Неопределен интеграл

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Правила за интегриране

1. $\int \left[f(x) \pm g(x) \right] dx = \int f(x)\,dx \pm \int g(x)\,dx$

2. $\int kf(x)\,dx = k \int f(x)\,dx$ $\qquad k = const$

3. $\int f(x)\,dx = \int f(x)\,d(x \pm a)$ $\qquad a = const$

4. $\int f(x)\,dx = \frac{1}{a} \int f(x)\,d(ax)$ $\qquad a \neq 0,\ a = const$

5. $\int d\,F(x) = F(x) + C$

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Таблица на основните интеграли

1. $\int dx = x + C$

2. $\int x^p dx = \dfrac{x^{p+1}}{p+1} + C$ $\quad p \neq -1$

3. $\int \dfrac{dx}{x} = \ln|x| + C$

4. $\int e^x dx = e^x + C$

5. $\int a^x dx = \dfrac{a^x}{\ln(a)} + C$

6. $\int \sin x dx = -\cos x + C$

7. $\int \cos x dx = \sin x + C$

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Таблица на основните интеграли

8. $$\int \frac{1}{\cos^2 x}\,dx = \operatorname{tg} x + C$$

9. $$\int \frac{1}{\sin^2 x}\,dx = -\operatorname{cotg} x + C$$

10. $$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\operatorname{arctg}\left(\frac{x}{a}\right) + C$$

11. $$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$$ $a > 0$

12. $$\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

13. $$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

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Да се пресметне интеграла:

Непосредствено пресмятане на интеграл

от (2) $\int x^p dx = \frac{x^{p+1}}{p+1} + C$

Задача 1 $\int x^2 dx = \frac{x^3}{3} + C$

Задача 2 $\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$

Задача 3 $\int \left(5x^7 - 4x^3 + \frac{2}{x^2} - \frac{3}{\sqrt[3]{x^2}} \right) dx =$

$= 5\int x^7 dx - 4\int x^3 dx + 2\int \frac{1}{x^2} dx - 3\int \frac{1}{x^{\frac{2}{3}}} dx =$

$= 5\frac{x^8}{8} - 4\frac{x^4}{4} + 2\frac{x^{-1}}{-1} - 3\frac{x^{-\frac{2}{3}+1}}{\frac{1}{3}} + C = \frac{5x^8}{8} - x^4 - \frac{2}{x} - 9x^{\frac{1}{3}} + C$

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Задача 4

$$\int \frac{1}{\sqrt{5-x^2}}dx = \arcsin\frac{x}{\sqrt{5}} + C$$

от (11) $\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\left(\frac{x}{a}\right) + C$

Задача 5

$$\int\left(\frac{1}{3+x^2} + \frac{1}{\sqrt{x^2+3}}\right)dx = \int \frac{1}{\left(\sqrt{3}\right)^2 + x^2}dx + \int \frac{1}{\sqrt{x^2+3}}dx =$$

$$= \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}} + \ln\left|x + \sqrt{x^2+3}\right| + C$$

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Задача 6

$$\int\left(\sqrt{x^7}+\frac{2}{x}-3^x\right)dx=$$

$$=\int x^{\frac{7}{2}}dx+2\int\frac{1}{x}dx-\int 3^x dx=\frac{2x^{\frac{9}{2}}}{9}+2\ln|x|-\frac{3^x}{\ln 3}+C$$

Задача 7

$$\int\frac{x^2}{1-x^2}dx=\int\frac{x^2-1+1}{1-x^2}dx=-\int\frac{x^2-1}{x^2-1}dx+\int\frac{1}{1-x^2}dx=$$

$$=-\int dx-\int\frac{1}{x^2-1}dx=-x-\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+C=-x+\frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|+C$$

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Задача 8

$$\int \frac{x^2}{x^2+2}dx = \int \frac{x^2+2-2}{x^2+2}dx = \int dx - 2\int \frac{1}{x^2+2}dx =$$

$$= x - 2\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x}{\sqrt{2}} + C$$

