Plane and Simple: A Dynamic Probabilistic Model for Locating Missing Aircraft

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Abstract

Recent commercial airplane crashes, such as the loss of flight MH-370, have demonstrated the need for effective mathematical models to aid in locating a missing aircraft. Such a model must consider not only the dynamics of the crash itself, but also the capabilities of search and rescue vehicles. In this work, we present a comprehensive probabilistic model which accounts for these components in addition to other effects such as ocean currents, ocean depth, and the cost of utilizing search vehicles. Ultimately, our model identifies regions which are the most cost-effective to search at a given time, allowing searchers to maximize the chance of finding the wrecked aircraft with finite resources.

We model the initial phase of the crash process with two distinct processes: controlled emergencies and uncontrolled emergencies. In the controlled case, the pilot attempts to steep the plane to the nearest airport but is not successful, creating wreckage along the way. In the uncontrolled case, the plane travels in random direction until crashing. Each of these crashes can create surface debris or sunken debris. One key strength of this work is model the drift of surface debris due to the ocean currents, and account for that drift in our recommendations.

Because of the detection limitations between different types of search vehicles, simply knowing the most probable locations of the wreckage isn't useful enough for searchers. Our model accounts for these differences by taking into consideration the contrast between a search plane's superior ability to visually spot floating wreckage versus a search vessel's superior ability to detect sunken wreckage. In this sense, the model uses the debris probability maps in conjunction with ocean depth data to determine the locations where a search plane would be most likely to find debris and to where a search vessel would be most likely to find debris.

Additionally, our model aims at quantifying the monetary implications of conducting a search. We construct equations for operational costs of search vehicles and relate them back to the computed wreckage probabilities in order to approximate the expected costs of deploying a search to the most promising areas.

Ultimately, this model is able to apply a probabilistic methods to data from a wide variety of sources to produce recommendations. These recommendations include a diverse range of realistic effects including pilot behavior during a crash, types of crashes, the drifting of debris due to the current, and the utility of different sensors. The method is validated on a realistic case study, and founded to give reasonable predictions with valuable insight.

Contents

1 Introduction

1.1 Problem Background

The disappearance of Malaysia Airlines Flight 370 triggered an international response that outlined how truly difficult it can be to find a lost plane feared to have crashed in open water. In this work, we present a mathematical model to be used for estimating the location of a downed aircraft and planning a search which makes effective use of finite resources.

We consider four primary considerations when addressing this problem:

- Estimating the initial location of the crash knowing nothing other than the planned route
- Determining potential movement of wreckage due to ocean currents
- Understanding the limitations of available search vehicles and their tools
- Quantifying monetary implications of conducting a search

1.2 Previous Research

There exists literature discussing the usage of Bayesian statistics in an effort to locate a crashed plane. One successful recent application of such a model for determining the most probable locations of a plane crash was by Lawrence D. Stone [1]. Stone's approach assumed that the crash occurred shortly after a loss of communication with the plane, and his model allowed incorporating ocean current information provided when floating debris was found in order to help determine the original crash location.

While there is extremely limited publicly available research that explicitly focuses on the search capabilities of search vehicles, there is an abundance of raw data available on the vehicles themselves and the tools that they would use when conducting searches.

1.3 Our Work

We seek to create a generic mathematical model that could **assist searchers in planning a useful search for a lost plane feared to have crashed in open water**. We begin by stating our assumptions, symbols, and definitions. We design a modular mathematical framework to:

1. Approximate the current location of the plane wreckage
2. Generate probabilities of locating the wreckage using search vehicles
3. Evaluate the expected costs of searching with different vehicles

2 Assumptions, Symbols, and Definitions

2.1 Problem Assumptions

In order to make the problem tractable, several key assumptions are made.

- **Contact is lost along the plane route**
  We assume to have been provided the flight's origin and destination, but no further information regarding possible crash times or locations (e.g., radar data, black box pings, satellite communication, etc.).

- **The plane crash occurs over a large body of water**
  We do not consider land crashes or crashes that occur on small bodies of water (e.g., rivers or lakes).

- **There are only two types of search vehicles**
  We assume that the only two types of vehicles that can aid in the finding of a crashed plane are search planes and search vessels. Additionally, search planes excel at locating floating debris, whereas search vessels excel at locating wreckage below ocean surfaces.

2.2 Symbols and Definitions

Our model utilizes several variables and functions which we define in Table 1.

