机器学习第三次实验实验报告

实验要求

在这个练习中，需要用 Python 实现 LeNet5 来完成对 MNIST 数据集中 0-9 10 个手写数字的分类。代码只能使用 python 实现，不能使用 PyTorch 或 TensorFlow 框架。

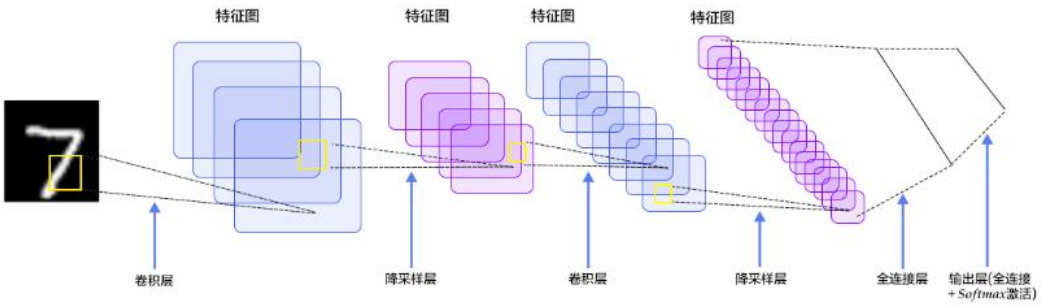
实验环境

Python3.10

Vs Code

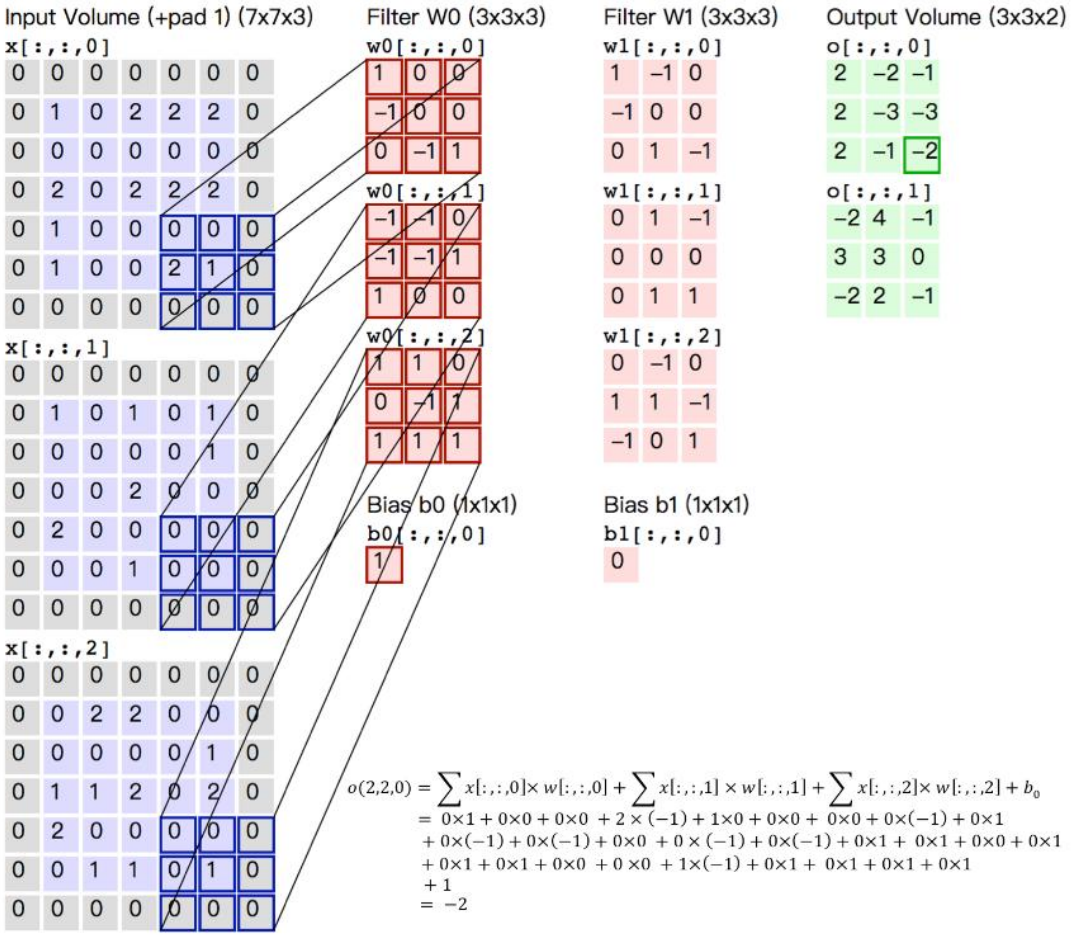
实验准备

LeNet-5 是一个较简单的卷积神经网络。下图显示了其结构：输入的二维图像，先经过两次卷积层到池化层，再经过全连接层，最后使用 softmax 分类作为输出层。



卷积层

卷积层是卷积神经网络的核心基石。在图像识别里我们提到的卷积是二维卷积，即离散二维滤波器（也称作卷积核）与二维图像做卷积操作，简单的讲是二维滤波器滑动到二维图像上所有位置，并在每个位置上与该像素点及其领域像素点做内积。卷积操作被广泛应用与图像处理领域，不同卷积核可以提取不同的特征，例如边沿、线性、角等特征。在深层卷积神经网络中，通过卷积操作可以提取出图像低级到复杂的特征。

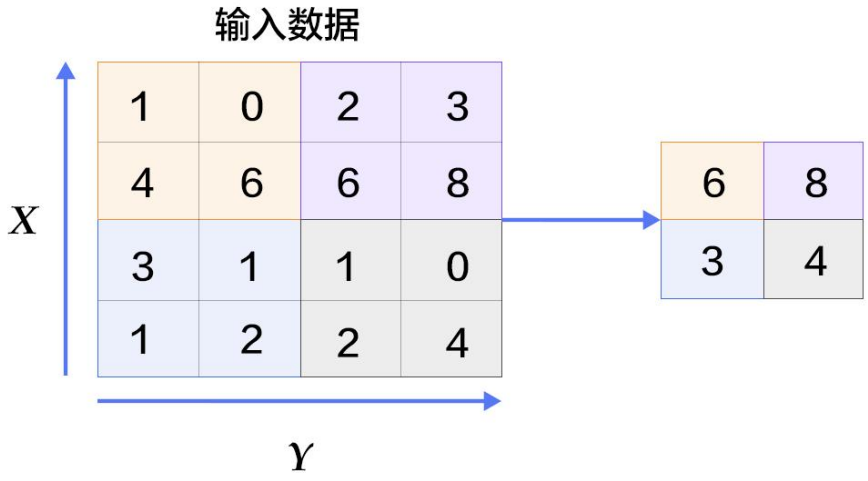


计算过程如上图示例。输入图像大小为 H=5，W=5，D=3，这个示例图中包含两（用 K 表示）组卷积核，即图中滤波器 W0 和 W1。

在卷积计算中，通常对不同的输入通道采用不同的卷积核，如图示例中每组卷积核包含（D=3）个 3×3（用 F×F 表示）大小的卷积核。另外，这个示例中卷积核在图像的水平方向（W 方向）和垂直方向（H 方向）的滑动步长为 2（用 S 表示）；对输入图像周围各填充 1（用 P 表示）个 0，即图中输入层原始数据为蓝色部分，灰色部分是进行了大小为 1 的扩展，用 0 来进行扩展。经过卷积操作得到输出为 3×3×2（用 Ho×Wo×K 表示）大小的特征图，即 3×3 大小的 2 通道特征图，其中 Ho 计算公式为：Ho=(H−F+2×P)/S+1，Wo 同理。 而输出特征图中的每个像素，是每组滤波器与输入图像每个特征图的内积再求和，再加上偏置 bo，偏置通常对于每个输出特征图是共享的。输出特征图 o[:,:,0]中的最后一个−2 计算如上图右下角公式所示。