Задача 9

$$\int \frac{2x^4-5x^2+3}{x^2-1}dx = \int \frac{\left(2x^2-3\right)\left(x^2-1\right)}{x^2-1}dx =$$

$$= 2\int x^2 dx - 3\int dx = \frac{2}{3}x^3 - 3x + C$$

Задача 10

$$\int \frac{2+\sqrt{x^3-x^5}}{\sqrt{1-x^2}}dx = \int \frac{2}{\sqrt{1-x^2}}dx + \int \frac{\sqrt{x^3-x^5}}{\sqrt{1-x^2}}dx =$$

$$= 2\arcsin x + C + \int \frac{\sqrt{x^3}\sqrt{1-x^2}}{\sqrt{1-x^2}}dx = 2\arcsin x + C + \int x^{\frac{3}{2}}dx =$$

$$= 2\arcsin x + \frac{2}{5}x^{\frac{5}{2}} + C$$

Задача 11

$$\int \operatorname{tg}^2 x dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx =$$

$$= \int \left(\frac{1}{\cos^2 x} - 1 \right) dx = \int \frac{1}{\cos^2 x}dx - \int dx = \operatorname{tg} x - x + C$$

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Задача 12

$$\int \frac{2+3x^2}{x^4+x^2}dx = \int \frac{2}{x^2\left(x^2+1\right)}dx + 3\int \frac{x^2}{x^2\left(x^2+1\right)}dx =$$

$$= \int \frac{2}{x^2\left(x^2+1\right)}dx + 3\int \frac{1}{\left(x^2+1\right)}dx =$$

$$= \int \left(\frac{2}{x^2} - \frac{2}{x^2+1}\right)dx + 3\int \frac{1}{\left(x^2+1\right)}dx =$$

$$= \int \frac{2}{x^2}dx - \int \frac{2}{x^2+1}dx + 3\operatorname{arctg} x + C =$$

$$= -\frac{2}{x} - 2\operatorname{arctg} x + 3\operatorname{arctg} x + C = -\frac{2}{x} + \operatorname{arctg} x + C$$

Интегриране чрез внасяне на константа под знака на диференциала

$$\boxed{\int f(ax+b)\,dx = \frac{1}{a}\int f(ax+b)\,d(ax+b)}$$

Задача 13

$$\int \frac{1}{x-3}\,dx = \int \frac{1}{x-3}\,d(x-3) = \ln|x-3| + C$$

Задача 14

$$\int (x+7)^{17}\,d \quad = \int (x+7)^{17}\,d(x+7) = \frac{1}{18}(x+7)^{18} + C$$

Задача 15

$$\int \frac{1}{\sqrt{1-25x^2}}\,dx = \frac{1}{5}\int \frac{1}{\sqrt{1-(5x)^2}}\,d(5x) = \frac{1}{5}\arcsin 5x + C$$

Задача 16

$$\int \frac{1}{9+4x^2}dx = \int \frac{1}{3^2+(2x)^2}dx = \frac{1}{2}\int \frac{1}{3^2+(2x)^2}d(2x) =$$

$$= \frac{1}{2}\frac{1}{3}\operatorname{arctg}\frac{2x}{3} + C$$

Задача 17

$$\int \frac{1}{e^{3x-2}}dx = \frac{1}{-3}\int e^{-3x+2}d(-3x+2) = -\frac{1}{3}e^{-3x+2} + C$$

Задача 18

$$\int \sin 5x dx = \frac{1}{5}\int \sin 5x d(5x) = -\frac{\cos 5x}{5} + C$$

Задача 19

$$\int \frac{1}{\sqrt{5x^2-4}}dx = \int \frac{1}{\sqrt{\left(\sqrt{5}x\right)^2 - 2^2}}dx = \frac{1}{\sqrt{5}}\int \frac{1}{\sqrt{\left(\sqrt{5}x\right)^2 - 2^2}}d\sqrt{5}x =$$

$$= \frac{1}{\sqrt{5}}\ln\left|\sqrt{5}x + \sqrt{5x^2-4}\right| + C$$

Задача 20

$$\int \cos^2 x dx = \int \frac{1+\cos 2x}{2} dx = \frac{1}{2}\int dx + \frac{1}{2}\frac{1}{2}\int \cos 2x d(2x) =$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

