Solid-State Refrigeration

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Abstract

Our capstone project consists of two experiments:

- Verifying the effectiveness of handmade thermocouples which exploit the Seebeck Effect
- Analyzing how the behavior of Peltier plates changes when they are placed in series

We found that thermocouples using chromel and alumel as well as copper and brass had results that aligned best with expectation. We also found that Peltier plates have a high heating efficiency, low cooling efficiency, and best cooling effect when placed in parallel as opposed to series.

Motivation

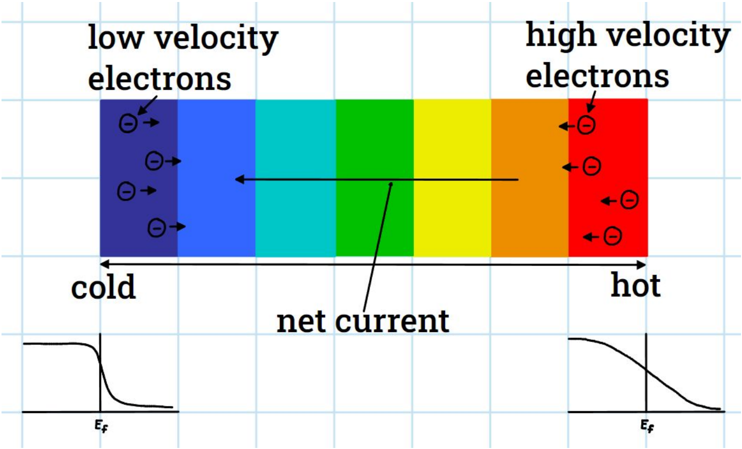
Originally, we wanted to build a heat-engine refrigerator. However, we did not have sufficient time to do so, so we decided to investigate solid state cooling instead.

This led us to explore the thermoelectric effect.

The Seebeck Effect

In a conductor with no thermal gradient, the electrons move **randomly**, so there is **no net current** in any direction.

However, if we place a thermal gradient across a conductor, the electrons at the hot side will have **more kinetic energy** than those at the cold side.



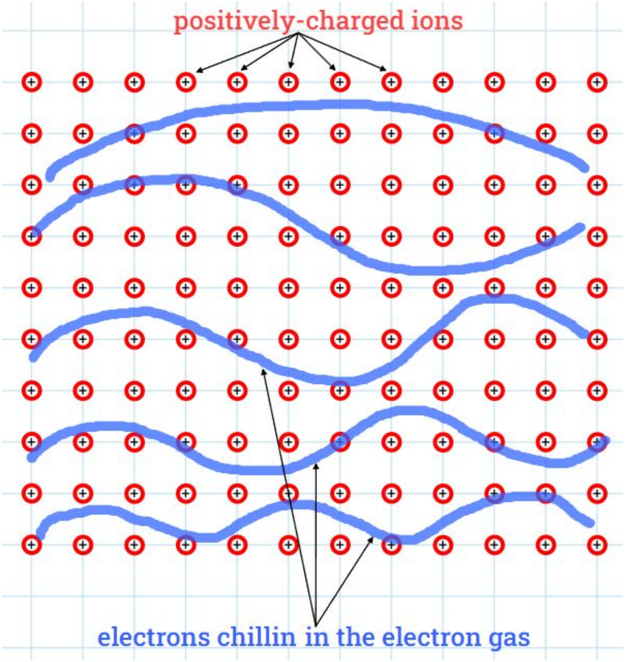
The electrons at either side will tend to migrate to the other side, but the hot ones will do so faster than the cold ones, leading to a **net current** from hot to cold. This causes a charge buildup and thus a voltage across the material. This is the **Seebeck Effect**.

Note that this obeys our thermodynamic expectations of energy transferring from hot to cold.

The Fermi Gas

The **free electron model** is one way of describing how charge carriers behave in a conductor.

The electrons are modeled as a **Fermi gas**, i.e. an electron gas. These electrons **do not interact** with the underlying metallic lattice of positively-charged ions (which is there just to preserve charge neutrality).

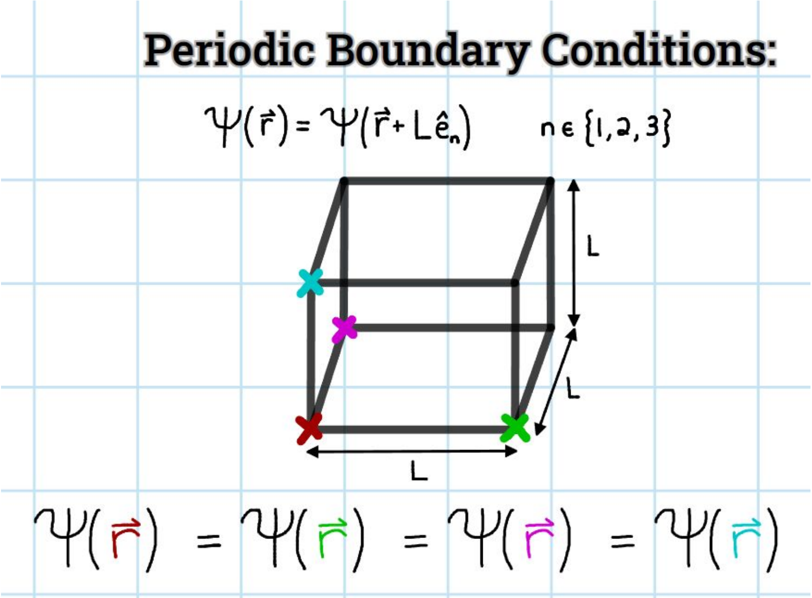


The Fermi Gas Cont

The allowed electron energy (and momentum) states in such a gas are **continuous**.

Intuitively, this is because the electron wavefunctions are **delocalized**: because they can spread to arbitrary extent in the crystal, they can have arbitrary wavevectors.

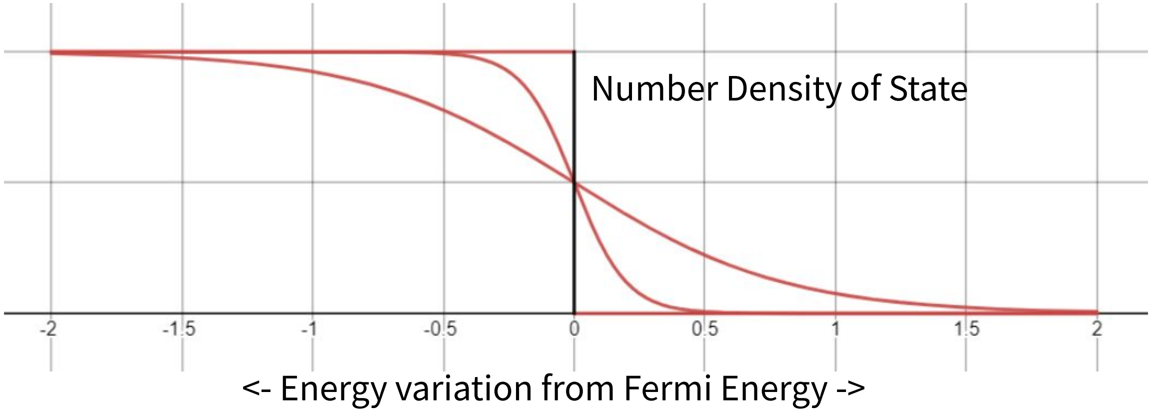
This can be shown by modeling the electrons as being trapped within a 3D torus (i.e. a box with periodic boundary conditions) with side length L, and then limiting L towards infinity.



