Optomechanical Isolator for Two Coupled Cavities

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With the increasing applications of cavity optomechanics for quantum information, addressing the challenge of creating an optical isolator is important for protecting devices from unwanted noise. In this project, I review a proposal [1] for an ideal optical isolator in a two-cavity system and explore some consequences of larger detunings and interaction strengths than shown in the paper. I also rederive the transmission amplitude calculations and comment on potential applications of this device, along with desirable qualities in the mechanical resonator and cavity coupling strength.

I. INTRODUCTION

An ideal optical isolator for a two-cavity system would allow only one way signal transmission across our entire system. This would prevent noise from processes like readout at the output end, and suppress unwanted signals. In this project I review and expand on the theoretical performance and conditions needed for an ideal optical isolator in a two-cavity optomechanical system in which the damping rates of its mechanical (x1) and optical (x2) modes are unequal. As we will see, optical isolators can be achieved at a variety of mechanical damping rates and detunings, though with different effects and limitations.

In reviewing the optical isolator, I split this paper into three sections. The first (Section I) reviews the two-cavity optomechanical system itself and describes the system Hamiltonian starting from the Jaynes-Cummings model. After this, I restate (Section II) the derivation of the transmission amplitudes without some approximations made in the paper [1]. Finally, I explore some new parameters (Section III) by shifting the phase difference between the effective optomechanical coupling and the relative interaction strengths between the optomechanical modes. The paper concludes with some discussion of practical implementations of this ideal optical isolator and challenges that will emerge.

II. OPTOMECHANICAL SYSTEM

The optomechanical system we are analyzing consists of three modes. The first two come from the two cavities, and the third comes from the mechanical resonator coupled to both. Before further analysis, it is useful to introduce the terms used in the paper [1]. Our mechanical resonator (Labeled b) has a frequency ω_m, damping rate γ, and annihilation operator b. For sake of simplicity, our two optical modes are set to the same frequency with damping rates κ_1,κ_2, and annihilation operators c_1 and c_2. We assume that our cavity interaction has linear coupling described by $\hbar J\left(c_1^\dagger c_2 + c_2^\dagger c_1\right)$, with J being the coupling strength. We also have strong coupling fields with frequency ω_c and amplitudes ε_c and ε_d. Likewise, we have two probe fields of frequency ω_p with amplitudes ε_L and ε_R. The analysis of this paper will focus on obtaining equations for ε_L and ε_R, which correspond to the input and output field strengths.

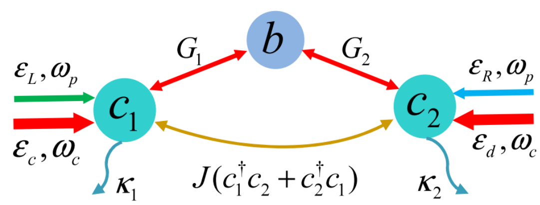

FIG. 1. A schematic of the two-cavity optomechanical system with a mechanical resonator mediating an interaction between the two cavities. Strong coupling fields (red) and probe fields (blue, green) are used to drive the linearly coupled cavities.

We can justify the full Hamiltonian provided in the paper [1] by breaking it down. As a reference, we can observe the Hamiltonian of a single cavity coupled to a mechanical resonator.

$$\begin{aligned} H_{single} = \omega_c c^\dagger c + \omega_m b^\dagger b - g_0 c^\dagger c(b + b^\dagger) \\ + g_1(c^\dagger + c)(b + b^\dagger) \end{aligned}$$

We can easily add a second cavity interacting with the mechanical oscillator with single photon coupling constant g_2. Adding in some other terms and omitting $\hbar$ (set it as 1), we observe that the free Hamiltonian of two cavity modes [2] is H_0, the interaction terms can be contained in H_1, and the coherent pumping action can be written as H_2. The paper ignores photon losses, which I will also assume. This removes the higher order terms of the Jaynes-Cumming Hamiltonian.

