Assignment 8

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1 Introduction

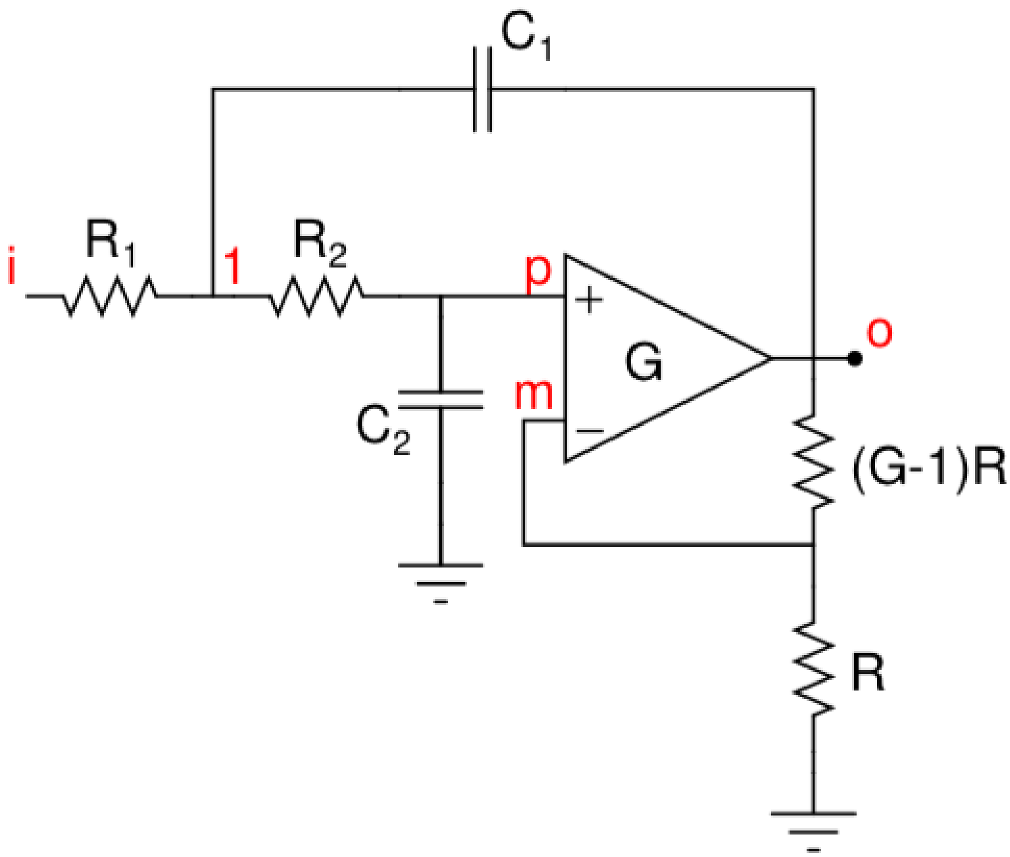
In this assignment simulations have been carried out by solving the circuits symbolically. We just have to feed in a matrix and we can can simulate the circuit.In this assignment a Highpass and a Lowpass circuit has been simulated.

2 Import Libraries

```
In [1]: from sympy import *
        import numpy as np
        import matplotlib.pyplot as plt
        import scipy.signal as sp
```

3 Lowpass filter

In the circuit given below we write the nodal equations and form a matrix.



The circuit equations are:
$V_m = \frac{V_m}{G_1}$
$V_p = V1 \frac{1}{1+j\omega R_2.C_2}$
$V_0 = G_2(V_p - V_m)$
$\frac{V_i - V_1}{R1} + \frac{V_p - V_1}{R2} + j\omega C1(V_0 - V_1) = 0$
Therefore the matrix becomes:

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{1}{G_1} \\ -\frac{1}{1+sC_2R_2} & 1 & 0 & 0 \\ 0 & -G_2 & G_2 & 1 \\ -\frac{1}{R_1} - \frac{1}{R_2} - sC_1 & \frac{1}{R_2} & 0 & sC_1 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_p \\ V_m \\ V_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ V_i(s)/R_1 \end{pmatrix}$$

* Assuming G_2 is the gain of the opamp
* Image courtesy:The Art of Electronics by Paul Horowitz and Winfield Hill

```
In [3]: s=symbols('s')
        def lowpass(R1,R2,C1,C2,G_1,G_2,Vi):
            s=symbols('s')
            A=Matrix([[0,0,1,-1/G_1],[-1/(1+s*R2*C2),1,0,0],[0,-G_2,G_2,1],[-1/R1-1/R2-s*C1,1/R2
            b=Matrix([[0],[0],[0],[-Vi/R1]])
            V=A.inv()*b
```

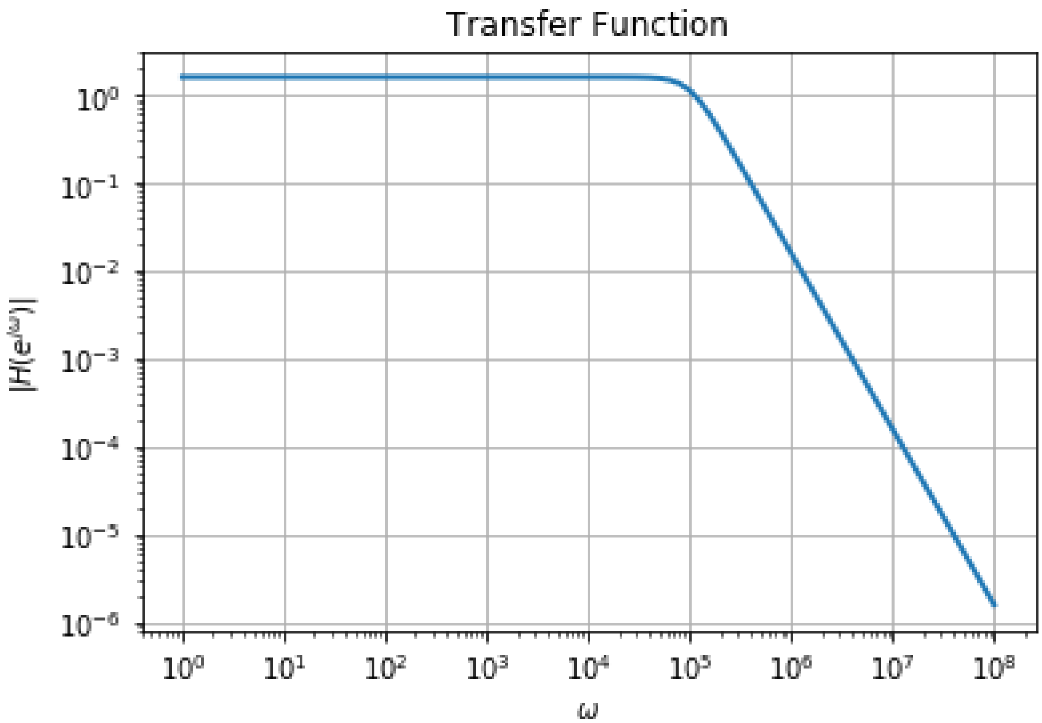
```
        return A,b,V

    A,b,V=lowpass(10000,10000,1e-9,1e-9,1.586,1000,1)
    VO=V[3]
```

```
In [4]: def plot(VO,upperlimit,title):
            '''
            Plots the transfer function of VO
            upperlimit=maximum omega
            title= title for the plot
            '''
            w=np.logspace(0,upperlimit,upperlimit*10+1)
            ss=1j*w
            s=symbols('s')
            hf=lambdify(s,VO,'numpy')
            v=hf(ss)
            plt.loglog(w,abs(v),lw=2)
            plt.xlabel('$\omega$')
            plt.ylabel('$|H(e^{j \omega})|$')
            plt.title(title)
            plt.grid(True)
            plt.show()
```

