Content-Based Recommendation System Developed from the MovieLens 10M Dataset

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Introduction

The Big Tech companies use recommendation systems to provide customized suggestions of products to users (Koren 2008, 426; Rocca 2019). These algorithms aim to enhance user experience and build customer loyalty (Koren 2008, 426). They are commonly developed using content-based methods founded on user and product information and collaborative filtering methods based on user-product interactions (Rocca 2019).

In 2006, the streaming service company Netflix offered USD 1,000,000 to any team that managed to come up with an algorithm that improved on their internal recommendation system, Cinematch, by 10% or more. The hybrid team of KorBell, Big Chaos, and Pragmatic Chaos - Bell-Kor's Pragmatic Chaos - won the challenge (Van Buskirk 2009). The team utilized advanced methods of collaborative filtering, restricted Boltzmann machine, and gradient-boosted decision trees, yet Koren (2008, 434; 2009, 9) of team KorBell emphasized the significance of content information from the baseline predictors in capturing the main biases in the dataset and refining the prediction of the algorithm.

Following this lead, this project aims to develop a modest recommendation system that is based on content information from baseline predictors in a similar dataset of the movie recommender MovieLens and that predicts future movie ratings of users with a root mean squared error (RMSE) rate of less than 0.86490. In the succeeding sections, we examine the dataset and user preferences then subsequently build algorithms to develop the recommendation system.

This undertaking is part of the capstone in the Professional Certificate Program in Data Science of Harvard Online.

MovieLens 10M Dataset

The MovieLens 10M Dataset was composed of 10,000,054 movie ratings from users with 20 ratings or more (GroupLens, n.d.; Harper and Konstan 2015). The dataset was downloaded and subjected to wrangling using an initial code in the language R provided by Harvard Online, which standardized its formatting and partitioning into 90% edx set and 10% final holdout test set across all projects.

The edx set was used to examine user preferences and to train and test content-based algorithms to develop the recommendation system. It contained 9,000,055 movie ratings, each with six variables: userId, movieId, rating, timestamp, title, and genres (fig. 1). UserIds and movieIds were unique to each of its 69,878 users and 10,677 movies, respectively. Ratings ranged from 0.5 to 5 stars, and timestamps were rendered in Unix time. Titles were captured manually according to how they appear in the movie database IMDb and included the years of movie release. Lastly, the genres were comprised of 18 individual ones (GroupLens, n.d.).

On the other hand, the final holdout test set was reserved to evaluate the recommendation system. It contained 999,999 movie ratings, each with the aforementioned six variables. We circle back to this set in a later section.

userId	movieId	rating	timestamp				title	genres
1	122	5	838985046			Boomerang	(1992)	Comedy Romance
1	185	5	838983525			Net, The	(1995)	Action Crime Thriller
1	292	5	838983421			Outbreak	(1995)	Action Drama Sci-Fi Thriller
1	316	5	838983392			Stargate	(1994)	Action Adventure Sci-Fi
1	329	5	838983392	Star	Trek:	Generations	(1994)	Action Adventure Drama Sci-Fi

Figure 1: First rows of the edx set.

User Preferences

The interactions of the contents of the edx set were visualized using ggplot2 functions to examine user preferences.

MovieId and Genre

A plot of user ratings against movieIds of a random sample of the data showed an aggregation of the ratings on a number of movieIds (fig. 2). This indicated the tendency of users to rate certain movies more than others and consequently imparted which movies are popular.

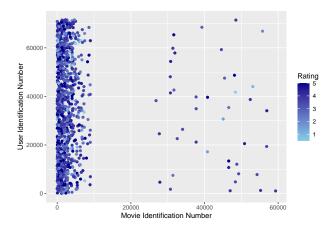


Figure 2: Scatterplot of user ratings vs movies of a random sample of the edx set.

A similar plot of user ratings against genres likewise exhibited the preference of users for certain genres such as Comedy and Drama (fig. 3).

