Mystery Motor Data Sheet

Motor manufacturers make the motor characteristics below available to you to help you decide which motor to purchase. When you buy a motor second-hand or surplus, you may need to measure these properties yourself. We will use all SI units (which is not the case on most motor data sheets).

Motor Property	Symbol	Value	Units	Comments
Nominal voltage	V_{nom}		V	Chosen so the no-load speed is safe for brushes, commutator, and bearings.
Power rating	P		W	The max continuous mechanical power output at the nominal voltage without overheating.
Max mechanical power	P_{max}		W	The max mechanical power output at the nominal voltage (including short-term operation).
No-load speed	ω_0		rad/s	Speed when no load and powered by V_{nom}. Usually given in rpm (revs/min, sometimes m^{-1}).
No-load current	I_0		A	Nonzero because of friction torque.
Max continuous current	I_c		A	Max continuous current without overheating.
Starting current	I_s		A	Same as stall current, V_{nom}/R.
Max continuous torque	T_c		Nm	Directly proportional to max continuous current.
Stall torque	T_s		Nm	Same as starting torque.
Max efficiency	η_{max}		%	Approximately $\left(1 - \sqrt{\frac{I_0}{I_s}}\right)^2$, occurring at approximately $1/7$ of stall torque.
Terminal resistance	R		Ω	May change as brushes slide over commutator segments. Increases with heat.
Terminal inductance	L		H	Due to the coils.
Electrical time constant	τ_e		s	The time for the motor current to reach 63% of its final value. Equal to L/R.
Torque constant	k_t		Nm/A	Also called the motor constant.
Electrical constant	k_e		Vs/rad	Same numerical value as the torque constant (in SI units). Also called voltage or back-emf constant.
Speed constant	k_s		rad/(Vs)	Inverse of electrical constant.
Mechanical time constant	τ_m		s	The time for the motor to go from rest to 63% of its final speed under constant voltage.
Rotor inertia	J		kgm^2	Often given in units gcm^2.
Short-circuit damping	B		Nms/rad	Not included in some data sheets, but useful for motor braking (and haptics).
Friction				Not included in most data sheets. See explanation.

All of our motor modeling derives from the power equality

$$\underbrace{IV}_{\text{electrical power in}} = \underbrace{T\omega}_{\text{mech power out}} + \underbrace{I^2 R}_{\text{coil heating}} + \underbrace{I\,L\,dI/dt}_{\text{charging coils}} + \text{friction losses}$$

and the relation, due to Lorentz's force law,

$$T = k_t I.$$

Dividing the power equation by I, and ignoring friction for now, we get the common voltage equation

$$V = k_t\,\omega + IR + L\,dI/dt,$$

where it is more common to see k_t written as k_e. They are equivalent, but in the torque equation this constant converts amps to Newton-meters, and in the voltage equation it converts radians/s to volts.

Getting to Know Your Brushed DC Motor

Here are some things to try to get to know your motor and encoder:

1. Spin the motor shaft by hand. Get a feel for the rotor inertia and friction.

2. Now short the motor terminals by plugging the wires into the same row of a breadboard. Spin again by hand. Do you notice a difference?

3. Work with a partner. Connect your two motors together so the + terminals of the motors are attached to each other, and the − terminals are attached to each other. (Doesn't matter which terminal you call + or −. Connect the wires through the breadboard.) No batteries or external power. Now try spinning one of the motor shafts by hand. What do you predict will happen to the other motor, if anything? What actually happens?

4. Try measuring your motor's resistance using your multimeter. Notice that it varies with the angle of the shaft, and it may not be easy to get a steady reading. What is the minimum value you can get reliably?

5. Attach your motor's encoder cable to +5 V (red wire) and GND (black) of your nuScope. Attach the other two encoder wires to two of your scope's digital inputs. Attach one of your motor terminals to channel A on your scope, and the other to scope ground. Spin the shaft by hand and observe the encoder pulses, including their relative phase. Also observe the back-emf on channel A.

6. Now disconnect your two motor terminals from the scope, and instead power your motor using a 6 V battery pack. Your motor's encoder has 100 lines. Given this, and the rate of the encoder pulses you observe on your scope, calculate the motor's no-load speed and enter it on your data sheet.