4 Plot of transfer function of lowpass filter

This filter has a cutoff at $\omega = 10^5$. After this it falls at 20dB/dec.

```
In [5]: plot(VO,8,'Transfer Function')
```

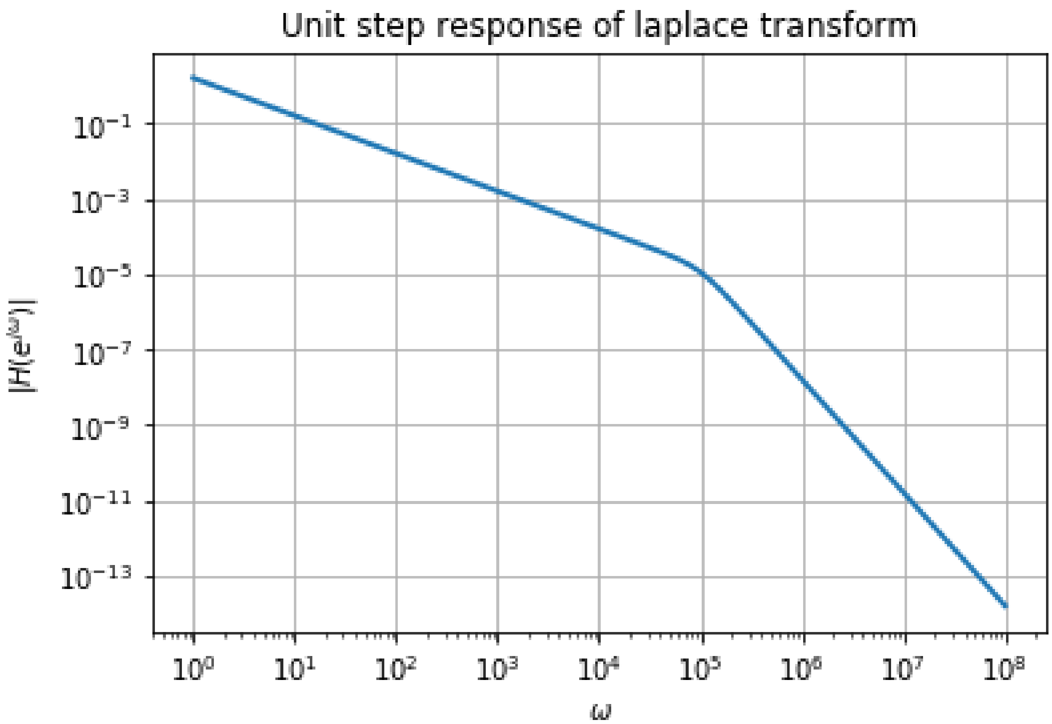



```
In [6]: def plot_graph(V0,t,V_i,V_i_name):
            '''
            Plots the time response
            V0=transfer function
            t=time array
            V_i=input voltge
            V_i_name= name of input voltage to be shown in the graph
            '''
            expr_num, expr_den = V0.as_numer_denom()
            Hnum = Poly(expr_num).coeffs()
            Hden = (Poly(expr_den)).coeffs()
            transferX = sp.lti(np.array(Hnum, dtype=float), np.array(Hden, dtype=float))
            t,x,svec = sp.lsim(transferX,V_i,t)
            plt.plot(t,x)
            plt.title('Response to input %s'%V_i_name)
            plt.xlabel('$t$')
            plt.ylabel('$V_0(t)$')
            plt.grid()
            plt.show()
```

The Laplace transform of the unit response for the lowpass filter falls initially at 20dB/dec till $\omega = 10^5$ and then it falls at 40 dB/dec