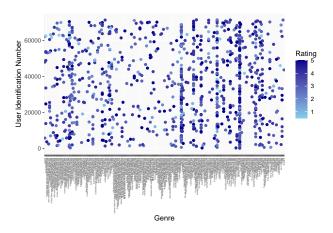


Figure 3: Scatterplot of user ratings vs genres of a random sample of the edx set.

Title and Year of Release

Titles were not unique to each movie unlike movields and were therefore not good movie identifiers. However, they contained important information: the years of movie release. Hence, the years were extracted from the ends of the titles utilizing stringr functions, and a plot of user ratings per year of a random sample of the data was produced (fig. 4). This revealed the inclination of users toward movies from the 1990s to 2000s.

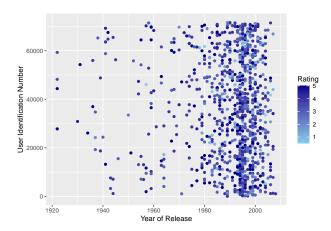


Figure 4: Scatterplot of user ratings per year of movie release of a random sample of the edx set.

UserId

A simple bar chart of average user ratings of a random sample of the data showed the tendencies for certain users to give a generous average rating of 5 (fig. 5). On the other hand, some users gave a stingy average rating of 1.

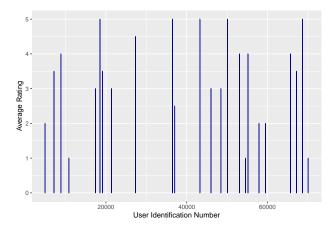


Figure 5: Bar chart of average user ratings of a random sample of the edx set.

Timestamp and Relative Age

Timestamps in Unix time implicitly contained the dates and times when the ratings were made. Using lubridate functions, the hour, day of the week, day of the month, and month of the ratings were obtained. Plots of user ratings per hour, day of the week, day of the month, and month of a random sample of the data were constructed but showed no readily observable bias (figs. 6-9).

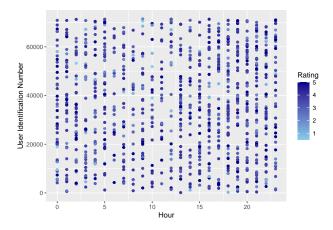


Figure 6: Scatterplot of user ratings per hour of the day of a random sample of the edx set.

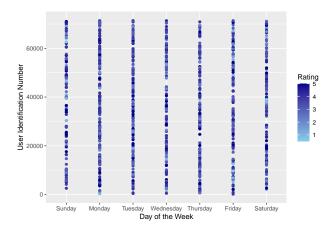


Figure 7: Scatterplot of user ratings per day of the week of a random sample of the edx set.

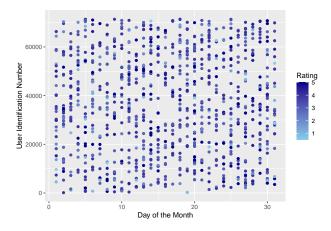


Figure 8: Scatterplot of user ratings per day of the month of a random sample of the edx set.

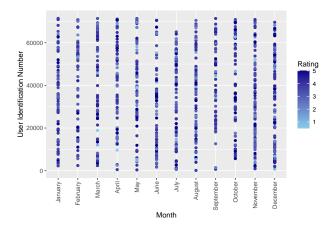


Figure 9: Scatterplot of user ratings per month of the year of a random sample of the edx set.

Unlike hours, days, and months, the years of rating are not recurring units of time that we expect in future data. Instead of using them as is, the relative ages of movies were calculated from the years of rating and release through stringr and lubridate functions. A plot of user ratings against relative ages of a random sample of the data exhibited the preference of users to rate movies within 10 years of their release (fig. 10).

