Generating Single Photons with Neutral Atoms

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With the recent explosion of progress on neutral atoms as a platform for quantum information, single photon generation within an optical cavity has become essential for efficient transfer of information between computing nodes or for use in a standalone approach to quantum computing. In this review paper, I discuss current progress towards a bright, high-fidelity single photon source over the past three decades by covering the basic theory of CQED spontaneous emission, experimental realizations of such sources, metrics used to evaluate single photon quality, and methods to simulate decay dynamics. I then explore attempts to use many-body interactions to generate single photons and shape collective emissions with direct analogue to the Jaynes-Cumming Hamiltonian of neutral atom setups.

I. INTRODUCTION

Over the last three decades, proposals to use individual atoms as single photon sources have shown great experimental progress. Most of these ideas revolve around STIRAP (Stimulated Raman scattering involving adiabatic passage) [1-3,6-9], which takes advantage of the cavity field mediating interactions with a two level atom. Recent ideas have taken this a step further by adding Rydberg blockade effects [1] or frequency conversion [2]. The main appeal of a single photon source lies in its potential applications for a quantum computation platform. A direct application would be in an entirely optical quantum computer [3] where information is stored entirely on the state of photons. Due to highly limited technical capabilities however, this platform remains far away from near term fault tolerant algorithms. A more practical approach might be to use these sources as a quantum transducer - This has far looser requirements on the technical capabilities of our photon. Information from an atom-based quantum computer could be converted into photonic form, sent through an optical fiber, and received at a separate node. Both applications rely on a consistent, high-fidelity, bright source of photons.

Recent developments in neutral atoms mean that the second application is of direct interest for NISQ devices. With two and three qubit gates mediated by Rydberg blockade interactions implemented at $> 99\%$ fidelity and dozens of logical qubits implemented in Lukin's recent paper [4], neutral atoms are quickly developing towards, if not a viable option for quantum computing, a robust platform for simulations. The challenge for a single photon source *in the context of neutral atoms* is then twofold. Not only do we want a fast, coherent generator of single photons, but we would also like to reliably store information in the state of said photons with high fidelity.

The first task of creating said photons is very well explored. The second, not so much. In this review paper I will focus solely on that first task by providing a broad overview of the field. The contents of this paper can be split into two main sections. Starting from the bread and butter theory of spontaneous emission, 'Current Progress' walks the reader through the current theory and experimental methods of single photon generation. The second portion of 'Future Directions' is less well-defined, and highlights the potential applications of coherent interactions and superradiance. The recent explosion of progress in atomic control - Local addressing, optical tweezer grids, collective interactions - enables potential improvements in photon generation. I hope this review paper gathers current progress into a coherent resource that will help with that task.

II. CURRENT PROGRESS

A. Spontaneous Emission of a Single Atom

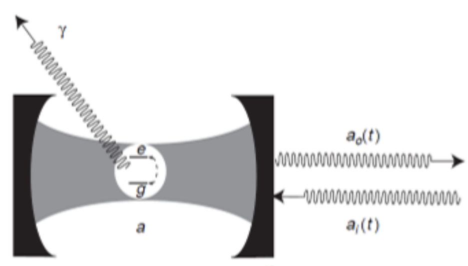

FIG. 1. Two level atom system in a Fabry-Perot cavity, from Wall's textbook [5]. The right mirror is lossy, as defined by κ.

Calculating the spontaneous emission of a photon from a two level atom inside a cavity means we must solve for the master equations of such a system. Thankfully, Walls does it for us in chapter 11 of his textbook [5]. While most of our results going forward will work with three-level atoms in cavities, Wall's derivation still provides good theory. We write the Linbladian:

$$\begin{aligned}\frac{d\rho}{dt} &= -i\delta[\hat{a}^\dagger\hat{a},\rho] - i\frac{\Delta}{2}[\sigma_z,\rho] - i[\mathcal{E}^*\hat{a} + \mathcal{E}\hat{a}^\dagger,\rho] \\ &-ig[\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-,\rho] + \frac{\kappa}{2}(2\hat{a}\rho\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\rho - \rho\hat{a}^\dagger\hat{a}) \\ &+\frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-)\end{aligned} \tag{1}$$

With $\mathcal{E}$ containing our driving laser field, $\delta = \omega_c - \omega_L$ being the cavity and driving field detuning, and $\Delta = \omega_a - \omega_L$ being the two-level atom and driving field detuning. Defining $p_{emission}(t) = 1 - p_e(t)$, where $p_e(t) = \langle e|\rho|e\rangle$, we can directly obtain the spontaneous emission probability of our system over time. Given an input driving field of the correct form, this would give us persistent emission of single photons from just one atom. This is better seen in subsection D with numerics.