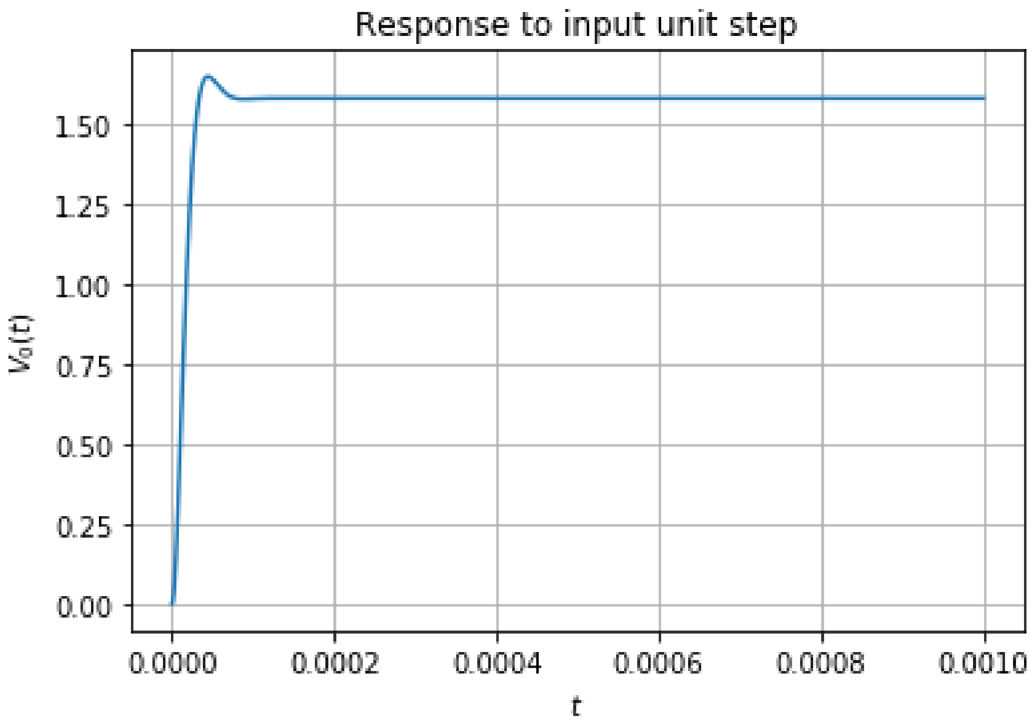
```
In [7]: V_i=1/s
        A,b,V=lowpass(10000,10000,1e-9,1e-9,1.586,1000,V_i)
```

```
VO=V[3]
plot(VO,8,'Unit step response of laplace transform')
```



The unit response of the lowpass filter reaches it's steady state after a small overshoot.This overshoot is because of the quality factor being greater than $\frac{1}{\sqrt{2}}$ in the second oreder system.

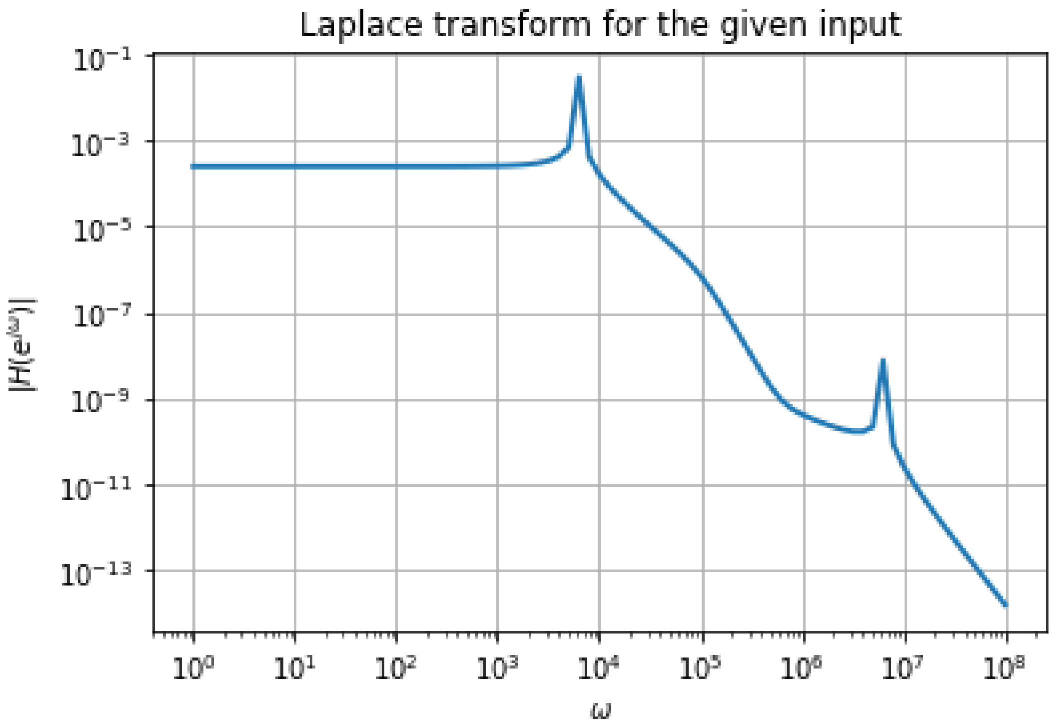
```
In [8]: t=np.arange(0,1e-3,1e-8)
        V_i = np.ones(t.size) #unit step
        plot_graph(VO,t,V_i,'unit step')
```



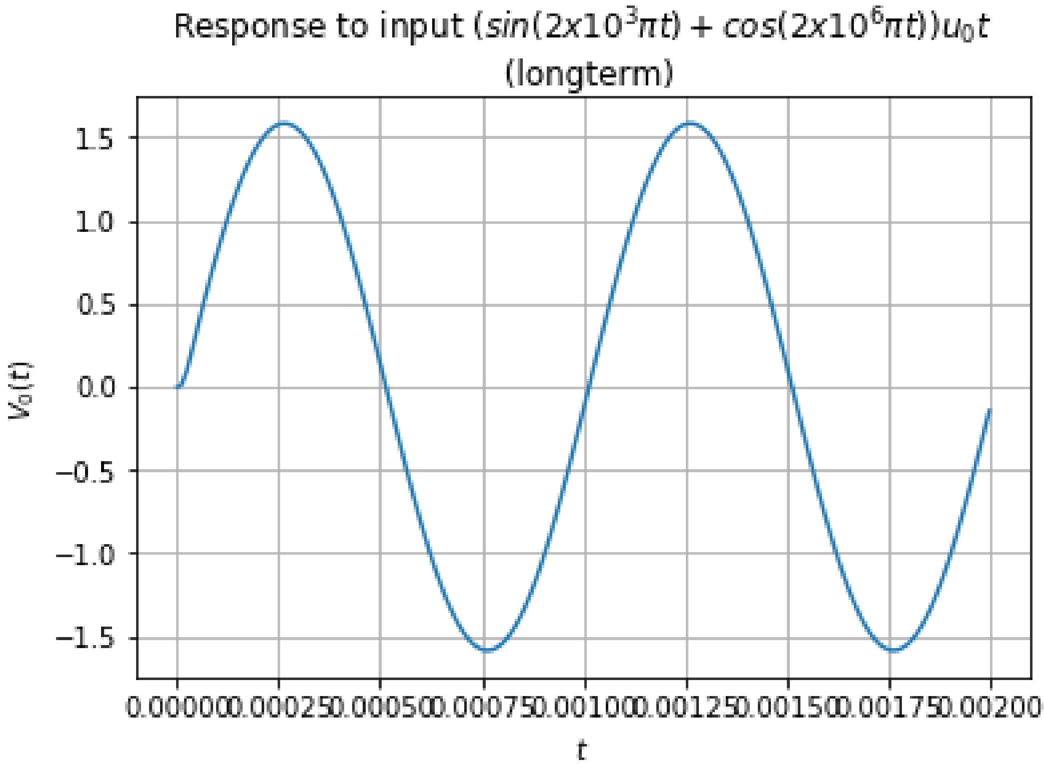
5 Sinusoidal input voltage

$V_i(t) = (sin(2000\pi t) + cos(2 * 10^6 \pi t))u_0 t$ In the input we have to frequency components:10^3 and 10^6. This causes the system to respond to the 10^3 component and 10^6 component but the second component is attenuated heavily.