Symbol	Definition	Units
$p_{directed}$	Probability that the pilot attempted to head to nearest airport	–
σ_{dist}	Variance of the normally distributed distance to nearest airport	km^2
α_{snkn}	Probability that the wreckage primarily sinks to the ocean floor	–
$p_{sfrc}(x,y,t)$	Probability that at time t there is wreckage floating on the surface at location (x,y)	–
$p_{snkn}(x,y)$	Probability that there is sunken wreckage at location (x,y)	–
$p_{sp,srfc}(x,y)$	Probability of search plane locating wreckage floating on ocean surface	–
$p_{sp,snkn}(x,y,d)$	Probability of search plane locating sunken wreckage in location (x,y) at depth d	–
$p_{sv,srfc}$	Probability of search vessel locating wreckage floating on ocean surface	–
$p_{sv,snkn}$	Probability of search vessel locating sunken wreckage	–
sp_{alt}	Altitude at which search plane flies	m
sp_{lat}	Effective lateral search range of search plane	m
$d(x,y)$	Depth of the ocean at point (x,y)	m
$N(d)$	Number of unit grid squares that a plane can search in a single excursion	–
$C_{operation}$	Cost of search plane's operation	USD
D	Distance of search location from nearest airbase	km
R_o	Maximum distance plane can travel in one excursion	km
$C_{sp}(d)$	Cost for search plane to search N(d) contiguous unit grid squares	USD

Table 1: Symbols and Definitions

3 Modeling Methodology

We consider the modeling challenge to have three components: first generating relative probabilities that the wreckage is at specific locations, then estimating probabilities of actually locating the wreckage using search vessels and planes, and finally approximating the expected costs of deploying a search to the most promising areas.

We recommend reviewing the Model Assumptions in Sec. 5.1.

A graphical representation of our model is given by Fig. 1.

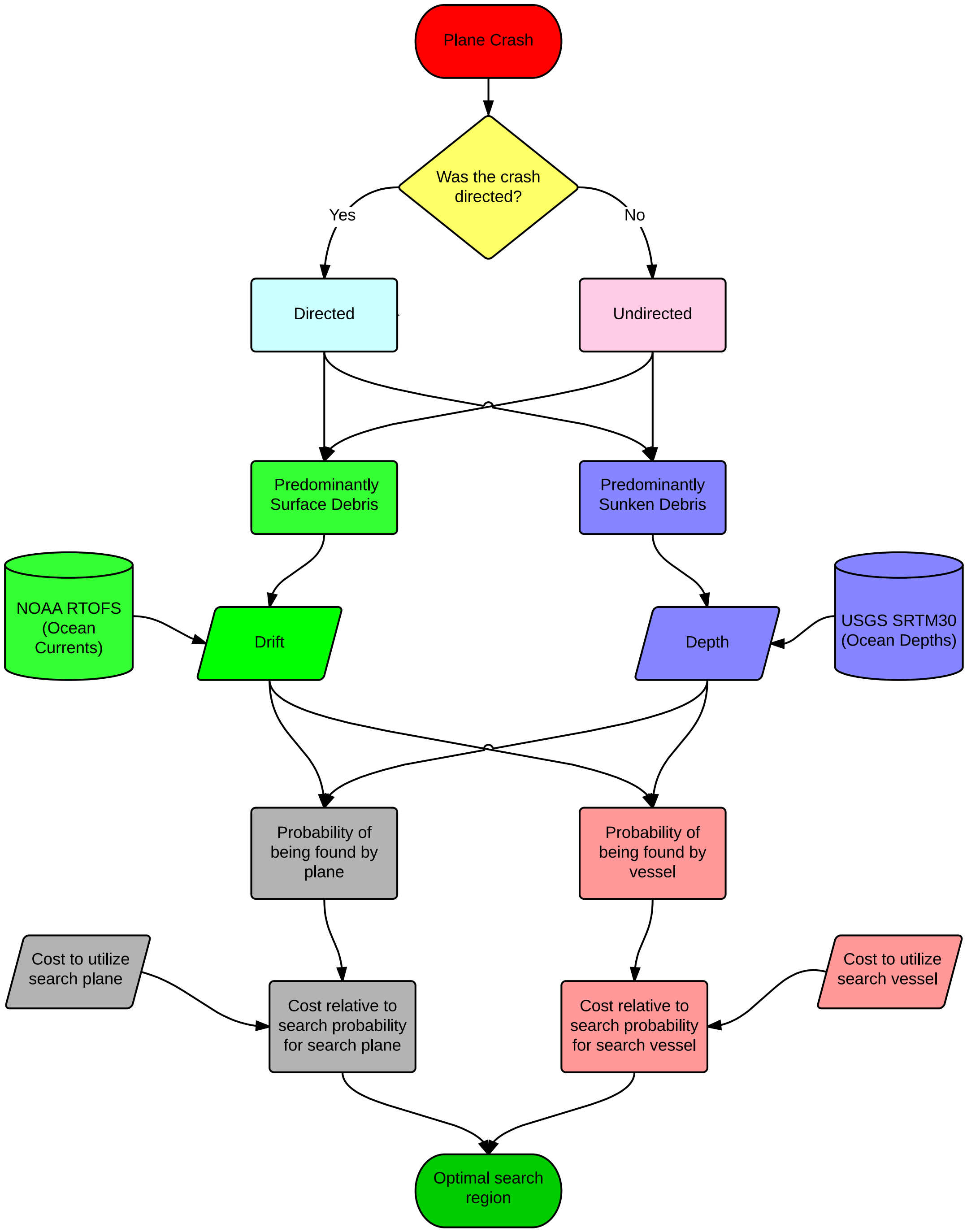


Figure 1: Model process, where unknown but mutually exclusive events are probabilistically combined

3.1 Location of the wreckage

When the only information available is the start and intended end of the aircraft's route, we assume that an emergency occurs at any point along this route with uniform probability. The emergency is modeled as either *directed* or *undirected*; what happens next depends on the type of the emergency.

