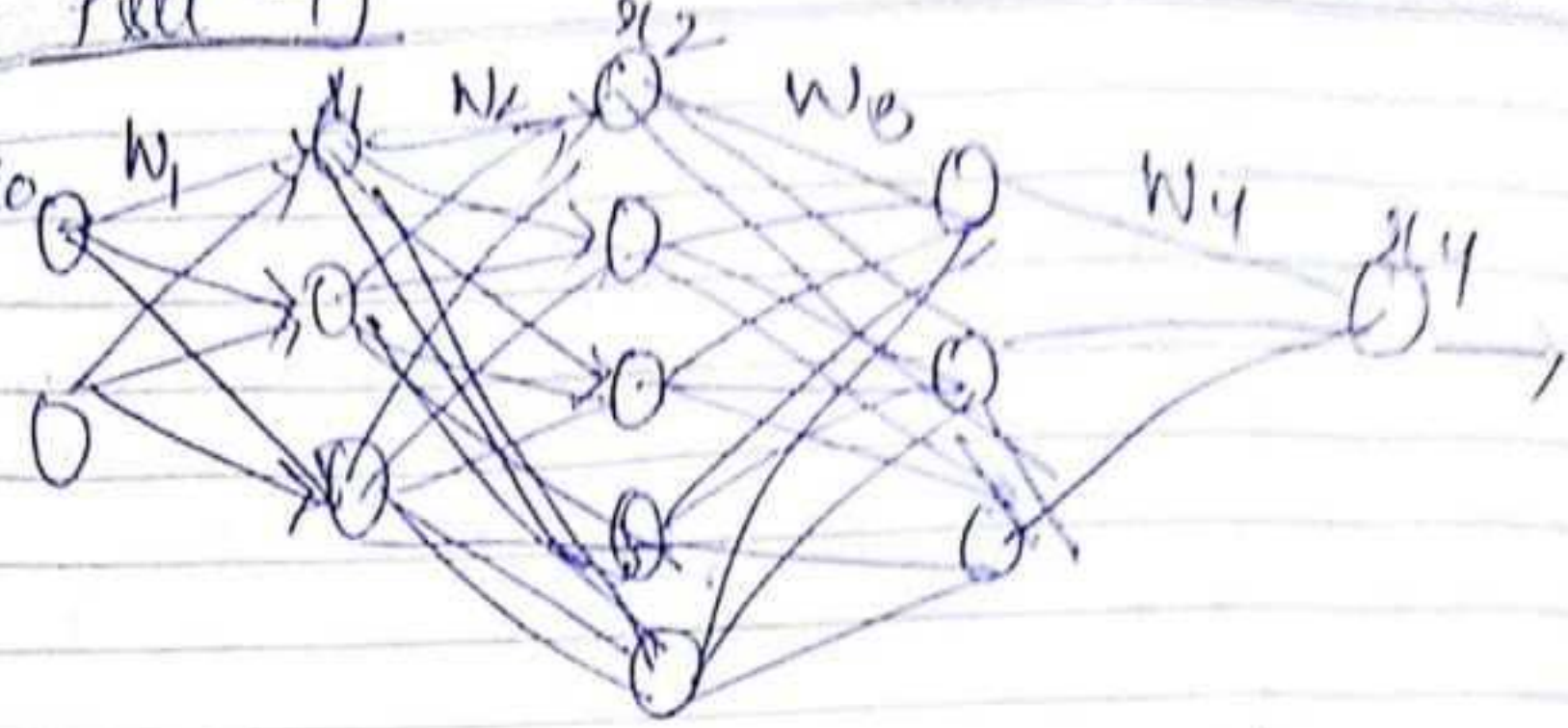
Part - 17

Ans = 2



Input : x_0

Hidden layer 1 output : $x_1 = f_1(w_1 x_0)$

" " 2 " $x_2 = f_2(w_2 x_1)$

" " 3 " $x_3 = f_3(w_3 x_2)$

Output : $x_4 = f_4(w_4 x_3)$

Here f_1, f_2, f_3, f_4 are activation functions eg sigmoid.

For Loss $L = \frac{1}{2} \sum_{i \in \text{Output}} \|o_i - t\|^2$

$$w_i^{k+1} \leftarrow w_i^k - \alpha_w \frac{\partial L(k)}{\partial w_i^{(k)}} \quad \forall\ w_i\text{'s}$$

Back propagation eqn's are

$$\frac{\partial L}{\partial w_4} = \frac{\partial}{\partial w_4}\left(\frac{1}{2}(f_4(w_4 x_3) - t)^2\right)$$

$$= (f_4(w_4 x_3) - t)\, f_4'(w_4 x_3) \frac{\partial w_4 x_3}{\partial w_4}$$

$$= \underbrace{(f_4(w_4 x_3) - t)\, f_4'(w_4 x_3)}_{\delta_4} x_3^T$$

$$\Rightarrow \frac{\partial E}{\partial w_4} = \delta_4 x_3^T$$

Dimensions: $(y_4 - t)\, w_4 \rightarrow 1 \times 3$

$(f_4(w_4 x_3) - t) \rightarrow 1 \times 1$

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$\Rightarrow$ $f_4(W_4 X_3) \rightarrow 1\times1$