B. Experimental Realizations

To see spontaneous emission in action, let's take a look at STIRAP-based generation of single photons. The first setup is from Max-Planck Institute, dubbed 'A single-photon server with just one atom' [6]. Here, the authors setup a Rb^{85} trap with a MOT loading into a running-wave 1032 nm dipole trap that served as a carrier belt into a high-finesse optical cavity. After loading, atoms were captured with 780 nm beams. The cavity was defined by $(g, \kappa, \gamma) = 2\pi\cdot(5,5,3)$ MHz, where g is the maximum atom-cavity coupling constant, κ the cavity-field decay rate, and γ the atomic dipole decay rate. To encourage spontaneous emission onto one side of the cavity, one of the mirrors was selected to have 50 times higher transmittance than the other. A rough schematic is shown in Figure 2.

With this setup, the authors were able to keep atoms in the cavity for 10.3(1) s on average, with 8.3(2) s of that time being available for use as a single-photon source. Due to large optical losses from mirror defects in the cavity, propogation loss, and low detector efficiency, the photon-generation probability was a meager 0.93%. However, this was with a trigger rate of 100 kHz and an ideal case efficiency of 9%. The authors attributed this low efficiency to the large number of Zeeman states and noise from the atom moving inside its optical tweezer. The measured antibunching visibility was 94.0%, also not great, but not terrible for a setup from 2007.

The suggested protocol for single photon generation using this scheme was as follows. First, initialize the system by trapping a few atoms. By measuring the light coming out of the cavity during the recycling and cooling pulses, a signal for the population of the cavity can be obtained. Once all atoms except one scatter away, photons emitted during the next 1.5 seconds are recorded and the cross-correlation calculated. When the average number of correlations at zero time difference is confirmed to be below a set amount, this signals that single photon generation is truly underway. The beam can then be redirected

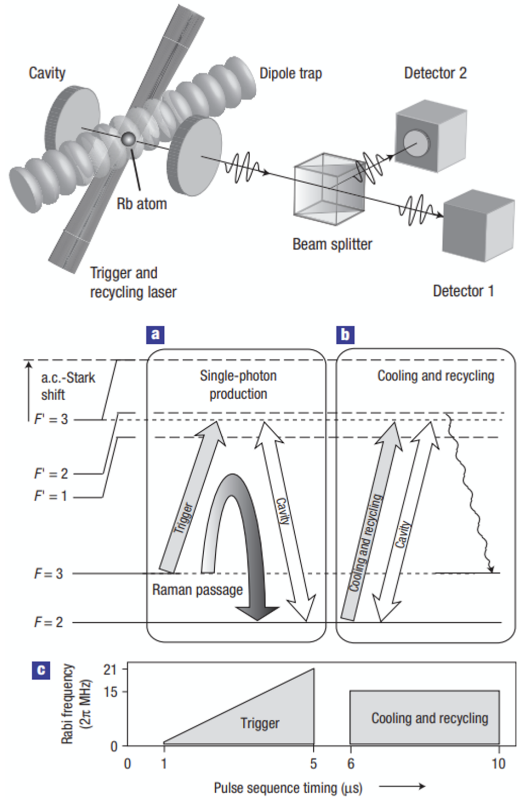

FIG. 2. Top: Schematic of the Rb^{85} trap setup [6], along with a Hanbury-Brown-Twiss configuration made to measure the second order coherence function. Bottom: Illustration of the single-photon production process, with each arrow being a laser beam with their pulse sequence below. In a.) The Rb^{85} atom is excited with a 4 μs long pulse on resonance with the $F = 3 \rightarrow F' = 3$ transition. The cavity is on resonance with the $F = 2 \rightarrow F' = 3$ transition, so this drives STIRAP from $F = 3$ to $F = 2$ while emitting a photon into the cavity mode. In b.) the atom is then cooled adiabatically by a new 4 μs long pulse used to return the atom to the $F = 3$ state while cooling it down with multiple photons scattering into the cavity. This resets the atom and lets us repeat the cycle.

