Class-Activity-3

$$W^1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}, \quad W^2 = \begin{bmatrix} 0.2 \\ 1 \\ -3 \end{bmatrix}, \quad W^3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Forward Propagation

For data points $x = 2$, $y = 1$

$$s^1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}^T \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 1 \end{bmatrix}$$

$$x^1 = \begin{bmatrix} 1 \\ \tanh 0.7 \\ \tanh 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.604 \\ 0.762 \end{bmatrix}$$

$$s^2 = \begin{bmatrix} 0.2 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.604 \\ 0.762 \end{bmatrix} = \begin{bmatrix} -1.482 \end{bmatrix}$$

$$x^2 = \begin{bmatrix} 1 \\ \tanh(-1.482) \end{bmatrix} = \begin{bmatrix} 1 \\ -0.902 \end{bmatrix}$$

$$s^3 = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -0.902 \end{bmatrix} = -0.804$$

$$x^3 = \tanh(-0.804) = -0.666$$

<u>Backpropagation</u>:

$$\delta^{(L)} = \frac{\partial e}{\partial s^{(L)}} = \frac{\partial}{\partial s^{(L)}} (x^{(L)} - y)^2$$

$$= 2(x^{(L)} - y)\,\theta'(s^{(L)})$$

So,

$$\delta^3 = [2 \cdot (-0.66 - 1) \cdot (1 - (-0.66)^2)) = -1.855$$

$$\delta^2 = [(1 - 0.9^2) \cdot 2 \cdot (-1.855)] = -0.69$$

$$\delta^1 = \begin{bmatrix} -0.44 \\ 0.88 \end{bmatrix}$$

To obtain partial derivatives that we need for gradient:

$$\frac{\partial e}{\partial W^1} = x^0 (\delta^1)^T = \begin{bmatrix} -0.44 & 0.88 \\ -0.88 & 1.75 \end{bmatrix}$$

$$\frac{\partial e}{\partial w^2} = x^1 (\delta^2)^T = \begin{bmatrix} -0.69 \\ -0.42 \\ -0.53 \end{bmatrix}$$

$$\frac{\partial e}{\partial w^3} = x^2 (\delta^3)^T = \begin{bmatrix} -1.85 \\ 1.67 \end{bmatrix}$$