In [9]:
```
s=symbols('s')
w_0=2000*np.pi
sin_s=w_0/((s**2+w_0**2))
w_0=2*pi*1e6
cos_s=s/((s**2+w_0**2))
V_i=cos_s+sin_s
A,b,V=lowpass(10000,10000,1e-9,1e-9,1.586,1000,V_i)
V0=V[3]
plot(V0,8,'Laplace transform for the given input')
```

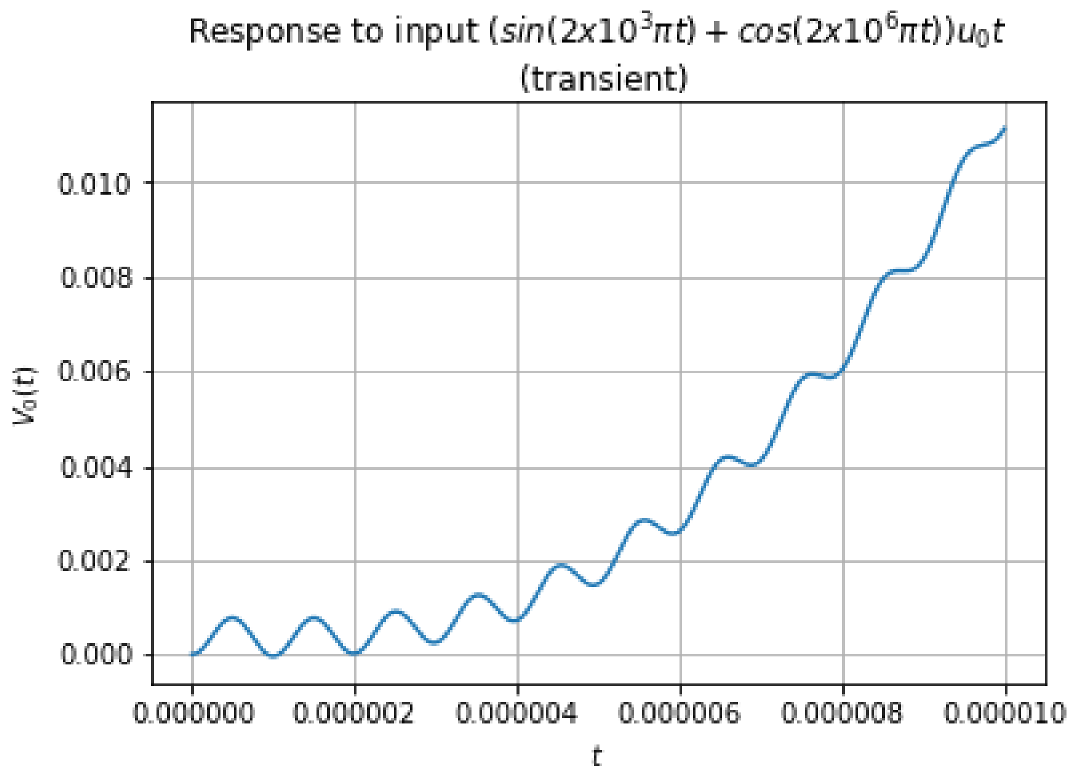


In [10]:
```
A,b,V=lowpass(10000,10000,1e-9,1e-9,1.586,1000,1)
V0=V[3]
t=np.arange(0,2*1e-3,1e-7)
u = np.ones(t.size)
V_i=(np.sin(2000*np.pi*t)+np.cos(2*1e6*np.pi*t))*u
plot_graph(V0,t,V_i,'$(sin(2x10^3\pi t)+cos(2x10^6\pi t))u_0{t}$ \n (longterm)')
```



The small ripples in th transient correspond to the frequency component with $\omega = 10^6$

```
In [11]: t=np.arange(0,1e-5,1e-8)
         u = np.ones(t.size)
         V_i=(np.sin(2000*np.pi*t)+np.cos(2*1e6*np.pi*t))*u
         plot_graph(V0,t,V_i,'$(sin(2x10^3\pi t)+cos(2x10^6\pi t))u_0{t}$ \n (transient)')
```