3.1.1 Directed emergencies

In an emergency situation where a pilot still has control of the aircraft, he or she will divert to the nearest airport to attempt an emergency landing. We call this a *directed emergency*. The nearest airport may be one of the airports planned for departure or arrival, but it also may be a third airport in another location.

In our model a directed emergency happens with constant probability p_{directed}. The craft then crashes at any location along the trajectory towards the nearest airport according to a truncated (half) normal distribution with variance σ_{dist}.

To visualize possible trajectories in a directed emergency, we consider a Voronoi diagram, which partitions the plane into polygons according to the closest airport. The Voronoi diagram can be efficiently calculated for any set of points in a plane as the dual of the Delauney triangulation. Figure 2 diagrams possible directed emergency trajectories along a route using a Voronoi diagram.

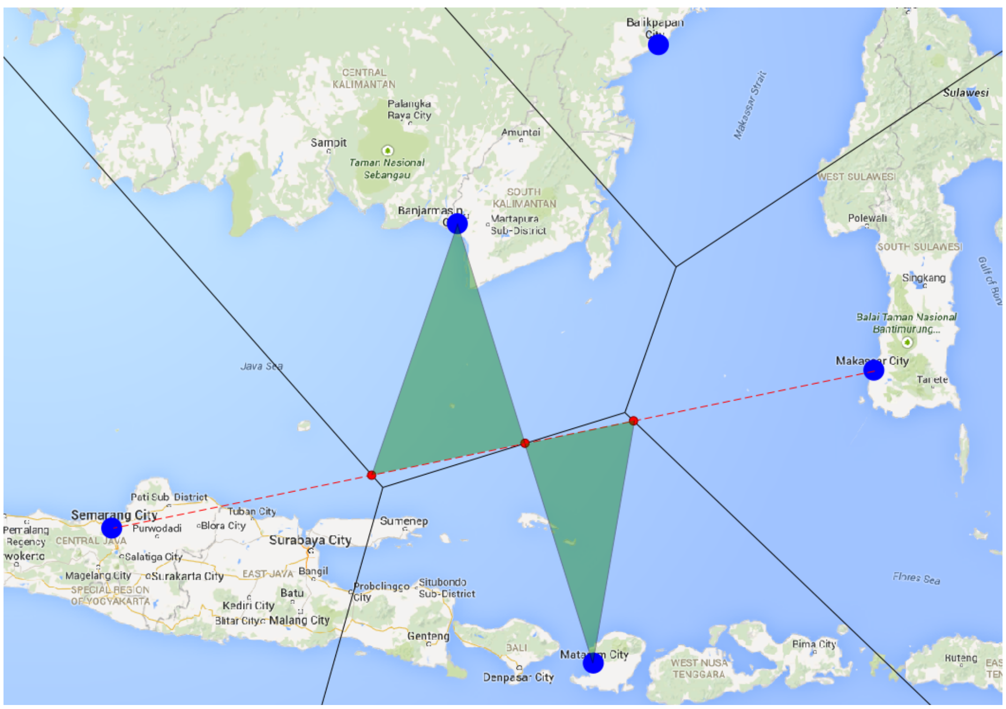


Figure 2: The Voroni diagram for five airports (blue dots) around the Java sea. A route is shown in red, and the two green triangles indicate potential crash sites that might result from diverting to the nearest airport due to a controlled emergency.

3.1.2 Undirected emergencies

Emergencies which are not directed are *undirected*, and occur with constant probability $p_{\text{undirected}} = 1 - p_{\text{directed}}$. In these emergencies, the plane is unable to locate or steer towards the nearest airport, and travels in any direction with equal probability. The craft ultimately crashes along this direction according to a (half) normal distribution with variance $\sigma_{\text{dist long}}$.

3.1.3 Sunken wreckage

Sunken wreckage occurs when the aircraft impacts smoothly and sinks to the ocean floor. These crashes do not create significant breakup which could be identified on the surface. Sunken wrecks do not drift in the ocean current, but the ability to detect them is affected by the ocean depth (see below). In this model, a crash results in a sunken wreck with constant probability α_{snkn}. The probability density of observing a sunken wreckage at any location is given by $p_{snkn}(x, y)$.

3.1.4 Surface wreckage and drift

Surface wreckage occurs when the aircraft impacts destructively and breaks in to many small pieces which float on the surface. In this model, a crash results in a surface wreck with constant probability $\alpha_{srfc} = 1 - \alpha_{snkn}$.

After the wreck occurs, surface wreckage begins to drift according to the surface velocity of the ocean current. As such, the probability density of finding a surface wreckage at any given location must be taken as a function of time $p_{sfrc}(x, y, t)$.

