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1) ~~Once we obtain the DH parameters $(\theta_k, d_k, a_k, \alpha_k)$~~

Once we set the co-ordinate frames according to the ~~DH parameters~~ con. DH - convention,

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

let

$$R_1' = \begin{bmatrix} C\theta_k & -S\theta_k & 0 & 0 \\ S\theta_k & C\theta_k & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_1' = \begin{bmatrix} 1 & 0 & 0 & a_k \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_k & -S\alpha_k & 0 \\ 0 & S\alpha_k & C\alpha_k & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{k-1}^{k} = [T_2' R_2' R_1' T_1'].$$

Now T_{k-1}^{k} is unique as it represents the position of joint 'k' w.r.t 'k-1'.

$\therefore$ Suppose we have another set of DH parameters, $T_1^2, T_2^2, R_1^2, R_2^2$ with parameters $\alpha_k^2, \theta_k^2, a_k^2, d_k^2$

$$T_{k-1}^{k} = T_2^2 R_2^2 . R_1^2 . T_1^2$$

$$\therefore T_2^2 R_2^2 . R_1^2 T_1^2 = T_2' R_2' R_1' T_1'.$$

as they are equal ~~for~~ we need each of the matrices to be equal, i.e $T_2' = T_2^2$, $T_1' = T_1^2$, $R_2' = R_2^2$, $R_1 = R_1^2$

$\therefore \alpha_k' = \alpha_k^2$; $\theta_k' = \theta_k^2$; $a_k' = a_k^2$, $d_k' = d_k^2$.

$\therefore$ The DH parameters are unique.

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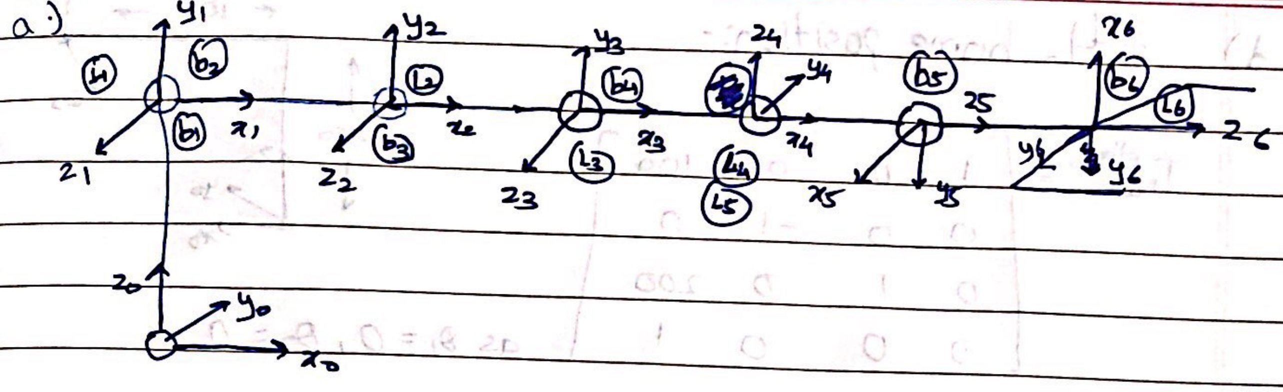
2.) a.)



b.)

	θ	d	a	α
1	θ_1	200	0	90°
2	θ_2	0	100	0
3	θ_3	0	150	0
4	θ_4	0	100	-90°
5	θ_5	0	0	-90°
6	θ_6	250	0	0

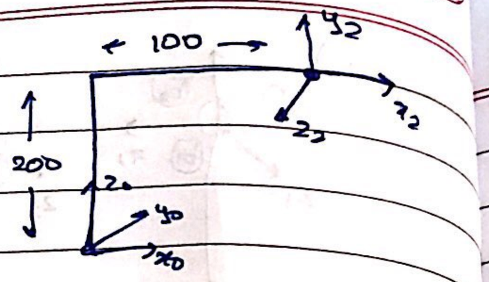
c.) $T_{base}^{elbow} = T_{base}^{shoulder} \times T_{shoulder}^{elbow}$

$$T_{base}^{shoulder} = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{shoulder}^{elbow} = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 100C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & 100S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore T_{base}^{elbow} = \begin{bmatrix} C\theta_1C\theta_2 & -C\theta_1S\theta_2 & S\theta_1 & 100C\theta_1C\theta_2 \\ S\theta_1S\theta_2 & -S\theta_1S\theta_2 & -C\theta_1 & 100S\theta_1S\theta_2 \\ S\theta_2 & C\theta_2 & 0 & 100S\theta_2+200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) soft- home position:-

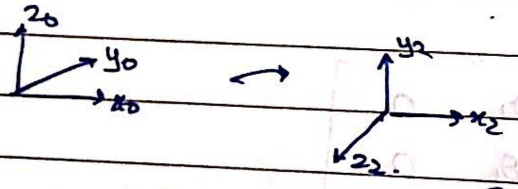


$$T_{base}^{elbow} = \begin{bmatrix} 1 & 0 & 0 & 100 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ as $\theta_1 = 0, \theta_2 = 0$

$\therefore$ translation ~~matrix~~ co-ordinates = $\begin{bmatrix} 100 \\ 0 \\ 200 \\ 1 \end{bmatrix}$ → matches with the co-ordinates

as $p_x = 100$, $p_y = 0$, $p_z = 200$.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$



→ rotation about 'x_1' by 'θ'=90°'

$\therefore$ rotation matrix = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

as $\theta = 90°$ → $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ → matches with R

$\therefore$ both translation and rotation matrix match.