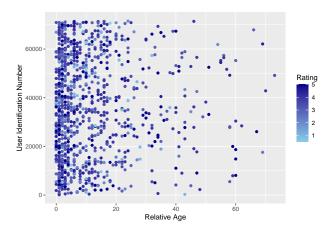


Figure 10: Scatterplot of user ratings vs relative ages of movies of a random sample of the edx set.

Content-Based Algorithms

Given the biases observed in the contents of the edx set, the following were considered as possible predictors of the actual ratings in the recommendation system: movieId, genre, year of movie release, userId, and relative age of the movie. Algorithms based on these predictors were progressively trained on 90% of the edx set (train set) by calculating the deviation caused by the predictor from the average using the penalized least squares estimation (Irizarry 2022, Koren 2009). Thereafter, the algorithms were tested on the remaining 10% of the edx set (test set) for the the RMSE of their predicted ratings from the actual ones. The RMSE was the preferred measure of error as it results in the same units (Irizarry 2022).

There is a more convenient lm function that can automatically train the algorithms and produce the least squares estimates for their linear modeling. However, it comes with the constraint of time and/or computer capability with a dataset of this scale and was not be used for this project (Irizarry 2022).

Baseline Algorithm: Average Rating μ

The Baseline Algorithm was constructed on the basic assumption that the average rating μ of 3.5124 in the train set predicts ratings with the least error from the actual ratings in the testing set. This gave an RMSE of 1.0593, which served as the basis of comparison for the error rate of the succeeding algorithms (tab. 1).

```
mu <- mean(train_set$rating)
mu
## [1] 3.5124
baseline_rmse <- rmse(test_set$rating, mu)
baseline_rmse
## [1] 1.0593</pre>
```

Algorithm 1: Average Rating μ + Movie Bias b_i

The Baseline Algorithm predicted rating was improved by adding the deviation from the average caused by the observed movie bias. The bias of each movie b_i was estimated from the average of the ratings less the effect of the μ (Irizarry 2022). This lowered the RMSE of the prediction to 0.94292 (tab. 1).

```
bi_tibble <- train_set |>
  group_by(movieId) |>
  summarize(bi = mean(rating - mu))
head(bi_tibble, n = 5)
## # A tibble: 5 x 2
##
     movieId
                 hi
##
       <int> <dbl>
## 1
           1 0.419
## 2
           2 - 0.309
## 3
           3 -0.366
## 4
           4 -0.645
## 5
           5 -0.442
algorithm1_rating <- test_set |>
  left_join(bi_tibble, by = "movieId") |> # adds the bi column
 mutate(algorithm1_rating = mu + bi) |>
  pull(algorithm1_rating)
algorithm1_rmse <- rmse(test_set$rating, algorithm1_rating)</pre>
algorithm1_rmse
## [1] 0.94292
```

Algorithm 2: Average Rating μ + Movie Bias b_i + Genre Bias b_q

The Algorithm 1 predicted ratings were amended with the deviation from the average caused by the observed genre bias. The bias of each genre combination b_g was estimated from the average of the ratings less the effects of the μ and b_i . However, this did not significantly affect the RMSE of the prediction, which remained at 0.94292 (tab. 1).

```
bg_tibble <- train_set |>
  left_join(bi_tibble, by = "movieId") |>
  group_by(genres) |>
  summarize(bg = mean(rating - mu - bi))
head(bg_tibble, n = 5)
## # A tibble: 5 x 2
```

```
genres
##
     <chr>
                                                                <db1>
## 1 (no genres listed)
                                                             0
## 2 Action
                                                            -8.97e-17
## 3 Action/Adventure
                                                            -1.91e-15
## 4 Action | Adventure | Animation | Children | Comedy
                                                            -4.81e-16
## 5 Action | Adventure | Animation | Children | Comedy | Fantasy 4.47e-17
algorithm2_rating <- test_set |>
  left join(bi tibble, by = "movieId") |>
  left_join(bg_tibble, by = "genres") |>
  mutate(algorithm2_rating = mu + bi + bg) |>
  pull(algorithm2_rating)
algorithm2_rmse <- rmse(test_set$rating, algorithm2_rating)</pre>
algorithm2_rmse
## [1] 0.94292
```