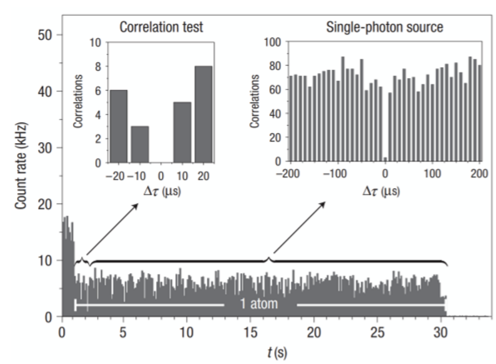

FIG. 3. Single atom signal [6]. Loss of multiple atoms is observed until only a single atom remains, with the correlations at $\tau = 0$ approaching 0 signalling the generation of single photons.

to any purpose we want, and if the atom is lost it can be detected within 30 ms with 98% probability. While this is not ideal for error correction purposes (30 ms is enough time for hunereds of operations!) techniques have thankfully improved since this paper was released. Similar results can be found in [7,8,9], with the experimental procedures and results being almost identical to this one.

The second setup [1] is much more recent (2020) and incorporates a blockaded Rydberg ensemble as a generator of large optical nonlinearity. By having atoms spaced close enough for the Van der Waals interactions to create Rydberg blockade, the authors suppressed excitation of more than one atom. But since the setup is inherently quantum, this single atom excitation manifested itself on these multiple atoms as a spin wave. By maintaining the phase coherence of this spin wave, the eventual de-excitation of this Rydberg state onto an intermediate state allows for the creation of high quality single photons. For the main highlights: This system produced single photons at rates of up to 400 kHz with a generation probability of up to 0.40(4), a $g^{(2)}$ of $5.0(1.6) \cdot 10^{-4}$, and indistinguishability of 0.980(7). A bit more realistically though, the single mode rate was $1.18(2) \cdot 10^4 \mathrm{s}^{-1}$.

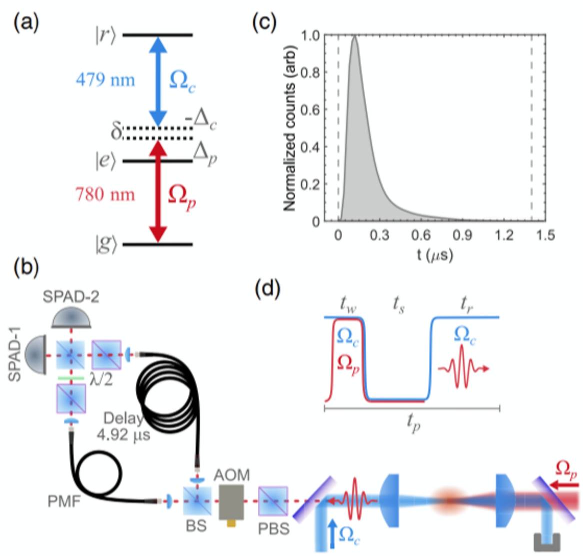

FIG. 4. a.) Atomic levels and input driving beam wavelengths, along with detunings. In the paper, they set the single photon detuning $\Delta_p/(2\pi) = 50$MHz, with a two-photon detuning $\delta = \Delta_p + \Delta_c$, and $\delta/(2\pi) = -2$MHz. b.) Experimental setup. All photons pass through a PBS that projects them onto a single polarization, then an AOM that shutters incoming photons off during the Rydberg pulse, and then a beamsplitter that allows a set time delay to be added onto the system. The photons then proceed to a Hanbury-Brown-Twiss and Hong-Ou-Mandle interferometer setup. c.) Temporal envelope of a single photon. Peak is shown near $t = 0.2\mu$s. d.) Pulse sequence, where the Rydberg excitation pulse lasts for $t_w = 370$ns.

