Math 5344, Iterative Solvers, Fall 2020
Programming problem #3: Due Wednesday, 2 Dec, 2020

Problem 1. Repeat the convergence study from problem 2 with preconditioned CG in place of preconditioned GMRES.

Problem 2. Modify the PCG function to accept the known solution x^* as an optional argument with default value `None`. Then, add code that computes the A norm and 2 norm of the error at each step when $x^* \neq$ `None`. Then, for each of the DH matrices 6, 6, 10, 12, 14 (or as large as you can go on your system), do the following:

1. Use `np.linalg.cond` to compute the condition number κ. (This will be difficult beyond matrix #14).

2. Set the `maxIters` parameter to a large number, say 10^4. Run *unpreconditioned* CG (with the exact solution x^* as an argument) until $\|r_{n+1}\| \,/\, \|b\| \leq 10^{-14}$. At each step, record the 2 norm of the relative residual $\|r_{n+1}\|_2 \,/\, \|r_0\|_2$ and the A norm of the relative error, $\|e_n\|_A \,/\, \|e_0\|_A$.

   Using the same figure for all matrices, plot $\|e_n\|_A \,/\, \|e_0\|_A$ and the theoretical bound

   $$\left(\frac{\sqrt{\kappa}-1}{\sqrt{\kappa}+1}\right)^n$$

   as functions of n using different symbols/colors for each matrix. Since these quantities will vary over many orders of magnitude, use a semilog scale. Verify that the error bound is always obeyed.