RTP Exercise Sheet

Series 5

Exercise 5.1

Simulations are key to validate models. Thus, we would like to use this exercise to simulate several time series by means of an ARMA model.

a) AR(2) model with coefficients $\alpha_1 = 0.9$ and $\alpha_2 = -0.5$.

b) MA(3) model with coefficients $\beta_1 = 0.8$, $\beta_2 = -0.5$ and $\beta_3 = -0.4$.

The innovation E_t shall follow a standard normal distribution $\mathcal{N}(0;1)$ in every model. For each of the three models, do the following:

i) First think of how the autocorrelations should behave theoretically.

ii) Use the procedure ARMAacf() to compute the theoretical autocorrelations of the models and plot them.

```r
## Theoretical autocorrelations for X_t = 0.9 X_t-1
## - 0.5 X_t-2 + e_t
plot(0:30, ARMAacf(ar = c(0.9, -0.5), lag.max = 30),
    type = "h", ylab = "ACF")
## Theoretical partial autocorrelations
plot(1:30, ARMAacf(ar = c(0.9, -0.5), lag.max = 30,
    pacf = TRUE), type = "h", ylab = "PACF")
plot(0:30, ARMAacf(ma = c(..., ..., ...), ...), ...)
```

iii) Now simulate a realisation of length $n = 200$ for the models a) - c). Repeat each simulation several times to develop some intuition on what occurs by chance and what is structure.

Hint: You may use the procedure arima.sim() to simulate the time series. The length of the simulated series is choosen by setting the argument **n**. The model is set by the parameter **model** (to a list!).

```r
r.sim1 <- arima.sim(n = ..., model = list(ar = c(0.9,
    -0.5)))
```

iv) Inspect the time series plot and the correlograms with the ordinary and partial autocorrelations.

Exercise 5.2

In this exercise we consider some examples of AR(p) models and check their stationarity.

a) Test the models

   i) $X_t = 0.5X_{t-1} + 2X_{t-2} + E_t$

   ii) $Y_t = Y_{t-1} + E_t$

   with the innovation E_t on stationarity with the help of the R function **polyroot**.

   Hint: During the lectures you saw that only roots with absolute value greater than 1 lead to stationary models. Confer the lecture notes at page 21.

b) For which value of the coefficient α_2 of X_{t-2} is the model $X_t = 0.5X_{t-1} + \alpha_2 X_{t-2} + E_t$ stationary?

c) Why is the model $Y_t = \alpha Y_{t-1} + E_t$ not stationary for $|\alpha| \geq 1$? Calculate the characteristic function and determine its roots to confirm this observation.

   Hint: Confer the hint at part a).

Disclaimer: Parts of the exercises are adopted from 'Applied Time Series Analysis' course at ETHZ by Marcel Dettling.