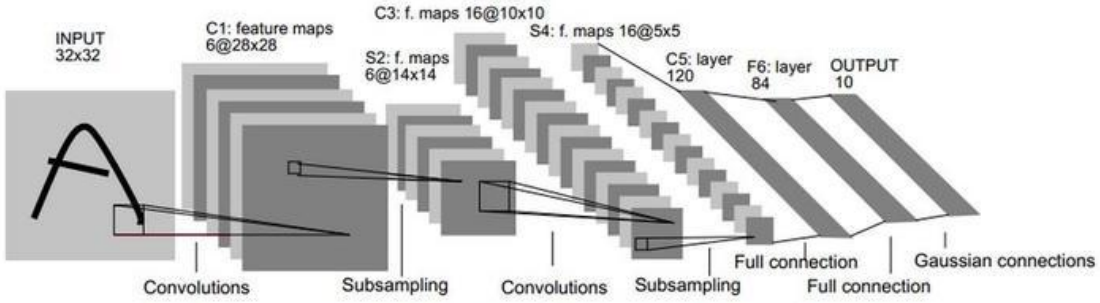
在卷积操作中卷积核是可学习的参数，经过上面示例介绍，每层卷积的参数大小为 D×F×F×K。卷积层的参数较少，这也是由卷积层的主要特性即局部连接和共享权重所决定。

池化层



池化是非线性下采样的一种形式，主要作用是通过减少网络的参数来减小计算量，并且能够在一定程度上控制过拟合。通常在卷积层的后面会加上一个池化层。池化包括最大池化、平均池化等。其中最大池化是用不重叠的矩形框将输入层分成不同的区域，对于每个矩形框的数取最大值作为输出层，如上图所示。

Lenet-5



LeNet-5 共有 7 层，不包含输入，每层都包含可训练参数；每个层有多个 Feature Map，每个 FeatureMap 通过一种卷积滤波器提取输入的一种特征，然后每个 FeatureMap 有多个神经元。

INPUT 层-输入层

首先是数据 INPUT 层，输入图像的尺寸统一归一化为 32*32。

C1 层-卷积层

输入图片：32*32

卷积核大小：5*5

卷积核种类：6

输出 featuremap 大小：28*28 （32-5+1）=28

神经元数量：28*28*6

可训练参数:（5*5+1) * 6（每个滤波器 5*5=25 个 unit 参数和一个 bias 参数，一共 6 个滤波器）

连接数：（5*5+1）*6*28*28=122304

详细说明：对输入图像进行第一次卷积运算（使用 6 个大小为 5*5 的卷积核），得到 6 个 C1 特征图（6 个大小为 28*28 的 feature maps, 32-5+1=28）。我们再来看看需要多少个参数，卷积核的大小为 5*5，总共就有 6*（5*5+1）=156 个参数，其中+1 是表示一个核有一个 bias。对于卷积层 C1，C1 内的每个像素都与输入图像中的 5*5 个像素和 1 个 bias 有连接，所以总共有 156*28*28=122304 个连接（connection）。有 122304 个连接，但是我们只需要学习 156 个参数，主要是通过权值共享实现的。

S2 层-池化层（下采样层）

输入：28*28

采样区域：2*2

采样方式：4 个输入相加，乘以一个可训练参数，再加上一个可训练偏置。结果通过 sigmoid

采样种类：6

输出 featureMap 大小：14*14（28/2）

神经元数量：14*14*6

连接数：（2*2+1）*6*14*14

S2 中每个特征图的大小是 C1 中特征图大小的 1/4。

详细说明：第一次卷积之后紧接着就是池化运算，使用 2*2 核 进行池化，于是得到了 S2，6 个 14*14 的 特征图（28/2=14）。S2 这个 pooling 层是对 C1 中的 2*2 区域内的像素求和乘以一个权值系数再加上一个偏置，然后将这个结果再做一次映射。同时有 5x14x14x6=5880 个连接。

C3 层-卷积层

输入：S2 中所有 6 个或者几个特征 map 组合

卷积核大小：5*5

卷积核种类：16

输出 featureMap 大小：10*10 (14-5+1)=10

C3 中的每个特征 map 是连接到 S2 中的所有 6 个或者几个特征 map 的，表示本层的特征 map 是上一层提取到的特征 map 的不同组合

存在的一个方式是：C3 的前 6 个特征图以 S2 中 3 个相邻的特征图子集为输入。接下来 6 个特征图以 S2 中 4 个相邻特征图子集为输入。然后的 3 个以不相邻的 4 个特征图子集为输入。最后一个将 S2 中所有特征图为输入。