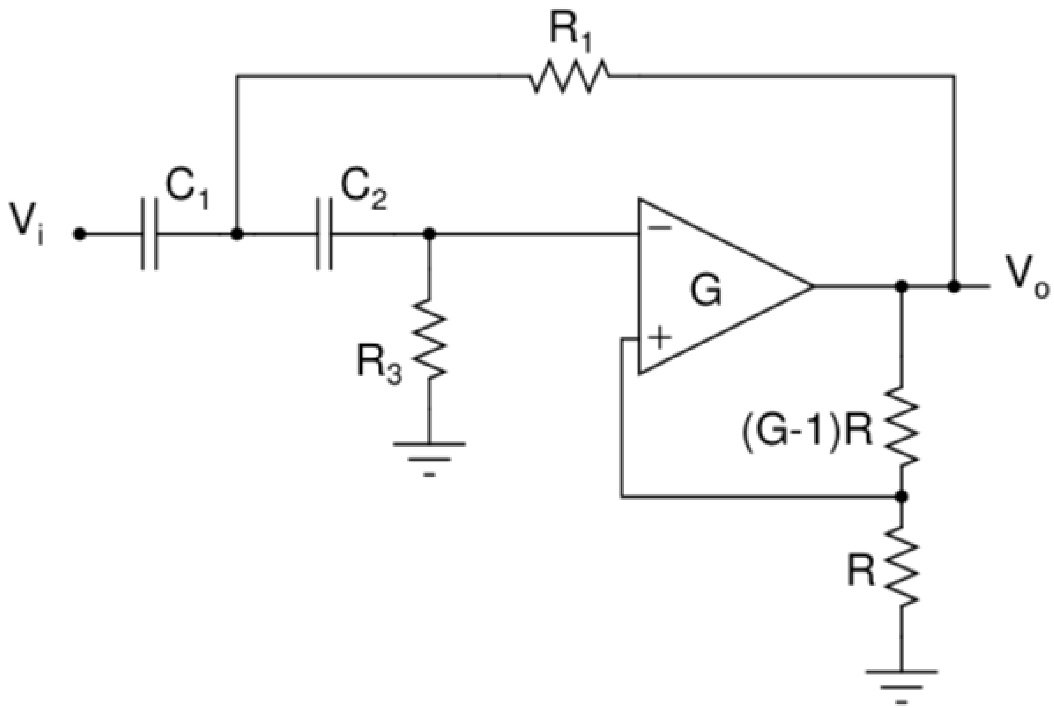


6 Highpass Filter

In the circuit given below we write the nodal equations and form a matrix.

In [12]:
```
from IPython.display import Image
Image(filename='/Users/siddharthnayak/Desktop/highpass.png')
```

Out[12]:



The circuit equations are:
$V_m = \frac{V_m}{G_1}$ $V_p = V1\frac{1}{1+j\omega R_2.C_2}$
$V_0 = G_2(V_p - V_m)$
$\frac{V_i - V_1}{R1} + \frac{V_p - V_1}{R2} + \frac{(V_0 - V_1)}{j\omega C1} = 0$
Therefore the matrix becomes:

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{1}{G_1} \\ -\frac{1}{1+sC_2R_2} & 1 & 0 & 0 \\ 0 & -G_2 & G_2 & 1 \\ -sC_1 - sC_2 - \frac{1}{R_1} & sC_2 & 0 & \frac{1}{R_1} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_p \\ V_m \\ V_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ V_i(s)sC_1 \end{pmatrix}$$

* Assuming G_2 is the gain of the opamp.
* Image courtesy:The Art of Electronics by Paul Horowitz and Winfield Hill

```
In [13]: def highpass(R1,R2,C1,C2,G1,G2,Vi):
             s=symbols('s')
             A = Matrix([[0, 0, 1, -1/G1],[0, -G2, G2, -1],[s*C2*R2, -(s*C2*R2+1), 0, 0],[s*C1+s
             b = Matrix([0, 0, 0, Vi*s*C1])
             V=A.inv()*b
             return A,b,V[3]
```

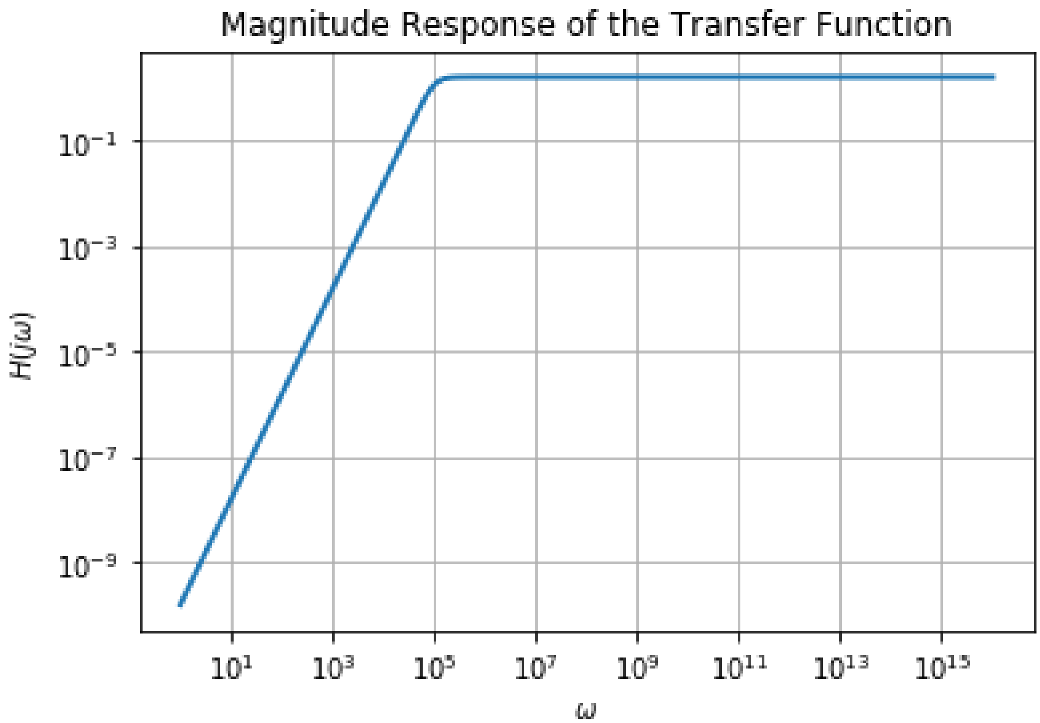
7 Obtaining the transfer function and extracting the coefficients from the Sympy polynomial.

```
In [14]: s = symbols('s')
         t = np.linspace(0,1e-3,1001)
         A, b, Vo = highpass(10000,10000,1e-9,1e-9,1.586,1000,1) # Vo consists of the transfer f

         num, den = simplify(Vo).as_numer_denom()
         p_num_den = poly(num,s), poly(den,s) # Polynomials #
         c_num_den = [expand(p).all_coeffs() for p in p_num_den] # Coefficients #
         l_num, l_den = [lambdify((),c)() for c in c_num_den] # Convert to floats #
         H = sp.lti(l_num, l_den) # LTI transfer function of the circuit #

         w = np.logspace(0, 16,1001)
         ss = 1j*w # Obtaining <jw> values to replace <s> in the laplace transform #
         f = lambdify(s, Vo, 'numpy')
         Vo = f(ss)
```

8 Plot the magnitude response

This Highpass filter has a cutoff at $\omega = 10^5$. i.e. till $\omega = 10^5$ the frequency components will be attenuated and after that we have a gain of 1 for higher frequencies.

```
In [15]: plt.loglog(w, np.abs(Vo),lw=2)
         plt.xlabel("$\omega$")
         plt.ylabel("$H(j\omega)$")
         plt.grid(True)
         plt.title("Magnitude Response of the Transfer Function")
         plt.show()
```