7. Work with a partner. Couple your two motor shafts together using some tubing. Now plug one terminal of one of the motors (we'll call it the *passive* motor) into channel A of a scope, and plug the other terminal of the passive motor into GND of the same scope. Now power the other motor (the *driving* motor) with 6 V so that both motors spin. Measure the speed of the passive motor by looking at its encoder count rate. Also measure its back-emf on channel A. With this information, calculate the passive motor's electrical constant k_e. (**Note:** You may need to use your multimeter to measure the back-emf, instead of the scope, if the back-emf exceeds the range of the scope.)

You can now continue with the assignment at the end of this document. Some methods for completing the assignment are suggested below.

Characterizing a Brushed DC Motor

Below are some ideas for characterizing a motor using mostly your nuScope, multimeter, encoder (or any way to measure the motor speed), and resistors and capacitors. There is more than one way to measure most of these motor parameters. Feel free to come up with your own methods.

Nominal voltage   This voltage is just a recommendation. Exceeding the nominal voltage is fine, though damage to the brushes and commutator, or to the bearings, is possible at high speeds. Nominal voltage cannot be measured, but a typical no-load speed for a brushed DC motor is between 3000 and 10,000 rpm, so the nominal voltage will usually give a no-load speed in this range. For our data sheet, we'll use a nominal voltage of 6 V. (The motor is rated for 24 V, but we want to keep current low.)

Power rating   It is not easy to come up with thermal ratings without permanently damaging the motor. Assume that the motor coils can dissipate up to 5 W continuously before overheating.

Max mechanical power   The max mechanical power occurs at $\frac{1}{2}T_s$ and $\frac{1}{2}\omega_0$. For most motor data sheets, the max mechanical power occurs outside the continuous operation regime.

No-load speed   You can determine ω_0 by measuring the unloaded motor speed when powered with the nominal voltage. The amount this is less than V_{nom}/k_e can be attributed to friction torque.

No-load current   You can determine I_0 by using a current-sensing resistor and an oscilloscope/multimeter, or by using a multimeter in current measurement mode. Friction torque is $k_t I_0$.

Max continuous current   For our motor, assume that the motor coils can dissipate up to 5 W continuously before overheating. Use your estimate of motor resistance to then estimate the max continuous current.

Starting current   Starting current is equivalent to stall current. You can estimate this using your estimate of R. Since R may be difficult to measure with a multimeter, you can instead put a current sensing resistor in series with the motor, stall the motor, and measure the current flowing through the motor and resistor by measuring the voltage across the resistor. Make sure your estimate accounts for the voltage drop across the resistor. Alternatively, dispense with the current sensing resistor and use your multimeter in current sensing mode, provided the multimeter can handle the current.

Max continuous torque   This is determined by thermal considerations. We will assume that the motor coils can dissipate up to 5 W continuously.

Stall torque   This can be obtained from k_e and I_s.

Max efficiency   Efficiency is defined as the power out divided by the power in, $T\omega/(IV)$. The wasted power is due to coil heating and friction losses. Friction torque is proportional to I_0, so the smaller I_0 is, the more efficient the motor is. Motors are most efficient at converting electrical power to mechanical power at high speeds, or approximately $(1/7)T_s$.

Terminal resistance   You can measure R with a multimeter. The resistance may change as you rotate the shaft by hand, as the brushes move to new positions on the commutator. You should record the minimum resistance you can reliably find. A better choice, however, may be to measure the current when the motor is stalled. See the description of starting current, above.

Terminal inductance   There are a number of ways we could measure inductance, for example based on an L/R time constant we can observe with an oscilloscope, or based on the oscillation frequency of an RLC circuit if we add a capacitor. We propose the latter method. (If you'd like to try something else, be our guest, but (1) make sure to document your method, and (2) if you are driving the motor with a current from your nuScope, be sure to use a 1 kΩ resistor in series with the motor, to limit the current the scope has to provide.)