则：可训练参数：6*(3*5*5+1)+6*(4*5*5+1)+3*(4*5*5+1)+1*(6*5*5+1) =1516

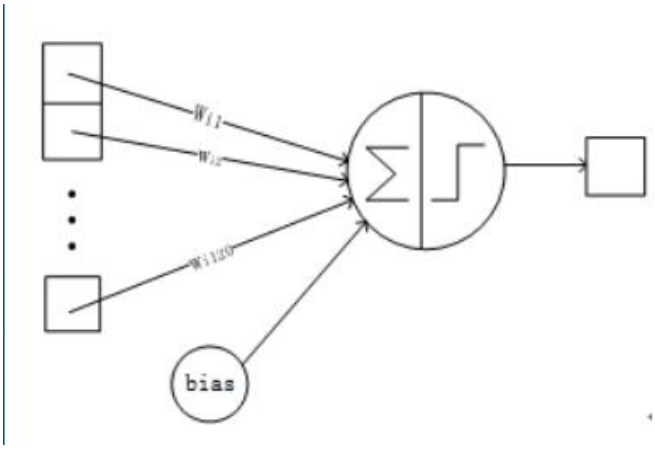
连接数：10*10*1516=151600

详细说明：

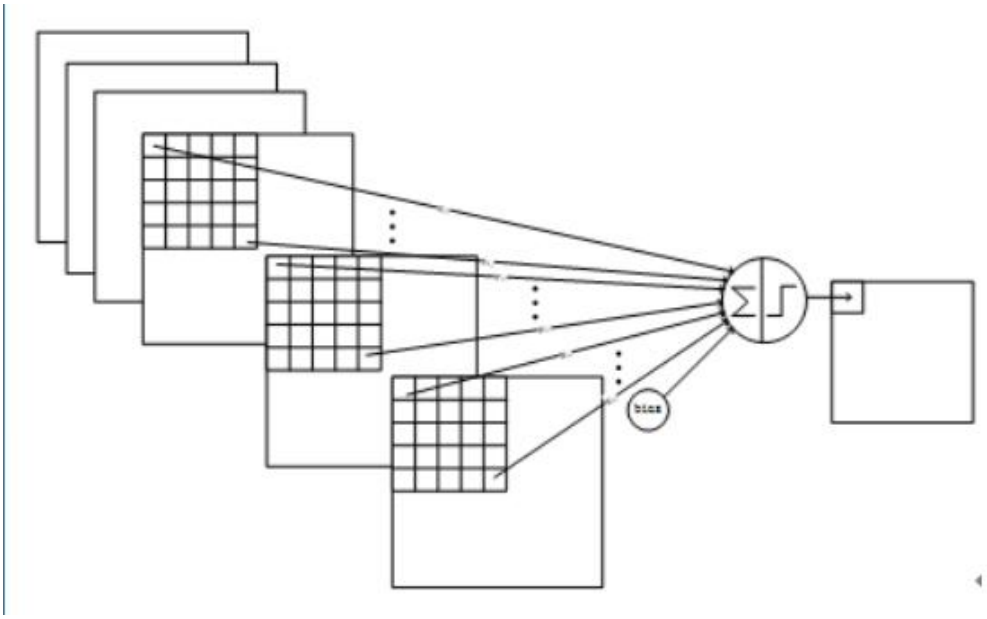
第一次池化之后是第二次卷积，第二次卷积的输出是 C3，16 个 10x10 的特征图，卷积核大小是 5*5. 我们知道 S2 有 6 个 14*14 的特征图，怎么从 6 个特征图得到 16 个特征图了？这里是通过对 S2 的特征图特殊组合计算得到的 16 个特征图。具体如下：

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	X				X	X	X			X	X	X	X		X	X
1	X	X				X	X	X			X	X	X	X		X
2	X	X	X				X	X	X			X		X	X	X
3		X	X	X			X	X	X	X			X		X	X
4			X	X	X			X	X	X	X		X	X		X
5				X	X	X			X	X	X	X		X	X	X

C3 的前 6 个 feature map（对应上图第一个红框的 6 列）与 S2 层相连的 3 个 feature map 相连接（上图第一个红框），后面 6 个 feature map 与 S2 层相连的 4 个 feature map 相连接（上图第二个红框），后面 3 个 feature map 与 S2 层部分不相连的 4 个 feature map 相连接，最后一个与 S2 层的所有 feature map 相连。卷积核大小依然为 5*5，所以总共有 6*(3*5*5+1)+6*(4*5*5+1)+3*(4*5*5+1)+1*(6*5*5+1)= 1516 个参数。而图像大小为 10*10，所以共有 151600 个连接。



C3 与 S2 中前 3 个图相连的卷积结构如下图所示：



上图对应的参数为 3*5*5+1，一共进行 6 次卷积得到 6 个特征图，所以有 6*（3*5*5+1）参数。 为什么采用上述这样的组合了？论文中说有两个原因：1）减少参数，2）这种不对称的组合连接的方式有利于提取多种组合特征。

S4 层-池化层（下采样层）

输入：10*10

采样区域：2*2

采样方式：4 个输入相加，乘以一个可训练参数，再加上一个可训练偏置。结果通过 sigmoid

采样种类：16

输出 featureMap 大小：5*5（10/2）

神经元数量：5*5*16=400

连接数：16*（2*2+1）*5*5=2000

S4 中每个特征图的大小是 C3 中特征图大小的 1/4

详细说明：S4 是 pooling 层，窗口大小仍然是 2*2，共计 16 个 feature map，C3 层的 16 个 10x10 的图分别进行以 2x2 为单位的池化得到 16 个 5x5 的特征图。有 5x5x5x16=2000 个连接。连接的方式与 S2 层类似。

C5 层-卷积层

输入：S4 层的全部 16 个单元特征 map（与 s4 全相连）

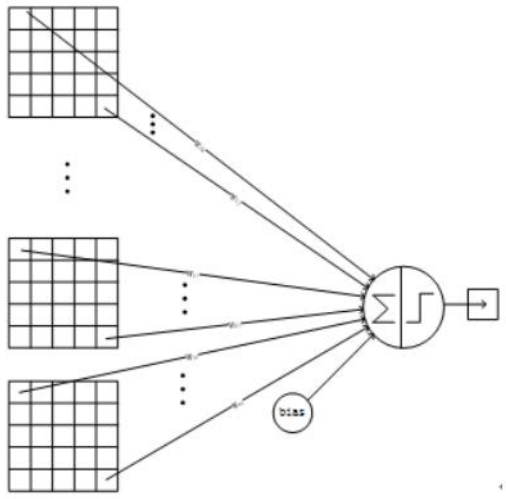
卷积核大小：5*5

卷积核种类：120

输出 featureMap 大小：1*1（5-5+1）

可训练参数/连接：120*（16*5*5+1）=48120

详细说明：C5 层是一个卷积层。由于 S4 层的 16 个图的大小为 5x5，与卷积核的大小相同，所以卷积后形成的图的大小为 1x1。这里形成 120 个卷积结果。每个都与上一层的 16 个图相连。所以共有 (5x5x16+1)x120 = 48120 个参数，同样有 48120 个连接。C5 层的网络结构如下：



F6 层-全连接层

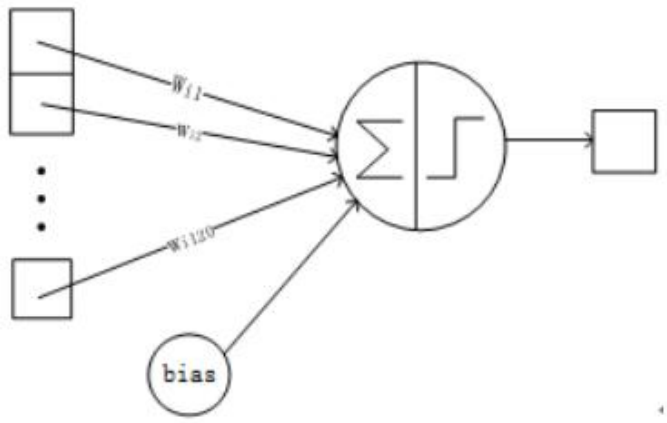
输入：c5 120 维向量

计算方式：计算输入向量和权重向量之间的点积，再加上一个偏置，结果通过 sigmoid 函数输出。

可训练参数:84*(120+1)=10164

详细说明：6 层是全连接层。F6 层有 84 个节点，对应于一个 7x12 的比特图，-1 表示白色，1 表示黑色，这样每个符号的比特图的黑白色就对应于一个编码。该层的训练参数和连接数是(120 + 1)x84=10164。ASCII 编码图如下：