The Fermi Gas Cont Cont

At T = 0, the highest occupied electron energy state is called the **Fermi Energy**. The electron energy states are distributed about the Fermi energy according to the **Fermi-Dirac Distribution.**

The higher the temperature, the more the electron energy levels vary from the Fermi energy, and specifically there are more electrons at energy levels higher than the Fermi energy.



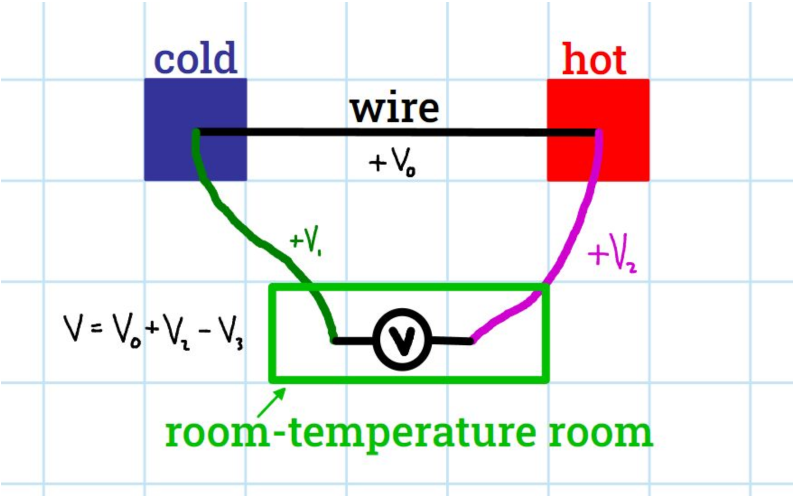
Absolute Seebeck Coefficient = Pain

The induced voltage is **proportional** to the magnitude of the temperature gradient:

$$\mathbf{E}_{\mathrm{emf}} = -S\nabla T,$$

Measuring S directly is difficult, because the measurement device is also affected by the Seebeck effect: this hinders isolation of the wire under investigation.

One solution: make the measurement wires superconducting (superconductors do not experience the Seebeck effect).



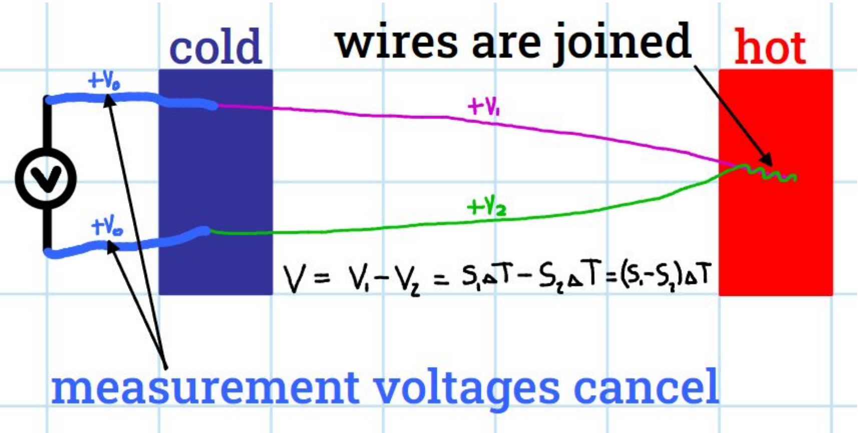
Relative Seebeck Coefficients and Thermocouples

Alternate solution: measure relative Seebeck coefficients with a **thermocouple**.

A thermocouple is effectively two wires joined together at one end. By placing the same temperature gradient over each wire, a Seebeck voltage is generated in each.

Because the materials are different, the generated voltages will be different, and a net voltage will be measured across the wire ends.

The voltages generated in the measurement wires will exactly cancel if they are made of the same material, solving our earlier problem.



The Peltier Effect (arc(arc(Seebeck Effect^{-1})))

The **Peltier Effect** is the inverse of the Seebeck effect: instead of a temperature gradient inducing a voltage, an applied voltage induces a temperature gradient.

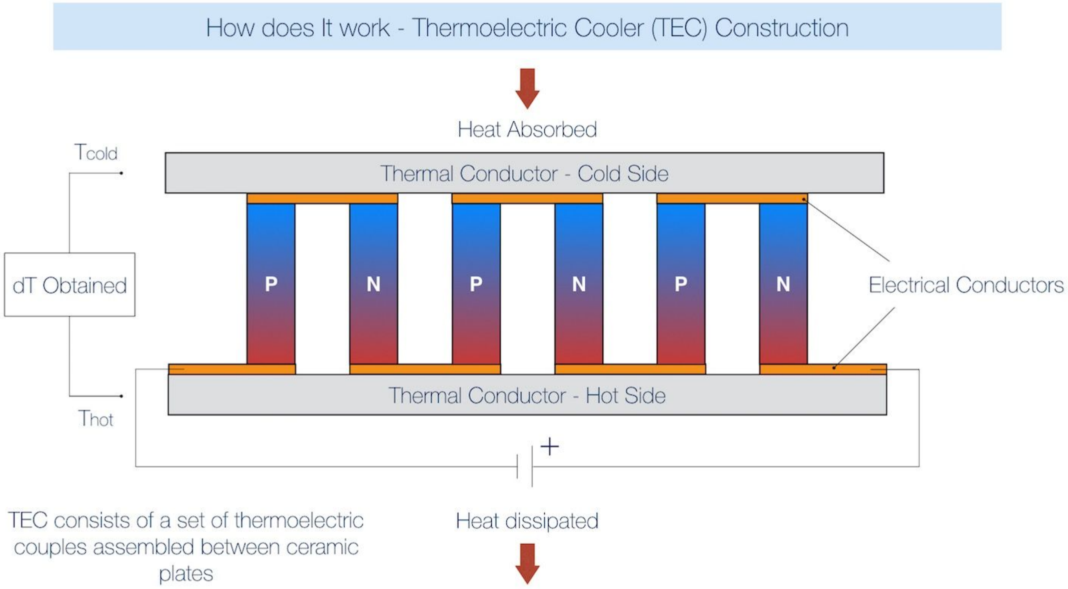
This is through the exact opposite process: the voltage causes electrons to migrate across the conductor, and those with more energy tend to migrate much faster, causing a net flow of energy across the conductor.

Macroscopically, this translates to heat being transferred across the conductor as the electrons deposit their thermal energy at the hot end.

Peltier Plates

Several semiconductors are placed in series electrically such that they are in parallel thermally.

The Seebeck effect in P and N type semiconductors are inverse of each other due to the type of charge carrier (electrons vs holes).