The authors started with a MOT of Rb^{87} atoms cooled with a molasses trap down to 10μK. Like before, the atoms were loaded into a 1003 nm optical dipole trap, and then excited by coupling the Rydberg state $|r\rangle = |139S_{1/2}, m_J = 1/2\rangle$ to the ground state $|g\rangle = |5S_{1/2}, F = 2, m_F = 2\rangle$ via the intermediate state $|e\rangle = |5P_{3/2}, F = 3, m_F = 3\rangle$ with an intermediate detuning $\Delta_p/(2\pi)$. By turning on the $|e\rangle \to |r\rangle$ beam and shining it on the ensemble of atoms as a whole, the authors could guarantee that a spin wave was written onto the atoms corresponding to a single photon excitation. A neat advantage was that the effective two-photon Rabi frequency $\Omega_{2ph} = \frac{\Omega_p \Omega_c}{2\Delta_p}$ was enhanced by a factor of $\sqrt{N} = 20$ from the N atoms participating in this excitation. Note that there is no 'trapping' 780 nm light here - there are hundreds of atoms in the dipole trap, all excited by single laser pulses. After waiting for the spin wave to be written ($t_s > 350$ns, the control field is turned back on to induce spontaneous emission of our ensemble. Detection of the generated photon was done with two single-photon avalanche detectors (SPAD) with quantum efficiencies of 70%.

Further information on the $g^{(2)}(\tau)$, detection efficiency, and fidelity of the single photon source can be found in the paper. Of primary interest to us is the future improvements possible. The authors state that removing atoms in pollutant states at the start of generation might increase the generation rate up an order of magnitude to $1.2 \cdot 10^5 \mathrm{s}^{-1}$ without impacting the photon fidelity (The single-mode fidelity observed in this paper was the highest as of 2020). Some further theory on the collective emission of a Rydberg-blockaded single photon source can be found here [10] for an ensemble of thermal atoms.

C. Evaluating a Photon Source

The metric of goodness for single photon sources is widely accepted as the second-order coherence function $g^{(2)}(\tau)$. This function measures the probability of two-photons being found together within a set time τ, and ideally we would like $g^{(2)}(0) = 0$ in order to ensure there is **no chance** of two photons being found together at a single point in time. We define $g^{(2)}(\tau)$ as:

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle^2} \qquad (2)$$

Experimentally, this quantity is typically measured with a Hanbury-Brown-Twiss setup which uses a 50/50 beam splitter and two photodetectors with a variable time delay in between them to act as τ [6]. When graphing $g^{(2)}(\tau)$, we always want to see a trough at $\tau = 0$, and it does not matter what the shape is away from zero time. Counterintuitively, it turns out that having near-zero $g^{(2)}(\tau)$ does not necessarily surpress $g^{(n)}(\tau)$ for all $n > 2$. The reason for this is explored in more detail here [10], and it turns out to have a nice link to the Monty Hall problem.

D. Simulation

Tying the above ideas together, Walls [5] provides an excellent reference to a paper simulating a single-photon source [12]. Though this paper is designed to simulate a solid-state system, it is easily generalizable to a three-level atom model. The authors start with the basic Hamiltonian for this system,

$$\hat{H}/\hbar = \omega\hat{a}^\dagger\hat{a} + \sum_{j=1}^{3}[\omega_j + A_j(t)]|j\rangle\langle j| + ig\langle\hat{a}||2\rangle\langle 1| - H.c.) \\ +\Omega(t)\cos(\omega_{31}t)(|3\rangle\langle 1| + H.c.), \tag{3}$$

Where $\hat{a}$ is the annihilation operator for a cavity field with frequency ω. The authors assume that the atomic levels $|3\rangle$ and $|2\rangle$ are coupled incoherently, and by also making the rotating wave approximation this reduces to:

$$\hat{H}/\hbar = A(t)\hat{\sigma}_z^{21} + \Omega(t)(\sigma_+^{31} + H.c.) + ig(\hat{a}\sigma_+^{21} - H.c.), \tag{4}$$

Where the first term contains the time-dependent Stark pulse and the second the coherent driving pulse term. Loss terms are included with the Linblad master equation:

$$L = \frac{1}{i}[\hat{H},\hat{\rho}] + \sum_{i=1}^{3}\hat{C}_i\hat{\rho}\hat{C}_i^\dagger - \frac{1}{2}(\hat{C}_i^\dagger\hat{C}_i\hat{\rho} + \rho\hat{C}_i^\dagger\hat{C}_i). \tag{5}$$

