Homework 1

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Problem 1:

Katz centrality us defined as

$$\boldsymbol{c}_{Katz} = \beta(\boldsymbol{I} - \alpha \boldsymbol{A})^{-1}\overrightarrow{\mathbf{1}} \tag{1}$$

and the expansion of the RHS in the above equation is defined as

$$\boldsymbol{c} = (\mathbf{I} - \alpha A)^{-1}\mathbf{1} = \mathbf{1} + \alpha A\mathbf{1} + \alpha^2 A^2\mathbf{1} + \dots \tag{2}$$

where β is constant. The divergence of the series would happen when $\det(\mathbf{I} - \alpha A) = 0$, whose roots are α^{-1} which are eigen value of A.
The limit at which series would diverge is when $\alpha = 1/\kappa$ where κ is the largest principle eigen value of A. Hence equation 1 will converge for $\alpha \in \left[0, \kappa^{-1}\right)$ as for α when 0 would make katz centrality uniform throughout the graph based on constant β.

Problem 2:

For finding common neighbours we can use the concept of walk with length 2. Hence, relation to compute total number of common neighbours $|N(v_i) \cap N(v_j)|$ between nodes v_i and v_j is given as

$$\begin{aligned} N_{ij}^{(2)} &= \sum_{k=1}^{n} A_{ik} A_{kj} \quad \text{...From lec 1 slides} \\ &= \left[A^2\right]_{ij} \end{aligned} \tag{3}$$

Problem 3:

Jaccard's local overlap similarity is given as

$$S_{ij}^{\text{Jaccard}} = \frac{|N(v_i) \cap N(v_j)|}{|N(v_i) \cup N(v_j)|} \tag{4}$$

Where numerator $|N(v_i) \cap N(v_j)|$ is given by equation 3 and the denominator $|N(v_i) \cup N(v_j)| = d_i + d_j - |N(v_i) \cap N(v_j)|$. Here d_i is the degree of vertex v_i.

Coding for the same, one gets similarity matrix as,

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	1.000000	0.000000	0.000000	0.000000	0.000000	0.500000	0.333333	0.333333	0.333333	0.500000	0.000000	0.000000	0.000000	0.000000	0.000000
1	0.000000	1.000000	0.125000	0.000000	0.111111	0.000000	0.125000	0.125000	0.000000	0.000000	0.166667	0.000000	0.250000	0.166667	0.000000
2	0.000000	0.125000	1.000000	0.200000	0.166667	0.000000	0.200000	0.000000	0.000000	0.000000	0.000000	0.500000	0.000000	0.000000	0.000000
3	0.000000	0.000000	0.200000	1.000000	0.400000	0.250000	0.200000	0.000000	0.000000	0.000000	0.000000	0.200000	0.166667	0.000000	0.000000
4	0.000000	0.111111	0.166667	0.400000	1.000000	0.200000	0.000000	0.166667	0.000000	0.000000	0.000000	0.166667	0.142857	0.000000	0.000000
5	0.500000	0.000000	0.000000	0.250000	0.200000	1.000000	0.250000	0.250000	0.250000	0.333333	0.000000	0.000000	0.000000	0.000000	0.000000
6	0.333333	0.125000	0.200000	0.200000	0.000000	0.250000	1.000000	0.200000	0.200000	0.250000	0.000000	0.200000	0.166667	0.000000	0.000000
7	0.333333	0.125000	0.000000	0.000000	0.166667	0.250000	0.200000	1.000000	0.500000	0.250000	0.000000	0.200000	0.000000	0.000000	0.333333
8	0.333333	0.000000	0.000000	0.000000	0.000000	0.250000	0.200000	0.500000	1.000000	0.250000	0.000000	0.200000	0.000000	0.000000	0.333333
9	0.500000	0.000000	0.000000	0.000000	0.000000	0.333333	0.250000	0.250000	0.250000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000
10	0.000000	0.166667	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000
11	0.000000	0.000000	0.500000	0.200000	0.166667	0.000000	0.200000	0.200000	0.200000	0.000000	0.000000	1.000000	0.000000	0.000000	0.333333
12	0.000000	0.250000	0.000000	0.166667	0.142857	0.000000	0.166667	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.250000	0.000000
13	0.000000	0.166667	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.250000	1.000000	0.000000
14	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.333333	0.333333	0.000000	0.000000	0.333333	0.000000	0.000000	1.000000

Figure 1: Similarity matrix

and the labels for similarity matrix 1 is given by figure 2.

and similarity between "Ginori" family and other families in the Florentine Families graph is given by figure 3,

```
00: 'Acciaiuoli'
01: 'Medici'
02: 'Castellani'
03: 'Peruzzi'
04: 'Strozzi'
05: 'Barbadori'
06: 'Ridolfi'
07: 'Tornabuoni'
08: 'Albizzi'
09: 'Salviati'
10: 'Pazzi'
11: 'Bischeri'
12: 'Guadagni'
13: 'Ginori'
14: 'Lamberteschi'
```

Figure 2: Label for similarity matrix

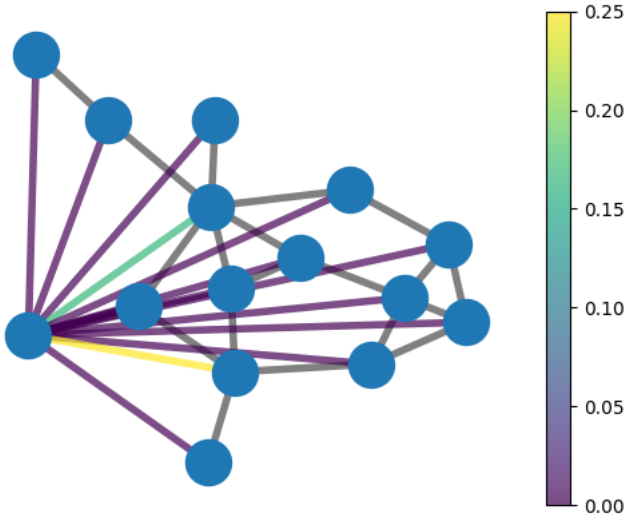

Figure 3: similarity between "Ginori" family and other families