We'll use the nuScope to generate a 1 kHz square wave to pulse an RLC circuit constructed of the motor, which acts as a resistor and inductor, and an added capacitor. Build the circuit shown below, where a good choice for C may be 0.01 μF or 0.1 μF:
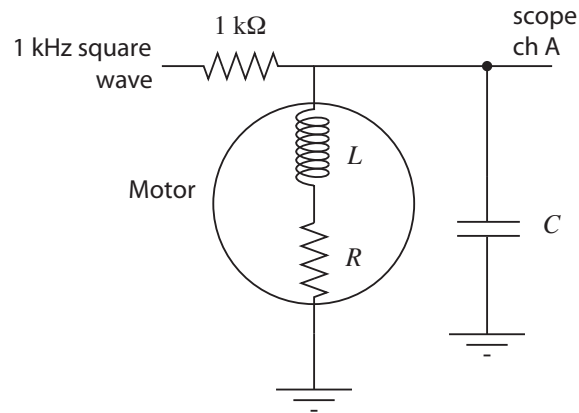
Use the nuScope to put a 1 kHz square wave at the point indicated. The 1 kΩ resistor limits the current from the scope. The motor is modeled as a resistor and inductor. (Back-emf will not be an issue, because the motor will not move, as it is being driven with tiny current at high frequency.) Measure the voltage with your nuScope where indicated. You should be able to see a decaying oscillatory response to the square wave transitions when you choose the right V/div and s/div on your scope. Measure the frequency of the oscillatory response. Knowing C and that the natural frequency of an RLC circuit is $\omega_n = 1/\sqrt{LC}$ in rad/s, estimate L.

Let's think about why we see this response. Say the input to the circuit has been at 0 V for a long time. Then your scope will also read 0 V. Now the input steps up to 5 V. After some time, in steady state, the capacitor will be an open circuit and the inductor will be a closed circuit (wire), so the voltage on the scope will settle to 5 V $\times$ $(R/(1000 + R))$—the two resistors in the circuit set the final voltage. Right after the voltage step, however, all current goes to charge the capacitor (as the zero current through the inductor cannot change discontinuously). If the inductor continued to enforce zero current, the capacitor would charge to 5 V. As the voltage across the capacitor grows, however, so does voltage across the inductor, and therefore so does the rate of change of current that must flow through the inductor (by the relation $V_L + V_R = V_C$ and the consitutive law $V_L = L \ dI/dt$). Eventually the integral of this rate of change dictates that all current is redirected to the inductor, and in fact the capacitor will have to provide current to the inductor, discharging itself. As the voltage across the capacitor drops, though, the voltage across the inductor will eventually become negative, and therefore the rate of change of current across the inductor will become negative. And so on, to create the oscillation. If R were large, i.e., the circuit were heavily damped, the oscillation would die quickly, but you should be able to see it.

(Note that you are seeing a damped oscillation, so you are actually measuring a damped natural frequency. But the damping is low if you are seeing at least a couple of cycles of oscillation, so the damped natural frequency is indistinguishable from the undamped natural frequency.)

Electrical time constant   The electrical time constant is just L/R.

Torque constant   You can measure this by spinning the shaft of the motor, measuring the back-emf at the motor terminals, and measuring the rotation rate using the encoder. If friction losses are negligible, a good approximation is V_{nom}/ω_0. This eliminates the need to spin the motor externally.

Electrical constant   Identical to the torque constant. When written in SI units, k_t and k_e have the same numerical values, but often k_t will be given in mNm/A or English units like oz-in/A, and often k_e will be given in V/rpm.

Speed constant   Just the inverse of the electrical constant.

Mechanical time constant   The mechanical time constant is JR/k_t^2, the time it takes the unloaded motor to reach 63% of its final speed under a constant voltage input and in the absence of friction. The time constant can be measured by applying a constant voltage to the motor, measuring the velocity, and determining the time it takes to reach 63% of final speed.

Rotor inertia   The rotor inertia can be estimated from measurements of the mechanical time constant, k_t, and R. Alternatively, a constant current could be applied to the motor and the motor's position differentiated twice to get acceleration α. Then the inertia can be estimated using the relation $J = T/\alpha = k_t I/\alpha$. Note that constant current/torque/acceleration is not possible for long periods of time.