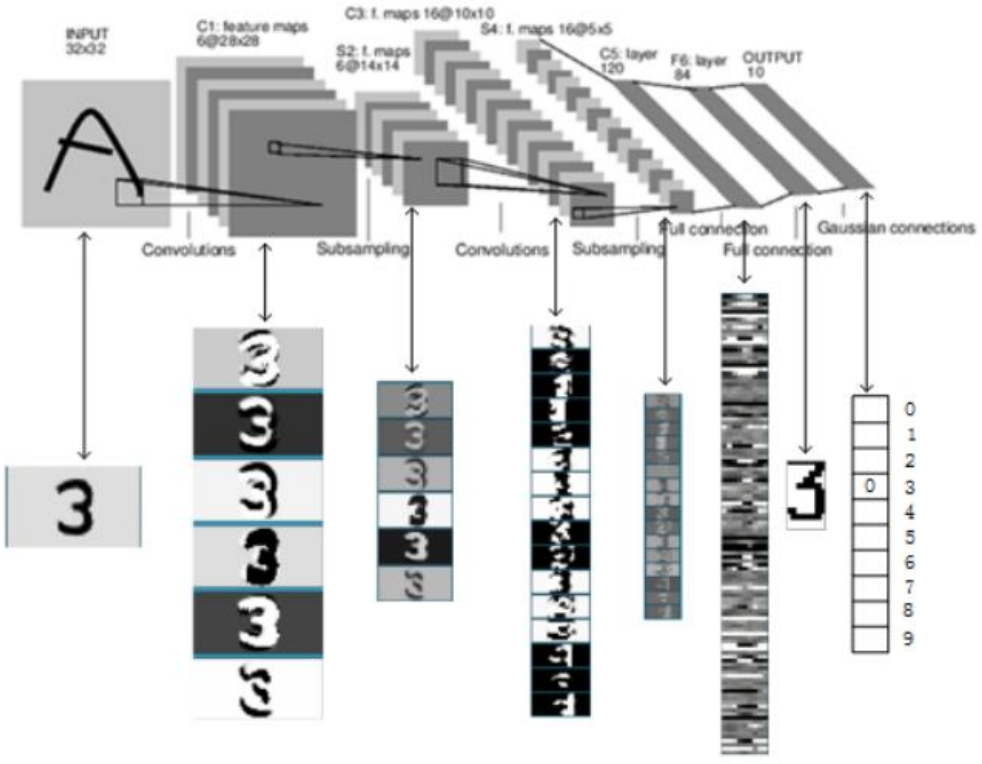
F6 层的连接方式如下：



Output 层-全连接层

Output 层也是全连接层，共有 10 个节点，分别代表数字 0 到 9，且如果节点 i 的值为 0，则网络识别的结果是数字 i。采用的是径向基函数（RBF）的网络连接方式。假设 x 是上一层的输入，y 是 RBF 的输出，则 RBF 输出的计算方式是：$y_i = \sum_j \left(x_i - w_{ij}\right)^2$

上式 w_ij 的值由 i 的比特图编码确定，i 从 0 到 9，j 取值从 0 到 7*12-1。RBF 输出的值越接近于 0，则越接近于 i，即越接近于 i 的 ASCII 编码图，表示当前网络输入的识别结果是字符 i。该层有 84x10=840 个参数和连接。



实验内容

Main

mian 函数中，首先是对数据的读取和正则化，由于 mnist 数据集中的图片为 28 * 28，需要 padding 到 32 * 32。

然后，将处理之后的数据输入 train 函数，开始训练。

训练完成后，在测试集上测试训练网络的性能。

main.py > ...

```python
# coding=utf-8
from data_processing import *
from train import *
from LeNet5 import *

if __name__ == '__main__':
    # get the data
    train_images, train_labels, test_images, test_labels = load_data()

    print("Got data...\n")

    # data processing, normalization&zero-padding
    print("Normalization and zero-padding...\n")
    train_images = normalize(zero_pad(train_images[:, :, :, np.newaxis], 2), 'LeNet5')
    test_images = normalize(zero_pad(test_images[:, :, :, np.newaxis], 2), 'LeNet5')
    print("The shape of training image with padding: ", train_images.shape)
    print("The shape of testing image with padding: ", test_images.shape)
    print("Finish data processing...\n")

    # train LeNet5
    LeNet5 = LeNet5()
    print("Start training...")
    start_time = time.time()
    train(LeNet5, train_images, train_labels)
    end_time = time.time()
    print("Finished training, the total training time is {}s \n".format(end_time - start_time))

    # read model
    # with open('model_data_13.pkl', 'rb') as input_:
    #     LeNet5 = pickle.load(input_)

    # evaluate on test dataset
    print("Start testing...")
    error01, class_pred = LeNet5.Forward_Propagation(test_images, test_labels, 'test')
    print("error rate:", error01 / len(class_pred))
    print("Finished testing, the accuracy is {} \n".format(1 - error01 / len(class_pred)))

```

数据处理（data_processing）

data_processing.py > ...