Задача 21

$$\int \left(\frac{1}{\cos^2 5x} + \frac{1}{\sqrt{x^2-4}} \right) dx = \int \frac{1}{\cos^2 5x} dx + \int \frac{1}{\sqrt{x^2-4}} dx =$$

$$= \frac{1}{5}\int \frac{1}{\cos^2 5x} d(5x) + \ln\left| x + \sqrt{x^2-4} \right| + C =$$

$$= \frac{1}{5}\operatorname{tg} 5x + \ln\left| x + \sqrt{x^2-4} \right| + C$$

Задача 22

$$\int \sin^2 x dx = \int \frac{1-\cos 2x}{2} dx = \int \left(\frac{1}{2} - \frac{\cos 2x}{2} \right) dx =$$

$$= \int \frac{1}{2} dx - \int \frac{\cos 2x}{2} dx = \frac{1}{2} x + C - \frac{1}{2} \frac{1}{2} \int \cos 2x d(2x) =$$

$$= \frac{1}{2} x - \frac{1}{4} \sin 2x + C$$

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Задача 23

$$\int \frac{1}{\sqrt{x+2}-\sqrt{x-3}}\,dx = \int \frac{1}{\sqrt{x+2}-\sqrt{x-3}}\cdot\frac{\sqrt{x+2}+\sqrt{x-3}}{\sqrt{x+2}+\sqrt{x-3}}\,dx =$$

$$= \frac{1}{5}\int\left(\sqrt{x+2}+\sqrt{x-3}\right)dx = \frac{1}{5}\left[\int\sqrt{x+2}\,d\left(x+2\right)+\int\sqrt{x-3}\,d\left(x-3\right)\right] =$$

$$= \frac{2}{15}\left[\left(x+2\right)^{\frac{3}{2}}+\left(x-3\right)^{\frac{3}{2}}\right]+C$$

Задача 24

$$\int\left[\left(3x+5\right)^7+e^{2x}\right]dx = \int\left(3x+5\right)^7dx+\int e^{2x}dx =$$

$$= \frac{1}{3}\int\left(3x+5\right)^7 d\left(3x+5\right)+\frac{1}{2}\int e^{2x}d\left(2x\right) = \frac{1}{3}\frac{\left(3x+5\right)^8}{8}+\frac{e^{2x}}{2}+C$$

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Интегриране чрез внасяне на функция под знака на диференциала

$$\boxed{\int f(x)g(x)dx = \int g(x)d\left(\int f(x)dx\right)}$$

Задача 25

$$\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}dx = \int \sin^{-\frac{2}{3}} x d\sin x = \frac{(\sin x)^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C = 3\sqrt[3]{\sin x} + C$$

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Задача 26

$$\int \frac{\ln^2 x}{x} dx = \int \ln^2 x \frac{1}{x} dx = \int \ln^2 x d\left(\ln x\right) =$$

$$= \int \left(\ln x\right)^2 d\left(\ln x\right) = \frac{\left(\ln x\right)^3}{3} + C$$

Задача 27

$$\int \left(\frac{x}{\sqrt{1-x^2}} \right) dx = \int \frac{1}{\sqrt{1-x^2}} d\frac{x^2}{2} = \frac{1}{2} \int \left(1-x^2\right)^{-\frac{1}{2}} dx^2 =$$

$$= -\frac{1}{2} \int \left(1-x^2\right)^{-\frac{1}{2}} d\left(-x^2+1\right) = -\frac{1}{2} \frac{\left(1-x^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = -\sqrt{1-x^2} + C$$

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Задача 28

$$\int \frac{1}{\left(1+x^2\right)\operatorname{arctg} x}dx = \int \frac{1}{\left(1+x^2\right)} \cdot \frac{1}{\operatorname{arctg} x}dx = \int \frac{1}{\operatorname{arctg} x}d\left(\operatorname{arctg} x\right) =$$

$$= \ln\left|\operatorname{arctg} x\right| + C$$

Задача 29

$$\int \frac{5x-3}{7+2x^2}dx = 5\int \frac{x}{7+2x^2}dx - 3\int \frac{1}{\left(\sqrt{7}\right)^2+\left(\sqrt{2}x\right)^2}dx =$$

$$= 5\frac{1}{2}\frac{1}{2}\int \frac{1}{7+2x^2}d\left(2x^2+7\right) - \frac{3}{\sqrt{2}}\int \frac{1}{\left(\sqrt{7}\right)^2+\left(\sqrt{2}x\right)^2}d\left(\sqrt{2}x\right) =$$

$$= \frac{5}{4}\ln\left(7+2x^2\right) - \frac{3}{\sqrt{14}}\operatorname{arctg}\frac{\sqrt{2}x}{\sqrt{7}} + C$$