Hence, individual genres were arranged from highest to lowest rating. Then, the top genre was extracted from each genre combination with the str_detect function. The b_g was recalculated for each top genre from the average of the ratings less the effects of the μ and b_i . Still, this did not significantly affect the error rate of the prediction, which remained at 0.94292 (tab. 1).

```
bg_tibble <- train_set |>
  left_join(bi_tibble, by = "movieId") |>
  mutate(t_genre = case_when(str_detect(genres, "Film-Noir") ~ "Film-Noir",
                             str_detect(genres, "Documentary") ~ "Documentary",
                             str_detect(genres, "Drama") ~ "Drama",
                             str_detect(genres, "War") ~ "War",
                             str_detect(genres, "Western") ~ "Western",
                             str_detect(genres, "Thriller") ~ "Thriller",
                             str_detect(genres, "Fantasy") ~ "Fantasy",
                             str_detect(genres, "Musical") ~ "Musical",
                             str_detect(genres, "Romance") ~ "Romance",
                             str_detect(genres, "Comedy") ~ "Comedy",
                             str_detect(genres, "Crime") ~ "Crime",
                             str_detect(genres, "Mystery") ~ "Mystery",
                             str_detect(genres, "Animation") ~ "Animation",
                             str_detect(genres, "Adventure") ~ "Adventure",
                             str_detect(genres, "Action") ~ "Action",
                             str_detect(genres, "Sci-Fi") ~ "Sci-Fi",
                             str_detect(genres, "Horror") ~ "Horror",
                             str_detect(genres, "Children") ~ "Children")) |>
  group_by(t_genre) |>
  summarize(bg = mean(rating - mu - bi))
head(bg_tibble, n = 5)
## # A tibble: 5 x 2
     t_genre
                      bg
     <chr>
##
                   <dbl>
## 1 Action
               2.34e-16
## 2 Adventure 1.59e-15
## 3 Animation 1.12e-15
## 4 Children -3.04e-16
## 5 Comedy
                2.83e-15
algorithm2_rating <- test_set |>
```

```
left_join(bi_tibble, by = "movieId") |>
  mutate(t_genre = case_when(str_detect(genres, "Film-Noir") ~ "Film-Noir",
                             str_detect(genres, "Documentary") ~ "Documentary",
                             str_detect(genres, "Drama") ~ "Drama",
                             str_detect(genres, "War") ~ "War",
                             str_detect(genres, "Western") ~ "Western",
                             str_detect(genres, "Thriller") ~ "Thriller",
                             str_detect(genres, "Fantasy") ~ "Fantasy",
                             str_detect(genres, "Musical") ~ "Musical",
                             str_detect(genres, "Romance") ~ "Romance",
                             str_detect(genres, "Comedy") ~ "Comedy",
                             str_detect(genres, "Crime") ~ "Crime",
                             str_detect(genres, "Mystery") ~ "Mystery",
                             str_detect(genres, "Animation") ~ "Animation",
                             str_detect(genres, "Adventure") ~ "Adventure",
                             str_detect(genres, "Action") ~ "Action",
                             str_detect(genres, "Sci-Fi") ~ "Sci-Fi",
                             str_detect(genres, "Horror") ~ "Horror",
                             str_detect(genres, "Children") ~ "Children")) |>
  left_join(bg_tibble, by = "t_genre") |>
  mutate(algorithm2_rating = mu + bi + bg) |>
  pull(algorithm2_rating)
algorithm2_rmse <- rmse(test_set$rating, algorithm2_rating)</pre>
algorithm2_rmse
## [1] 0.94292
```

