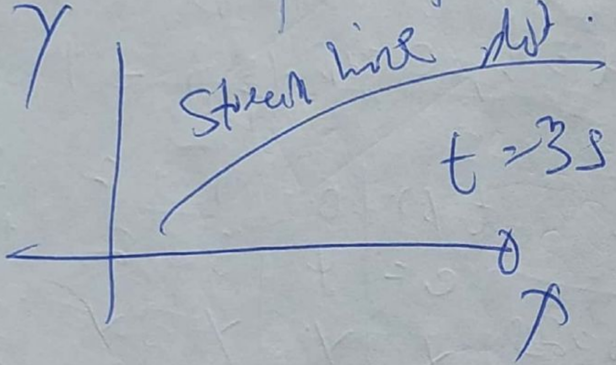
* Streakline of particles that passed through point (x_0, y_0) at $t = 3$ s.

t_0(s)	t(s)	X(m)	Y(m)
0	3	9.25	4.25
1	3	6.67	4.00
2	3	3.58	3.25
3	3	1.0	2.0



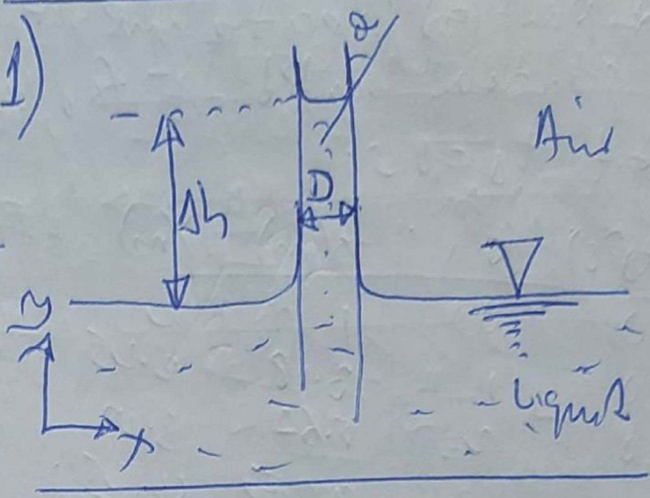
5) 1.38 N

(6) 9.15×10^{-3} N

(7) $1461 \, ms^{-1}$.

Tutorial sheet 2.

1)



$$\Sigma F_y = \sigma \pi D \cos\theta - \rho g \Delta V = 0$$

Neglecting the volume of the liquid above Δh, we obtain

$$\Delta V = \frac{\pi}{4} D^2 \Delta h$$

$$\Rightarrow \sigma \pi D \cos\theta - \rho g \frac{\pi}{4} D^2 \Delta h = 0$$

$$\Rightarrow \Delta h = \frac{4\sigma \cos\theta}{\rho g D}.$$

2) Equating pressures on either side of level 2, we have

a) $\Delta P = \rho_l g (h + H)$

– To eliminate H, we recognise that the volume of manometer liquid remains constant i.e the volume displaced from the reservoir must be equal to the volume rise in the tube

$$\Rightarrow \frac{\pi}{4} D^2 H = \frac{\pi}{4} d^2 L$$

$$H = L\left(\frac{d}{D}\right)^2$$

$$\Rightarrow \Delta P = \rho_l g\left\{L\sin\theta + L\left(\frac{d}{D}\right)^2\right\}$$

$$\Delta P = \rho_l g L\left(\sin\theta + \left(\frac{d}{D}\right)^2\right) \quad \text{--- (1)}$$

$$\Rightarrow L = \frac{\Delta P}{\rho_l g\left\{\sin\theta + \left(\frac{d}{D}\right)^2\right\}}$$

b) To obtain an expression for sensitivity, express ΔP in terms of an equivalent water column height, h_e