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Задача 30

$$\int \frac{2x+3}{3+4x^2}dx = 2\int \frac{x}{3+4x^2}dx + 3\int \frac{1}{3+4x^2}dx =$$

$$= 2\int \frac{1}{3+4x^2}d\frac{x^2}{2} + 3\int \frac{1}{\left(\sqrt{3}\right)^2 + (2x)^2}dx =$$

$$= \frac{2}{2}\frac{1}{4}\int \frac{1}{3+4x^2}d\left(4x^2+3\right) + \frac{3}{2}\int \frac{1}{\left(\sqrt{3}\right)^2 + (2x)^2}d\left(2x\right) =$$

$$= \frac{1}{4}\ln\left|3+4x^2\right| + \frac{3}{2}\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x}{\sqrt{3}} + C =$$

$$= \frac{1}{4}\ln\left|3+4x^2\right| + \frac{\sqrt{3}}{2}\operatorname{arctg}\frac{2x}{\sqrt{3}} + C$$

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Задача 31

$$\int \frac{3x+4}{\sqrt{5-x^2}}dx = 3\int \frac{x}{\sqrt{5-x^2}}dx + 4\int \frac{1}{\sqrt{\left(\sqrt{5}\right)^2 - x^2}}dx =$$

$$= \frac{3}{2}\int \frac{1}{\sqrt{5-x^2}}d\left(x^2\right) + 4\arcsin\frac{x}{\sqrt{5}} + C =$$

$$= -\frac{3}{2}\int \frac{1}{\sqrt{5-x^2}}d\left(-x^2+5\right) + 4\arcsin\frac{x}{\sqrt{5}} + C =$$

$$= -\frac{3}{2}\int \left(5-x^2\right)^{-\frac{1}{2}}d\left(5-x^2\right) + 4\arcsin\frac{x}{\sqrt{5}} + C =$$

$$= -3\sqrt{5-x^2} + 4\arcsin\frac{x}{\sqrt{5}} + C$$

Задача 32

$$\int \operatorname{tg} x dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{1}{\cos x}d\left(\cos x\right) = -\ln\left|\cos x\right| + C$$

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Задача 33

$$\int \frac{1}{\sin x}dx = \int \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx = \int \frac{1}{\sin\frac{x}{2}\cos\frac{x}{2}}d\left(\frac{x}{2}\right) = \int \frac{1}{\left(\sin\frac{x}{2}\cos\frac{x}{2}\right)\frac{\cos x/2}{\cos x/2}}d\left(\frac{x}{2}\right) =$$

$$= \int \frac{1}{\operatorname{tg}\frac{x}{2}\left(\cos\frac{x}{2}\right)^2}d\left(\frac{x}{2}\right) = \int \frac{1}{\operatorname{tg}\frac{x}{2}}d\left(\operatorname{tg}\frac{x}{2}\right) = \ln\left|\operatorname{tg}\frac{x}{2}\right| + C$$

Задача 34

$$\int \frac{1}{\cos x}d \;\; = -\int x \frac{1}{\sin\left(\pi/2 - x\right)}d\left(\frac{\pi}{2} - x\right) = -\int \frac{1 + \left(\operatorname{tg}\left(\pi/4 - x/2\right)\right)^2}{2\operatorname{tg}\left(\pi/4 - x/2\right)}d\left(\frac{\pi}{2} - x\right) =$$

$$= -\int \frac{1 + \left(\frac{\sin\left(\pi/4 - x/2\right)}{\cos\left(\pi/4 - x/2\right)}\right)^2}{\operatorname{tg}\left(\pi/4 - x/2\right)}d\left(\frac{\pi}{4} - \frac{x}{2}\right) == -\int \frac{\frac{1}{\left(\cos\left(\pi/4 - x/2\right)\right)^2}}{\operatorname{tg}\left(\pi/4 - x/2\right)}d\left(\frac{\pi}{4} - \frac{x}{2}\right) =$$

$$= -\int \frac{1}{\operatorname{tg}\left(\pi/4 - x/2\right)}d\left(\operatorname{tg}\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) = -\ln\left|\operatorname{tg}\left(\frac{\pi}{4} - \frac{x}{2}\right)\right| + C$$

$$\boxed{\sin x = \frac{2\operatorname{tg} x/2}{1 + \left(\operatorname{tg} x/2\right)^2}}$$

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Интегриране по части

$$\boxed{\int u(x)\,d\left(v(x)\right) = u(x)\,v(x) - \int v(x)\,d\left(u(x)\right)}$$