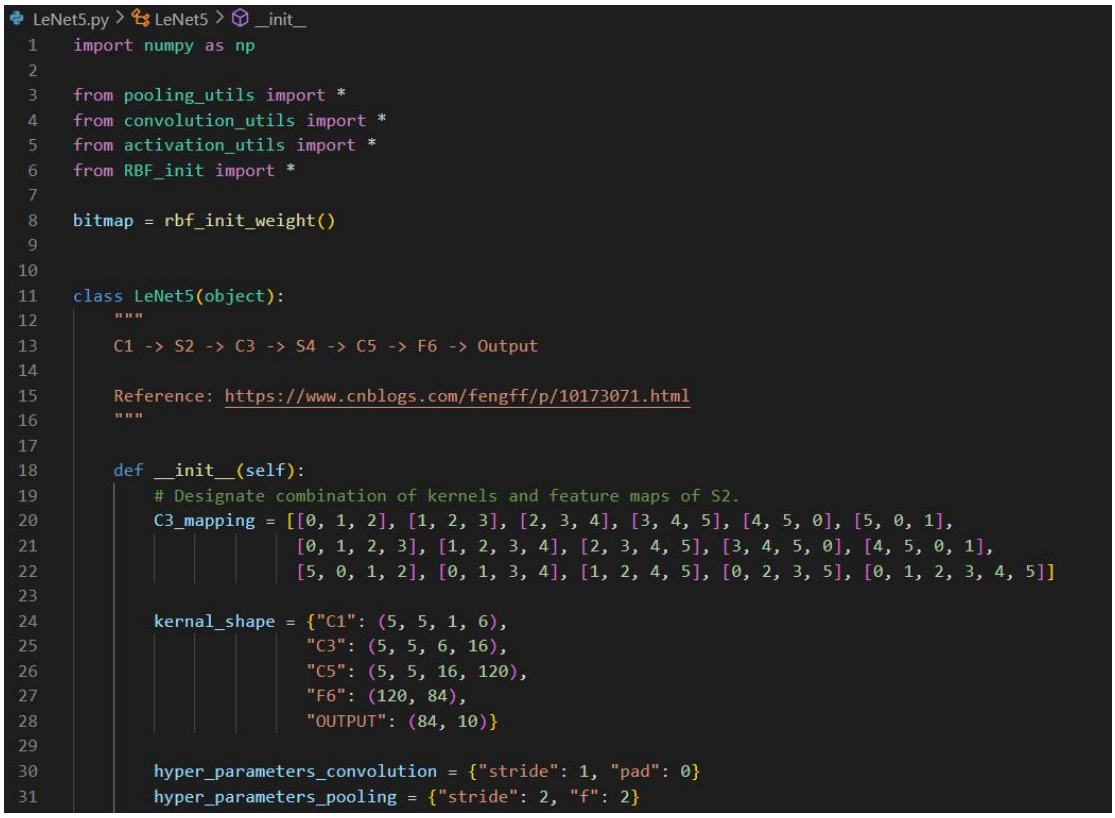
```python
# coding=utf-8
import numpy as np
import struct
import os

data_dir = "D:/wtx/machine-learning/ex1/mnist_data/"
train_data_dir = "train-images-idx3-ubyte"
train_label_dir = "train-labels-idx1-ubyte"
test_data_dir = "t10k-images-idx3-ubyte"
test_label_dir = "t10k-labels-idx1-ubyte"


# Load the MNIST data for this exercise
def load_mnist(file_dir, is_images='True'):
    # Read binary data
    bin_file = open(file_dir, 'rb')
    bin_data = bin_file.read()
    bin_file.close()
    # Analysis file header
    if is_images:
        # Read images
        fmt_header = '>iiii'
        magic, num_images, num_rows, num_cols = struct.unpack_from(fmt_header, bin_data, 0)
        data_size = num_images * num_rows * num_cols
        mat_data = struct.unpack_from('>' + str(data_size) + 'B', bin_data, struct.calcsize(fmt_header))
        mat_data = np.reshape(mat_data, [num_images, num_rows, num_cols])
    else:
        # Read labels
        fmt_header = '>ii'
        magic, num_images = struct.unpack_from(fmt_header, bin_data, 0)
        mat_data = struct.unpack_from('>' + str(num_images) + 'B', bin_data, struct.calcsize(fmt_header))
        mat_data = np.reshape(mat_data, [num_images])
    print('Load images from %s, number: %d, data shape: %s' % (file_dir, num_images, str(mat_data.shape)))
    return mat_data
```

```python
# call the load_mnist function to get the images and labels of training set and testing set
def load_data():
    print('Loading MNIST data from files...')
    train_images = load_mnist(os.path.join(data_dir, train_data_dir), True)
    train_labels = load_mnist(os.path.join(data_dir, train_label_dir), False)
    test_images = load_mnist(os.path.join(data_dir, test_data_dir), True)
    test_labels = load_mnist(os.path.join(data_dir, test_label_dir), False)
    return train_images, train_labels, test_images, test_labels


# transfer the image from gray to binary and get the one-hot style labels
def data_convert(x, y, m, k):
    x[x <= 40] = 0
    x[x > 40] = 1
    ont_hot_y = np.zeros((m, k))
    for t in range(m):
        ont_hot_y[t, y[t]] = 1
    return x, ont_hot_y
```

```python
# padding for the matrix of images
def zero_pad(X, pad):
    X_pad = np.pad(X, ((0, 0), (pad, pad), (pad, pad), (0, 0)), 'constant', constant_values=(0, 0))
    return X_pad


# normalization of the input images
def normalize(image, mode='LeNet5'):
    image -= image.min()
    image = image / image.max()
    if mode == '0p1':
        return image  # range = [0,1]
    elif mode == 'n1p1':
        image = image * 2 - 1  # range = [-1,1]
    elif mode == 'LeNet5':
        image = image * 1.275 - 0.1  # range = [-0.1,1.175]
    return image
```