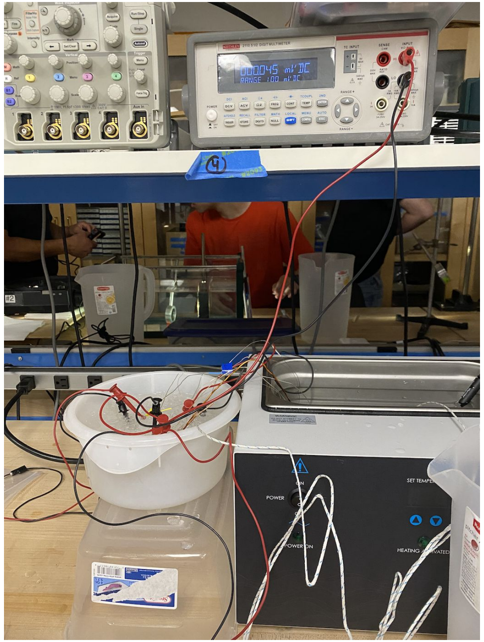
Design and Methods - Experiment 1

- Tested four different thermocouples:
  - Thermocouple A - Commercial Type K (chromel and alumel)
  - Thermocouple B - Thick Copper and Brass
  - Thermocouple C - Tungsten and Nichrome
  - Thermocouple D - Iron and thin copper
- Chose the combinations of wires randomly
- Twisted one end of the two metals together

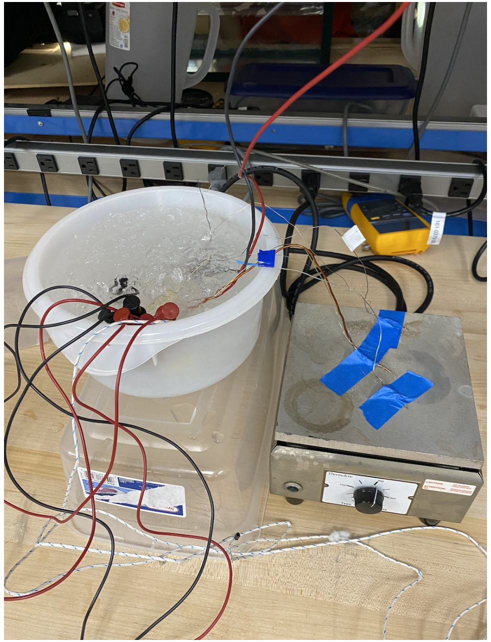
Thermocouple D

Design and Methods - Experiment 1 (cont.)

- Set up all 4 thermocouples:
  - Place joined end in a heat bath
  - Placed separated ends in an ice-water slurry
- Started the heat bath at 49.7 °C and lowered it by about 3 °C for each subsequent trial
- We also used a hot plate instead of a heat bath to test hotter temperatures



heat bath configuration



hot plate configuration

Challenges

- Initially, we saw negligible voltage differences between the two ends of our homemade thermocouples
  - Sanded off some of the insulation on the wires
- Finding an efficient way to test all four thermocouples at the same temperature
  - Created setup using 4 pairs of voltage probes
- Measuring the temperature of the hot plate
  - At first, we made a mistake by placing the probe vertically to the plate instead of horizontally flat

Data and Analysis - Introduction

The statement of Seebeck effect is as follows:

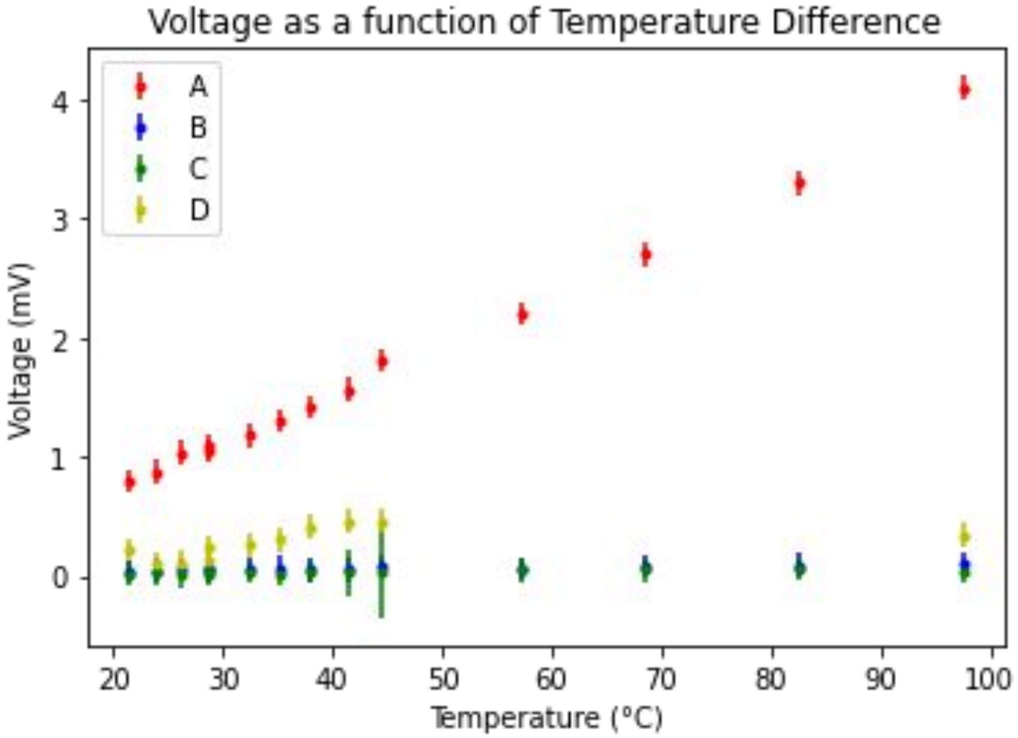
$$\mathbf{E}_{\mathrm{emf}} = -S\nabla T,$$

where **E** is the measured emf, S is the Seebeck coefficient, and ∇T is the temperature difference.

Since we placed the separated ends of our thermocouples in an ice-water slurry at 0 °C, the statement reduced to $\mathbf{E}_{emf}$ = -S T, where T is the temperature of the hot source.

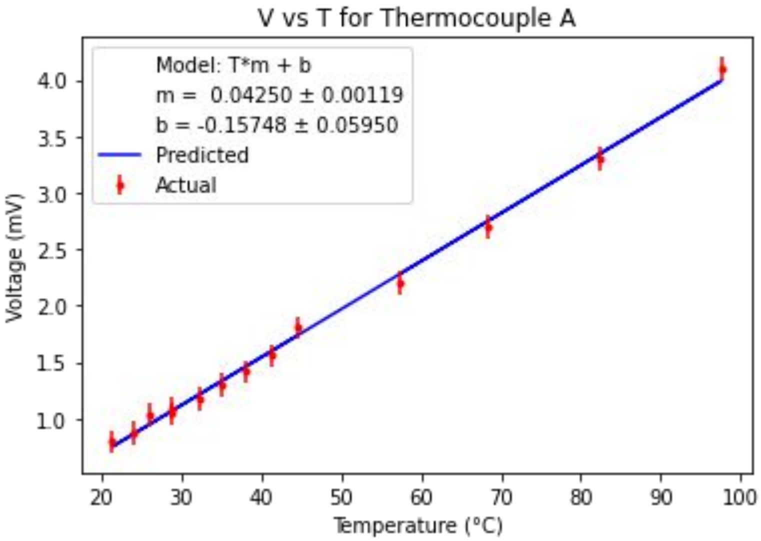
We graphed the voltage versus temperature to observe our experimental values for S. For each thermocouple, S = S_1 - S_2.

