8 Fourier Series

8.1 Periodic Signals

- **Fundamental Period** T_0
- **Fundamental Frequency** $F_0 = 1/T_0$
  Any periodic signal with a fundamental period can be composed as a sum of sinusoidal signals.
  The frequency of these sinusoidal signals must divide the fundamental period.
  Thus when given various sinusoidal signals, the fundamental frequency is the Greatest Common Divisor.

A real signal $x(t)$ which consists of a DC + a sum of N different frequencies $f_1 \dots f_n$ with $f_1 < \dots < f_N$ is periodic with fundamental period T_0 when it has a fundamental frequency $F(0) = gcd(f1, \dots, f_N)$

Thus we can write a periodic signal $x(t)$ as a weighted sum of phasor components as follows:

$$x(t) = \sum_{k=-M}^{M} \alpha_k e^{j2\pi F_0 t}$$

- a_k : the amplitude and phase of frequency components at frequency $k * F_0$, usually in the form of $C_1 \cdot e^{j*C_2}$
- For only N out of M values a_k is not equal to 0 if we have N frequencies

8.2 Fourier series synthesis

In theory, M can be infinitely large. Resulting in the general expression:

$$x(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{j2\pi F_0 t}$$