Algorithm 3: Average Rating μ + Movie Bias b_i + Release Bias b_r

The Algorithm 1 predicted ratings were updated with the deviation from the average caused by the observed bias from the years of movie release in place of that from the genres. The bias of each year of release b_r was estimated from the average of the ratings less the effects of the μ and b_i . However, this did not significantly affect the RMSE of the prediction as well, which remained at 0.94292 (tab. 1).

```
br_tibble <- train_set |>
  left join(bi tibble, by = "movieId") |>
  mutate(year_rel = str_extract(title, "\\(\\d{4}\\)$")) |>
  mutate(year_rel = str_replace_all(year_rel, "[:punct:]", "")) |>
  mutate(year_rel = as.integer(year_rel)) |>
  group_by(year_rel) |>
  summarize(br = mean(rating - mu - bi))
head(br_tibble, n = 5)
## # A tibble: 5 x 2
##
   year_rel
                    br
##
       <int>
## 1
       1915 -5.71e-17
## 2
        1916 -5.48e-17
## 3
       1917 -1.53e-17
## 4
     1918 -3.22e-18
## 5
        1919 2.25e-17
algorithm3_rating <- test_set |>
 left_join(bi_tibble, by = "movieId") |>
 mutate(year rel = str extract(title, "\((\d{4}\\)$")) |>
 mutate(year_rel = str_replace_all(year_rel, "[:punct:]", "")) |>
```

```
mutate(year_rel = as.integer(year_rel)) |>
left_join(br_tibble, by = "year_rel") |>
mutate(algorithm3_rating = mu + bi + br) |>
pull(algorithm3_rating)
algorithm3_rmse <- rmse(test_set$rating, algorithm3_rating)
algorithm3_rmse
## [1] 0.94292</pre>
```

Algorithm 4: Average Rating μ + Movie Bias b_i + User Bias b_u

The Algorithm 1 predicted ratings were enhanced by adding the deviation from the average caused by the observed user bias in place of that from the years of movie release. The bias of each user b_u was estimated from the average of the ratings less the effects of the μ and b_i (Irizarry 2022). This further lowered the RMSE of the prediction to 0.86458, which narrowly breached the required error rate of 0.86490 (tab. 1).

```
bu_tibble <- train_set |>
  left_join(bi_tibble, by = "movieId") |>
  group_by(userId) |>
  summarize(bu = mean(rating - mu - bi))
head(bu_tibble, n = 5)
## # A tibble: 5 x 2
     userId
##
##
      <int> <dbl>
## 1
         1 1.66
## 2
          2 -0.130
## 3
          3 0.168
## 4
          4 0.659
## 5
          5 0.160
algorithm4_rating <- test_set |>
 left_join(bi_tibble, by = "movieId") |>
  left_join(bu_tibble, by = "userId") |>
  mutate(algorithm4_rating = mu + bi + bu) |>
  pull(algorithm4_rating)
algorithm4_rmse <- rmse(test_set$rating, algorithm4_rating)</pre>
algorithm4_rmse
## [1] 0.86458
```

Algorithm 5: Average Rating μ + Movie Bias b_i + User Bias b_u + Relative Age Bias b_a

