First results : Michelson Lambda meter

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April 2021

Table of Content

Introduction

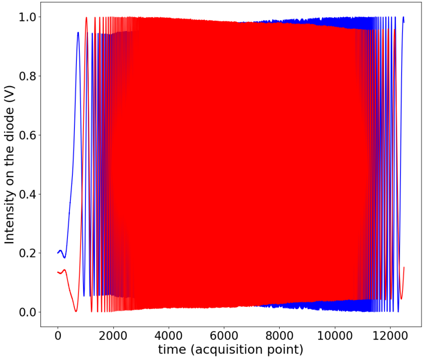
We give ourselves a Michelson interferometer with two input wavelength : $\lambda_{IR} = 1529nm$ and $\lambda_{NIR} = 780nm$ (precisely known).

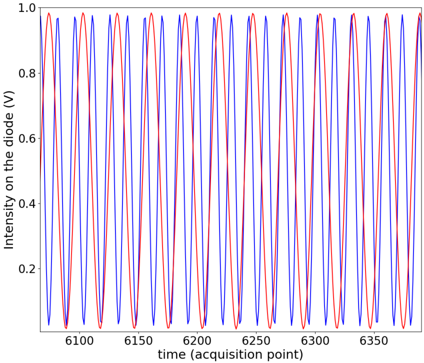
The protocol will be explained : we can compute the value of $\lambda_{IR} = 1529nm$ measuring the way both rays (IR and NIR laser) interfere with themselves in the Michelson modifying the optical path.

We scan the optical path difference with a given length $L = 120\mu m$, each scan permit the computation of λ_{IR} and thus define a "measurement". All error bars are the estimated standard error of the mean. Until we say the contrary, one measurement has a duration of 50 ms and acquire 12.500 points.

Computation Protocol

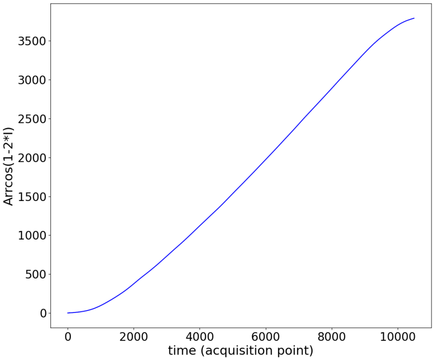
We measure the intensity of the interfering rays of the Michelson on a photodiode.

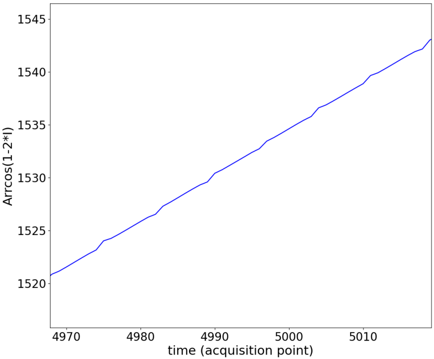




The signal have the form $I_0 sin^2(k\delta(t) + \phi_0)$ with $\delta(t)$ the optical dephasing path of the Michelson scanned.

We can then extract the phase applying an arrcos to $2I - 1$ (being of the form cos(x), we have to re normalize between -1 and 1) with I the signal. We do that for both signals. We then apply $np.unwrap((2I-1)sign(\frac{d(2I-1)}{dx})$ in order to reconstruct the phase :





Here we explain how we unwrap a phase from a cos(x) function.

