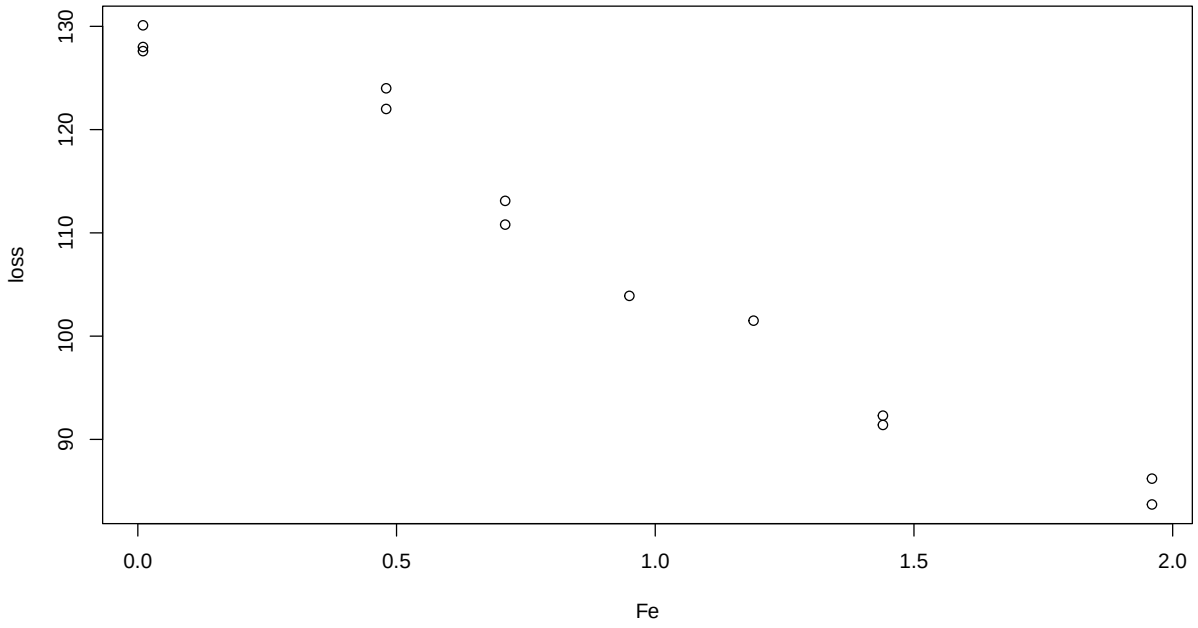
Confidence interval for simple linear regression

The Data

```
library(faraway)
data(corrosion)
help(corrosion)
plot(corrosion)
```



Simple Linear Model

```
fit = lm(loss ~ Fe, data = corrosion)
summary(fit)
```

```
Call:
lm(formula = loss ~ Fe, data = corrosion)

Residuals:
    Min      1Q  Median      3Q     Max
-3.7980 -1.9464  0.2971  0.9924  5.7429

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  129.787      1.403   92.52  < 2e-16 ***
Fe           -24.020      1.280  -18.77 1.06e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.058 on 11 degrees of freedom
Multiple R-squared:  0.9697,    Adjusted R-squared:  0.967
F-statistic: 352.3 on 1 and 11 DF,  p-value: 1.055e-09
```

Iron prevents corrosion. For 1.5% of iron we would expect 129.8 - 1.5 * 24.0 weight loss.

```
129.8 - 1.5 * 24
```

```
[1] 93.8
```

Hypothesis testing

We want to test

$$\psi(\beta) = c^T\beta$$

for

$$c^T = (1, 1.5)$$

The test statistics is

$$\frac{\widehat{\psi} - 95}{\widehat{SE}_{\widehat{\psi}}}$$

```
library(car)
```

```
Loading required package: carData
```

```
Attaching package: 'car'
```

```
The following objects are masked from 'package:faraway':

    logit, vif
```

```
predict(fit, data.frame(Fe = 1.5), interval = 'confidence')
```

```
       fit      lwr     upr
1 93.75676 91.18722 96.3263
```

```
c = c(1, 1.5)
linearHypothesis(fit, c, 95)
```

```
Linear hypothesis test:
(Intercept)  + 1.5 Fe = 95

Model 1: restricted model
Model 2: loss ~ Fe

  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     12 113.45
2     11 102.85  1    10.603 1.1341 0.3097
```

Test by hand

```
n = nrow(corrosion)
df = n - ncol(model.matrix(fit))
target = 95
estimate = predict(fit, data.frame(Fe = 1.5))
sigma = sigma(fit)

test = (estimate - target) / sigma

alpha = 0.05
ifelse(abs(test) > qt(1 - alpha / 2, df), "Reject", "Don't reject")
```

```
            1
"Don't reject"
```