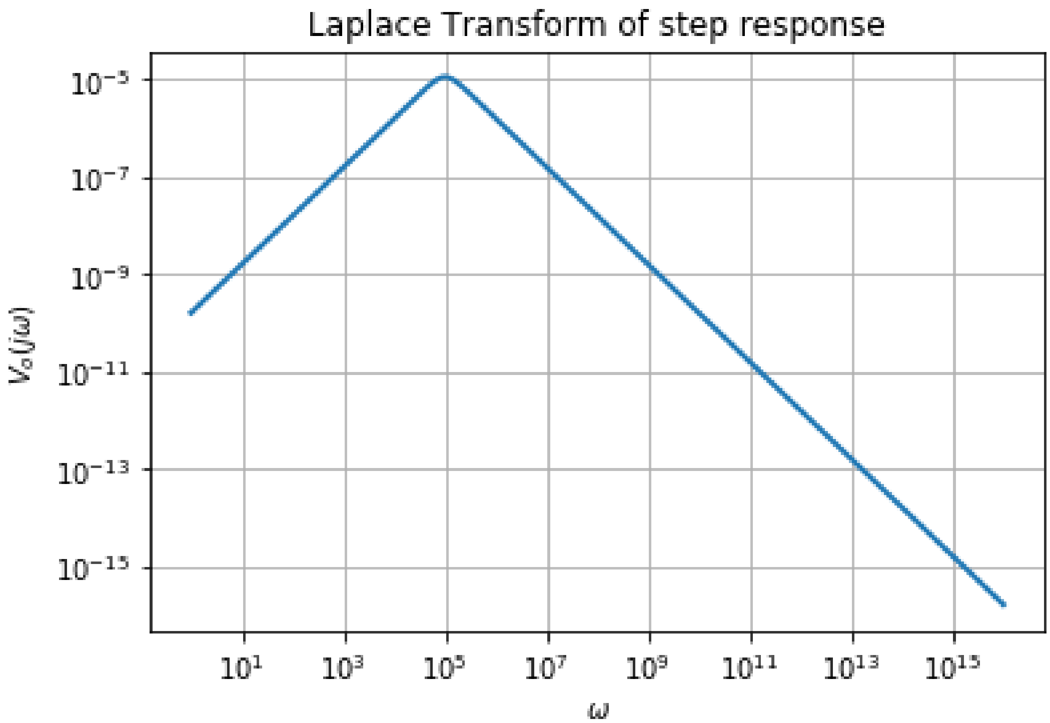


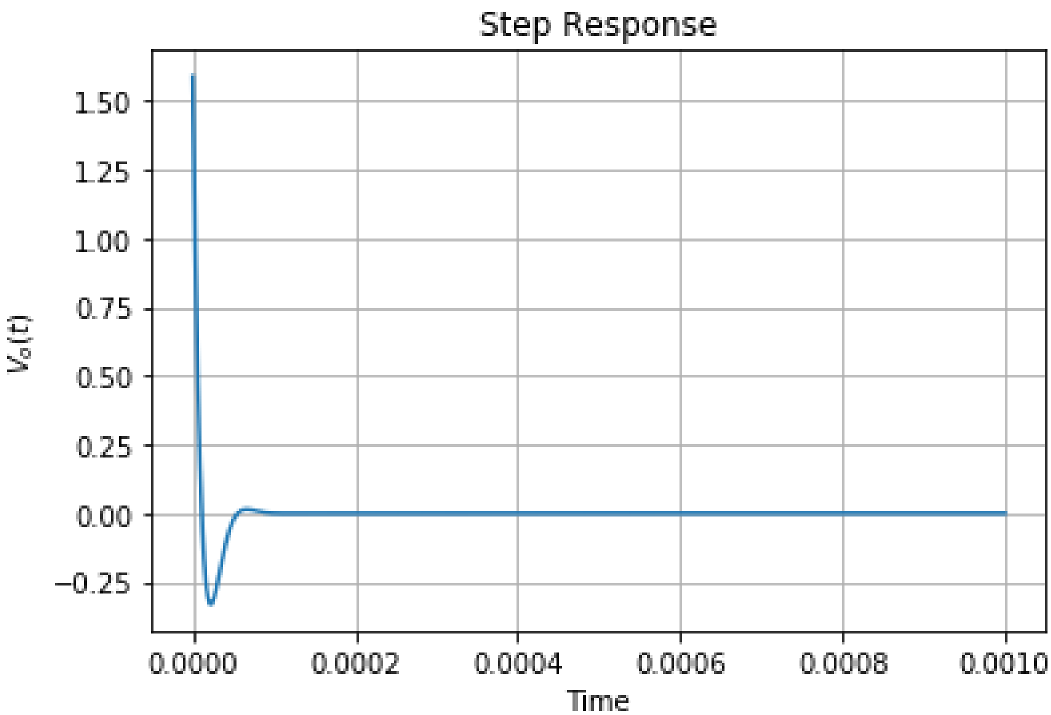
9 Obtaining the step response of the circuit

The unit response has some initial values which decreases to zero with a little overshoot.This is because the quality factor of the second order system is greater than $\frac{1}{\sqrt{2}}$. Also in a unit step all the frequency components are present at t=0.At this point the low frequency components are attenuates and the high frequency components are amplified. After some time the frequency component is zero thus output is zero.

```
In [16]: Vi_step = 1/s
         step = np.heaviside(t, 1) # Step input #
         time, step_response, svec = sp.lsim(H, step, t)
         A, b, Vo = highpass(10000,10000,1e-9,1e-9,1.586,1000,Vi_step)
         f = lambdify(s, Vo, 'numpy')
         Vo = f(ss)


         plt.loglog(w, np.abs(Vo), lw=2)
         plt.xlabel("$\omega$")
         plt.ylabel("$V_o(j\omega)$")
         plt.grid(True)
         plt.title("Laplace Transform of step response")
         plt.show()
```

```
plt.plot(t, step_response)
plt.xlabel("Time")
plt.ylabel("$V_o(t)$")
plt.title("Step Response")
plt.grid(True)
plt.show()
```