Sketch of an arrccos(cos(x)) function

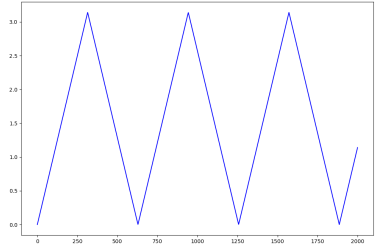


Figure: arrcos(cos(x)))

Sketch of an $arrcos(cos(x))sign(\frac{dcos(x)}{dx})$ function : we can apply the unwrap function "jumps" of π

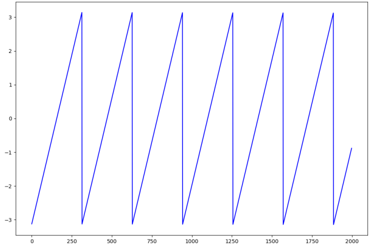


Figure: $arrcos(cos(x))sign(\frac{dcos(x)}{dx})$

We then plot $\phi_{IR}(\phi_{NIR})$ and fit a linear curve because :

$\phi_{IR} = k_{IR}\delta(t) + \phi_{0,IR}$ and $\phi_{NIR} = k_{NIR}\delta(t) + \phi_{0,NIR}$

And so that means that $\phi_{NIR} = \frac{k_{IR}}{k_{NIR}}\phi_{IR} + C$

Given a value of k_{IR}, we can determine the value of k_{IR} which contain λ_{IR} with a linear fit.

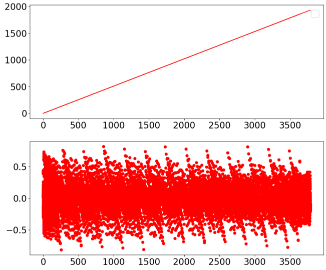


Figure: $\phi_{IR}(\phi_{NIR})$ and the "dispersion" of the linear fit

Relative Precision

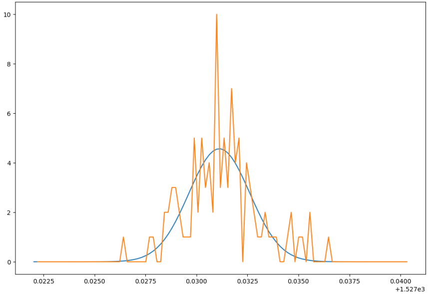
Precision on a single measurement :
We have a mean standard deviation on 16 different subset of 100 measurement each of 0.0016 nm, so a relative precision of $0.0016/1527{=}1.047.10^{-6}$ (Can be seen in next slides)

Precision on a 100 measurement set :

The Gaussian fit of a 100 measurement gives us a σ=0.0015 nm which have to be divided by $\sqrt{N}$ with N=100.
So we have a relative precision $= \frac{\sigma/\sqrt{N}}{\lambda} = \frac{0.0015/10}{1527} = 9.8x10^{-8}$ on a 100 measurement set.

Gaussian fit

Precision on a single measurement :
We take 1600 measurement and divide it in subsets of 100 measurements.
The Gaussian fit of a 100 measurement gives us a $\sigma = 1.494.10^{-3}$ nm which have to be divided by $\sqrt{N}$ with N=100.
So we have a relative precision $= \frac{\sigma/\sqrt{N}}{\lambda} = \frac{0.0015/10}{1527} = 9.8x10^{-8}$ on a 100 measurement set.



Precision on a single measurement

We locked the NIR laser (at 780.246452 nm in void, with a Rb cell) in this 3200 measurement (divided in 32 subsets). The error is the SE_M. The mean and sigma are from statistical process.

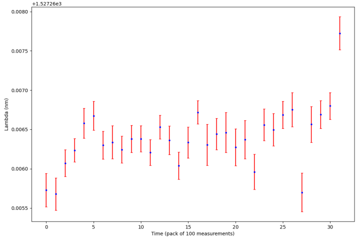


Figure: λ_{IR} (nm) function of the subset's number

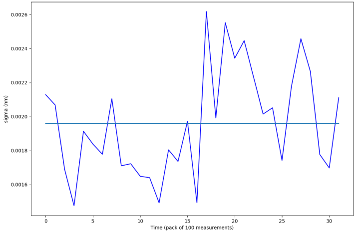


Figure: $sigma_{IR}$ (nm) function of the subset's number

Mean standard deviation on 32 different subset (100 measurement each) of 0.0020 nm, so a relative precision of a single measurement of $0.0020/1527{=}1.30976.10^{-6}$