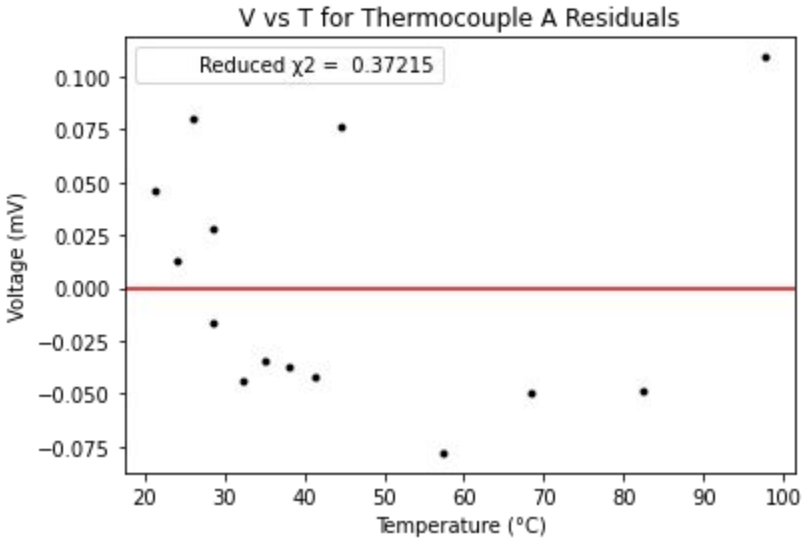
Data for all 4 thermocouples



Data and Analysis - Thermocouple A

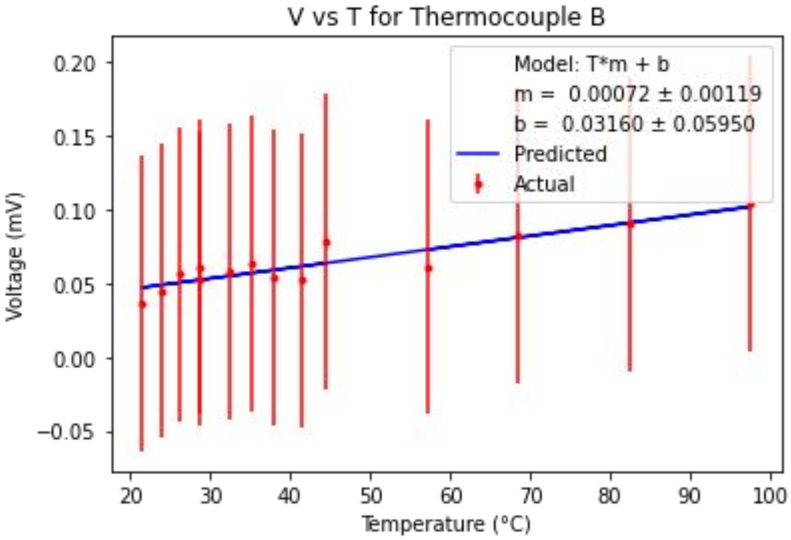
Theoretical values of S (relative to Platinum):
Chromel: 21.7 μV/K
Alumel: -17.3 μV/K
Effective S for thermocouple: 21.7 - (-17.3) μV/K = 39 μV/K
Expected slope lies slightly below the experimental slope (42.50 ± 1.19 μV/K), but the data falls very close to the predicted line.

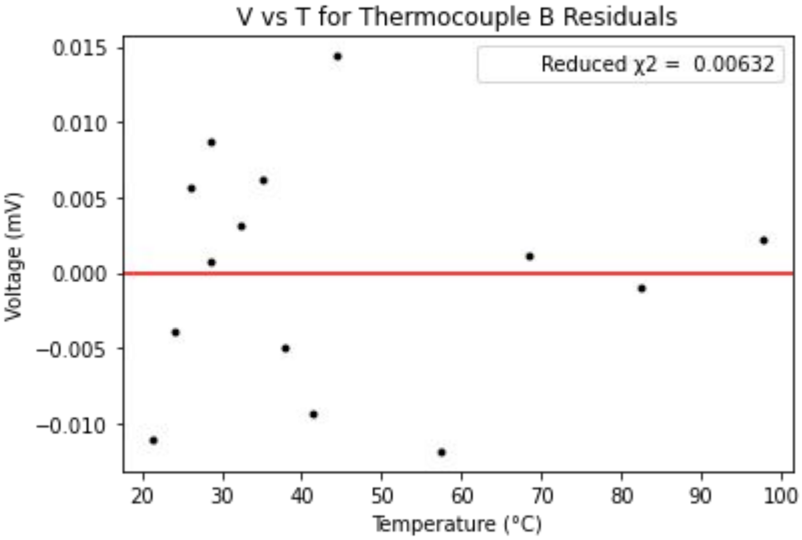




Data and Analysis - Thermocouple B

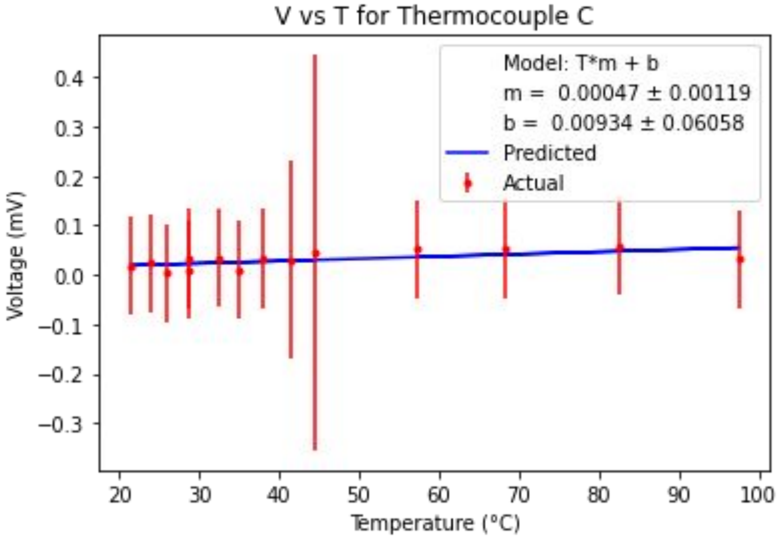
Theoretical values of S (relative to Platinum):
Copper: 6.5 μV/K
Brass: 6 μV/K
Effective S for thermocouple: 6.5 - 6 μV/K = 0.5 μV/K
Expected slope again lies slightly below the experimental slope (0.72 ± 1.19 μV/K), but the errors are much larger with a chi-squared value that is much smaller than 1.