Where $\hat{H}$ is our Hamiltonian and our loss terms are $\hat{C}_1 = \sqrt{\kappa}\hat{a}, \hat{C}_2 = \sqrt{\gamma_{21}}\hat{\sigma}_-^{21}, \hat{C}_3 = \sqrt{\gamma_{32}}\hat{\sigma}_-^{32}$. Physically, these loss terms correspond to (respectively) the cavity coupling to external modes, spontaneous emission of the $\hat{\sigma}_+^{32}$ transition, and the incoherent population of $|2\rangle$ post-pump pulse.

Solving this Lindblad equation then, the probability of our system being in the excited state as a function of time in the same way shown in subsection A. For these results, the authors simulated two cases. The first (a) simulated an 'active' source where the atom was kept off resonance long enough to allow for de-excitation from the upper excited state, then brought back into resonance with a π pulse. Note that there was no intermediate cooling in this simulation as needed for STIRAP. The second (b) used a 'passive' source that induced an A.C. Stark shift to keep the atom on-resonance with the cavity.

Note that this simulation was done assuming we are in the 'strong' Purcell regime ($\kappa/g = 1.5, \gamma_{21}/g = 0.2$. There is a slight 'bump' in the probability graph of case (a), while there is none for case (b) - This is due to the small chance of the atom decaying once more to the lowest energy state allowed. Further graphs from the authors show strong antibunching and low $g^{(2)}$ values for both pulse schemes, along with further tips on pulse shaping that are better covered in the paper.

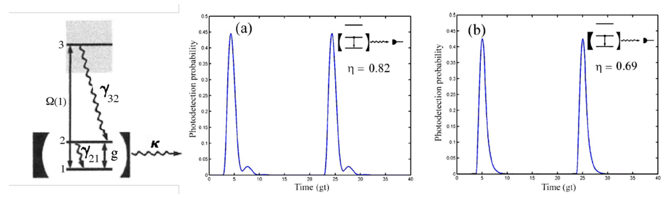

FIG. 5. Left: Diagram of three level system [12], shaded region corresponding to a continuum of allowed excited states. Photon emission is detected upon de-excitation to the state $|2\rangle$. Right: Numerical results showing the photodetection probability on a device performing a direct measurement immediately outside of the cavity.

III. FUTURE WORKS

With such progress already made on single photon sources, where do we go from here? Certainly, further progress can be made on experimental control and pulse optimization of single atoms. But as the Rydberg ensemble experiment suggests, there is much more room to develop by going into the many body regime. Whether that is with multiple atoms in a lattice, coherent excitations, Rydberg blockade, or superradiance - There's a whole new set of toys available in the many body regime.

A. Quantum Emitter Chains

Let's take some inspiration from the quantum dot community. A recent theory paper [13] suggests that indistinguishable single photons can be generated from a collection of noisy quantum emitters, the formal name for this being a Su-Schrieffer-Heeger chain. The Hamiltonian for this model is:

$$H = \sum_i \mathcal{E}_i(t)\sigma_i^+\sigma_i^- + \sum_i J_i(\sigma_i^+\sigma_{i+1}^- + \sigma_{i+1}^+\sigma_i^-), \tag{6}$$

The idea is to excite the entire chain, and collect single photon emission from the edge sites. Due to the topologically nontrivial chain (quantum dots are spaced unevenly with different on-site and hopping energies as a result), unintuitively, the system becomes quite robust to noise near the phase transition point - Becoming a stable source of single photons. Applying some standard techniques from solid-state physics the authors proceed derive the spontaneous emission spectrum of this chain:

$$C_i(t,t') = \langle\sigma_i^+(t)\sigma_i^-(t')\rangle, \tag{7}$$

Assuming that our system is initially in an excited state at t = 0, an ideal detector can be used to capture emissions from the edge sites. This would give a single-photon spectrum of

$$S_i(\omega) = \frac{\int_0^\infty dt \int_0^\infty dt' C_i(t,t')e^{-i\omega(t-t')}}{2\pi\int_0^\infty dt C_i(t,t)} \tag{8}$$