3.2 Discovery probabilities

Even if searchers were checking the right areas where the wreckage actually was, there's still a possibility that the wreckage could be overlooked. For instance, due to its vantage point, a search vessel would have limited visibility for spotting floating debris. Similarly, a search plane's sensors may not be powerful enough to detect sunken wreckage in deep water. Thus, we consider the problem of actually locating wreckage in different component:

$$\begin{aligned}
&\mathbb{P}(t)[\text{search vehicle locating wreckage at}(x, y)] &(1)\\
&\quad = \mathbb{P}[\text{search vehicle locating wreckage at } (x, y) \mid \text{surface debris is at } (x, y)] * \mathbb{P}(t)[\text{surface debris is at}(x, y)] &(2)\\
&\quad + \mathbb{P}[\text{search vehicle locating wreckage at } (x, y) \mid \text{sunken debris is at } (x, y)] * \mathbb{P}[\text{sunken debris is at } (x, y)]
\end{aligned}$$

3.2.1 Search plane discovering wreckage

To derive $\mathbb{P}(t)[\text{search plane locating wreckage at}(x, y)]$, we multiply our previous estimates of the probable locations of wreckage floating on the surface at a given time

$$p_{srfc}(x, y, t) = \mathbb{P}(t)[\text{surface debris is at}(x, y)]$$

with a constant representing the probability that a search plane would successfully locate the floating wreckage if it were at the location it is searching

$$p_{sp,srfc} = \mathbb{P}[\text{search plane locating wreckage at } (x, y) \mid \text{surface debris is at } (x, y)]$$

to obtain the probability that the search plane will locate surface wreckage.
Similarly, we multiply our previous estimates of the probable locations of sunken wreckage

$$p_{snkn}(x, y) = \mathbb{P}[\text{sunken debris is at}(x, y)]$$

with a function representing the probability that a search plane would successfully locate the sunken wreckage if it were at the location it is searching

$$p_{sp,snkn}(x, y, d) = \mathbb{P}[\text{sunken debris is at } (x, y)]$$

to obtain the probability that the search plane will locate sunken wreckage. We assumed that search planes' ability to detect sunken wreckage was provided by a magnetic anomaly detector, whose detection range is known to drop at a rate proportional to the third power of the object's distance. Thus, we fit simple inverse cubic function with parameter c_{MAD} to researched magnetic anomaly detector data to provide a search plane's sunken detection probability:

$$p_{sp,snkn}(x, y, d) = \frac{p_{sp,srfc}}{1 + c_{MAD} * (sp_{alt} + d(x, y))^3}$$

Combining these using Eq. 1, **the overall probability of a search plane successfully locating the wreckage at time t in location (x, y) is**:

$$p_{sp}(x, y) = p_{sp,srfc} * p_{srfc}(x, y, t) + \frac{p_{sp,srfc}}{1 + c_{MAD} * (sp_{alt} + d(x, y))^3} * p_{snkn}(x, y)$$

3.2.2 Search vessel discovering wreckage

To derive $\mathbb{P}(t)$[search vessel locating wreckage at(x, y)], we multiply our previous estimates of the probable locations of wreckage floating on the surface at a given time

$$p_{srfc}(x, y, t) = \mathbb{P}(t)[\text{surface debris is at}(x, y)]$$

with a constant representing the probability that a search vessel would successfully locate the floating wreckage if it were at the location it is searching

$$p_{sv,srfc} = \mathbb{P}[\text{search vessel locating wreckage at } (x, y) \mid \text{surface debris is at } (x, y)]$$

to obtain the probability that the search plane will locate surface wreckage.
Similarly, we multiply our previous estimates of the probable locations of sunken wreckage

$$p_{snkn}(x, y) = \mathbb{P}[\text{sunken debris is at}(x, y)]$$

with a constant representing the probability that a search vessel would successfully locate the sunken wreckage if it were at the location it is searching

$$p_{sv,snkn} = \mathbb{P}[\text{sunken debris is at } (x, y)]$$

to obtain the probability that the search vessel will locate sunken wreckage.

Combining these using Eq. 1, **the overall probability of a search plane successfully locating the wreckage at time t in location (x, y) is**:

$$p_{sv}(x, y) = p_{sv,srfc} * p_{srfc}(x, y, t) + p_{sv,snkn} * p_{snkn}(x, y)$$

3.3 Search cost models

The two types of search vehicles, search planes and search vessels, have different models for their typical operational costs. These are vastly simplified formulations of operational costs, but capture the general scaling of the expenses.

3.3.1 Search plane operation costs

The search process for an aerial vehicle consists of traveling from an airbase to the search zone, scanning the search zone, and returning to the airbase. One such trip is called an *excursion*.