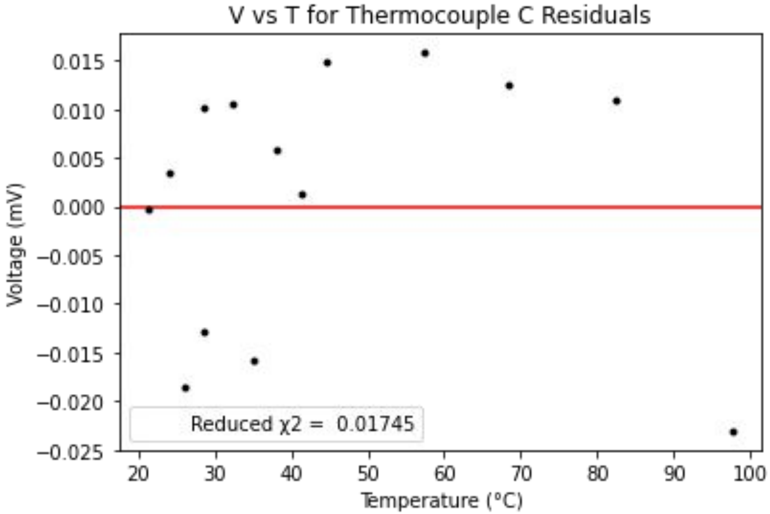




Data and Analysis - Thermocouple C

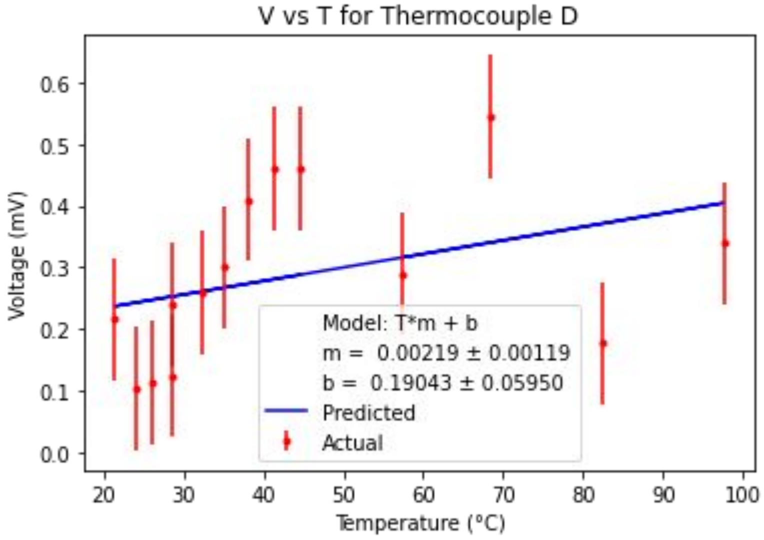
Theoretical values of S (relative to Platinum):
Tungsten: 7.5 μV/K
Nichrome: 25 μV/K
Effective S for thermocouple: 7.5 - 25 μV/K = -17.5 μV/K (we took the absolute value when graphing our data)
Expected slope has a significantly higher magnitude than the experimental value (0.47 ± 1.19 μV/K). We saw that the voltage measurements were all very close to 0, which led us to believe there might have been some experimental error such as a short for this thermocouple.

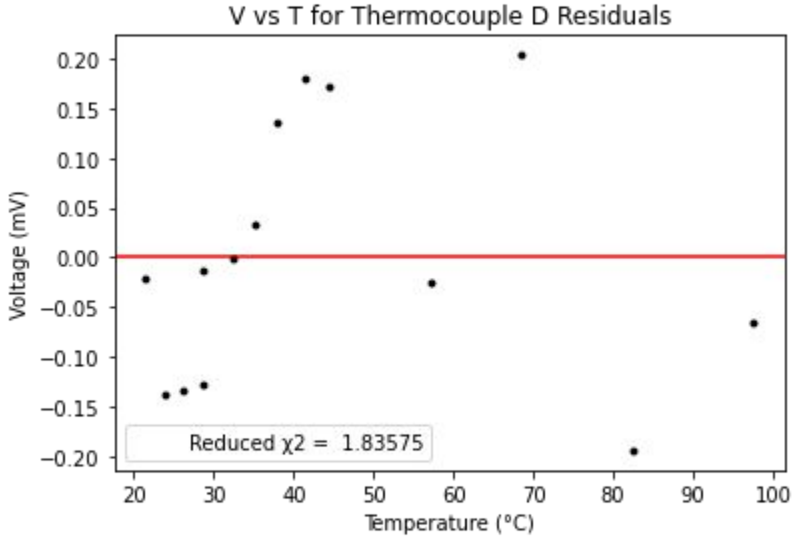




Data and Analysis - Thermocouple D

Theoretical values of S (relative to Platinum):
Iron: 19 μV/K
Copper: 6.5 μV/K
Effective S for thermocouple: 19 - 6.5 μV/K = 12.5 μV/K
Expected slope is significantly greater than the experimental slope (2.19 ± 1.19 μV/K). The data points do not lie very close to the linear model, which means we likely had some significant experimental error.





Summary of Results & Discussion of Error

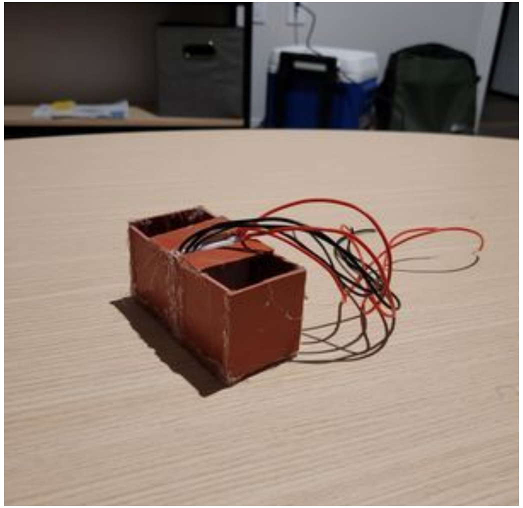
A and B	C and D
- Data roughly aligns with expectation	- Data deviates significantly from expectation
- Experimental Seebeck coefficients slightly greater than theoretical values	- Experimental Seebeck coefficients much smaller than theoretical values
- Small differences could be due to: imperfections in the multimeter, the temperature of the heat bath being higher than we measured (imperfection in the temperature probe), differences in lengths between where we probed the voltage on the two wires. Impurities of metals.	- We likely had large experimental errors for these two thermocouples. We suspect that there might have been a short because the separated ends of the thermocouple with the voltage probes were submerged in water. For D, the materials might have been a poor choice to make a thermocouple.

Future Directions

- Test the thick copper and thin copper against the same material to observe the effect of wire thickness on Seebeck coefficients
- Find a way to standardize the lengths of the wires that we used while maintaining a stable way to have the thermocouples reach between the hot and cold baths
- Use materials with very different Seebeck coefficients -> observe a greater voltage difference between the separated ends

Design and Methods - Experiment 2

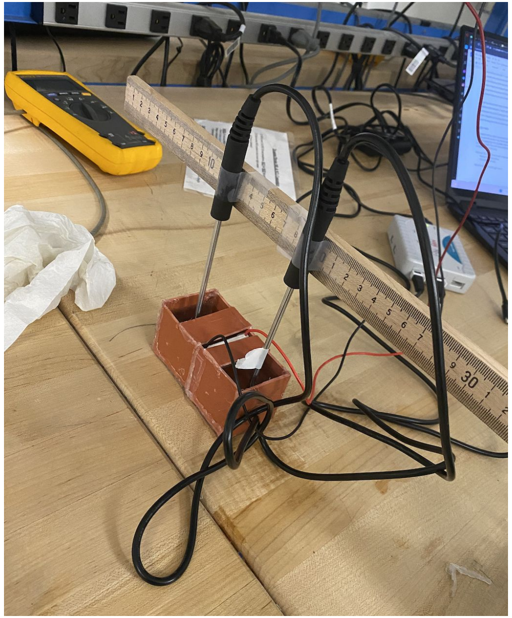
- For each side of the Peltier plate:
  - Cut up silicone gasket pieces to create an insulated box
  - Hot glue the box together to make it watertight
- Connect the two boxes together to either side of the Peltier plate using tape
- Measure a mass of water to pour into each side
- Continually measure the temperature of the water using a temperature probe and Logger Pro



:) creative name for box ^^

Design and Methods - Experiment 2 (cont.)

- Run a 2 A current through the plate using DC power supply
- Hold temperature probes steady by taping them to a meter stick over the Peltier box
- Take temperature readings over 600 seconds
- Repeat the process using two and three Peltier plates in series
    - Increased the voltage on the DC power supply as necessary to keep a current of approximately 2 A
    - 12 V, 16 V, and 20 V for single, double, and triple plate respectively