With this in mind, the authors proceed to present the master equations for this system in order to derive the second order coherence function. Both are shown below:

$$\text{Setting} \quad H_\rho = H + P(t) \sum_i (\sigma_i^+ + \sigma_j^-), \tag{9}$$

$$i\hbar \frac{d\rho}{dt} = [H_\rho, \rho] + \frac{i\gamma_0}{2} \sum_i (2\sigma_i^- \rho \sigma_i^+ - \sigma_j^+ \sigma_j^- \rho - \rho \sigma_j^+ \sigma_i^i)$$

$$\text{and} \quad g_i^{(2)}(t,t') = \frac{\langle \sigma_i^+(t) \sigma_i^+(t') \sigma_i^-(t) \sigma_j^-(t') \rangle}{\langle n_i(t) \rangle \langle n_i(t') \rangle} \tag{10}$$

Predicted graphs of the second order correlation function are in Figure 6. Surprisingly, the authors found that the quality of the single photon source **increases** with the number of additional quantum dots used in the system. This suggests, perhaps, a direct application for a neutral atom system - Why not create an analogous Hamiltonian with a 1D chain of neutral atoms? With local addressing beams and tunable tweezer positions, it may be possible to engineer a system with identical topology to the quantum dots proposed in this paper.

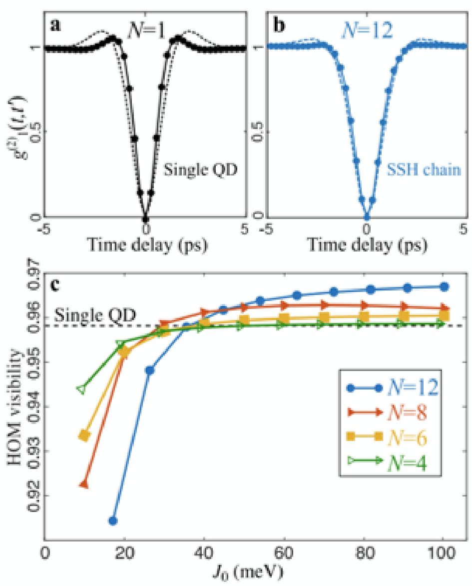

FIG. 6. Top: Predicted $g^{(2)}(\tau)$ behavior of the topologically nontrivial quantum dot chain's photon emission. Observe that the chain becomes more robust to error with large N. A similar improvement can be found in Bottom: HOM visibility, HOM being the Hong-Ou-Mandel effect which represents the indistinguishability of generated photons. [13]

IV. COOPERATIVE EMISSION OF ATOMIC CHAINS

We can also be inspired by progress in atomic arrays trapped near nanoscale waveguides. A 2021 paper [14] explores the decay dynamics of atoms arranged in a linear array. Only a segment of these atoms is excited to a super radiant mode, triggering emission of light into the waveguide. Additional atoms on the sides of this segment act as reflectors or absorbers of this light, and the authors show that this 1D system can be used to redirect emitted light or even localize it inside of a pseudo cavity of atomic 'mirrors. The derivation of these results is well-laid out in the actual paper, so I will only focus on the main points here.

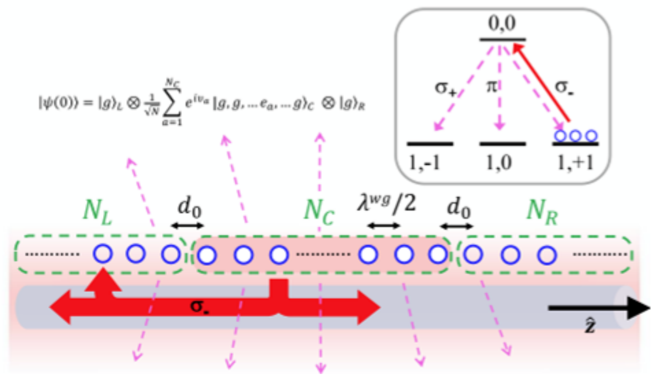

FIG. 7. Schematic of the proposed system [14]. An atomic array is trapped near a dielectric waveguide, with all atoms initially spin polarized in the $\hat{\mathbf{z}}$ direction. The central segment is excited and decays superradiantly, and the left and right segments serve as pseudo mirrors for this decay to be directionally enhanced or suppressed.

