Phys 103 – Spring 2013

Homework 4

Due on Monday 05/06 by 6 PM in box outside Daly 312

Problems 3-8 in this assignment are to be solved with pencil and paper. You can use computer to check your results, but I will not accept computer based solutions.

1. Consider the following revision to the epidemic model discussed in class: Suppose that after recovery, there is a loss of immunity that causes recovered individuals to become susceptible. This re-infection mechanism can be represented as ρR, where ρ is the re-infection rate. Modify the model to include this mechanism and plot $S(t), I(t), R(t)$ on the same figure using $\rho = 0.03\,[1/day]$. Assume there are 10,000 people, all of whom are initially susceptible. The parameters $a = 0.002\,[1/(person \times week)$ and $r = 0.15\left[1/day\right]$ are the same as in the original model.

2. **Bonus question:** Problem 3.25. Use $\rho = 28$, time step of $\tau = 5 \cdot 10^{-3}$, and 10,000 steps.

3. For the following sets of equations write and row reduce the augmented matrix to find the solutions and state whether there is exactly one solution, no solutions, or an infinite set of solutions.

$$\begin{cases} -x + y - z = 4 \\ x - y + 2z = 3 \\ 2x - 2y + 4z = 6 \end{cases}, \quad \begin{cases} x + 2y - z = 1 \\ 2x + 3y - 2z = 1 \\ 3x + 4y - 3z = -4 \end{cases}, \quad \begin{cases} 2x + 5y + z = 2 \\ x + y + 2z = 1 \\ x + 5z = 3 \end{cases}$$

4. Find the rank of the matrices $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -3 & 5 & 3 \\ 4 & -1 & 1 & 1 \\ 3 & -2 & 3 & 4 \end{pmatrix}$,

$$C = \begin{pmatrix} 1 & 0 & 1 & 0 \\ -1 & -2 & -1 & 0 \\ 2 & 2 & 5 & 3 \\ 2 & 4 & 8 & 6 \end{pmatrix}$$

5. Evaluate the following determinant using a minimum number of operations

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$

6. Show that the following sets of functions are linearly independent:

$\left\{x, e^x, xe^x\right\}$, $\left\{1, x^2, x^4, x^6\right\}$, $\left\{\sin x, \cos x, x\sin x, x\cos x\right\}$

7. Simplify $\left(AB\right)^{-1}\left(ABA^{-1}\right)^3\left(BA^{-1}\right)^{-1}$

8. Given the matrices $A = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 0 & -1 \\ 4 & -2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$

   a. Find A^{-1}, B^{-1}, $B^{-1}AB$, $B^{-1}A^{-1}B$

   b. Show that the last two matrices are inverses, that is, that their product is the unit matrix.

   c. Part (b) is a special case of the general theorem that the inverse of a product of matrices is the product of the inverses in reverse order. Prove this.