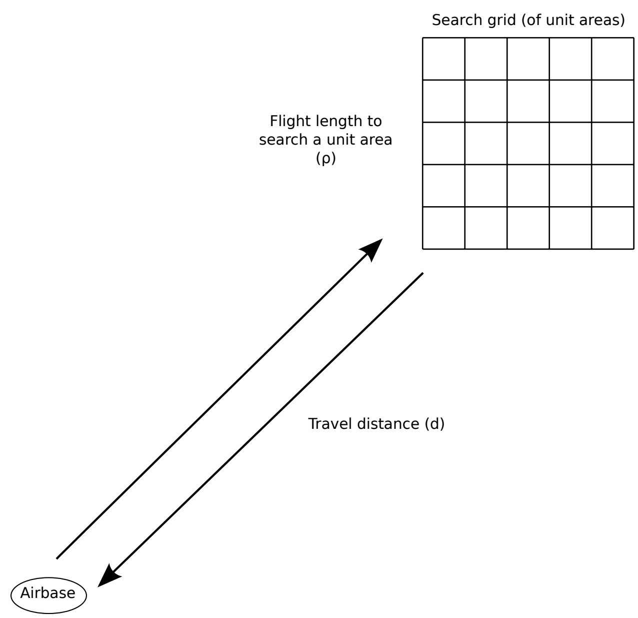


Figure 3: A diagram of the rudimentary model used to estimate search costs when using aerial vehicles.

For a trip of this form, the number of unit grid squares which can be searched in an excursion is given by

$$N(d) = \frac{R_o - 2D}{\rho}$$

where D is the distance of the search location from the airbase, R_o is the maximum distance that can be traveled in an excursion, and ρ is the distance which must be traveled to scan a unit grid square.

Thus the cost per unit grid square when searching $N(D)$ contiguous grid squares is given by

$$C_{\text{sp}}(D) = \frac{C_{\text{operation}}}{N(D)} = \frac{\rho C_{\text{operation}}}{R_o - 2D}$$

where $C_{\text{operation}}$ is the monetary cost of running an excursion.

While simple, this relation has all of the properties we would expect for the search cost. The cost of searching an area increases dramatically near the limit of the operational range, and it is impossible to search an area beyond the operational range.

We consider the cost of searching a unit area to be the minimum of the costs from any airport. Note that this may be ∞ if the location is not within the operational range of any airport.

3.3.2 Search vessel operations costs

The cost for a oceangoing search vessel to search a unit area is taken as a constant. Because vessels engage in long-term voyages, the distance from port is not a significant factor in their search cost. The constant C_{sv} gives the cost of searching a unit area with a vessel.

3.4 Summary of model constants

Several constants of varying forms were assumed in the previous derivation of the model. For some of these constants, we were able to confidently research exact values, for others we made an informed estimate drawing on related data. In nearly all cases, the constants could be made more accurate via consultation with a domain expert.

Table 2 contains a summary of all constants, the values used in our implementation of the model, and the sources thereof.

Constant	Meaning	Value	Justification
α_{snkn}	Probability that the wreckage primarily sinks to the ocean floor	.13	[2]
$p_{sp,srfc}$	Probability of search plane locating floating debris in search area	.90	[3]
c_{MAD}	Parameter for magnetic anomaly detector range	3.435×10^{-8}	[4]
$p_{sv,srfc}$	Probability of search vessel locating floating debris in search area	.66	[3]
$p_{sv,srfc}$	Probability of search vessel locating sunken debris in search area	.98	[5]
sp_{lat}	Effective lateral search range of a search plane	830m	[4]
sp_{alt}	Altitude that the search plane flies at	400m	[6]
$C_{operation}$	The cost to make a full-range flight in a search plane	$33,463	(Sec. 3.4.1)
C_{sv}	The cost to utilize a search vessel per km	$34.23	(Sec. 3.4.2)

Table 2: The values assigned to the constants in our implementation of the model.

3.4.1 Search plane cost estimation

As search missions are generally conducted by militaries, we used the P-3 Orion, one of the primary planes used to search for MH370, as a basis for calculating search plane costs. The P-3 Orion runs on JP-5 fuel and has a total fuel capacity of 9194 gallons [7]. Assuming the standard price for a gallon of JP-5 fuel [8], we calculate the total cost to make a full-range flight in a search plane to be $C_{operation} = \$33,463$.

3.4.2 Search vessel cost estimation

We estimate the cost of operating a search vessel to be the cost of a Coast Guard patrol boat, $\$1,147$ per hour, divided by the average speed of a large coast guard vessel: $1147 per hour/33.5 km per hour = $34.23 per km [9].

4 Data Sources

The process of designing models must begin with considering the available data and an assessment of its quality. A significant amount of data is required to reasonably model potential oceanic crash locations. However, the scale of this relatively uncharted geography poses significant challenges with respect to data acquisition and utilization

The process of designing models must begin with considering the available data and an assessment of its quality. A significant amount of data is required to reasonably model potential oceanic crash locations. The scale of the problem forces us to intelligently choose relevant data in aiding how we scope our model.

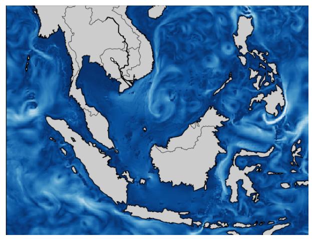
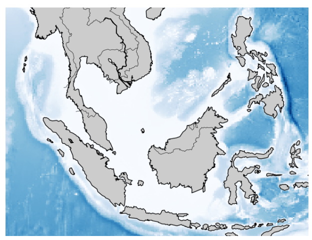
Data	Resolution	Visualization	Source	Ref.
Land/Ocean mask	*variable*		SOEST GSHHG Dataset (via `Basemap`)	[10]
Ocean surface velocities	30 arcsec		NOAA Global RTOFS Dataset	[11]
Ocean floor depth	30 arcsec		USGS EROS SRTM30 Dataset	[12]

Table 3: Sources for datasets over the Earth's surface.