$$\begin{aligned} H_0 =& (\omega_0 - \omega_c)\left(c_1^\dagger c_1 + c_2^\dagger c_2\right) \\ H_1 =& \left(g_1 c_1^\dagger c_1 + g_2 c_2^\dagger c_2\right)\left(b^\dagger + b\right) + \hbar J\left(c_1^\dagger c_2 + c_2^\dagger c_1\right) \\ H_2 =& i\left(\varepsilon_c c_1^\dagger - \varepsilon_c^* c_1\right) + i\left(\varepsilon_d c_2^\dagger - \varepsilon_d^* c_2\right) \\ &+ i\varepsilon_L(c_1^\dagger e^{-i\delta t} - c_1 e^{i\delta t}) + i\varepsilon_R\left(c_2^\dagger e^{-i\delta t} - c_2 e^{i\delta t}\right) \end{aligned}$$

This can be simplified by writing $\Delta_c = w_0 - w_c$ and

$\delta_p = w_p - w_c$, giving a total Hamiltonian of:

$$\begin{aligned}H =&\Delta_c(c_1^\dagger c_1 + c_2^\dagger c_2) + \omega_{\mathrm{m}} b^\dagger b + (g_1 c_1^\dagger c_1 + g_2 c_2^\dagger c_2)\\ &(b^\dagger + b) + J(c_1^\dagger c_2 + c_2^\dagger c_1) + i(\varepsilon_c c_1^\dagger - \varepsilon_c^* c_1) +\\ &i(\varepsilon_d c_2^\dagger - \varepsilon_d^* c_2) + i\varepsilon_L(c_1^\dagger e^{-i\delta t} - c_1 e^{i\delta t}) +\\ &i\varepsilon_R(c_2^\dagger e^{-i\delta t} - c_2 e^{i\delta t})\end{aligned}$$

At this point, it is now worth asking - What is required to create a true optical isolator? It does not suffice to merely find modes with full transmission in either direction [2], we must create a transmission matrix which suppresses any backpropagation. To accomplish this, we must satisfy certain conditions in our system. We will derive those conditions in the next section.

III. DERIVATIONS

The following comes almost directly from the derivation in the main paper. Writing the quantum Langevin equations for the cavity and optical operators, we obtain:

$$\begin{aligned}\dot{c}_1 &= -\left[i\Delta_c + \frac{\kappa_1}{2} + ig_1\left(b^\dagger + b\right)\right] c_1 + \varepsilon_c + \varepsilon_L e^{-i\delta t} - iJc_2\\ \dot{c}_2 &= -\left[i\Delta_c + \frac{\kappa_2}{2} + ig_2\left(b^\dagger + b\right)\right] c_2 + \varepsilon_d + \varepsilon_R e^{-i\delta t} - iJc_1\\ \dot{b} &= -i\omega_m b - \frac{\gamma}{2} b - i\left(g_1 c_1^\dagger c_1 + g_2 c_2^\dagger c_2\right)\end{aligned}$$

In order to proceed, let us make some more assumptions. First, assume that $\langle bc_i\rangle = \langle b\rangle\langle c_i\rangle$ which requires that $g_i << \omega_m$ Also, assume that our probe fields $\varepsilon_L, \varepsilon_R$ are sufficiently weak in comparison to the coupling fields $\varepsilon_c, \varepsilon_d$. With these approximations, we can treat the interaction of our probe fields with the system as a small fluctuation added to the mean value (The mean value coming from the system operating in the single-photon weak coupling regime). That is, $b = b_s + \delta b, c_1 = c_{1s} + \delta c_1, c_2 = c_{2s} + \delta c_2$. In the interaction picture, the fluctuation terms become $\delta b \to \delta b e^{-i\omega_m t}, \delta c_1 \to \delta c_1 e^{-i\Delta_1 t}, \delta c_2 \to \delta c_2 e^{-i\Delta_2 t}$. We then write out our mean values as:

$$\begin{aligned}b_s &= \frac{-i\left(g_1\left|c_{1s}\right|^2 + g_2\left|c_{2s}\right|^2\right)}{\frac{\gamma}{2} + i\omega_m},\\ c_{1s} &= \frac{\left(\frac{\kappa_2}{2} + i\Delta_2\right)\varepsilon_c - iJ\varepsilon_d}{J^2 + \left(\frac{\kappa_1}{2} + i\Delta_1\right)\left(\frac{\kappa_2}{2} + i\Delta_2\right)},\\ c_{2s} &= \frac{\left(\frac{\kappa_1}{2} + i\Delta_1\right)\varepsilon_d - iJ\varepsilon_c}{J^2 + \left(\frac{\kappa_1}{2} + i\Delta_1\right)\left(\frac{\kappa_2}{2} + i\Delta_2\right)}\end{aligned}$$