The following graphs in Figure 8 play, essentially, the same game. There are two probabilities to pay attention to: $p(t)$, the excitation probability shared by the whole atomic array and $p_0(t)$, which captures the decay of the originally excited state. These probabilities are graphed vs. the system time, with $p_0(t)$ shown as a magenta dash-dotted line while $p(t)$ is shown as a magenta line. Additional lines showing the unperturbed collective emitter decay (gray) and a single-atoms natural decay into free space (gray dashed) are shown as benchmarks. Note that the y-axis here is exponential, so the system is indeed undergoing exponential decay during spontaneous emission. The authors test simulations with different configurations of atoms and plot the dynamics of the system. The dashed green lines around the atoms are the region of initial excitation, while those outside are not touched at all.

Further elaborations about these graphs are best found in the paper itself. What we are interested in is the fact that emission from a central set of excited atoms can be delayed, sped up, or oscillated dependent on the configuration of atomic 'mirrors'. This is exciting, because the results for these collective emitters can be directly mapped onto the Jaynes-Cumming Hamiltonian. If not done already, exploring different configurations of 1D chains with superradiant excitations could prove fruitful experimentally. It would also be possible to extend these

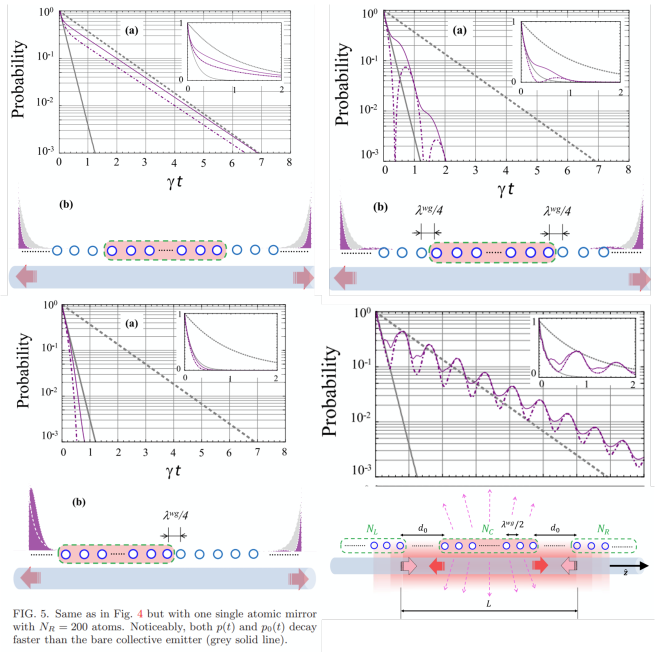

FIG. 8. Simulations from [14]. Upper left: Translationally-invariant configuartion in which the center atoms are excited. There are 100 atoms in each section (Left, Center, Right) for a total of 300 atoms. Upper right: Identical to Upper left, but with the direct spacing from the Center section to its neighbors being 1/4 the waveguide length. Bottom left: Same as Upper right, but with a single atomic mirror with 200 atoms. There remain 100 excited atoms all on the left. Bottom right: Excitation probabilities for a long resonator with direct spacing from the Center section to its neighbors being 20 cm, and the left and right atomic mirrors have 500 atoms each. The center still has 100 atoms.

experiments to 2D arrays. What sort of decay dynamics would a system experience if, say, only the center of a 2D array experienced superradiant decay? How might the lattice spacings of the surrounding grid influence the reflectivity of our atoms? It may be possible to create atomic configurations that enhance atom-atom coupling by forming a ring of 'mirrors' around a central zone, or that direct emitted light in a programmable direction.

V. CONCLUSION

This review paper provides a broad overview of single photon generation, its theory, past works, and current progress. I hope the inclusion of multiple experimental approaches (STIRAP, Rydberg ensembles) along with references pointing to the theory of spontaneous emission and simulation of such effects can form a useful resource for someone new to this field. I also hope to work on the future directions suggested here. Supperradiance and neutral atoms are hot topics as of 2024, and finding new ways to enhance coupling interactions or direct the emission of light are exciting pursuits. Discovering new features of many body physics will be critical to advancing neutral atoms as a platform for quantum simulation, and provide some guidance for fault-tolerant quantum computation.

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