5 Model Analysis

5.1 Model Assumptions and Limitations

– **Map Projections**
The spherical nature of Earth's surface cannot be perfectly mapped to a Cartesian plane, a wide variety of projections have been developed, each of which has its own shortcomings. In this work, we initially transform all longitude and latitude coordinates to a cartesian grid according to a Miller Cylindrical projection, and perform all subsequent calculations on this grid.

We note that the Miller Cylindrical projection is accurate near the equator, the region of interest for our case study. Alternate projections could be used to analyze wrecks near the poles.

5.1.1 Wreckage Location Assumptions

– **Route trajectories are straight lines**
In reality planes often follow curved trajectories to minimize distance of a sphere, or deviate further to utilize favorable wind currents.

– **There are only two behaviors of a crashing aircraft**
We assume that as soon as the crash-inducing malfunction occurs, the pilots either:

1. Lose primary control over the aircraft, in which case it begins to go down immediately
2. Retain enough control over the aircraft that they are able to head towards the nearest airport

– **Every crash is either a surface crash or a sunken crash**
In reality, most crashes will leave some amount debris on the surface as well as underwater. To facilitate our probabilistic interpretation, we assume that one one of these happens, according to a constant probability.

– **Ocean current is the only force that can act on the debris of a crashed plane**

– **The crash stays intact and resistant to scattering through the duration of the study**
Though our model takes into account ocean current for the moving of wreckage, we assume that the wreckage stays intact after the crash. In reality, remains of a plane crash are susceptible to scattering and further breaking.

5.1.2 Search Vehicle Assumptions

– **Search planes have only two methods of detecting wreckage**
In a search plane's feasible lateral range, the available methods of wreckage-detection are:

1. Visual recognition of surface debris by any means (e.g., binoculars, cameras, etc.). The probability of a true-positive visual wreckage detection is constant ($p_{sp,srfc}$).
2. Sensor detection of sunken debris, where the sensors are assumed to consume geomagnetic information (i.e., MAD: Magnetic Anomaly Detectors). The probability of a true-positive wreckage detection ($p_{sp,snkn}$) with these sensors is a function of only the vertical separation between the search plane and the wreckage.

– **Search vessels have only two methods of detecting wreckage**
In a search vessels' feasible lateral range, the available methods of wreckage-detection are:

1. Visual recognition of surface debris by any means (e.g., binoculars, cameras, etc.). The probability of a true-positive visual wreckage detection from a search vessel is constant ($p_{sv,srfc}$).
2. Sensor detection of sunken debris, where the senors are assumed to consume acoustic information (i.e., sonar tows). From the research done, we confidently assume that a search vessel's probability of a true-positive wreckage detection ($p_{sv,snkn}$) is equal to 1 when passing over a suspect location.

5.1.3 Search Cost Assumptions

– **The cost of utilizing search planes depends only on fuel expenditure**
We do not take into account additional operational costs that go into using search planes, such as the employment costs of pilots and other crew, maintenance costs, or vehicle/equipment rental costs during the efforts.

– **The fuel expenditure rate of a search plane is constant during a flight**
Typically, a single mission run by a search plane has three components: traveling to the search area, performing a grid-search over the area, and then traveling back. To maximize fuel efficiency on the way to and from the search area, the search plane will travel at its cruising altitude/speed. Once it reaches the search area, it will travel at a low altitude and slow speed in order to maximize the probability of spotting wreckage. In reality, these scenarios consume different amounts of fuel per unit traveled; our model assumes that an average of these consumption rates into a single value will suffice.

- **The cost of utilizing search vessels is constant for a fixed search area**
  While we acknowledge that there are many factors that may affect the cost of using a search vessel to locate crash wreckage (such as the size of the crew, the time at sea, the equipment being used, and fuel consumed), we assume that a single constant can sufficiently summarize these factors' costs.

5.2 Model Strengths

- We **make use of ocean current data** to determine where surface wreckage may be at any given time.
- Our usage of Voronoi diagrams allows us to **account for the possibility of planes altering their course to reach the nearest airport**. This incorporates the typical recovery behavior of a pilot in a partial emergency situation.
- Our model for search vehicles locating crash wreckage utilizes data collected from the past 15 years when calculating the probabilities of types of crashes.
- The results that our model yields goes beyond just probability estimates to point searchers in the most probable locations. It additionally **informs searchers of the costs associated with searching each location**, allowing them to immediately and directly plan desired search routes.