Assuming that each strong coupling field drives one cavity mode, we simplify our new Langevin equations with the rotating wave approximation.

$$\begin{aligned}\dot{c}_1 &= -\frac{\kappa_1}{2} c_1 - iG_1 b - iJc_2 + \varepsilon_L e^{-ixt},\\ \dot{c}_2 &= -\frac{\kappa_2}{2} c_2 - iG_2 e^{i\theta} b - iJc_1 + \varepsilon_R e^{-ixt},\\ \dot{b} &= -\frac{\gamma}{2} b - iG_1 c_1 - iG_2 e^{-i\theta} c_2\end{aligned}$$

Where $x = \delta - \omega_m$. We can solve these differential equations using an exponential wave approximation for b, c_1, c_2 which gives us:

$$\begin{aligned}c_{1+} &= \frac{\left(8G_2^2 + 2\gamma_x\kappa_{2x}\right)\varepsilon_L - \left(8G_1G_2 e^{-i\theta} + 4iJ\gamma_x\right)\varepsilon_R}{\Sigma - 16iG_1G_2J\cos\theta},\\ c_{2+} &= \frac{\left(8G_1^2 + 2\gamma_x\kappa_{1x}\right)\varepsilon_R - \left(8G_1G_2 e^{i\theta} + 4iJ\gamma_x\right)\varepsilon_L}{\Sigma - 16iG_1G_2J\cos\theta},\\ b_+ &= \frac{\left(iG_1\kappa_{2x} + 2G_2Je^{-i\theta}\right)\varepsilon_L + \left(2G_1J + iG_2\kappa_{1x}e^{-i\theta}\right)\varepsilon_R}{4iG_1G_2J\cos\theta - \Sigma/4}\end{aligned}$$

with $\gamma_x = \gamma - 2ix, \kappa_{jx} = \kappa_j - 2ix, s_- = 0$, and $\Sigma = 4G_2^2\kappa_{1x} + 4G_1^2\kappa_{2x} + 4J^2\gamma_x + \kappa_{1x}\kappa_{2x}\gamma_x$. Examining the input-output relation [2] we have:

$$\begin{aligned}\varepsilon_L^{\text{out}} + \varepsilon_L^{\text{in}} e^{-ixt} &= \sqrt{\kappa_1} c_1,\\ \varepsilon_R^{\text{out}} + \varepsilon_R^{\text{in}} e^{-ixt} &= \sqrt{\kappa_2} c_2,\end{aligned}$$

where $\varepsilon_{L,R}^{in} = \varepsilon_{L,R}/\sqrt{\kappa_{1,2}}$. Reusing the exponential wave approximation, we get that:

$$\begin{aligned}\varepsilon_{L+}^{\text{out}} &= \sqrt{\kappa_1} c_{1+} - \varepsilon_L/\sqrt{\kappa_1},\\ \varepsilon_{R+}^{\text{out}} &= \sqrt{\kappa_2} c_{2+} - \varepsilon_R/\sqrt{\kappa_2},\end{aligned}$$

and $\varepsilon_{L-}^{\text{out}} = \varepsilon_{R-}^{\text{out}} = 0$. We can now express our transmission amplitude conditions as:

$$T_{L\to R} = \left|\frac{\varepsilon_R^{\text{out}}}{\varepsilon_L^{in}}\right|_{\varepsilon_R^{in}=0} = 1, T_{R\to L} = \left|\frac{\varepsilon_L^{\text{out}}}{\varepsilon_R^{\text{in}}}\right|_{\varepsilon_L^{in}=0} = 0$$

This is a sufficient condition, as it means that signals can only propagate in a single direction. Of course, we can reverse the left-right labeling on these coefficients and achieve the same effect in the opposite direction.