LeNet5

LeNet5 类的 Forward_Propagation 和 Back_Propagation 函数会调用每一层对应的 forward_propagation 和 back_propagation 函数，可以看到，层与层之间的参数会一层层的进行传递。通过 SDLM 算法进行计算得到的每一轮的学习率。



```python
import numpy as np

from pooling_utils import *
from convolution_utils import *
from activation_utils import *
from RBF_init import *

bitmap = rbf_init_weight()


class LeNet5(object):
    """
    C1 -> S2 -> C3 -> S4 -> C5 -> F6 -> Output

    Reference: https://www.cnblogs.com/fengff/p/10173071.html
    """

    def __init__(self):
        # Designate combination of kernels and feature maps of S2.
        C3_mapping = [[0, 1, 2], [1, 2, 3], [2, 3, 4], [3, 4, 5], [4, 5, 0], [5, 0, 1],
                      [0, 1, 2, 3], [1, 2, 3, 4], [2, 3, 4, 5], [3, 4, 5, 0], [4, 5, 0, 1],
                      [5, 0, 1, 2], [0, 1, 3, 4], [1, 2, 4, 5], [0, 2, 3, 5], [0, 1, 2, 3, 4, 5]]

        kernal_shape = {"C1": (5, 5, 1, 6),
                        "C3": (5, 5, 6, 16),
                        "C5": (5, 5, 16, 120),
                        "F6": (120, 84),
                        "OUTPUT": (84, 10)}

        hyper_parameters_convolution = {"stride": 1, "pad": 0}
        hyper_parameters_pooling = {"stride": 2, "f": 2}
```

```python
        hyper_parameters_convolution = {"stride": 1, "pad": 0}
        hyper_parameters_pooling = {"stride": 2, "f": 2}

        self.C1 = ConvolutionLayer(kernal_shape["C1"], hyper_parameters_convolution)
        self.a1 = Activation("LeNet5_squash")
        self.S2 = PoolingLayer(hyper_parameters_pooling, "average")

        self.C3 = ConvolutionLayer_maps(kernal_shape["C3"], hyper_parameters_convolution, C3_mapping)
        self.a2 = Activation("LeNet5_squash")
        self.S4 = PoolingLayer(hyper_parameters_pooling, "average")

        self.C5 = ConvolutionLayer(kernal_shape["C5"], hyper_parameters_convolution)
        self.a3 = Activation("LeNet5_squash")

        self.F6 = FCLayer(kernal_shape["F6"])
        self.a4 = Activation("LeNet5_squash")

        self.Output = RBFLayer(bitmap)
```

```
    def Forward_Propagation(self, input_image, input_label, mode):
        self.label = input_label
        self.C1_FP = self.C1.forward_propagation(input_image)
        self.a1_FP = self.a1.forward_propagation(self.C1_FP)
        self.S2_FP = self.S2.forward_propagation(self.a1_FP)

        self.C3_FP = self.C3.forward_propagation(self.S2_FP)
        self.a2_FP = self.a2.forward_propagation(self.C3_FP)
        self.S4_FP = self.S4.forward_propagation(self.a2_FP)

        self.C5_FP = self.C5.forward_propagation(self.S4_FP)
        self.a3_FP = self.a3.forward_propagation(self.C5_FP)

        self.flatten = self.a3_FP[:, 0, 0, :]
        self.F6_FP = self.F6.forward_propagation(self.flatten)
        self.a4_FP = self.a4.forward_propagation(self.F6_FP)

        # output sum of the loss over mini-batch when mode = 'train'
        # output tuple of (0/1 error, class_predict) when mode = 'test'
        out = self.Output.forward_propagation(self.a4_FP, input_label, mode)

        return out
```

```
    def Back_Propagation(self, momentum, weight_decay):
        dy_pred = self.Output.back_propagation()

        dy_pred = self.a4.back_propagation(dy_pred)
        F6_BP = self.F6.back_propagation(dy_pred, momentum, weight_decay)
        reverse_flatten = F6_BP[:, np.newaxis, np.newaxis, :]

        reverse_flatten = self.a3.back_propagation(reverse_flatten)
        C5_BP = self.C5.back_propagation(reverse_flatten, momentum, weight_decay)

        S4_BP = self.S4.back_propagation(C5_BP)
        S4_BP = self.a2.back_propagation(S4_BP)
        C3_BP = self.C3.back_propagation(S4_BP, momentum, weight_decay)

        S2_BP = self.S2.back_propagation(C3_BP)
        S2_BP = self.a1.back_propagation(S2_BP)
        C1_BP = self.C1.back_propagation(S2_BP, momentum, weight_decay)

    # Stochastic Diagonal Levenberg-Marquaedt method for determining the learning rate before the beginning of each ep
    def SDLM(self, mu, lr_global):
        d2y_pred = self.Output.SDLM()
        d2y_pred = self.a4.SDLM(d2y_pred)

        F6_SDLM = self.F6.SDLM(d2y_pred, mu, lr_global)
        reverse_flatten = F6_SDLM[:, np.newaxis, np.newaxis, :]

        reverse_flatten = self.a3.SDLM(reverse_flatten)
        C5_SDLM = self.C5.SDLM(reverse_flatten, mu, lr_global)

        S4_SDLM = self.S4.SDLM(C5_SDLM)
        S4_SDLM = self.a2.SDLM(S4_SDLM)
        C3_SDLM = self.C3.SDLM(S4_SDLM, mu, lr_global)

        S2_SDLM = self.S2.SDLM(C3_SDLM)
        S2_SDLM = self.a1.SDLM(S2_SDLM)
        C1_SDLM = self.C1.SDLM(S2_SDLM, mu, lr_global)
```

卷积层前向传播和后向传播（convolution_utils）

```python
# Numpy version: compute with np.tensordot()
def conv_forward(A_prev, W, b, hyper_parameters):
    """
    Implements the forward propagation for a convolution function

    :param A_prev: output activations of the previous layer, numpy array of shape (m, n_H_prev, n_W_prev, n_C_prev)
    :param W: Weights, numpy array of shape (f, f, n_C_prev, n_C)
    :param b: Biases, numpy array of shape (1, 1, 1, n_C)
    :param hyper_parameters: python dictionary containing "stride" and "pad"

    :return: Z -- conv output, numpy array of shape (m, n_H, n_W, n_C)
             cache -- cache of values needed for the conv_backward() function
    """
    (m, n_H_prev, n_W_prev, n_C_prev) = A_prev.shape
    (f, f, n_C_prev, n_C) = W.shape

    stride = hyper_parameters["stride"]
    pad = hyper_parameters["pad"]

    n_H = int((n_H_prev + 2 * pad - f) / stride + 1)
    n_W = int((n_W_prev + 2 * pad - f) / stride + 1)

    # Initialize the output volume Z with zeros.
    Z = np.zeros((m, n_H, n_W, n_C))
    A_prev_pad = zero_pad(A_prev, pad)
    for h in range(n_H):  # loop over vertical axis of the output volume
        for w in range(n_W):  # loop over horizontal axis of the output volume
            # Use the corners to define the (3D) slice of a_prev_pad.
            A_slice_prev = A_prev_pad[:, h * stride:h * stride + f, w * stride:w * stride + f, :]
            # Convolve the (3D) slice with the correct filter W and bias b, to get back one output neuron.
            Z[:, h, w, :] = np.tensordot(A_slice_prev, W, axes=([1, 2, 3], [0, 1, 2])) + b

    assert (Z.shape == (m, n_H, n_W, n_C))
    cache = (A_prev, W, b, hyper_parameters)
    return Z, cache
```

```python
# Numpy version: compute with np.dot
def conv_backward(dZ, cache):
    """
    Implement the backward propagation for a convolution function

    :param dZ: gradient of the cost with respect to the output of the conv layer (Z), numpy array of shape (m, n_H, n_W, n_C)
    :param cache: cache of values needed for the conv_backward(), output of conv_forward()

    :return: dA_prev -- gradient of the cost with respect to the input of the conv layer (A_prev),
                numpy array of shape (m, n_H_prev, n_W_prev, n_C_prev)
             dW -- gradient of the cost with respect to the weights of the conv layer (W)
                numpy array of shape (f, f, n_C_prev, n_C)
             db -- gradient of the cost with respect to the biases of the conv layer (b)
                numpy array of shape (1, 1, 1, n_C)
    """
    (A_prev, W, b, hyper_parameters) = cache
    (m, n_H_prev, n_W_prev, n_C_prev) = A_prev.shape
    (f, f, n_C_prev, n_C) = W.shape
    (m, n_H, n_W, n_C) = dZ.shape
    stride = hyper_parameters["stride"]
    pad = hyper_parameters["pad"]

    dA_prev = np.zeros((m, n_H_prev, n_W_prev, n_C_prev))
    dW = np.zeros((f, f, n_C_prev, n_C))
    db = np.zeros((1, 1, 1, n_C))

    if pad != 0:
        A_prev_pad = zero_pad(A_prev, pad)
        dA_prev_pad = zero_pad(dA_prev, pad)
    else:
        A_prev_pad = A_prev
        dA_prev_pad = dA_prev
```

```
    for h in range(n_H):  # loop over vertical axis of the output volume
        for w in range(n_W):  # loop over horizontal axis of the output volume
            # Find the corners of the current "slice"
            vert_start, horiz_start = h * stride, w * stride
            vert_end, horiz_end = vert_start + f, horiz_start + f

            # Use the corners to define the slice from a_prev_pad
            A_slice = A_prev_pad[:, vert_start:vert_end, horiz_start:horiz_end, :]

            # Update gradients for the window and the filter's parameters
            dA_prev_pad[:, vert_start:vert_end, horiz_start:horiz_end, :] += np.transpose(np.dot(W, dZ[:, h, w, :].T), (3, 0, 1, 2))

            dW += np.dot(np.transpose(A_slice, (1, 2, 3, 0)), dZ[:, h, w, :])
            db += np.sum(dZ[:, h, w, :], axis=0)

    # Set dA_prev to the unpadded dA_prev_pad
    dA_prev = dA_prev_pad if pad == 0 else dA_prev_pad[:, pad:-pad, pad:-pad, :]

    # Making sure your output shape is correct
    assert (dA_prev.shape == (m, n_H_prev, n_W_prev, n_C_prev))

    return dA_prev, dW, db

