south-f}} \times SHGC_{window_{south-f}} \times IAC_{window_{south-f}} \times FF_3_{window_{south-f}}$$

$$\Rightarrow = 2.84 (7.9 - 0.46 \times 11.9) + 557 \times 0.54 \times 1 \times 0.47 \approx 149 \frac{W}{m^2}$$

$$\Rightarrow Q_{cooling} = 149 \times 3.6 = \boxed{536.4 \text{ W}}$$

$$Q_{heating_window_{south-o}} = HF_{window_{south-o}} \times A_{window_{south-o}}$$

$$HF_{window_{south-o}} = U_{window_{south-o}} \times \Delta T_{heating}$$

$$\Rightarrow = 2.87 \times 24.8 \approx 71.2 \frac{W}{m^2}$$

$$\Rightarrow Q_{heating_window_{south-f}} = 71.2 \times 3.6 \approx \boxed{257 \text{ W}}$$

$$Q_{cooling_window_{south-o}} = CF_{window_{south-o}} \times A_{window_{south-o}}$$

$$CF_{window_{south-o}} = U_{window_{south-o}} (\Delta T_{cooling} - 0.46DR) + PXI_{window_{south-o}} \times SHGC_{window_{south-o}} \times IAC_{window_{south-o}} \times FF_3_{window_{south-o}}$$

$$\Rightarrow = 2.87 (7.9 - 0.46 \times 11.9) + 557 \times 0.46 \times 1 \times 0.47 \approx 127.43 \frac{W}{m^2}$$

$$\rightarrow Q_{cooling} = 127.43 \times 3.6 = 458.8 \text{ W}$$

Now we can calculate the values for Aluminum frames:

Window_West_Fixed	
U-factor	3.61 (Fixed Double Glazed Window with Aluminum Frame)
PXI	$E_D + E_d = 559 + 188 = 747$ (Based on Latitude of Piacenza)
SHGC	0.56 (Fixed Double Glazed Window with Aluminum Frame)
IAC	1 (No Interior Shading)
FF ₃	0.56 (Single Family Detached House)

Window_South_Fixed	
U-factor	3.61 (Fixed Double Glazed Window with Aluminum Frame)
PXI	$E_D + E_d = 348 + 209 = 557$ (Based on Latitude of Piacenza)
SHGC	0.56 (Fixed Double Glazed Window with Aluminum Frame)
IAC	1 (No Interior Shading)
FF ₃	0.47 (Single Family Detached House)

Window_South_Operable	
U-factor	4.62 (Fixed Double Glazed Window with Aluminum Frame)
PXI	$E_D + E_d = 348 + 209 = 557$ (Based on Latitude of Piacenza)
SHGC	0.55 (Fixed Double Glazed Window with Aluminum Frame)
IAC	1 (No Interior Shading)
FF ₃	0.47 (Single Family Detached House)

We also know from the presentation that $\Delta T_{heating} = 24.8^{\circ}C$, $\Delta T_{cooling} = 7.9^{\circ}C$ and $DR = 11.9^{\circ}C$, then:

$$Q_{heating_window_{west}} = HF_{window_{west}} \times A_{window_{west}}$$

$$HF_{window_{west}} = U_{window_{west}} \times \Delta T_{heating} = 3.61 \times 24.8 \approx 90 \frac{W}{m^2}$$

$$\rightarrow Q_{heating_window_{west}} = 90 \times 14.4 \approx \boxed{1300 W}$$

$$Q_{cooling_window_{west}} = CF_{window_{west}} \times A_{window_{west}}$$

$$CF_{window_{west}} = U_{window_{west}} (\Delta T_{cooling} - 0.46DR) + PXI_{window_{west}} \times SHGC_{window_{west}} \times IAC_{window_{west}} \times FF_3_{window_{west}}$$

$$\rightarrow = 3.61 (7.9 - 0.46 \times 11.9) + 747 \times 0.56 \times 1 \times 0.56 \approx 243 \frac{W}{m^2}$$

$$\rightarrow Q_{cooling} = 190 \times 14.4 \approx \boxed{3500 W}$$

$$Q_{heating_window_{south-f}} = HF_{window_{south-f}} \times A_{window_{south-f}}$$

$$HF_{window_{south-f}} = U_{window_{south-f}} \times \Delta T_{heating}$$

$$\rightarrow = 3.61 \times 24.8 \approx 90 \frac{W}{m^2}$$

$$\rightarrow Q_{heating_window_{south-f}} = 90 \times 3.6 \approx \boxed{324 W}$$

$$Q_{cooling_window_{south-f}} = CF_{window_{south-f}} \times A_{window_{south-f}}$$

$$CF_{window_{south-f}} = U_{window_{south-f}} (\Delta T_{cooling} - 0.46DR) + PXI_{window_{south-f}} \times SHGC_{window_{south-f}} \times IAC_{window_{south-f}} \times FF_3_{window_{south-f}}$$

$$\Rightarrow = 3.61 (7.9 - 0.46 \times 11.9) + 557 \times 0.56 \times 1 \times 0.47 \approx 155.4 \frac{W}{m^2}$$

$$\Rightarrow Q_{cooling} = 155.4 \times 3.6 \approx \boxed{560 \text{ W}}$$

$$Q_{heating_window_{south-o}} = HF_{window_{south-o}} \times A_{window_{south-o}}$$

$$HF_{window_{south-o}} = U_{window_{south-o}} \times \Delta T_{heating}$$

$$\Rightarrow = 4.62 \times 24.8 \approx 114.58 \frac{W}{m^2}$$

$$\Rightarrow Q_{heating_window_{south-f}} = 114.58 \times 3.6 \approx \boxed{413 \text{ W}}$$

$$Q_{cooling_window_{south-o}} = CF_{window_{south-o}} \times A_{window_{south-o}}$$

$$CF_{window_{south-o}} = U_{window_{south-o}} (\Delta T_{cooling} - 0.46DR) + PXI_{window_{south-o}} \times SHGC_{window_{south-o}} \times IAC_{window_{south-o}} \times FF_3_{window_{south-o}}$$

$$\Rightarrow = 4.62 (7.9 - 0.46 \times 11.9) + 557 \times 0.55 \times 1 \times 0.47 \approx 155.21 \frac{W}{m^2}$$

$$\rightarrow Q_{cooling} = 155.21 \times 3.6 = 559 \text{ W}$$

The increase of heat loss through our window after substituting the wooden frame by aluminum one is very significant.