$\Rightarrow \delta_4 \left(f_4(W_4 X_3) - t\right) \cdot f_4'(W_4 X_3) \rightarrow 1\times1$

$\rightarrow \delta_4 \; X_3 \rightarrow 3\times1$

$\Rightarrow \delta_4 X_3^T = 1\times3$ which is same as W_4 dimension

$\Rightarrow \frac{\partial t}{\partial W_4}$ has same dimension as W_4

$$\frac{\partial t}{\partial W_4} = \frac{\partial}{\partial W_4}\left(\frac{1}{2}\left(f_4(W_4 X_3) - t\right)^2\right)$$

$$= \left(f_4(W_4 X_3) - t\right)\left(\frac{\partial}{\partial W_4} f(W_4 X_3)\right)$$

$$= \left(f_4(W_4 X_3) - t\right) f_4'(W_4 X_3) \times \frac{\partial}{\partial W_3}(W_4 X_3)$$

$$= W_4^T \left(f_4(W_4 X_3) - t\right) f_4'(W_4 X_3) \times \frac{\partial X_3}{\partial W_3}$$

$$= W_4^T \delta_4 \frac{\partial X_3}{\partial W_3} = W_4^T \delta_4 f_3'(W_3 X_2) \frac{\partial W_3 X_2}{\partial W_3}$$

$$= \delta_3 X_2^T$$

Dimensions. $W_3 \rightarrow 3\times5$ $\quad \delta_4 \rightarrow 1\times1$ $\quad W_4 \rightarrow 1\times3$

$\Rightarrow W_4^T \rightarrow 3\times1$ $\quad f_3'(W_3 X_2) \rightarrow 3\times1$

$X_2 \rightarrow 5\times1$ $\quad \Rightarrow X_2^T \rightarrow 1\times5$

$\delta_3 = W_4^T \delta_4 \cdot f_3'(W_3 X_2)$

$\rightarrow \delta_3 \rightarrow 3\times1$ $\quad f_3' \rightarrow \delta_3 X_2^T \rightarrow 3\times5$

$$\frac{\partial t}{\partial W_2} = \frac{\partial}{\partial W_2}\left(\frac{1}{2} f_4(W_4 X_3) - t\right)^2$$

$$= \left(f_4(W_4 X_3) - t\right) \times f_4'(W_4 X_3) \frac{\partial}{\partial W_2}(W_4 X_3)$$

$$= W_4^T \delta_4 \frac{\partial X_3}{\partial W_2} \left(= W_4^T \delta_4 \frac{\partial}{\partial W_2} f_3(W_3 X_2)\right)$$

$$= W_4^T \delta_4 \times f_3'(W_3 X_2) \times \frac{\partial}{\partial x}(W_3 X_2) \times \frac{\partial X_2}{\partial W_2}$$

$$= W_3^T W_4^T \delta_4 \cdot f_3'(W_3 X_2) \times \frac{\partial f_2(W_2 X_1)}{\partial W_2}$$

$$= W_3^T \delta_3 \frac{\partial f_2(W_2 X_1)}{\partial W_2} = W_3^T \delta_3 \cdot f_2'(W_2 X_1) \frac{\partial}{\partial W_2}$$

$$= \boxed{\delta_2 X_1^T}$$

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Dimension: $W_2 \rightarrow 1 \times 3$

$\delta_3 : 3 \times 1$ $\quad W_3 \rightarrow 3 \times 1 \rightarrow W_3^T$ $\quad \delta_3 \rightarrow 1 \times 3$

$\rightarrow \delta_2 = W_3^T \delta_3 \cdot f_2'(W_2 X_1) \rightarrow 1 \times 1$

$X_2 = f_2(W_2 X_1) \rightarrow 1 \times 1 \Rightarrow f_2'(W_2 X_1) \rightarrow (X)$

$\delta_2 X_1^T \rightarrow 1 \times 3$ same as W_2

$$\frac{\partial E}{\partial W_1} = \frac{\partial}{\partial W_1}\left(\frac{1}{2}(f_4(W_4 X_3) - t)^2\right)$$

$$= (f_4(W_4 X_3) - t)\frac{\partial}{\partial W_1} f_4(W_4 X_3)$$

$$= \delta_1 X_1^T \qquad \delta_1 = (W_2^T \delta_2) \cdot f_1'(W_1 X_0)$$

Dimensions:

$a_1 = f_1(W_1 X_0) \rightarrow 3 \times 1 \Rightarrow f(W_1 X_0) \rightarrow 3 \times 1$

$\rightarrow X_1^T = 1 \times 3$

$\delta_1 = (W_2^T \delta_2) \cdot f_1'(W_1 X_0) \rightarrow 3 \times 1$

$\delta_2 \rightarrow 5 \times 1 \qquad W_2 \rightarrow 5 \times 3 \Rightarrow W_2^T = 3 \times 5$

$W_2^T \delta_2 = 3 \times 1$

$\Rightarrow \delta_1 X_1^T = 3 \times 1 \times 1 \times 2 = 3 \times 2 \rightarrow$ same as W_1