UAVRP with Moving Targets

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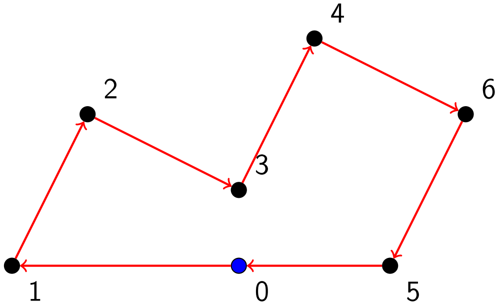
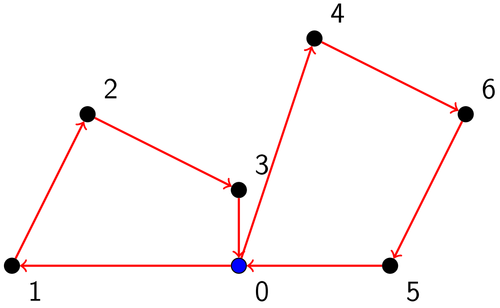
Traveling Salesman Problem

- number of vehicles: one;
- combinatorial objects: permutations;
- objective function: route length.



Vehicle Routing Problem

- number of vehicles: many;
- combinatorial objects: partition and permutations;
- objective function: sum of route lengths.



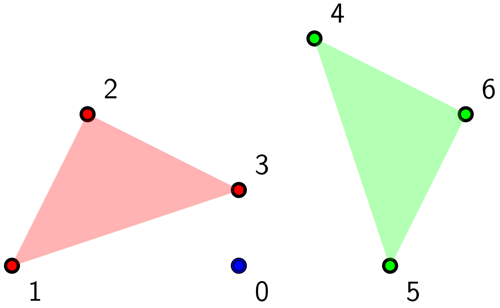
Moving Targets

Generalizes dynamic depots described in [1]. Instead of $x = (x_1, x_2, \ldots, x_d)$ we have $x(t) = f(t)$. In the linear case, $x(t) = x_0 + tv$, where $v = (v_1, v_2, \ldots v_d)$.

- Distance between moving points depends on time (order).
- In particular, 2-opt [2] does not apply.

Clustering

- what: optimal partition;
- why: divide and conquer;
- how: spacial or temporal proximity.



Two-Level Genetic Algorithm

1. Clustering moving objects. [3]
2. Find the shortest Hamiltonian *cycle* within each cluster.
3. Connect cycles between clusters by disconnecting one edge within each cycle.

For stationary targets this algorithm was introduced in [4].

Definition of the Ordering Power

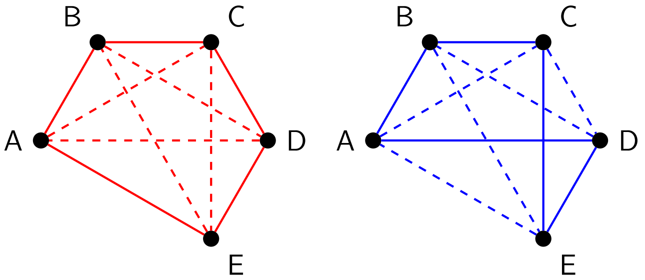
What is a combinatorial optimization problem?

- a set of meaningful instances S;
- a finite set of feasible solutions $C = \{c_1, c_2, \dots, c_n\}$ for each $x \in S$;
- and an objective function $f : C \to \mathbb{R}$ (wlog $f(c_i) \neq f(c_j)$ for $i \neq j$).

Definition

Define a mapping $g : S \to P_n$, where P_n is the set of permutations of $\{1, 2, \dots, n\}$ as follows: $g_i = |\{j : f(c_j(x)) < f(c_i(x))\}|$. In other words, the value of g is an ordering of feasible solutions by the value of the objective function. We call $\log |g(S)|$ an *ordering power of f*, as a function of instance size.

Example with Classical TSP



$$f(ABCDE) + f(ACEBD) = f(ABCED) + f(ACDBE).$$

Therefore, it is impossible to have $f(ABCDE) < f(ABCED)$ and $f(ACEBD) < f(ACDBE)$ simultaneously. The same argument does not hold for moving targets.

Weak Real-Valued Version

Definition

Define a mapping $g : S \to \mathbb{R}^n$ as follows: $g(x) = (f(c_1(x)), \dots, f(c_n(x)))$. We call $\dim g(S)$ a *weak ordering power of f*, as a function of instance size. **Claim**: the two notions are connected when g is linear.

For example, all $n!$ route lengths are continuous functions of $2dn$ variables describing a problem instance with n linearly moving points in d-dimensional Euclidean space. Here a point is defined by its position $(x_1, \dots, x_d)$ at time 0 and its velocity $(v_1, \dots, v_d)$. Position of a point at time t is $(x_1 + tv_1, \dots, x_d + tv_d)$. Therefore, $\dim g(S) = 2dn \ll n!$.

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