```

```
def conv_SDLM(dZ, cache):
    (A_prev, W, b, hparameters) = cache
    (m, n_H_prev, n_W_prev, n_C_prev) = A_prev.shape
源代码管理 (Ctrl+Shift+G) - 48 个挂起的更改 ) = W.shape
    stride = hparameters["stride"]
    pad = hparameters["pad"]
    (m, n_H, n_W, n_C) = dZ.shape
    dA_prev = np.zeros((m, n_H_prev, n_W_prev, n_C_prev))
    dW = np.zeros((f, f, n_C_prev, n_C))
    db = np.zeros((1, 1, 1, n_C))

    if pad != 0:
        A_prev_pad = zero_pad(A_prev, pad)
        dA_prev_pad = zero_pad(dA_prev, pad)
    else:
        A_prev_pad = A_prev
        dA_prev_pad = dA_prev

    for h in range(n_H):  # loop over vertical axis of the output volume
        for w in range(n_W):  # loop over horizontal axis of the output volume
            # Find the corners of the current "slice"
            vert_start, horiz_start = h * stride, w * stride
            vert_end, horiz_end = vert_start + f, horiz_start + f

            # Use the corners to define the slice from a_prev_pad
            A_slice = A_prev_pad[:, vert_start:vert_end, horiz_start:horiz_end, :]

            # Update gradients for the window and the filter's parameters
            dA_prev_pad[:, vert_start:vert_end, horiz_start:horiz_end, :] += np.transpose(
                np.dot(np.power(W, 2), dZ[:, h, w, :].T), (3, 0, 1, 2))

            dW += np.dot(np.transpose(np.power(A_slice, 2), (1, 2, 3, 0)), dZ[:, h, w, :])
    # Set dA_prev to the unpaded dA_prev_pad
    dA_prev = dA_prev_pad if pad == 0 else dA_prev_pad[:, pad:-pad, pad:-pad, :]
    assert (dA_prev.shape == (m, n_H_prev, n_W_prev, n_C_prev))
    return dA_prev, dW

```

池化层前向传播和后向传播（pooling_utils）

```python
def pool_forward(A_prev, hyper_parameters, mode):
    m, n_H_prev, n_W_prev, n_C_prev = A_prev.shape
    f = hyper_parameters["f"]
    stride = hyper_parameters["stride"]

    n_H = int(1 + (n_H_prev - f) / stride)
    n_W = int(1 + (n_W_prev - f) / stride)
    n_C = n_C_prev

    A = np.zeros((m, n_H, n_W, n_C))
    for h in range(n_H):  # loop on the vertical axis of the output volume
        for w in range(n_W):  # loop on the horizontal axis of the output volume
            # Use the corners to define the current slice on the ith training example of A_prev, channel c
            A_prev_slice = A_prev[:, h * stride:h * stride + f, w * stride:w * stride + f, :]
            # Compute the pooling operation on the slice. Use an if statement to differentiate the modes.
            if mode == "max":
                A[:, h, w, :] = np.max(A_prev_slice, axis=(1, 2))
            elif mode == "average":
                A[:, h, w, :] = np.average(A_prev_slice, axis=(1, 2))

    cache = (A_prev, hyper_parameters)
    assert (A.shape == (m, n_H, n_W, n_C))
    return A, cache
```

```python
def pool_backward(dA, cache, mode):

    A_prev, hyper_parameters = cache

    stride = hyper_parameters["stride"]
    f = hyper_parameters["f"]

    m, n_H_prev, n_W_prev, n_C_prev = A_prev.shape  # 256,28,28,6
    m, n_H, n_W, n_C = dA.shape  # 256,14,14,6

    dA_prev = np.zeros((m, n_H_prev, n_W_prev, n_C_prev))  # 256,28,28,6

    for h in range(n_H):  # loop on the vertical axis
        for w in range(n_W):  # loop on the horizontal axis
            # Find the corners of the current "slice"
            vert_start, horiz_start = h * stride, w * stride
            vert_end, horiz_end = vert_start + f, horiz_start + f

            # Compute the backward propagation in both modes.
            if mode == "max":
                A_prev_slice = A_prev[:, vert_start: vert_end, horiz_start: horiz_end, :]
                A_prev_slice = np.transpose(A_prev_slice, (1, 2, 3, 0))
                mask = A_prev_slice == A_prev_slice.max((0, 1))
                mask = np.transpose(mask, (3, 2, 0, 1))
                dA_prev[:, vert_start: vert_end, horiz_start: horiz_end, :] \
                    += np.transpose(np.multiply(dA[:, h, w, :][:, :, np.newaxis, np.newaxis], mask), (0, 2, 3, 1))

            elif mode == "average":
                da = dA[:, h, w, :][:, np.newaxis, np.newaxis, :]  # 256*1*1*6
                dA_prev[:, vert_start: vert_end, horiz_start: horiz_end, :] += np.repeat(np.repeat(da, 2, axis=1), 2, axis=2) / f / f

    assert (dA_prev.shape == A_prev.shape)
    return dA_prev
```

```python
def subsampling_forward(A_prev, weight, b, hparameters):
    A_, cache = pool_forward(A_prev, hparameters, 'average')
    A = A_ * weight + b
    cache_A = (cache, A_)
    return A, cache_A


def subsampling_backward(dA, weight, b, cache_A_):
    (cache, A_) = cache_A_
    db = dA
    dW = np.sum(np.multiply(dA, A_))
    dA_ = dA * weight
    dA_prev = pool_backward(dA_, cache, 'average')
    return dA_prev, dW, db
```