NIR locked Gaussian Fit

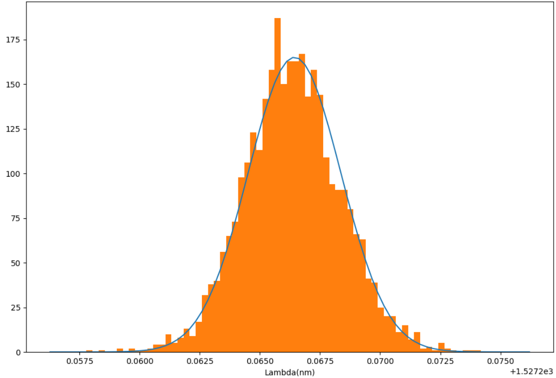


Figure: Histogramm λ_{IR} (nm) fitted with a Gaussian curve

$\lambda_{IR} = 1527.03112\text{nm}$, $\sigma_{IR} = 1.49404036.10^{-3}\text{nm}$

Systematic Errors

There are at least 2 systematic errors that we have to take into account :
1) The error on the optical index of air, which is computed with the Eldén formula.

We have $\delta n = 3.10^{-8}$ (of the Eldén formula) and $\delta n_{lab} = 10^{-6}$ (for the laboratory variations approx. -¿ pressure variation)
The algorithm gives us : $F(n_{IR}, n_{NIR}) = \frac{\lambda_{IR,air}(n_{IR})}{\lambda_{NIR,air}(n_{NIR})}$

$$\frac{\partial F}{\partial n_{IR}}(n_{IR}, n_{NIR}) = -\frac{\lambda_{IR,void}\, n_{IR}}{\lambda_{NIR,void}\, n_{NIR}^2}\ ,\ \frac{\partial F}{\partial n_{NIR}}(n_{IR}, n_{NIR}) = \frac{\lambda_{IR,void}}{\lambda_{NIR,void}\, n_{NIR}}$$

So we have : $\delta F = \sqrt{\left(\frac{\partial F}{\partial n_{IR}}(n_{IR},n_{NIR})\delta n_{IR}\right)^2+\left(\frac{\partial F}{\partial n_{NIR}}(n_{IR},n_{NIR})\delta n_{NIR}\right)^2}$

Numerical application : $\delta F = 2.77.10^{-6}$ approximately

2) The error on the misalignment of the IR and NIR rays in the Michelson that induce a different optical path and thus an error on the measurement. If we have $\delta_{IR}(t) = \delta_{NIR}(t) cos(\alpha)$, with α the angle between the two rays, given an uncertainty on α,

$F_0(\alpha) = \frac{\lambda_{IR,air} cos(\alpha)}{\lambda_{NIR,air}}$

$\delta F_0 = \mathrm{F}_0(\alpha = 0) - F_0(\alpha = \alpha_{max}) = (1 - cos(\alpha_{max}))\frac{\lambda_{IR,air} cos(\alpha)}{\lambda_{NIR,air}}$ so

$\delta F_0 = -\frac{\alpha_{max}^2}{2}\frac{\lambda_{IR,air} cos(\alpha)}{\lambda_{NIR,air}}$

with $\delta\alpha_{max} = 0.004$ rad, we have an uncertainty of 1 mm over 0.5m

Numerical application : $\delta F_0 = 3.136.10^{-5}$

What happen if we reduce the time of one measurement ? Acquisition card : maximum acquisition rate of 250.000 points per second

-50 ms scan : 12.500 points, approximately 18 points per period,
-25 ms scan : 6250 points, approximately 7 points per period,
-12.5 ms scan : 3125 points, approximately 3 points per period.