5.3 Model Extensions

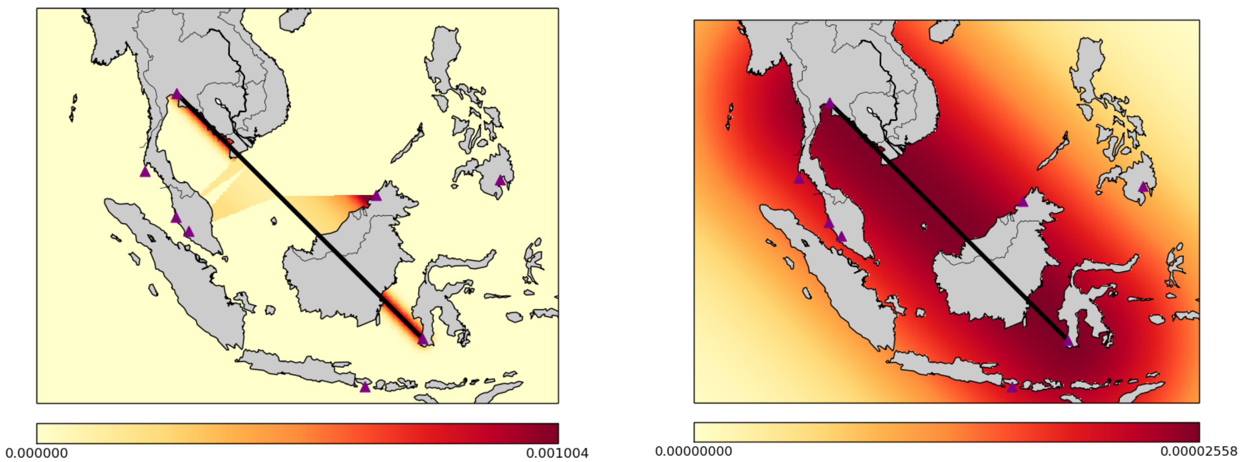
This model does not solve the scheduling problem of actually routing search vessels and planes. However, the results from this model could be easily used as the input into such a scheduling algorithm which could attempt to balance any combination of cost and probability criterion that is desired at that time.

Once such a scheduling algorithm is put into use, location probabilities of the wreckage could be updated as areas are searched via down-scaling the probabilities of already-searched locations through Bayesian updating. Moreover, our forward-drift calculations for wreckage could then be combined with the reverse-drift calculations of [1] to pinpoint the original crash location for any analysis that may need to be done there.

6 Model Results

To demonstrate the application of this model, we apply to a hypothetical crash incident, reminiscent of MH370. The model is implemented in the `Python` programming language and run using the datasets referenced above.

Figure 4 shows the initial distribution of potential crash locations broken down in to controlled and uncontrolled emergencies.