IV. ANALYSIS

At this point, we will analyze the cavity isolator system by keeping the angular term in our transmission amplitude equations (Rather than simplifying them to $\frac{\pi}{2}$ as in [1]). We can then express our optical output fields as:

$$\begin{aligned}\frac{\varepsilon_{R+}^{\text{out}}}{\varepsilon_L^{\text{in}}} &= \frac{-\sqrt{\kappa_1\kappa_2}\left(8G_1G_2e^{i\theta} + 4iJ\gamma_x\right)}{\Sigma - 16iG_1G_2J\cos\theta}\\ \frac{\varepsilon_{L+}^{out}}{\varepsilon_R^{\text{in}}} &= \frac{-\sqrt{\kappa_1\kappa_2}\left(8G_1G_2e^{-i\theta} + 4iJ\gamma_x\right)}{\Sigma - 16iG_1G_2J\cos\theta}.\end{aligned}$$

Considering that an ideal optical isolator can only be achieved with completely one-way transmission, a quick calculation shows that this one-way condition can only be achieved when:

$$x = -\frac{\gamma \cot\theta}{2},$$
$$J = \frac{2G_1G_2 \sin\theta}{\gamma}$$

We can now begin to independently analyze this system in Figure 2. In the top two graphs in Figure 2, we set $\theta = \frac{\pi}{2}$ with the stated parameters in order to provide a baseline for the transmission amplitudes. The rows below show graphs with $\theta = \frac{5\pi}{8}$ and $\theta = \frac{3\pi}{4}$.

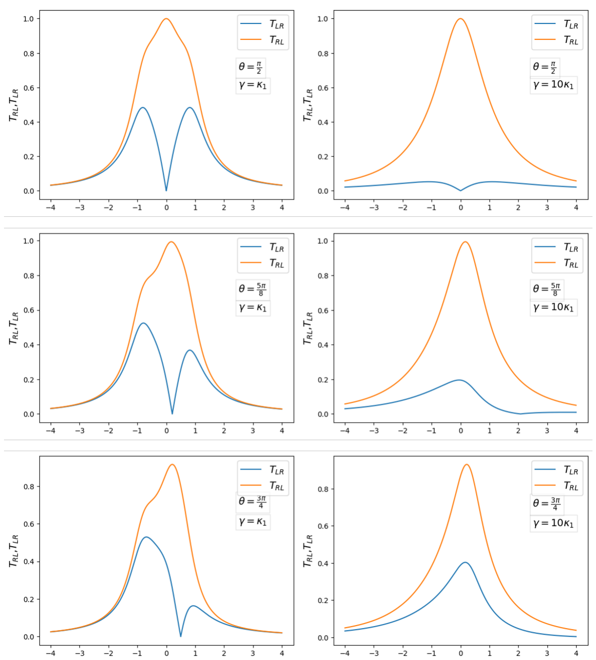

FIG. 2. Transmission amplitudes T_{LR} and T_{RL} are plotted vs normalized detuning $\frac{x}{\kappa_1}$ for different cavity damping rates and phase shifts. Other parameters: $\frac{\kappa_2}{\kappa_1} = 1$, $G_i = \frac{\sqrt{\kappa_i \gamma}}{2}$, and $J = \frac{\sqrt{\kappa_1 \kappa_2}}{2}$.

I observe clear evidence of nonideal optical isolation occuring, specifically when the phase difference becomes far away from $\frac{\pi}{2}$. The further away from $\frac{\pi}{2}$ we get, the further away we get from an ideal optical isolator. Indeed, in cases where the mechanical damping coefficient is much larger than one of the cavity couplings, we cannot achieve an ideal isolator at all. However, when we have our mechanical damping coefficient and cavity damping coefficient close to each other, we can find off-center points at which an ideal optical isolator can be achieved. However, the transmission profile becomes increasingly left skewed as we increase the angle, until it reaches *pi*, at which point the transmission in either direction is identical (This graph is not shown, but it has a Gaussian profile and a peak of 0.67).