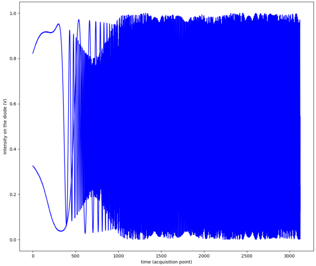


Figure: Intensity (V) as a function of the acquisition time for a 12.5 ms scan

Moreover, halving once the scan duration directly induce a significant increase in the standard deviation :

With a locked NIR laser : standard deviation : 0.0052 nm for a 25ms scan on a population of 320 measurement

As a reminder, we had standard deviation : 0.0021 nm for a 50 ms scan on a population of 1600 measurement with a locked NIR laser.

λ_{IR} function of the current of the IR laser

We measured λ_{IR} as a function of the current of the IR laser from 60 mA to 72.5 mA with steps of 0.5 mA. Each points is the mean of 320 measurements and the error is (np.std/$\sqrt{N}$) the estimated standard error of the mean.

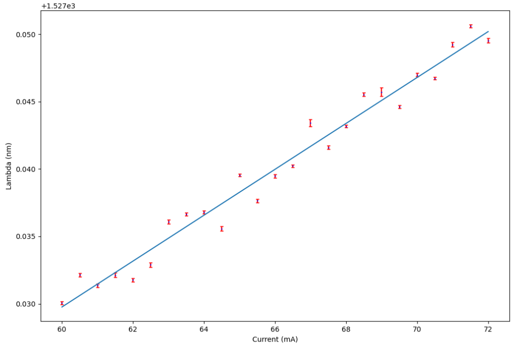


Figure: λ_{IR} (nm) function of the current (mA)

Parameters of a linear fit $ax + b$:

$a = 9.35.10^{-2} nm/mA$

$b = 1.52.10^{3} nm$

λ_{IR} function of the temperature of the IR laser

We measured λ_{IR} as a function of the temperature of the IR laser from 25.0°C to 29.7°C with steps of 0.1°C. Each points is the mean of 320 measurements and the error is (np.std/$\sqrt{N}$) the estimated standard error of the mean.

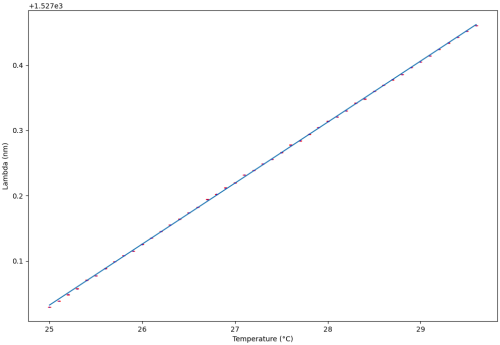


Figure: λ_{IR} (nm) function of the temperature (°C)

Parameters of a linear fit $ax + b$:

$a = 1.7.10^{-3} nm/C$

$b = 1.5269.10^{3} nm$

λ_{IR} function of the time

We took 1600 measurements (approx. 1 min) and divided it in 16 subsets. So each point : mean of 100 measurements. the error is the estimated standard error of the mean from the sigma of the fit. (mean,SE_M) of the 1600 measurements: (1527.0316 nm, 0.0021 nm) $->$ There is time fluctuation of lasers.

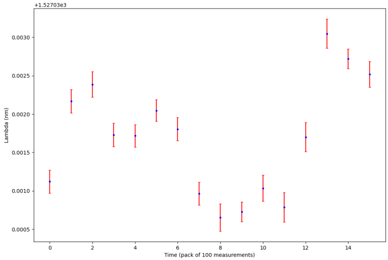


Figure: λ_{IR} (nm) function of the subset's number, sigma and mean from Gaussian fit

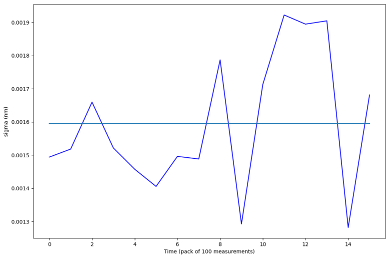


Figure: Sample standard deviation (Gaussian fit) (nm) function of the subset's number, constant curve = mean