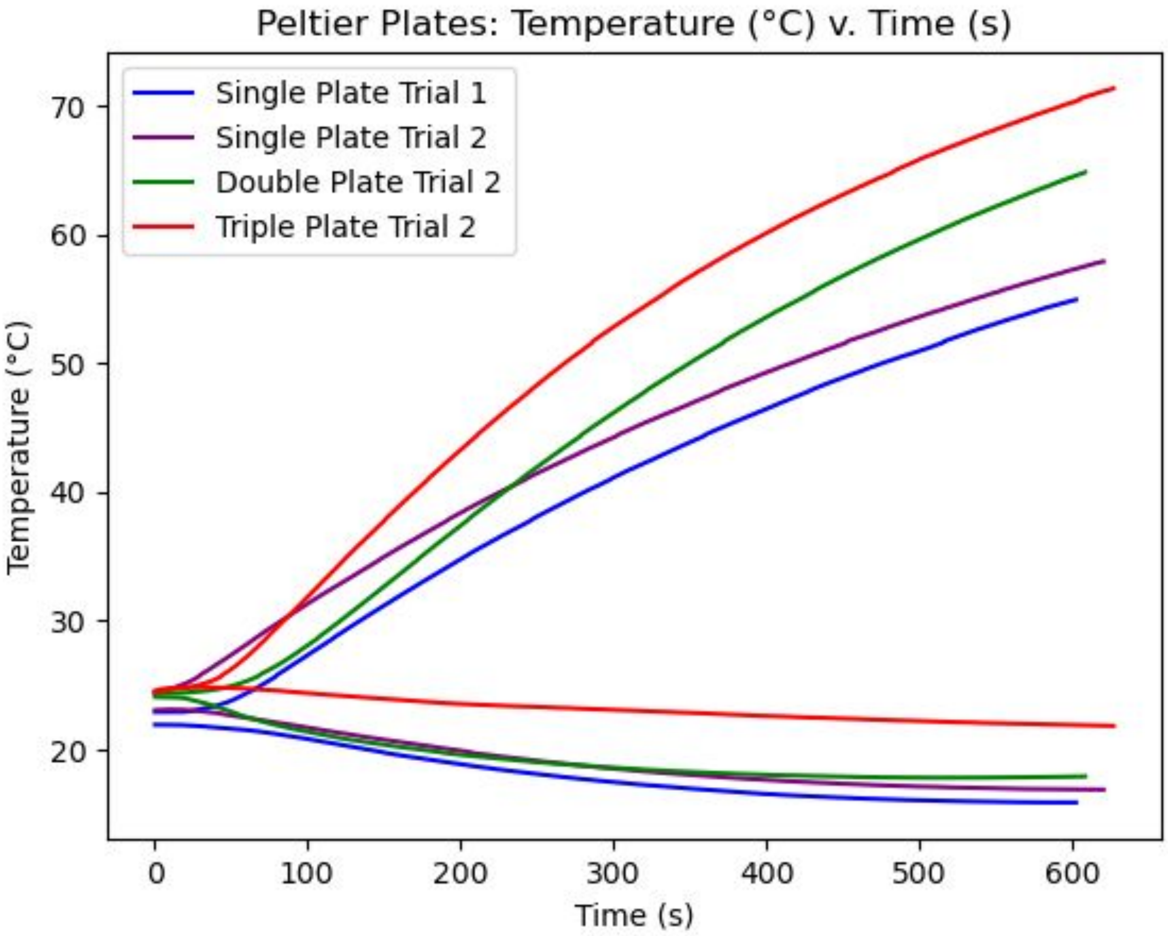
setup for Peltier box

Challenges

- Finding an insulating material to use
    - Original idea was styrofoam
    - Switched to silicon because store had it
- Making the Peltier box watertight
    - Used a lot of hot glue and took many tries to get it (mostly) watertight
- Tracking temperature in box
    - Taped probes onto a meter stick and dipped it in box.

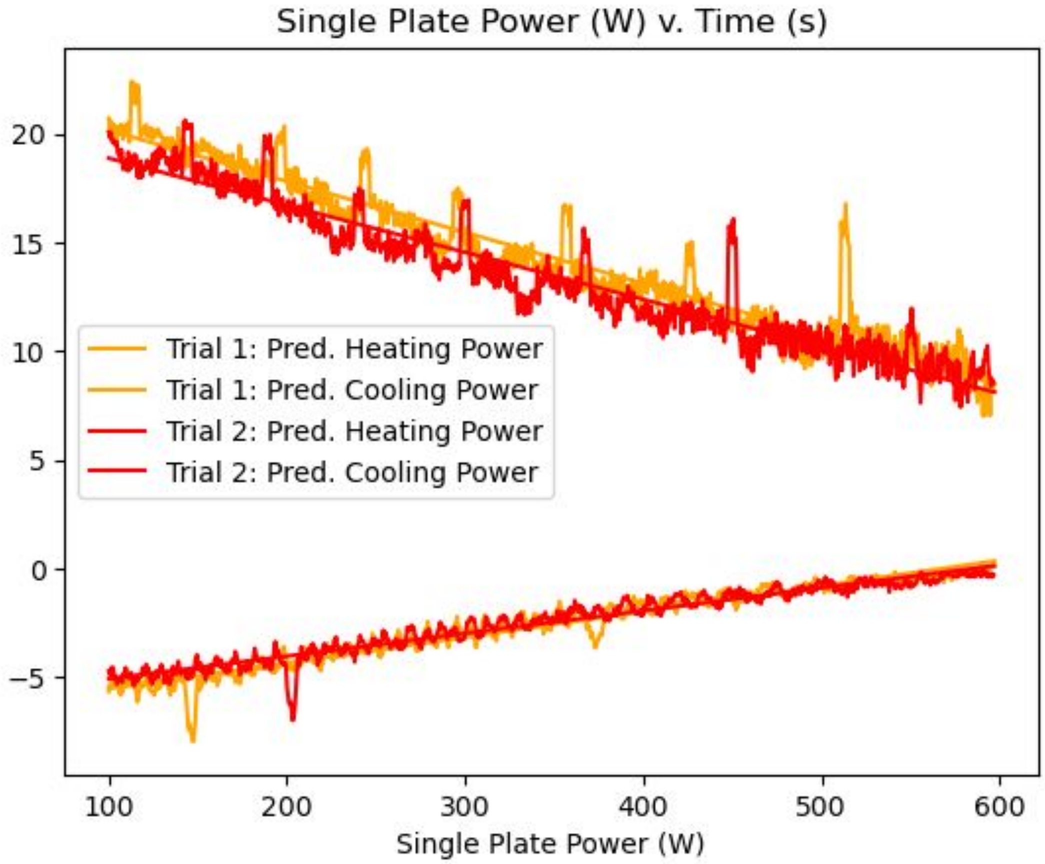
Highly sophisticated procedure . . .

Data



Peltier Plates: Temperature (°C) v. Time (s)
Single Plate Trial 1
Single Plate Trial 2
Double Plate Trial 2
Triple Plate Trial 2
Temperature (°C)
70
60
50
40
30
20
0
100
200
300
400
500
600
Time (s)

Analysis: Single Peltier Plate



Trial 1 (V0 = 12 V):

Heating

Power (W) = -0.0234±0.0002 (W/t)*T + 22.505±0.0791 (W)

Cooling

Power (W) = 0.0118±0.0001 (W/t)*T - 6.713±0.0343 (W)

Trial 2 (V0 = 12 V):

Heating

Power (W) = -0.0217±0.0002 (W/t)*T + 21.038±0.0779 (W)

Cooling

Power (W) = 0.0105±0.0001 (W/t)*T - 6.132±0.0313 (W)