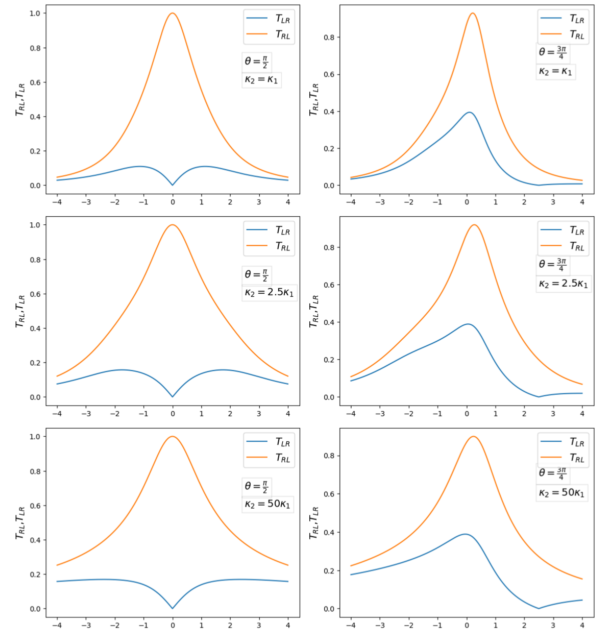

FIG. 3. Transmission amplitudes T_{LR} and T_{RL} are plotted vs normalized detuning $\frac{x}{\kappa_1}$ for different cavity coupling coefficients and phase shifts. Other parameters: $\frac{\gamma}{\kappa_1} = 5$, $G_i = \frac{\sqrt{\kappa_i \gamma}}{2}$, and $J = \frac{\sqrt{\kappa_1 \kappa_2}}{2}$.

As expected, if we keep one of the cavity's damping rates constant along with the damping rate of the mechanical resonator, then regardless of changes in the second cavity's damping rate we see that there is little difference in the transmission amplitudes past a certain point. When at $\frac{\pi}{2}$ we can consistently get ideal optical isolation, while at a detuning of $\frac{3\pi}{4}$ we cannot. For usage in experimental setups however, it would be more ideal to use cavities with similar damping rates to make calibration of the setup easier - Measuring larger changes in the transmission amplitudes versus changes in position is easier than measuring smaller changes.

V. CONCLUSION

When creating an optical isolator for a two-cavity system coupled by a mechanical resonator, it appears that there is no strict restriction on the mechanical damping rate of the resonator. In theory, we could achieve an ideal isolator even when the mechanical resonator is damped much more quickly than the cavity. However, the feasible range at which we can achieve optical isolation shrinks drastically, as seen in Figure 2. This poses challenges from an experimental standpoint, and it will likely be more practical to use resonators with a γ at most two orders of magnitude greater than κ. Additionally, for significant phase differences between the effective optomechanical coupling I observe that an ideal optical isolator cannot be achieved. Though one-way transmission can always be achieved, the efficiency of transmission falls drastically.

For a physical intuition of why this occurs, we should remember that the system exhibits a nonreciprocal response when θ is not an integer multiple of π, due to the interference between the mechanical resonator G_i and the cavity coupling J. So we must indeed satisfy strict conditions for the interaction terms in order to create an ideal optical insulator (Our conditions in Section IV. for x and J). This tells us that when implementing an optical isolator in the real world, we should phase match the phase difference to be as close to $\frac{\pi}{2}$ as possible. This can be done by adjusting the coupling field amplitudes ε_c and ε_d. In future works, demonstration of such an isolator might be achieved in existing setups that have auxiliary cavities. Such work would allow us to drastically reduce measurement and readout noise in our system, and potentially provide longer T_1 lifetimes for ultracold atoms.

VI. REFERENCES

1. Xia, Yan, Tian, Gao, ‘Ideal optical isolator with a two-cavity optomechanical system’, Optics Communications 451 197-201, 2019

2. Jalas et. al, ‘What is- and what is not - an optical isolator’, Nature Photonics Vol 7, August 2013

3. Wang et. al., ’Steady-state mechanical squeezing in a double-cavity optomechanical system’, Scientific Reports, August 2016