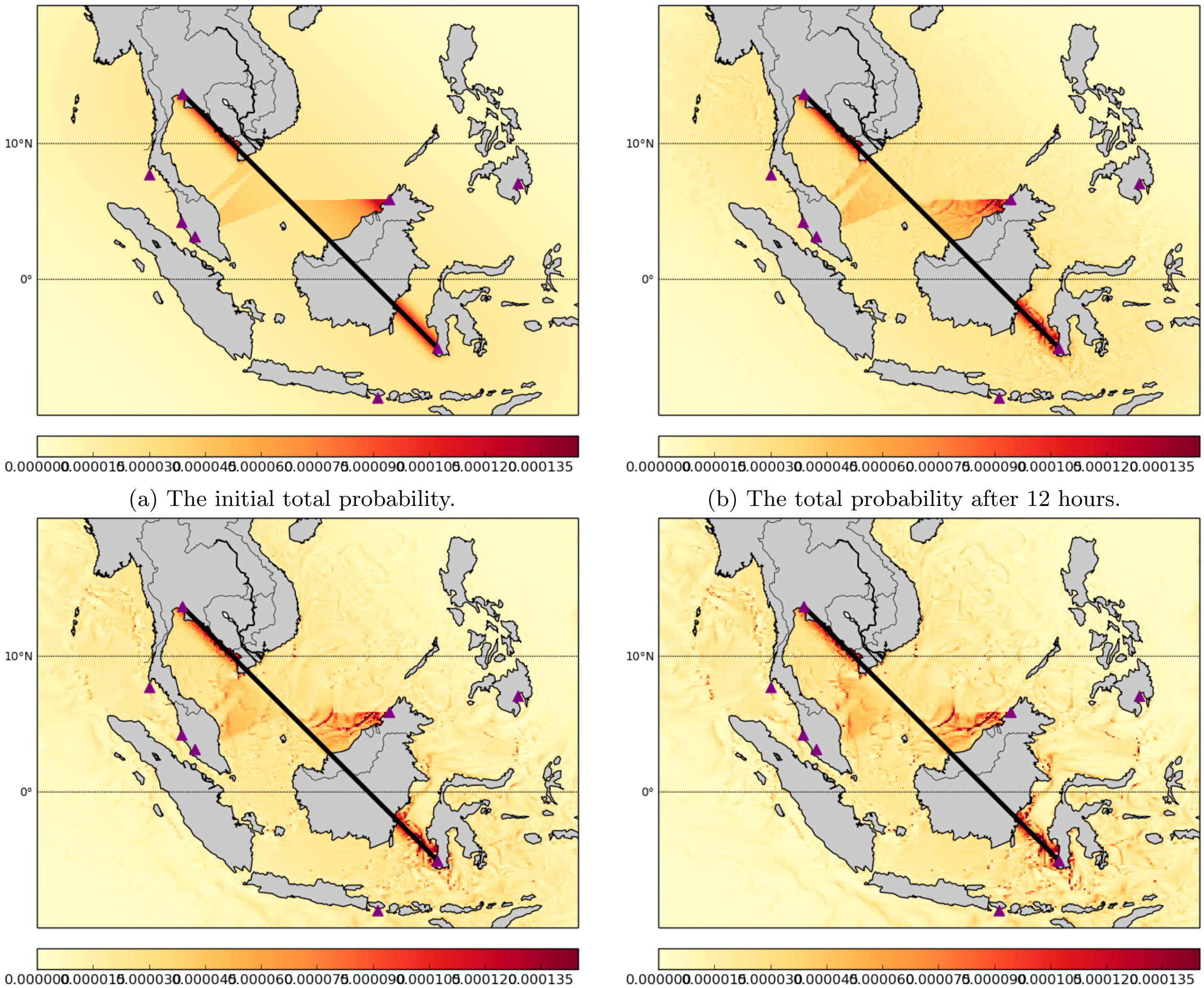


(a) The initial probability density of locations for a controlled crash. Note the clear triangular paths to nearby airports.

(b) The initial probability density of locations for an uncontrolled crash.

Figure 4

Figure 5 shows the composite initial distribution which includes both controlled and uncontrolled crashes. Additionally, it shows the change of this probability with time as the surface crashes drift with the current.



(a) The initial total probability.

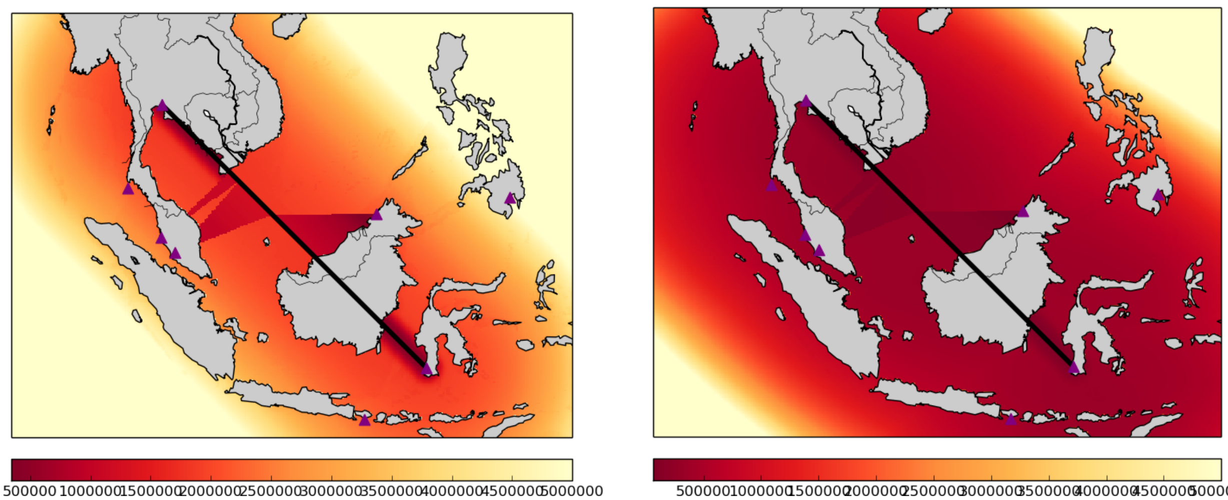
(b) The total probability after 12 hours.

(c) The total probability after 24 hours.

(d) The total probability after 36 hours.

Figure 5: The probability of sunken or surface residing at a given location over time. The surface probability is affect by the ocean currents.

Figure 6 shows the ultimate recommendations of the model, highlighting areas with the highest likelihood of finding the plane given the cost.



(a) The darker regions are the optimal areas to search with a plane.

(b) The darker regions are the optimal areas to search with vessel.

Figure 6

7 Conclusions

By taking into account what we considered to be the most influential factors involved in the search for a missing plane, we were able to build a general mathematical model for locating missing planes over large bodies of water and providing economic guidance to the associated search team.

With only the plane's origin and destination points, and assuming no additional knowledge about the individual plane crash except that it is feared to have crashed over water, our model is able to functionally consider many realistic factors that would make the potentially intractable search much more feasible.

Starting even before the crash itself, the model considers factors such as the pilot's ability to control the plane upon initial malfunction. It additionally takes into account how likely a "safe" water-landing was (as opposed to a crash-landing that destroys the plane on impact).

It then uses ocean drift and depth data to determine where floating wreckage may have drifted and how deep wreckage may have sunk.

Considering the differing capabilities of various search vehicles, the model determines where search planes would be most likely to find the wreckage versus where search vessels would be most likely to find it.

As seen in the MH370 search, it is not always feasible to simply send the best vehicle to where it is most likely to find the wreckage. With budget constraints, it may be more economical to utilize a more complex search strategy. This model determines costs of searching locations with the different search vehicles, which can be used by a scheduler (or scheduling algorithm) on the search team to economically optimally locate the wreckage.

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