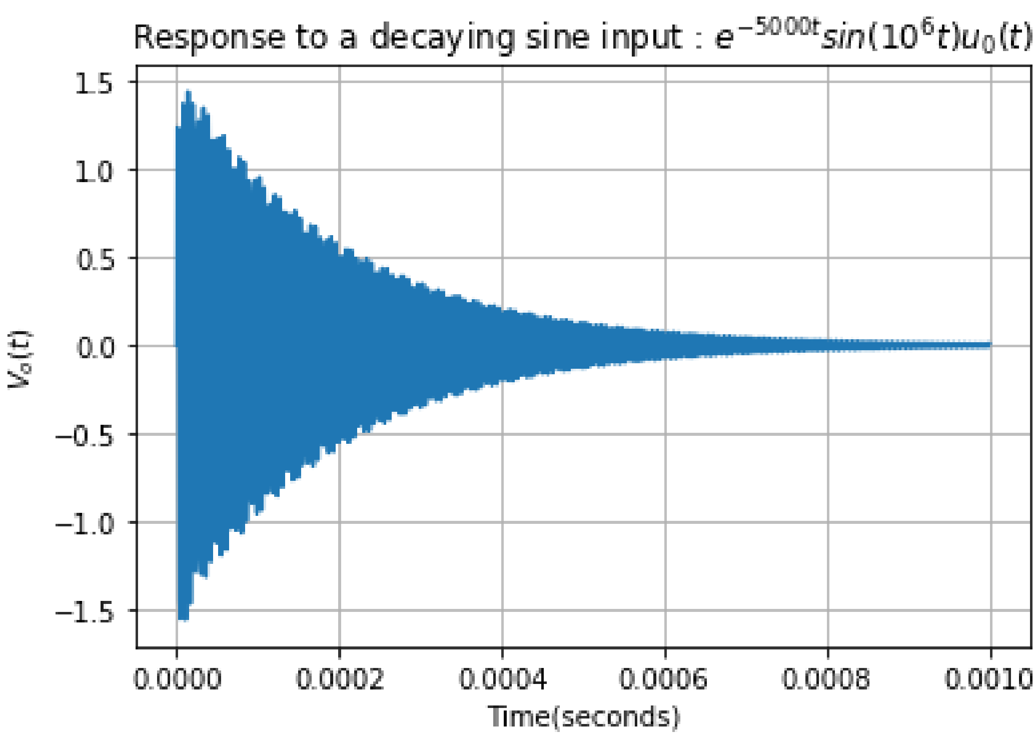


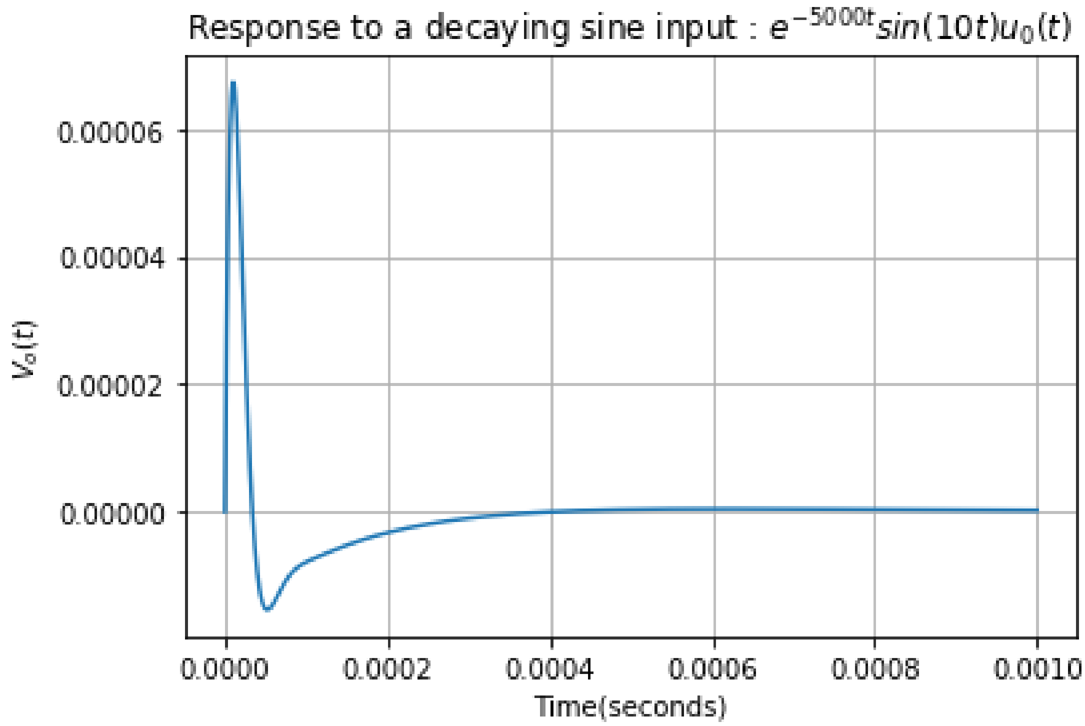
10 Decaying sinusoidal inputs to the high pass filter

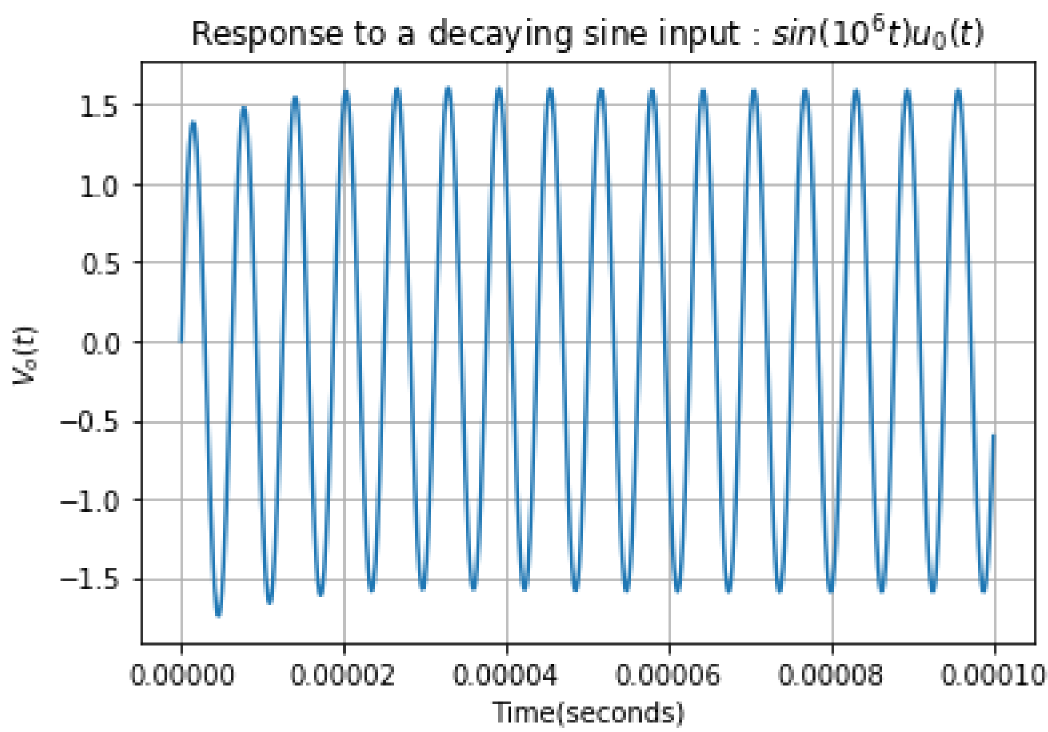
From the graphs it is clearly evident that the filter is responding only to high frequency components.