The Algorithm 4 predicted ratings were enhanced by adding the deviation from the average caused by the last observed bias from the relative ages of movies. The bias of each relative age b_a was estimated from the average of the ratings less the effects of the μ , b_i , and b_u . This slightly lowered the RMSE of the prediction to 0.86414, which is below the required error rate of 0.86490 (tab. 1).

```
ba_tibble <- train_set |>
  left_join(bi_tibble, by = "movieId") |>
  left_join(bu_tibble, by = "userId") |>
  mutate(year_rel = str_extract(title, "\\(\\d{4}\\\)$")) |> mutate(year_rel = str_replace_all(year_rel
  mutate(year_rel = as.integer(year_rel)) |>
  mutate(time = as_datetime(timestamp)) |>
  mutate(year_rat = year(time)) |>
  mutate(rel_age = year_rat - year_rel) |>
```

```
group_by(rel_age) |>
  summarize(ba = mean(rating - mu - bi - bu))
head(ba_tibble, n = 5)
## # A tibble: 5 x 2
##
     rel_age
                  ba
##
       <db1>
               <db1>
## 1
          -2 0.0228
          -1 0.147
## 3
           0 0.0759
## 4
           1 0.0271
## 5
           2 -0.0107
algorithm5_rating <- test_set |>
  left_join(bi_tibble, by = "movieId") |>
  left_join(bu_tibble, by = "userId") |>
  mutate(year_rel = str_extract(title, "\\(\\d{4}\\)$")) |>
  mutate(year_rel = str_replace_all(year_rel, "[:punct:]", "")) |>
  mutate(year_rel = as.integer(year_rel)) |>
  mutate(time = as_datetime(timestamp)) |>
  mutate(year_rat = year(time)) |>
  mutate(rel_age = year_rat - year_rel) |>
  left_join(ba_tibble, by = "rel_age") |>
  mutate(algorithm5_rating = mu + bi + bu + ba) |>
  pull(algorithm5_rating)
algorithm5_rmse <- rmse(test_set$rating, algorithm5_rating)</pre>
algorithm5 rmse
## [1] 0.86414
```

Algorithm 5 was reviewed for the plausibility of the top movies predicted by its movie bias b_i to ensure user confidence in the recommendations (Rocca 2019; Irizarry 2022). However, the top movies were questionable and came with very low numbers of ratings (fig. 11).

```
title bi n

Hellhounds on My Trail (1999) 1.4876 1

Satan's Tango (Sátántangô) (1994) 1.4876 1

Shadows of Forgotten Ancestors (1964) 1.4876 1

Fighting Elegy (Kenka erejii) (1966) 1.4876 1

Sun Alley (Sonnenallee) (1999) 1.4876 1

Maradona by Kusturica (2008) 1.4876 1

Blue Light, The (Das Blaue Litcht) (1932) 1.4876 1

Human Condition II, The (Ningen no joken II) (1959) 1.3296 3

Who's Singin' Over There? (a.k.a. Who Sings Over There) (Ko to tamo peva) (1980) 1.2376 4

Human Condition III, The (Ningen no joken III) (1961) 1.2376 4
```

Figure 11: Top movies predicted by the movie bias b_i of Algorithm 5.

Algorithm 6: Average Rating μ + Regularized (Movie Bias rb_i + User Bias rb_u + Relative Age Bias rb_a)

The movie bias and those of other predictors in Algorithm 5 were adjusted for the number of ratings using regularization, which penalizes big estimates from small sample sizes with λ and avoids over-fitting of the algorithm (Koren 2008, 427; Koren 2009, 2). Cross-validation resulted in an optimized λ of 5.3. This was used for the regularization of the biases, which refined the RMSE to 0.86353 (tab. 1).

```
r_bi_tibble <- train_set |>
group_by(movieId) |>
summarize(r_bi = sum(rating - mu)/(lambda + n()))
r_bu_tibble <- train_set |>
```

```
left_join(r_bi_tibble, by = "movieId") |>
  group_by(userId) |>
  summarize(r_bu = sum(rating - mu - r_bi)/(lambda + n()))
r_ba_tibble <- train_set |>
  left_join(r_bi_tibble, by = "movieId") |>
  left_join(r_bu_tibble, by = "userId") |>
  mutate(year_rel = str_extract(title, "\\(\\d{4}\\)$")) |>
  mutate(year_rel = str_replace_all(year_rel, "[:punct:]", "")) |>
  mutate(year_rel = as.integer(year_rel)) |>
  mutate(time = as_datetime(timestamp)) |>
  mutate(year_rat = year(time)) |>
  mutate(rel_age = year_rat - year_rel) |>
  group_by(rel_age) |>
  summarize(r_ba = sum(rating - mu - r_bi - r_bu)/(lambda + n()))
algorithm6_rating <- test_set |>
  left_join(r_bi_tibble, by = "movieId") |>
  left_join(r_bu_tibble, by = "userId") |>
  mutate(year_rel = str_extract(title, "\\(\\d{4}\\)$")) |>
  mutate(year_rel = str_replace_all(year_rel, "[:punct:]", "")) |>
  mutate(year_rel = as.integer(year_rel)) |>
  mutate(time = as_datetime(timestamp)) |>
  mutate(year_rat = year(time)) |>
  mutate(rel_age = year_rat - year_rel) |>
  left_join(r_ba_tibble, by = "rel_age") |>
  mutate(algorithm6_rating = mu + r_bi + r_bu + r_ba) |>
  pull(algorithm6_rating)
algorithm6_rmse <- rmse(test_set$rating, algorithm6_rating)</pre>
algorithm6_rmse
## [1] 0.86353
```

