1. Дефиниционна област и локални екстремуми на функцията

$$y = \frac{\ln x}{x}$$

$x \neq 0, \ x \geq 1$

$$y' = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1-\ln x}{x^2}$$

$$y' = 0 \rightarrow \frac{1-\ln x}{x^2} = 0$$

$$\Rightarrow 1 - \ln x = 0$$

$$\ln x = 1$$

$$x = e \ !$$

$$y'' = \frac{-\frac{1}{x}\cdot x^2 - 2x(1-\ln x)}{x^4} = \frac{-x-2x+2x\ln x}{x^4} =$$

$$= \frac{2x\ln x - 3x}{x^4}$$

$$y''(e) = \frac{2e\ln e - 3e}{e^4} = \frac{2e-3e}{e^4} = -\frac{1}{e^3} \neq 0 < 0$$

$\Rightarrow$ за $x = e$ y има максимум

$$y_{max} = \frac{\ln e}{e} = \frac{1}{e}$$

2.) a) $\int x^2 e^x\,dx = \int x^2\,de^x =$ (по част)

$= x^2e^x - \int e^x\,2x\,dx = x^2e^x - \int 2x\,de^x =$

$= x^2e^x - \left(2xe^x - \int e^x \cdot 2\,dx\right) =$

$= x^2e^x - 2xe^x + 2e^x + C =$

$= \underline{e^x(x^2 - 2x + 2) + C}$

б) $\int \frac{dx}{\cos x(\sin x + \cos x)}dx$ (полагаем $tg\,x = t$)

$tg\,x = t$

$dt = \frac{1}{\cos^2 x}dx$

$\frac{\sin x}{\cos x} = t$

$\sin x = t\cos x$

$\int \frac{dx}{\cos x(t\cos x + \cos x)} = \int \frac{\overbrace{dx}^{=dt}}{\cos^2 x(t+1)} =$

$= \int \frac{dt}{t+1} = \int \frac{d(t+1)}{t+1} = \ln(t+1) =$

$= \underline{\ln(tg\,x + 1) + C}$

3) екстремуми на функция и видът им (3)

$$z = f(x,y) = x^2 + y^2 + xy - 2x - y$$

$$\frac{\partial z}{\partial x} = 2x + y - 2$$

$$\frac{\partial z}{\partial y} = 2y + x - 1$$

$$\frac{\partial z}{\partial x} = 0 \to 2x + y - 2 = 0$$

$$\frac{\partial z}{\partial y} = 0 \to 2y + x - 1 = 0 \to x = 1 - 2y$$

$$2(1-2y) + y - 2 = 0$$

$$\cancel{2} - 4y + y - \cancel{2} = 0$$

$$-3y = 0$$

$$y = 0$$

$$x = 1 - 2.0 = 1.$$

$$\frac{\partial^2 z}{\partial x^2} = 2$$

$$\frac{\partial^2 z}{\partial y^2} = 2$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}(2x + y - 2) = 1$$

$$\Delta = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 =$$

$$= 2.2 - 1 = 3 \neq 0 > 0$$

$\to T(1,0)$ е екстремална!

4. Диференциално уравнение с отделени променливи

$$xy' + y = 3$$

$$xy' = 3 - y \;\Big|\cdot \frac{1}{x(3-y)}$$

$$y' \cdot \frac{1}{3-y} = \frac{1}{x}$$

$$\cancel{\int}\frac{dy}{dx} \cdot \frac{1}{3-y} = \frac{1}{x}$$

$$\frac{dy}{3-y} = \frac{dx}{x}$$

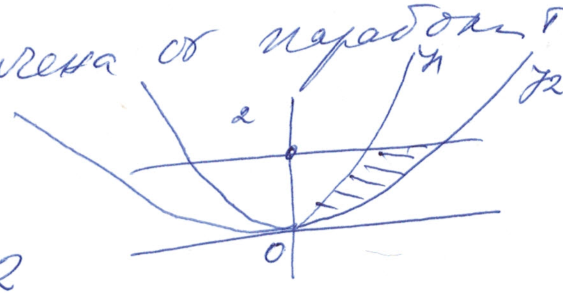
$$\int \frac{dy}{3-y} = \int \frac{dx}{x}$$

$$-\int \frac{d(3-y)}{3-y} = \ln x$$

$$-\ln|3-y| = \ln x$$

$$\frac{1}{3-y} = x$$

5. Лице на фигура, ограничена от параболите



$y = x^2$

$4y = x^2$

и правите $x = 0$, $x = 2$

$$S = \int_0^2 \left(x^2 - \frac{x^2}{4}\right)dx = \int_0^2 \frac{3x^2}{4}dx = \frac{3}{4}\frac{x^3}{3}\Big|_0^2 = 2$$

6 Сходимост на редя

$$\sum_{n=1}^{\infty} \frac{n!}{3^n}$$

$$\lim_{n\to\infty} \frac{\frac{(n+1)!}{3^{n+1}}}{\frac{n!}{3^n}} = \lim_{n\to\infty} \frac{\frac{\cancel{n!}\cdot(n+1)}{\cancel{3^n}\cdot 3}}{\frac{\cancel{n!}}{\cancel{3^n}}} = \lim_{n\to\infty} \frac{n+1}{3} = \infty$$

$\Rightarrow$ редът е разходящ