Задача 35

$$\int \frac{x \arcsin x}{\sqrt{1-x^2}}\,dx =$$

Внасяме $\frac{x}{\sqrt{1-x^2}}$ *под знака d.*

$$\boxed{\int \frac{x}{\sqrt{1-x^2}}\,d \quad = -\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,d\left(-x^2+1\right) = -\frac{1}{2}2\sqrt{1-x^2} + C = -\sqrt{1-x^2} + C}$$

$$= -\int \arcsin x\,d\sqrt{1-x^2} =$$

Интегрираме по части.

$$= -\sqrt{1-x^2}\arcsin x + \int \sqrt{1-x^2}\,d\arcsin x =$$

$$= -\sqrt{1-x^2}\arcsin x + \int \sqrt{1-x^2}\,\frac{1}{\sqrt{1-x^2}}\,dx =$$

$$= -\sqrt{1-x^2}\arcsin x + \int d \quad = -\sqrt{1-x^2}\arcsin x + x + C$$

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Задача 36

$$\int xe^{2x}dx = \frac{1}{2}\int xe^{2x}d\,2x = \frac{1}{2}\int xd\left(e^{2x}\right) = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx =$$

$$= \frac{x}{2}e^{2x} - \frac{1}{2}\frac{1}{2}e^{2x} + C = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C = e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right) + C$$

Задача 37

$$\int \operatorname{arctg} x\,dx = x\operatorname{arctg} x - \int xd\operatorname{arctg} x = x\operatorname{arctg} x - \int \frac{x}{1+x^2}dx =$$

$$= x\operatorname{arctg} x - \frac{1}{2}\int \frac{1}{1+x^2}d\left(1+x^2\right) = x\operatorname{arctg} x - \frac{1}{2}\ln\left(1+x^2\right) + C$$

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Задача 38

$$\int x^3 \operatorname{arctg} x dx = \frac{1}{4}\int \operatorname{arctg} x dx^4 = \frac{1}{4}x^4 \operatorname{arctg} x - \frac{1}{4}\int x^4 d \operatorname{arctg} x =$$

$$= \frac{1}{4}x^4 \operatorname{arctg} x - \frac{1}{4}\int \frac{x^4}{1+x^2} dx = \frac{1}{4}x^4 \operatorname{arctg} x - \frac{1}{4}\int \frac{x^4 - 1 + 1}{1+x^2} dx =$$

$$= \frac{1}{4}x^4 \operatorname{arctg} x - \frac{1}{4}\int \frac{\left(x^2-1\right)\left(x^2+1\right)}{1+x^2} dx - \frac{1}{4}\int \frac{1}{1+x^2} dx =$$

$$= \frac{1}{4}x^4 \operatorname{arctg} x - \frac{1}{4}\int \left(x^2-1\right) dx - \frac{1}{4}\operatorname{arctg} x + C =$$

$$= \frac{1}{4}x^4 \operatorname{arctg} x - \frac{1}{4}\int x^2 dx + \frac{1}{4}\int dx - \frac{1}{4}\operatorname{arctg} x + C =$$

$$= \frac{1}{4}x^4 \operatorname{arctg} x - \frac{1}{12}x^3 + \frac{1}{4}x - \frac{1}{4}\operatorname{arctg} x + C$$

Задача 39

$$\int x \operatorname{arctg}^2 x dx = \frac{1}{2}\int \operatorname{arctg}^2 x dx^2 = \frac{1}{2}x^2 \operatorname{arctg}^2 x - \frac{1}{2}\int x^2 d \operatorname{arctg}^2 x =$$

$$= \frac{1}{2}x^2 \operatorname{arctg}^2 x - \frac{2}{2}\int \frac{x^2}{1+x^2} \operatorname{arctg} x dx = \frac{1}{2}x^2 \operatorname{arctg}^2 x - \int \frac{x^2+1-1}{1+x^2} \operatorname{arctg} x dx =$$

$$= \frac{1}{2}x^2 \operatorname{arctg}^2 x - \int \operatorname{arctg} x dx + \int \frac{1}{1+x^2} \operatorname{arctg} x dx =$$

$$= \frac{1}{2}x^2 \operatorname{arctg}^2 x - x \operatorname{arctg} x + \int x d \operatorname{arctg} x + \int \operatorname{arctg} x d \operatorname{arctg} x =$$

$$= \frac{1}{2}x^2 \operatorname{arctg}^2 x - x \operatorname{arctg} x + \int \frac{x}{1+x^2} dx + \frac{1}{2}\operatorname{arctg}^2 x + C =$$

$$= \frac{1}{2}x^2 \operatorname{arctg}^2 x - x \operatorname{arctg} x + \frac{1}{2}\int \frac{1}{1+x^2} d\left(x^2+1\right) + \frac{1}{2}\operatorname{arctg}^2 x + C =$$

$$= \frac{1}{2}x^2 \operatorname{arctg}^2 x - x \operatorname{arctg} x + \frac{1}{2}\ln\left|x^2+1\right| + \frac{1}{2}\operatorname{arctg}^2 x + C$$