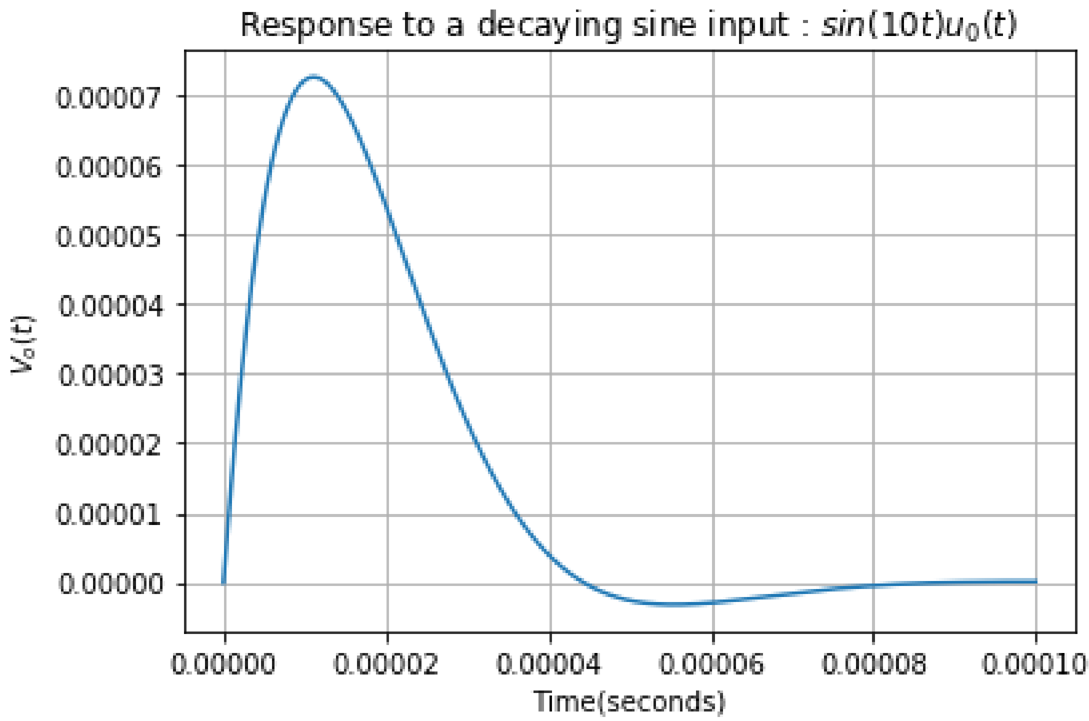
In [17]:
```
decay_sin_1=np.sin(1e6*t)*np.exp(-5000*t)*np.heaviside(t, 1) # Decaying sinusoid input
decay_sin_2=np.sin(10*t)*np.exp(-5000*t)*np.heaviside(t, 1)
t1=np.arange(0,1e-4,1e-7)
decay_sin_3=np.sin(1e6*t1)*np.heaviside(t1, 1)
t1=np.arange(0,1e-4,1e-7)
decay_sin_4=np.sin(10*t1)*np.heaviside(t1, 1)


time,sin_response_1,svec=sp.lsim(H,decay_sin_1,t) # Simulate the output using lsim #
time,sin_response_2,svec=sp.lsim(H,decay_sin_2,t)
time1,sin_response_3,svec=sp.lsim(H,decay_sin_3,t1)
time1,sin_response_4,svec=sp.lsim(H,decay_sin_4,t1)

plt.plot(t, sin_response_1)
plt.xlabel("Time(seconds)")
plt.ylabel("$V_o(t)$")
plt.title("Response to a decaying sine input : $e^{-5000t}sin(10^6t)u_0(t)$")
plt.grid(True)
```

```
plt.show()

plt.plot(t, sin_response_2)
plt.xlabel("Time(seconds)")
plt.ylabel("$V_o(t)$")
plt.title("Response to a decaying sine input : $e^{-5000t}sin(10t)u_0(t)$")
plt.grid(True)
plt.show()

plt.plot(t1, sin_response_3)
plt.xlabel("Time(seconds)")
plt.ylabel("$V_o(t)$")
plt.title("Response to a decaying sine input : $sin(10^6t)u_0(t)$")
plt.grid(True)
plt.show()

plt.plot(t1, sin_response_4)
plt.xlabel("Time(seconds)")
plt.ylabel("$V_o(t)$")
plt.title("Response to a decaying sine input : $sin(10t)u_0(t)$")
plt.grid(True)
plt.show()
```




Response to a decaying sine input : $e^{-5000t} sin(10t)u_0(t)$
0.00006
0.00004
0.00002
0.00000
$V_o(t)$
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010
Time(seconds)


Response to a decaying sine input : $sin(10^6 t)u_0(t)$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
$V_o(t)$
0.00000
0.00002
0.00004
0.00006
0.00008
0.00010
Time(seconds)



11 Conclusions

Any linear electrical circuit can be simulated using the `sympy` and `scipy.signal` module.
Two types of circuits namely Highpass filter and Lowpass Filter were simulated in this assignment.