Short-circuit damping   When the terminals of the motor are shorted together, you get a viscous damping torque T opposing the shaft angular velocity ω, satisfying the equation $T = -B\omega$, where $B > 0$. The damping B can be calculated from the torque constant and terminal resistance.

Friction   Friction torque arises from the motor shaft spinning in the bearings, and may depend on external loads. A typical model of friction includes both Coulomb friction and viscous friction, and may be written

$$T_{\text{fric}} = -b_0 \,\text{sgn}(\omega) - b_1 \omega,$$

where b_0 is the Coulomb friction torque, indicating that the friction torque opposes the direction of motion ($\text{sgn}(\omega)$ just returns the sign of ω), and b_1 is a viscous friction coefficient.

ME 333 Introduction to Mechatronics
Assignment 7: Characterizing a Brushed DC Motor
Electronic submission due before 12:30 PM Tuesday March 1

1. There are 20 entries on the data sheet on the first page of this document. Let's assume zero friction, so we ignore the last entry. (We will also assume that the motor coils can dissipate a maximum of 5 W continuously without overheating.) Of the 19 remaining entries, under the assumption of zero friction, how many independent entries are there? That is, what is the minimum number N of entries you need to be able to fill in the rest of the entries? Give a set of N independent entries from which you can derive the other $19 - N$ dependent entries. For each of the $19 - N$ dependent entries, give the equation in terms of the N independent entries. For example, V_{nom} and R will be two of the N independent entries, from which we can calculate the dependent entry $I_s = V_{\text{nom}}/R$.

2. Based on your experiments, create a data sheet with all 20 entries for $V_{\text{nom}} = 6$ V. For the remaining 19 entries, indicate how you calculated the entry. (Did you do an experiment for it? Did you calculate it from other entries? Or did you do estimate by more than one method to cross-check your answer?) For the friction entry, you can assume Coulomb friction only—the friction torque opposes the rotation direction ($b_0 \neq 0$), but is independent of the speed of rotation ($b_1 = 0$). For your measurement of inductance, turn in a screen snap of the scope trace you used to estimate ω_n and L, and indicate the value of C that you used.

   Note: If there are any entries you are unable to estimate experimentally, simply say so and leave that entry blank.

3. Based on your data sheet, draw the speed-torque curves on the next page, and answer the associated questions.

Speed-Torque Curves

Draw the speed-torque curve for your motor assuming a nominal voltage of 6 V. Indicate the stall torque and no-load speed. If the motor coils can dissipate a maximum of 5 W continuously before overheating, indicate the continuous operating regime. What is the power rating P for this motor? What is the max mechanical power P_{max}?

(Do not do any more experiments for the remainder of this problem; just extrapolate your previous results.) Draw the speed-torque curve for your motor assuming a nominal voltage of 24 V. Indicate the stall torque and no-load speed. If the motor coils can dissipate a maximum of 5 W continuously before overheating, indicate the continuous operating regime. What is the power rating P for this motor? What is the max mechanical power P_{max}?

Often DC motors spin at speeds that are too high, and torques that are too low, to be useful. If we put a $G = 10$, or 10:1, gearhead on the output of our motor, however, the speed of the motor is reduced by a factor of 10 ($\omega_{\text{out}} = \omega_{\text{in}}/G$) and the torque is increased by a factor of 10 ($T_{\text{out}} = GT_{\text{in}}$). Draw the speed-torque curve for the 24 V motor (the previous curve you drew) with a 10:1 gearhead and indicate the no-load speed and stall torque.

Gearheads are not 100% efficient; some power is lost due to friction and impact between gear teeth. Now assume our 10:1 gearhead from the previous example is $\eta = 80\%$ efficient. The relation $\omega_{\text{out}} = \omega_{\text{in}}/G$ must be preserved (it's enforced by the teeth), so we will use the relation $T_{\text{out}} = \eta GT_{\text{in}}$, giving $P_{\text{out}} = T_{\text{out}}\omega_{\text{out}} = \eta T_{\text{in}}\omega_{\text{in}} = \eta P_{\text{in}}$. Draw the speed-torque curve for the 24 V motor with an 80% efficient 10:1 gearhead.