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Задача 40

$$\int \frac{\arccos x}{x^2}dx = -\int \arccos x d\frac{1}{x} = = -\frac{1}{x}\arccos x - \int \frac{1}{x} d\arccos x =$$

$$= -\frac{1}{x}\arccos x - \int \frac{1}{x\sqrt{1-x^2}}dx = -\frac{1}{x}\arccos x - \int \frac{1}{x\sqrt{\left(1-x^2\right)\frac{x^2}{x^2}}}dx =$$

$$-\frac{1}{x}\arccos x - \int \frac{1}{x^2\sqrt{\frac{1}{x^2}-1}}dx = -\frac{1}{x}\arccos x + \int \frac{1}{\sqrt{\left(\frac{1}{x}\right)^2-1}}d\frac{1}{x} =$$

$$= -\frac{1}{x}\arccos x + \ln\left|\frac{1}{x} + \sqrt{\frac{1}{x^2}-1}\right| + C$$

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Задача 41

$$\int \frac{x\ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}dx$$

Внасяме $\frac{x}{\sqrt{1+x^2}}$ под знака d.

$$\boxed{\int \frac{x}{\sqrt{1+x^2}}d \;=\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}d\left(x^2+1\right)=\frac{1}{2}2\sqrt{1+x^2}+C=\sqrt{1+x^2}+C}$$

$$\int \ln\left(x+\sqrt{1+x^2}\right)d\sqrt{1+x^2}=$$

$$=\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)-\int \sqrt{1+x^2}\,d\ln\left(x+\sqrt{1+x^2}\right)=$$

$$=\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)-\int \sqrt{1+x^2}\,\frac{1}{x+\sqrt{1+x^2}}\left(1+\frac{1}{2}\frac{1}{\sqrt{1+x^2}}2x\right)dx=$$

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$$=\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)-\int\frac{\sqrt{1+x^2}}{x+\sqrt{1+x^2}}\left(1+\frac{x}{\sqrt{1+x^2}}\right)dx=$$

$$=\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)-\int\frac{\sqrt{1+x^2}}{x+\sqrt{1+x^2}}dx-\int\frac{x}{x+\sqrt{1+x^2}}dx=$$

$$-\int\frac{\sqrt{1+x^2}}{x+\sqrt{1+x^2}}dx=-\int\frac{\sqrt{1+x^2}}{x+\sqrt{1+x^2}}\frac{x-\sqrt{1+x^2}}{x-\sqrt{1+x^2}}dx=-\int\frac{x\sqrt{1+x^2}-1-x^2}{x^2-1-x^2}dx=$$

$$=\int\left(x\sqrt{1+x^2}-1-x^2\right)dx=\int x\sqrt{1+x^2}\,dx-\int dx-\int x^2dx$$

$$-\int\frac{x}{x+\sqrt{1+x^2}}dx=-\int\frac{x}{x+\sqrt{1+x^2}}\frac{x-\sqrt{1+x^2}}{x-\sqrt{1+x^2}}dx=-\int\frac{x^2-x\sqrt{1+x^2}}{x^2-1-x^2}dx=$$

$$=\int\left(x^2-x\sqrt{1+x^2}\right)dx=\int x^2dx-\int x\sqrt{1+x^2}\,dx$$

$$= \sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)+$$

$$\int x\sqrt{1+x^2}\,dx-\int dx-\int x^2dx+\int x^2dx-\int x\sqrt{1+x^2}\,dx=$$

$$= \sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)-\int dx=$$

$$= \sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)-x+C$$

Изчисление с Mathematica

Задача: Да се пресметне интеграла:

$$\int\left(5x^7-4x^3+\frac{2}{x^2}-\frac{3}{\sqrt[3]{x^2}}\right)dx$$

$$f[x\_] := \int\left(5x^7-4x^3+\frac{2}{x^2}-\frac{3}{\sqrt[3]{x^2}}\right)dx$$

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f[x]
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$$-\frac{2}{x}-x^4+\frac{5x^8}{8}-\frac{9\left(x^2\right)^{2/3}}{x}$$