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Abstract

1 Definitions

- M : Differentiable manifold representing the narrative space.
- g : Metric tensor on M .
- $c \in M$: A concept embedding, a point on the manifold.
- $\gamma : [0, 1] \rightarrow M$: A smooth, parameterized path on the manifold, representing a narrative.
- γ_T : Geodesic path representing the "ground truth" narrative.
- LCM_i : Large Concept Model of agent a_i , which generates narrative paths.
- a_i : Agent i .
- A_i : Tensor parameters encoding knowledge, beliefs, and influence, specific to agent a_i within the LCM.
- I : Influence tensor field over M .
- $V : M \rightarrow [0, 1]$: Veracity function.
- $T \subset M$: "Ground truth" region on the manifold.

2 Narrative Dynamics

Definition 1 (Narrative Path). *A narrative $N_{i,t}$ of agent a_i at time t is represented as a path $\gamma_{i,t}$ on the manifold M :*

$$\gamma_{i,t}(s) = (c_{i,1}, c_{i,2}, \dots, c_{i,T}) \quad (1)$$

where $\gamma_{i,t}(0) = c_{i,1}, \gamma_{i,t}(1/T) = c_{i,2}, \dots, \gamma_{i,t}(1) = c_{i,T}$, and $c_{i,k}$ are concept embeddings generated by LCM_i .

Definition 2 (Narrative Generation by LCM). *The next concept embedding $c_{i,k+1}$ in the narrative path is generated by the LCM as follows:*

$$c_{i,k+1} = LCM_i(c_{i,1:k}, K_{i,t}, B_{i,t}, C_t, A_i) \quad (2)$$

where $c_{i,1:k}$ is the sequence of preceding concept embeddings, $K_{i,t}$ is the agent's knowledge set, $B_{i,t}$ is the agent's belief set, C_t is the context at time t , and A_i are agent-specific parameters.

3 Deviation from Ground Truth

Definition 3 (Geodesic Deviation Score (GDS)). *The Geodesic Deviation Score measures the deviation of a narrative path $\gamma_{i,t}$ from the "ground truth" geodesic γ_T :*

$$GDS(\gamma_{i,t}, \gamma_T) = \int_0^1 \|\dot{\gamma}_{i,t}(s) - \dot{\gamma}_T(s)\| ds \quad (3)$$

where $\dot{\gamma}$ represents the tangent vector of the path.

Definition 4 (Torsion). *The torsion of a narrative path $\gamma_{i,t}$ is a measure of its "twisting":*

$$Torsion(\gamma_{i,t}) = \int_0^1 |\tau(s)| ds \quad (4)$$

where $\tau(s)$ is the torsion of the path at parameter s .

4 Veracity Function

The veracity function V can be redefined to incorporate geometric measures:

$$V(\gamma_{i,t}) = w_1 \cdot (1 - GDS(\gamma_{i,t}, \gamma_T)) + w_2 \cdot (1 - Torsion(\gamma_{i,t})) + \dots \quad (5)$$

where w_1, w_2, \dots are weights assigned to each factor.

5 Narrative Dynamics

Definition 5 (Narrative Path). *A narrative $N_{i,t}$ of agent a_i at time t is a path $\gamma_{i,t}$ on the manifold M .*

Definition 6 (Narrative Evolution). *The evolution of a narrative path is governed by a flow on the manifold:*

$$\frac{dN_{i,t}}{dt} = F(N_{i,t}, A_i, I(N_{i,t})) \quad (6)$$

where F is a function that describes the dynamics of the narrative flow, influenced by the agent's internal parameters A_i and the influence tensor field I at the current narrative point.

6 Influence Tensor Field

The influence tensor field I can be defined at each point $c \in M$ to represent the potential for influence or manipulation:

$$I(c) = \sum_{j \neq i} \alpha_{ij} \cdot G_{j(c)} \quad (7)$$

where:

- α_{ij} : Influence weight of agent a_j .
- $G_{j(c)}$: Influence tensor of agent a_j at point c , encoding the magnitude, direction, and source of influence.

7 Veracity and Geometry

$$V(\gamma_{i,t}) = w_1 \cdot (1 - GDS(\gamma_{i,t}, \gamma_T)) + w_2 \cdot (1 - \text{Torsion}(\gamma_{i,t})) + \dots \quad (8)$$

where:

- $GDS(\gamma_{i,t}, \gamma_T) = \int_0^1 \|\dot{\gamma}_{i,t}(s) - \dot{\gamma}_T(s)\| ds$ (Geodesic Deviation Score)
- $\text{Torsion}(\gamma_{i,t}) = \int_0^1 |\tau(s)| ds$ ($\tau(s)$ is the torsion of the path)