The top movies predicted by the regularized movie bias rb_i of Algorithm 6 were more rational and came with very high numbers of ratings (fig. 12).

```
title r_bi n
Shawshank Redemption, The (1994) 0.94196 25232
Godfather, The (1972) 0.89988 16017
Schindler's List (1993) 0.85313 20895
Usual Suspects, The (1995) 0.85301 19491
Rear Window (1954) 0.80803 7146
Casablanca (1942) 0.80539 10091
Sunset Blvd. (a.k.a. Sunset Boulevard) (1950) 0.80086 2609
Third Man, The (1949) 0.79883 2671
Double Indemnity (1944) 0.79689 1928
Seven Samurai (Shichinin no samurai) (1954) 0.79682 4658
```

Figure 12: Top movies predicted by the regularized movie bias rb_i of Algorithm 6.

Recommendation System

The final Algorithm 6 formed the recommendation system for this project. It predicted ratings (r_p) starting with an average rating of 3.5124 then additionally adjusting for the effects of the movie, user, and relative age biases that were regularized with a λ of 5.3: $r_p = 3.5124 + rb_i + rb_u + rb_a$. The movie and user biases accounted for majority of the effects, similar to what was observed in previous data (Koren 2008, 427). Nevertheless, the relative age bias helped achieve an RMSE of 0.86353.

Finally, the recommendation system was evaluated on the reserved final holdout test set to simulate the movie rating prediction of the recommendation system on future data. This resulted in a decent RMSE of 0.86469, which is below the required error rate of 0.86490 (tab. 1).

Table 1: Root mean squared errors (RMSEs) of the algorithms and the recommendation system.

	RMSE		
Baseline Algorithm: Average Rating			
Algorithm 1: Average Rating + Movie Bias	0.94292		
Algorithm 2: Average Rating + Movie Bias + Genre Bias	0.94292		
Algorithm 3: Average Rating + Movie Bias + Release Bias	0.94292		
(Requirement)	0.86490		
Algorithm 4: Average Rating + Movie Bias + User Bias	0.86458		
Algorithm 5: Average Rating + Movie Bias + User Bias + Relative Age Bias	0.86414		
Algorithm 6: Average Rating + Regularized (Movie Bias + User Bias + Relative Age Bias)	0.86353		
Recommendation System			

A previous project by McGinnis (2019) similarly reviewed the use of movie age and relative age in recommendation systems. However, the pattern used to extract the years of movie release from the titles yielded erroneous years, and both movie age and relative age were not used in the final algorithm.

Conclusion

In this project, a content-based recommendation system that predicted future movie ratings of users with an RMSE rate of 0.86469 was developed from the MovieLens 10M Dataset. As with the Netflix challenge, this error rate can be pushed down further using advanced methods, such as collaborative filtering of user-movie interactions. However, this goes beyond the scope of this project. It would be interesting to know if more predictor data to work with have been added to the MovieLens dataset from 2020 to 2021, when most people were confined to their homes and had more time to watch movies because of the COVID-19 pandemic.

References

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