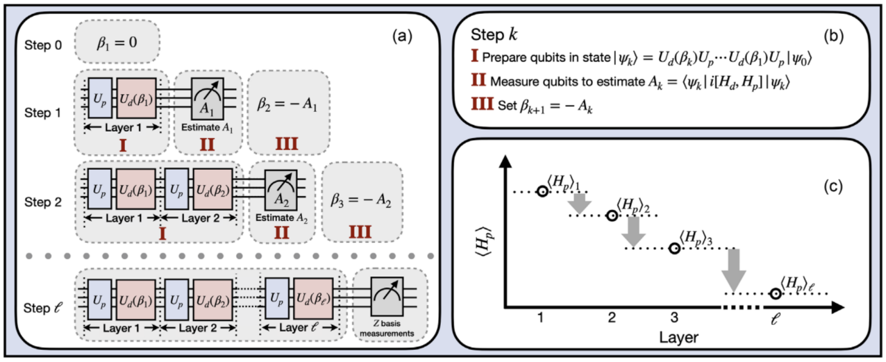
For my individual report, this is a removed addition to our group report on improving approximation algorithms that we left out because it wasn't too related.

Feedback Based Quantum Optimization

Magann et al. has recently proposed a twist on QAOA. By measuring the energy expectation after each layer of unitary gates applied to an initial state, we can monotonically approach the ground state. A diagram from their paper is shown below to illustrate this process.



Each step of the procedure uses the energy expectation (A) of the previous layer to parametrize the next layer.

Once convergence is reached, Z basis measurements on our state are made to form a bit-string probability distribution. We then select an optimal state.

As a brief derivation of why this works, consider the Schrodinger Equation:

$$i \frac{d}{dt} |\psi(t)\rangle = [H_p + H_d \beta(t)] |\psi(t)\rangle$$

Where Hp and Hd are the "drift" and "control" Hamiltonians. More precisely, Hp is a Hamiltonian expected to evolve with time, while Hd is a Hamiltonian expected to stay constant in time. We account for time evolution in the latter case by tacking on the beta term. In order to minimize our time-evolving Hamiltonian energy, we need to design our beta term such that:

$$\frac{d}{dt} \langle \psi(t) | H_p | \psi(t) \rangle (t) \leq 0, \quad \forall \ \ t \geq 0$$

From our Schrodinger equation, we observe that

$$\frac{d}{dt} \langle \psi(t) | H_p | \psi(t) \rangle = A(t)\beta(t)$$

with A(t) being defined as

$$\langle \psi(t) | i[H_d, H_p] | \psi(t) \rangle$$

We also set beta to a feedback loop

$$\beta(t) = -A(t - \tau)$$

However, this is not a set rule - There is a lot of room for our choice of β(t). By applying alternating layers of H_p and H_d based gates with this variation of β(t), we can obtain a Trotterized approximation of continuous time evolution for our system given small enough time steps. Glossing over minor details, our choice of β(t) as a feedback loop enforces the Schrodinger Equation and allows us to achieve monotonic convergence.

This algorithm is mainly interesting due to its simplicity. In contrast to QAOA which optimizes over a large set of parameters gamma and beta, Feedback-Based optimization minimizes our energy with a single function $\beta(t)$ without classical optimization. This makes it potentially implementable (on a small scale) on current Rydberg-atom cavity experiments. Given the right experimental settings to apply U-gates and perform measurements (Which are possible in the near-term for some Ultracold atom groups), it can be imagined that a 5-qubit system could be put under an arbitrary optical potential and allowed to evolve as we apply this algorithm.*

This feedback-based algorithm presents some key disadvantages and advantages. Though it is simple and easy to implement on a small number of qubits, scaling it upwards is a daunting task. It's single-variation of beta(t) means that circuits implementing Feedback-based optimization require deep circuits for convergence, as compared to shallow circuits used in QAOA. There is no classical optimization cost to Feedback-based optimization, which makes it useful in a niche setting. The prime use of this algorithm in the near future might be to give QAOA a better initial set of parameters.

**One of the authors (Mohan Sarvovar) brought this up at a group meeting for the Ultracold Atom group, and we thought it was an interesting application of near-term quantum experiments. Unfortunately, Dr. Stamper-Kurn mentioned this might be possible in >1 year. Many problems, particularly regarding quantum gates, will remain a problem for optically-trapped Rydberg atoms.*