Homework 1

NAME:

General Hints: When you are asked to "show" or "prove" something, you should make it a point to write down clearly the information you are given and what it is you are to show. One word of warning regarding the second part of Problem 1.1.22: To say that $\mathbf{v}$ is a linear combination of $\mathbf{v}_1, \dots, \mathbf{v}_k$ is to say that $\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k$ for some scalars $c_1, \dots, c_k$. These scalars will surely be different when you express a different vector $\mathbf{w}$ as a linear combination of $\mathbf{v}_1, \dots, \mathbf{v}_k$, so be sure you give the scalars for $\mathbf{w}$ different names.

("SA 1.1.21" means exercise 21 in Section 1.1 of textbook by **S**hifrin and **A**dams.)

Problem 1 (SA 1.1.21). Suppose $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and c is a scalar. Prove that $\mathrm{Span}(\mathbf{v} + c\mathbf{w}, \mathbf{w}) = \mathrm{Span}(\mathbf{v}, \mathbf{w})$. (See the blue box on p. 12 of the textbook.)

Problem 2 (SA 1.1.22). Suppose the vectors $\mathbf{v}$ and $\mathbf{w}$ are both linear combinations of $\mathbf{v}_1, \ldots, \mathbf{v}_k$.

1. Prove for any scalar c that $c\mathbf{v}$ is a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_k$.
2. Prove that $\mathbf{v} + \mathbf{w}$ is a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_k$.

Problem 3 (SA 1.1.25). Suppose $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ are nonparallel vectors. (Recall def, p.3 of text.)
1. Prove that if $s\mathbf{x} + t\mathbf{y} = 0$, then $s = t = 0$. (Hint: Show neither $s \neq 0$ nor $t \neq 0$ is possible.)
2. Prove that if $a\mathbf{x} + b\mathbf{y} = c\mathbf{x} + d\mathbf{y}$, then $a = c$ and $b = d$.