Initial Input Power = V*I -> 12 V * 2 A = **24 W**

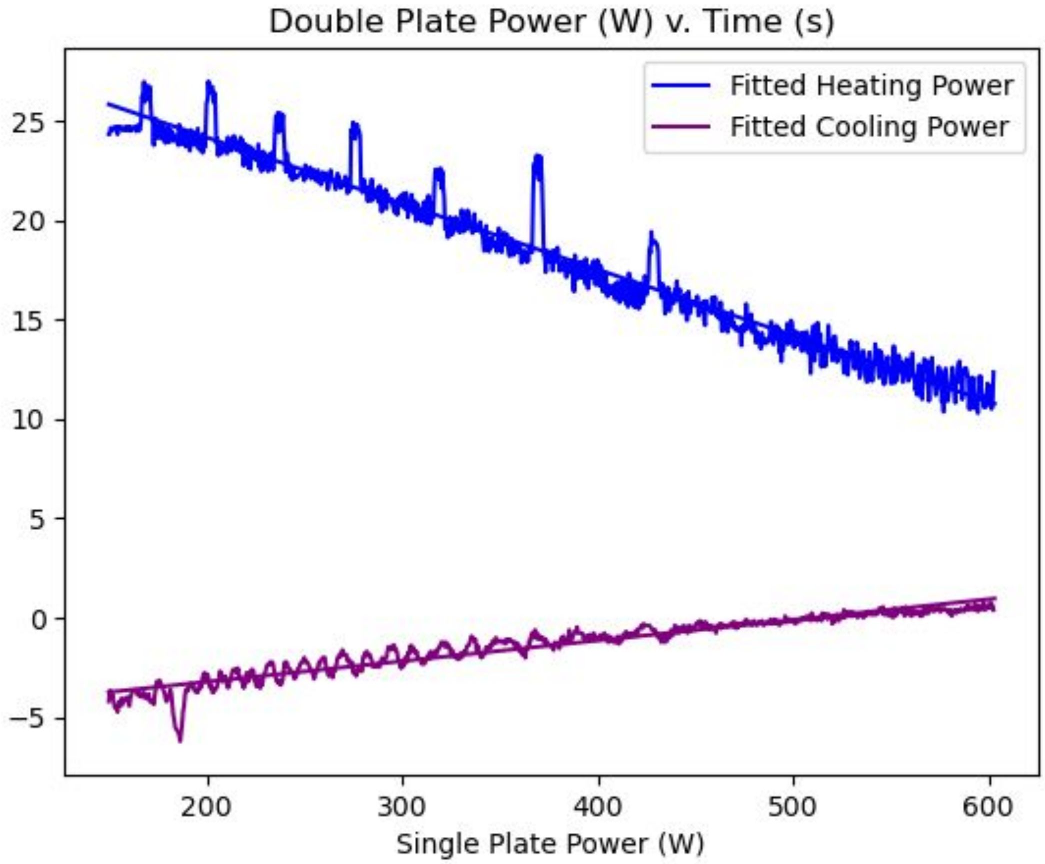
Calculated Heating Power -> **22.5 , 21.0 W**

Calculated Cooling Power -> **6.7 , 6.1 W**

Avg. Heating Efficiency -> **90.63%**

Avg. Cooling Efficiency -> **26.67%**

Analysis: Double Peltier Plate



Trial 1 (V0 = 16 V):

Heating

Power (W) = -0.0332±0.0002 (W/t)*T + 30.7976±0.0903 (W)

Cooling

Power (W) = 0.0104±0.0001 (W/t)*T - 5.3028±0.04142 (W)

Initial Input Power = V*I —> 16 V * 2 A = **32 W**

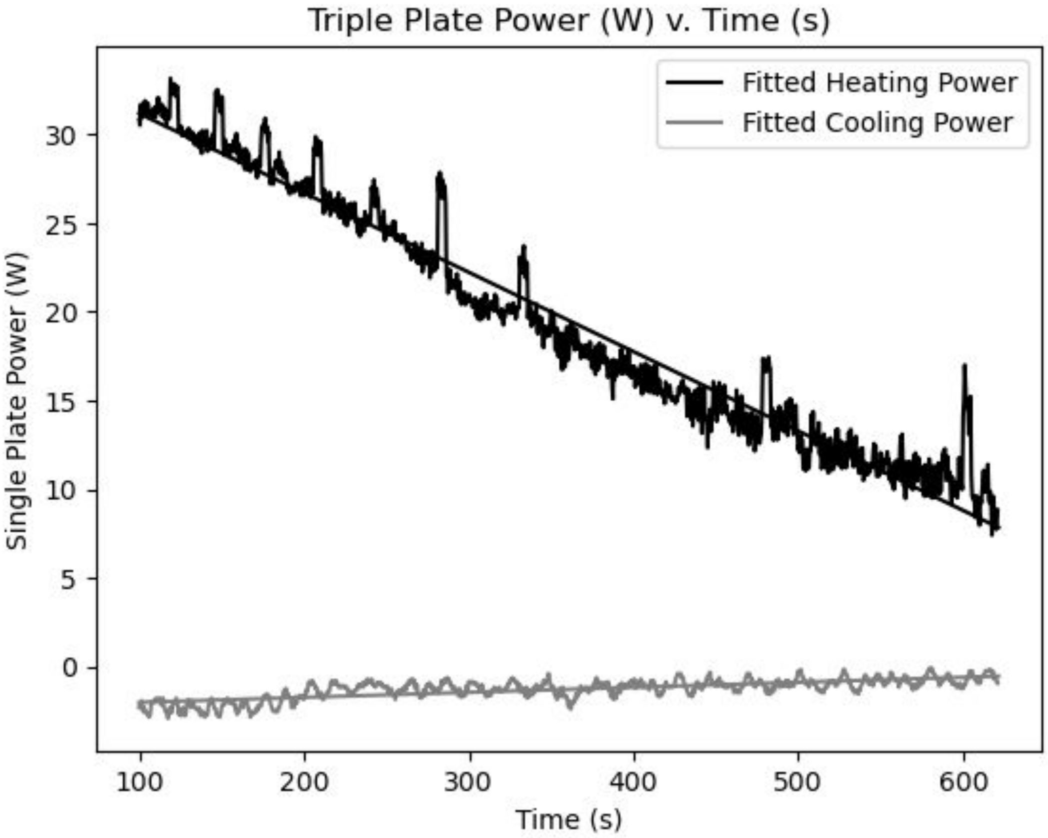
Calculated Heating Power -> **30.8 W**

Calculated Cooling Power -> **5.3 W**

Heating Efficiency —> **96.25%**

Cooling Efficiency —> **16.56%**

Analysis: Triple Peltier Plate



Trial 1 (V0 = 20 V):

Heating

Power (W) = -0.0447±0.0003 (W/t)*T + 35.6506±0.1146 (W)

Cooling

Power (W) = 0.0028±0.0001 (W/t)*T - 2.2933±0.0316 (W)