训练（train）

train.py > ...

```python
# coding=utf-8
import sys
import time
import math
import pickle
import numpy as np
import matplotlib.pyplot as plt


# Number of epochs & learning rate in the original paper
epochs_original, lr_global_original = 16, np.array([5e-4] * 2 + [2e-4] * 3 + [1e-4] * 3 + [5e-5] * 4 + [1e-5] * 8)
# Number of epochs & learning rate I used
epochs, lr_global_list = epochs_original, lr_global_original * 100
```

```python
def train(LeNet5, train_images, train_labels):
    momentum = 0.9
    weight_decay = 0
    batch_size = 256

    # Training loops
    cost_last, count = np.Inf, 0
    err_rate_list = []
    for epoch in range(0, epochs):
        print("---------------------------------- epoch{} begin ----------------------------------".format(epoch + 1))

        # Stochastic Diagonal Levenberg-Marquardt method for determining the learning rate
        batch_image, batch_label = random_mini_batches(train_images, train_labels, mini_batch_size=500, one_batch=True)
        LeNet5.Forward_Propagation(batch_image, batch_label, 'train')
        lr_global = lr_global_list[epoch]
        LeNet5.SDLM(0.02, lr_global)

        # print info
        print("global learning rate:", lr_global)
        print("learning rates in trainable layers:", np.array([LeNet5.C1.lr, LeNet5.C3.lr, LeNet5.C5.lr, LeNet5.F6.lr]))
        print("batch size:", batch_size)
        print("momentum:", momentum, ", weight decay:", weight_decay)
```

```python
        ste = time.time()
        cost = 0
        mini_batches = random_mini_batches(train_images, train_labels, batch_size)
        for i in range(len(mini_batches)):
            batch_image, batch_label = mini_batches[i]

            loss = LeNet5.Forward_Propagation(batch_image, batch_label, 'train')
            cost += loss

            LeNet5.Back_Propagation(momentum, weight_decay)

            # print progress
            if i % (int(len(mini_batches) / 100)) == 0:
                sys.stdout.write("\033[F")  # CURSOR_UP_ONE
                sys.stdout.write("\033[K")  # ERASE_LINE
                print("progress:", int(100 * (i + 1) / len(mini_batches)), "%, ", "cost =", cost, end='\r')
        sys.stdout.write("\033[F")  # CURSOR_UP_ONE
        sys.stdout.write("\033[K")  # ERASE_LINE

        print("Done, cost of epoch", epoch + 1, ":", cost, "                                                ")

        error01_train, _ = LeNet5.Forward_Propagation(train_images, train_labels, 'test')
        err_rate_list.append(error01_train / 60000)
        # error01_test, _ = LeNet5.Forward_Propagation(test_images, test_labels, 'test')
        # err_rate_list.append([error01_train / 60000, error01_test / 10000])
        print("0/1 error of training set:", error01_train, "/", len(train_labels))
        # print("0/1 error of testing set: ", error01_test, "/", len(test_labels))
        print("Time used: ", time.time() - ste, "sec")
        print(
            "---------------------------------- epoch{} end ----------------------------------\n".format(epoch + 1))

        # conserve the model
        # with open('model_data_' + str(epoch) + '.pkl', 'wb') as output:
        #     pickle.dump(LeNet5, output, pickle.HIGHEST_PROTOCOL)

    err_rate_list = np.array(err_rate_list).T
```

```python
# return random-shuffled mini-batches
def random_mini_batches(image, label, mini_batch_size=256, one_batch=False):
    m = image.shape[0]  # number of training examples
    mini_batches = []

    # Shuffle (image, label)
    permutation = list(np.random.permutation(m))
    shuffled_image = image[permutation, :, :, :]
    shuffled_label = label[permutation]

    # extract only one batch
    if one_batch:
        mini_batch_image = shuffled_image[0: mini_batch_size, :, :, :]
        mini_batch_label = shuffled_label[0: mini_batch_size]
        return mini_batch_image, mini_batch_label

    # Partition (shuffled_image, shuffled_Y). Minus the end case.
    num_complete_minibatches = math.floor(m / mini_batch_size)
    for k in range(0, num_complete_minibatches):
        mini_batch_image = shuffled_image[k * mini_batch_size: k * mini_batch_size + mini_batch_size, :, :, :]
        mini_batch_label = shuffled_label[k * mini_batch_size: k * mini_batch_size + mini_batch_size]
        mini_batch = (mini_batch_image, mini_batch_label)
        mini_batches.append(mini_batch)
    # Handling the end case (last mini-batch < mini_batch_size)
    if m % mini_batch_size != 0:
        mini_batch_image = shuffled_image[num_complete_minibatches * mini_batch_size: m, :, :, :]
        mini_batch_label = shuffled_label[num_complete_minibatches * mini_batch_size: m]
        mini_batch = (mini_batch_image, mini_batch_label)
        mini_batches.append(mini_batch)

    return mini_batches
```

实验结果

加载数据集并进行处理

```
Loading MNIST data from files...
Load images from D:/wtx/machine-learning/ex1/mnist_data/train-images-idx3-ubyte, number: 60000, data shape: (60000, 28, 28)
Load images from D:/wtx/machine-learning/ex1/mnist_data/train-labels-idx1-ubyte, number: 60000, data shape: (60000,)
Load images from D:/wtx/machine-learning/ex1/mnist_data/t10k-images-idx3-ubyte, number: 10000, data shape: (10000, 28, 28)
Load images from D:/wtx/machine-learning/ex1/mnist_data/t10k-labels-idx1-ubyte, number: 10000, data shape: (10000,)
Got data...

Normalization and zero-padding...

The shape of training image with padding:  (60000, 32, 32, 1)
The shape of testing image with padding:  (10000, 32, 32, 1)
Finish data processing...
```

训练模型

```
Start training...
---------------------------------- epoch1 begin ----------------------------------
global learning rate: 0.05
learning rates in trainable layers: [1.31954867e-05 1.22353197e-05 1.88510729e-05 1.94846469e-05]
batch size: 256
Done, cost of epoch 1 : 670873.0466392617
0/1 error of training set: 3915 / 60000617
Time used:  291.01013588905334 sec
---------------------------------- epoch1 end ----------------------------------

---------------------------------- epoch2 begin ----------------------------------
global learning rate: 0.05
learning rates in trainable layers: [2.40907961e-04 2.03305658e-05 1.65692959e-05 4.80155308e-06]
batch size: 256
Done, cost of epoch 2 : 220625.76046242146
0/1 error of training set: 1982 / 600002146
Time used:  313.1585421562195 sec
---------------------------------- epoch2 end ----------------------------------
```

测试模型

```
---------------------------------- epoch6 begin ----------------------------------
global learning rate: 0.01
learning rates in trainable layers: [5.70590825e-04 7.32549595e-06 4.68012735e-06 7.17054754e-07]
batch size: 256
Done, cost of epoch 6 : 119039.96486393375
0/1 error of training set: 1317 / 600003375
Time used:  289.01421427726746 sec
---------------------------------- epoch6 end ----------------------------------

Finished training, the total training time is 1804.8934412002563s

Start testing...
error rate: 0.022
Finished testing, the accuracy is 0.978
```

正确率为 97.8%，正确率较低的原因可能会训练模型时训练论述较低。但由于电脑配置较低，在轮数较高时会因为系统内存不足导致整个程序卡死，因此不得不降低轮数。（在只运行 vcode 的情况下，每一轮中，系统内存使用最高达到了 96%）