TDT4171: Exercise 2

Artificial Intelligence Methods

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Part A, Umbrella Domain as an HMM

X_t is a set of **unobsorvable** variable(s) and E_t is a set of **obsorvable** variable(s) for a given time slice t.
X_t : In the textbook's "Umbrella_world" example, weather(rain) is the unobservable variable.
E_t : Appearance of umberella is the sensor value, and also the observable variable in the umbrella model.

Dynamic model/transition model:

$$P(X_t|X_{t-1}) = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$$

Observation model/sensor model:

$$P(E_t|X_t) = \begin{pmatrix} 0.9 & 0.0 \\ 0.0 & 0.2 \end{pmatrix}$$

Probability distribution over the state variables is decided by the transition model which is the conditional probability $P(X_t|X_0 : X_{t-1}) = P(X_t|X_{t-1})$. In the umbrella model the probability of rain today depends on state of the previous day. This model solves the problem using the first-order **Markov assumption**. This assumption gives only an approximated prediction of the future day weather. The location and the season are two important dependencies to decide the accuracy of the approximation. This model could be improved by either increasing the order of Markov process model or increasing the set of state variables(such as, season, temperature, humidity, ...). But in the case of predicting an approximation of next day's weather, the assumption is reasonable.
Likely, the same order of Markov process applies to evidence variables. Where E_t depends only on previous variables. In order to solve the problem with infinit size of t time steps, the process of changes in the world state are assumed to be caused by a **stationary process** [**?**]. By that, the conditional probability of rain is same for all time steps t. This assupmtion is reasonable because the world state in umbrella model is nature and the rainy weather is decided by nature rules. endhomeworkProblem

Part B, Filtering Using FORWARD Operation

By the FORWARD operation the probability of rain at day 2 and 5 are calculated.

$$P(X_2|e_{1:2}) = 0.88335704$$

$$P(X_5|e_{1:5}) = 0.86733889$$

Forward messages are listed below:

Listing 1: Part B verification

```
Probability of rain at day 2:
------------------------------------------
forward_msg(0):  [ 0.5  0.5]
forward_msg(1):  [ 0.81818182  0.18181818]
forward_msg(2):  [ 0.88335704  0.11664296]

Probability of rain at day 5:
------------------------------------------
forward_msg(0):  [ 0.5  0.5]
forward_msg(1):  [ 0.81818182  0.18181818]
forward_msg(2):  [ 0.88335704  0.11664296]
forward_msg(3):  [ 0.19066794  0.80933206]
forward_msg(4):  [ 0.730794    0.269206  ]
forward_msg(5(:  [ 0.86733889  0.13266111]
```

Part C, FORWARD _ BACKWARD Algorithm

The result of calculation of by the forward_backward algorithm for $t = 2$ (Listing 2)is

$$P(X_2|e_{1:2}) = [0.88335704, 0.11664296]$$

Listing 2: Part C verification. Filtering and smoothing for probability of rain at day 2

```
Probability of rain at day 2:
-------------------------------------------
forward_msg( 0 ):  [ 0.5  0.5]
forward_msg( 1 ):  [ 0.81818182  0.18181818]
forward_msg( 2 ):  [ 0.88335704  0.11664296]


backward_msg( 2 ):  [1 1]
backward_msg( 1 ):  [ 0.69  0.41]
backward_msg( 0 ):  [ 0.4593  0.2437]


smooth( 0 ): [ 0.65334282  0.34665718]
smooth( 1 ): [ 0.88335704  0.11664296]
smooth( 2 ): [ 0.88335704  0.11664296]
```

And $P(X_5|e_{1:5})$ is calculated and shown in the Listing 3.

Listing 3: Part C verification. Filtering and smoothing for probability of rain at day 5

```
Probability of rain at day 5:
--------------------------------------------
forward_msg( 0 ):  [ 0.5  0.5]
forward_msg( 1 ):  [ 0.81818182  0.18181818]
forward_msg( 2 ):  [ 0.88335704  0.11664296]
forward_msg( 3 ):  [ 0.19066794  0.80933206]
forward_msg( 4 ):  [ 0.730794  0.269206]
forward_msg( 5 ):  [ 0.86733889  0.13266111]


backward_msg( 5 ):  [1 1]
backward_msg( 4 ):  [ 0.69  0.41]
backward_msg( 3 ):  [ 0.4593  0.2437]
backward_msg( 2 ):  [ 0.090639  0.150251]
backward_msg( 1 ):  [ 0.06611763  0.04550767]
backward_msg( 0 ):  [ 0.04438457  0.02422283]


smooth( 0 ): [ 0.64693556  0.35306444]
smooth( 1 ): [ 0.86733889  0.13266111]
smooth( 2 ): [ 0.82041905  0.17958095]
smooth( 3 ): [ 0.30748358  0.69251642]
smooth( 4 ): [ 0.82041905  0.17958095]
smooth( 5 ): [ 0.86733889  0.13266111]
```

Part D, Code

The code file `umbrella_world.py` with the output file of the program is attached to assignment delivery.

References

[1] Artificial Intelligence A Modern Approach, Third Edition, *Stuart J. Russell and Peter Norvig*