Initial Input Power = V*I -> 20 V * 2 A = **40 W**

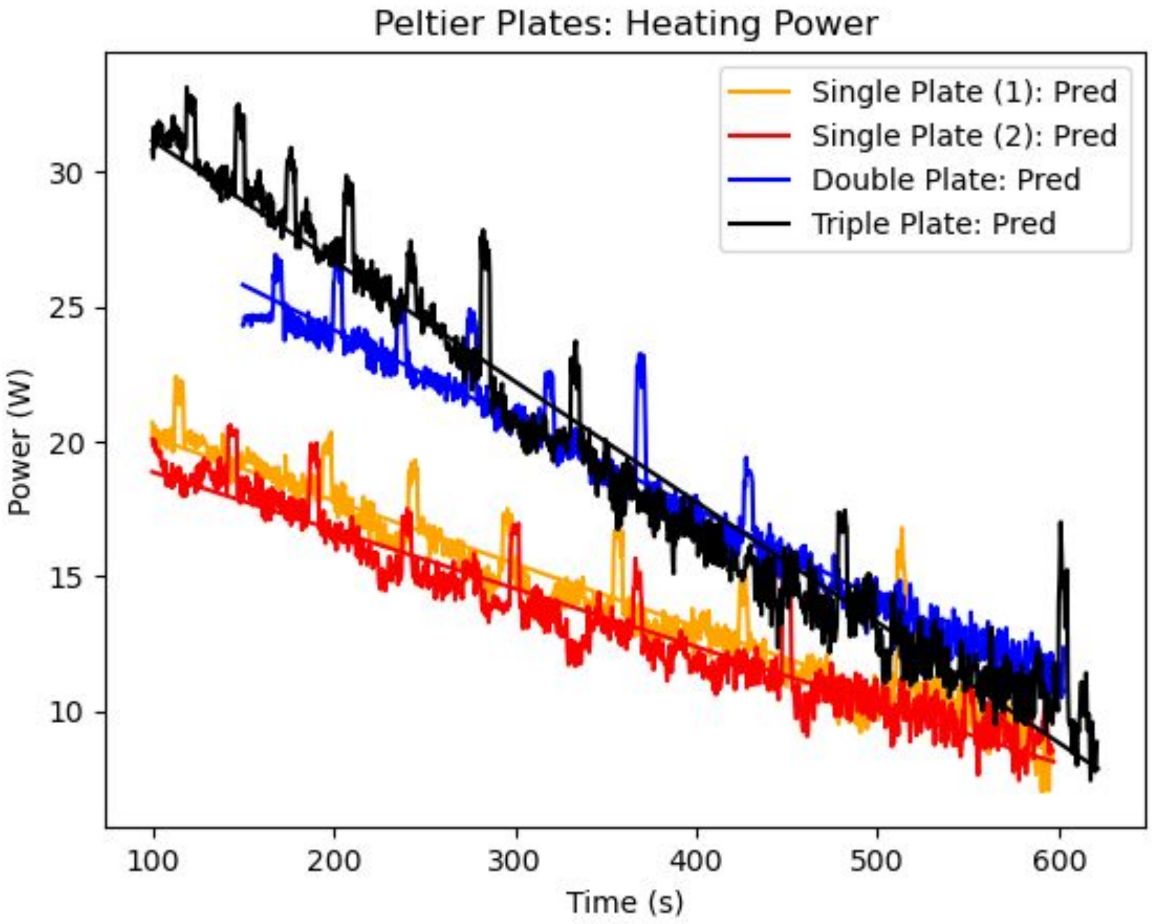
Calculated Heating Power -> **35.7 W**

Calculated Cooling Power -> **2.3 W**

Heating Efficiency -> **89.25%**

Cooling Efficiency -> **5.75%**

Analysis: Peltier Plates

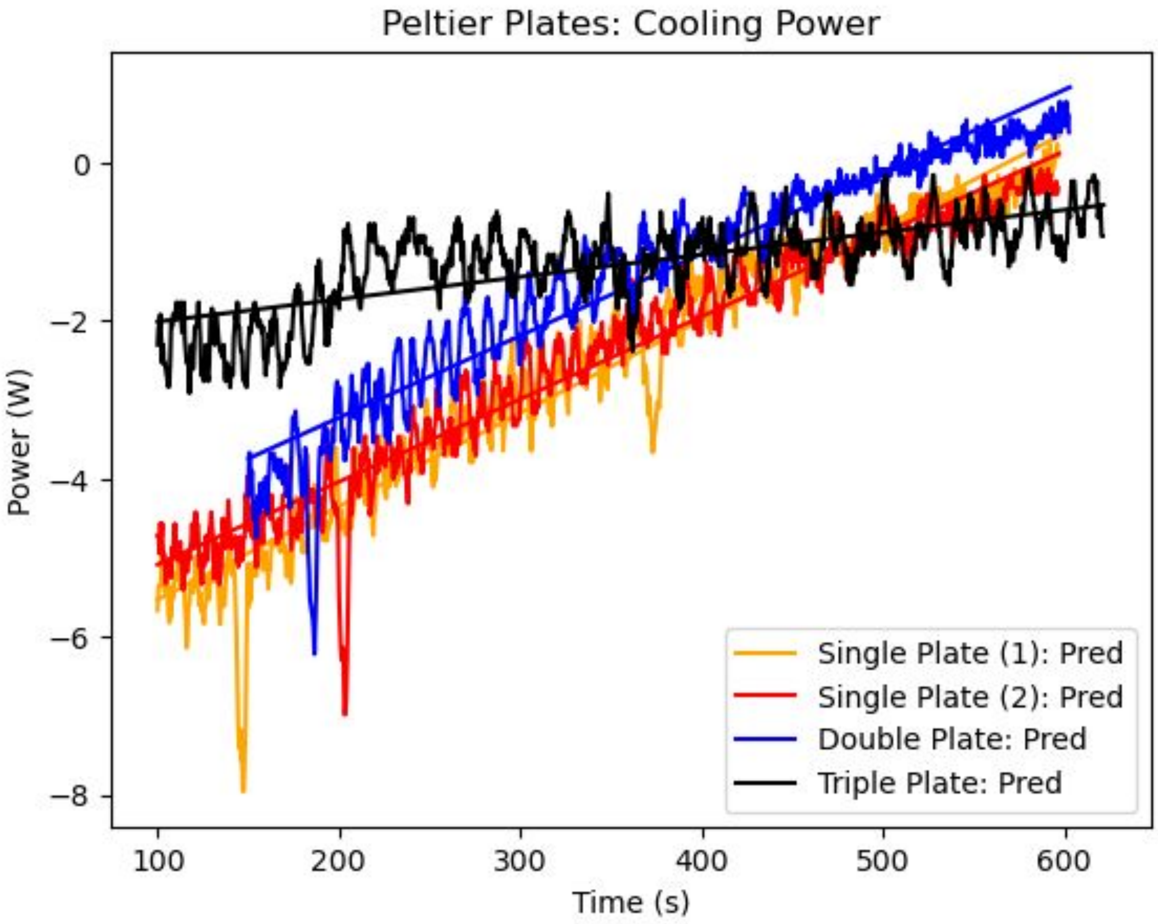


Peltier Plate Efficiencies

	Heating
Single	**90.63%**
Double	**96.25%**
Triple	**89.25%**

-No obvious pattern, *but*
-High efficiencies for heating

Analysis: Peltier Plates



Peltier Plate Efficiencies

	Cooling
Single	**26.67%**
Double	**16.56%**
Triple	**5.75%**

-Very low efficiencies
-Adding more plates decreases efficiency

Summary of Results & Discussion of Error

Heating Efficiency: ~90% **Cooling Efficiency: 27% -> 6%**

- Cooling efficiency decreases drastically with more plates in series.

- Heating efficiency has no obvious correlation

- Primary source of error: silicone is not a perfect insulator
    - Heat escapes through the walls of the box, which means the temperature change of the water does not measure the entire effect of the Peltier plate's heating
- Not perfectly watertight -> water leaked through the bottom of the box
    - Mass of water was decreasing throughout the 600 seconds of measurement
    - Made the Peltier plate seem more efficient than it was because it was heating up a smaller amount of water

Future Directions

- 3D print a box template and cover the insides with silicon/rubber in order to create thermal insulation.
- Increase box dimensions, and create more secure seals for the Peltier plate
- **Track Current and Voltage** at time increments (they changed over time, most likely due to changes in resistivity of the wires as they heated)
- Use smaller / more stable temperature probes

Summary and Conclusion

Experiment 1:

The given thermocouple and the copper + brass thermocouples behave almost as expected, while the tungsten + nichrome and iron + copper thermocouples do not, likely due to experimental errors.

Experiment 2:

Heating efficiencies are quite high (~90%), while cooling efficiency drops drastically with additional plates.

For Peltier plates in fridges - DON’T put them in series!!!

References

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