(e) Data Stream $x_i \in R^d$

$$\mu_i = \frac{(i-1)\mu_{i-1} + x_i}{i} \quad \text{Mean till } i^{th} \text{ sample}$$

$$\sigma_{i,mn} = \frac{(i-1)(\sigma_{(i-1)mn} + \mu_{(i-1)m}\mu_{(i-1)n}) + x_{im}x_{in}}{i} - \mu_{im}\mu_{in}$$

$\sigma_{i,mn} \rightarrow$ Covariance (if $m \neq n$) and variance $(m = n)$ till the i^{th} same.

For linear reg. closed soln, $\omega = (X^TX)^{-1}X^TY$

$$X^TX = i\left(\underbrace{\begin{bmatrix} \sigma_{i,11} & \sigma_{i,12} & \cdots \\ \sigma_{i,21} & & \\ \vdots & & \\ \sigma_{i,d1} & \cdots & \sigma_{i,dd} \end{bmatrix}}_{\text{Covariance Matrix } (C)} + \underbrace{\begin{bmatrix} (\mu_{i,1})^2 & \mu_{(i,1)}\mu_{(i,2)} & \\ (\mu_{i,2})(\mu_{i,1}) & \vdots & \\ \vdots & & \\ (\mu_{i,d})(\mu_{i,1}) & & \mu_{i,d}\mu_{i,d} \end{bmatrix}}_{\text{Mean Matrix } (M)}\right)$$

As for each term ~~[illegible]~~ $\sum_i x_{im}x_{ij} = [\sigma_{i,11} + (\mu_{i,1})^2]\, i$

$$X^TY = \sum_{k=1}^{i} x_k y_k = i\begin{bmatrix} \sigma_{x_1y} + \mu_{x_1}\mu_y \\ \sigma_{x_2y} + \mu_{x_2}\mu_y \\ \vdots \\ \sigma_{x_dy} + \mu_{x_d}\mu_y \end{bmatrix} = iU$$

$$\sum_{k=1}^{i} x_{1k}y_k = (\sigma_{x_1y} + \mu_{x_1}\mu_y)\, i$$

The covariance b/w each component of x & the y can be calculated using corresponding previous terms as above in formula

$$\omega = (i(C+M))^{-1} i(U)$$
$$= (C+M)^{-1}U$$

where all entires of C, M, U are in the form of covariances, variances and means which can be calculated by previous values using formula given above.

$x \in R$

let μ_i denote mean till i^{th} sample

$$\mu_i = \frac{(\mu_{i-1})(i-1) + x_i}{i} \quad \forall\, i > 0 \quad \text{and for } \mu_0 = x_0$$

$\sigma_i = E(x^2) - E(x)^2$

$\sigma_i = s_i - (\mu_i)^2$

where s_i denote the mean of square of samples till i and μ_i denote the mean of samples and s_i is calculated as

$$s_i = \frac{s_{i-1}(i-1) + x_i^2}{i} = \frac{[\sigma_{(i-1)} + \mu_{(i-1)}^2](i-1) + x_i^2}{i}$$

$\sigma_i = \sigma_{(i-1)} +$

$x \in R^d$

Mean till i^{th} sample

$x_i^T x_i$

$\mu_i = \mu_{i-1}$

$$E(XY) = \frac{(i-1)\, E(XY)^{i-1} + x_i y_i}{i}$$

σ_x^2

$(\sigma_x \sigma_y)^i = E(XY) - E(X)E(Y)$

$= (i-1)[$

$$(\sigma_{xy})^i = \frac{(i-1)[(\sigma_{xy})^{i-1} + x_{i-1}y_{i-1}] + x_i y_i}{i} - \mu.$$

Ans 2 Let us consider $x_i, y_i \in R^2$

Let the regression line be $y_i = m x_i + c$

error $= \bar{y}_i - y_i$

$$E = \sum_{1}^{N} (\bar{y}_i - y_i)^2$$

$$\Rightarrow E = \sum_{1}^{N} (\bar{y}_i - m x_i - c)^2$$

Differentiating wrt c

$$\frac{\partial E}{\partial c} = \sum_{1}^{N} 2(\bar{y}_i - m x_i - c) = 0$$

$$\Rightarrow \sum(\bar{y}_i - m x_i - c) = 0$$

$$\Rightarrow \sum \bar{y}_i - m \sum x_i = Nc$$

$$\Rightarrow c = \frac{\sum y_i - M \sum x}{N}$$

$$= \frac{\sum y_i}{N} - M \frac{\sum x}{N}$$

$$= \mu y - M \mu x$$

→ $\therefore$ $c = \mu y - M \mu x$

Differentiating wrt m.

$$\frac{\partial E}{\partial M} = 0 \Rightarrow \sum 2(\bar{y}_i - m x_i - c)(-x_i) = 0$$

$$\Rightarrow \sum x_i (y_i - m x_i - c) = 0$$

$$\Rightarrow \sum x_i y_i - m \sum x_i^2 - c \sum x_i = 0$$

Now substituting c.

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$\Sigma xy - m\Sigma x^2 + (M\mu x - \mu y)\Sigma x = 0$

$\Rightarrow \Sigma xy - \mu y \Sigma x + M(-\Sigma x^2 + \mu x \Sigma x) = 0$

$$\Rightarrow M = \frac{\mu y \Sigma x - \Sigma xy}{\mu x \Sigma x - \Sigma x^2}$$

$$\Rightarrow M = \frac{\Sigma y \Sigma x}{N} - \Sigma xy \Big/ \frac{(\Sigma x)^2}{N} - \Sigma x^2$$

$$\Rightarrow M = \frac{\Sigma y \Sigma x}{N^2} - \frac{\Sigma xy}{N} \Big/ \left(\frac{\Sigma x}{N}\right)^2 - \frac{\Sigma x^2}{N}$$

$$\Rightarrow M = \frac{\Sigma xy}{N} - \frac{\Sigma y \Sigma x}{N^2} \Big/ \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$$

$$\Rightarrow M = \frac{E(xy) - E(x)E(y)}{E(x^2) - E(x)^2} = \frac{6xy}{6x^2}$$

Hence equation of line is $y = mx + c$

$$\Rightarrow y = \left(\frac{6xy}{6x^2}\right)x + \left(\mu y - \frac{6xy}{6x^2}\mu x\right)$$

Hence we can express regression line in terms of μ & 6.

ns3 Let 4 points be

$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$$\text{mean} = \begin{bmatrix} \Sigma x_i / N \\ \Sigma y_i / N \end{bmatrix} = \begin{bmatrix} 9/2 \\ 4 \end{bmatrix}$$

$$C = \Sigma(x - \mu)(x - \mu)^T$$