$$\Delta P = \rho_w g h_e \quad \text{--- (2)}$$

Combining (1) and (2), we have

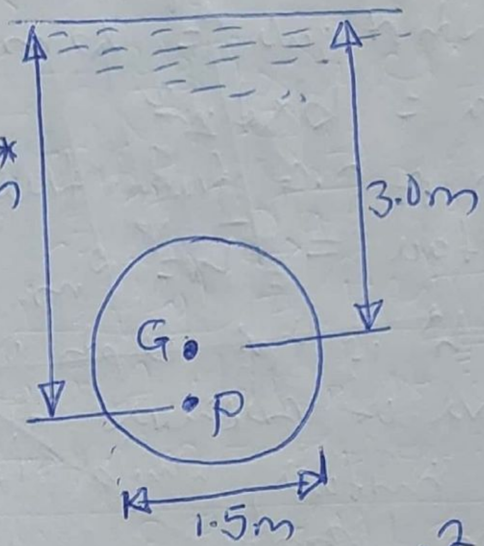
$$\rho_l g L\left\{\sin\theta + \left(\frac{d}{D}\right)^2\right\} = \rho_w g h_e$$

$$\Rightarrow S = \frac{L}{h_e} = \frac{1}{SG\left\{\sin\theta + \left(\frac{d}{D}\right)^2\right\}}$$

where $SG = \rho_l / \rho_w$.

c) The expression S for sensitivity shows that to increase sensitivity SG, $\sin\theta$ and d/D should be made as small as possible.

3



$d = 1.5\text{m}$, $A = \frac{\pi}{4}(1.5)^2 = 1.767\text{m}^2$

$\bar{h} = 3.0\text{m}$

Total pressure $F = \rho g A\bar{h}$

$F = 1000 \times 9.81 \times 1.767 \times 3.0\text{N}$

$= 52002.81\text{N}$

Position of center of pressure (h^*) is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

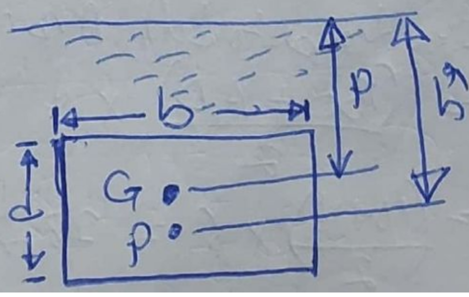
where $I_G = \frac{\pi d^4}{64} = \frac{\pi(1.5)^4}{64}$

$= 0.2485\text{m}^2$

$$h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0$$

$= 3.0468\text{m}$.

4



depth of vertical gate = d m

let width of gate = b m

Area = $b \times d \, m^2$

- Depth of C.G from free surface = $\bar{h} = p \, m$

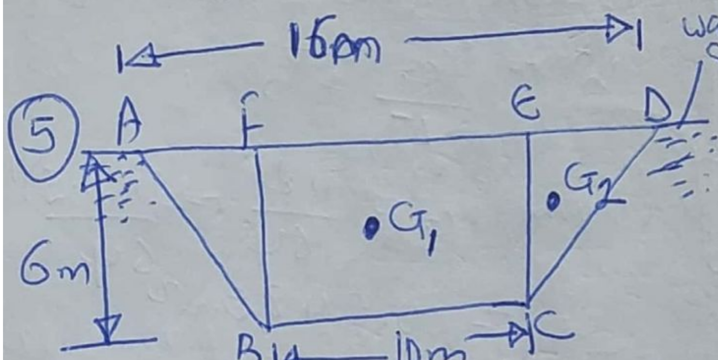
- Let h^* is the depth of centre of pressure from free surface, which is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}, \quad \text{where } I_G = \frac{bd^3}{12}$$

$$h^* = \left\{ \frac{bd^3/12}{b \times d \times P} \right\} + P$$

$$h^* = \frac{d^2}{12P} + P$$

5



width at top = 16m

width at bottom = 10m

Depth, d = 6m

Area of trapezoidal ABCD

$$A = \frac{(BC + AD)}{2} \times d$$

$$= \frac{(10+16)}{2} \times 6 = 78 \, m^2$$

Depth of C.G. of trapezoidal area ABCD from free surface of water,

$$\bar{h} = \frac{10 \times 6 \times 3 + \frac{(16-10)}{2} \times 6 \times \frac{1}{3} \times 6}{78}$$

$$= \frac{180 + 36}{78} = 2.769 \, m$$

from water surface.

i) Total pressure F is given by

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 78 \times 2.769$$

$$= 2118783 \, N.$$

ii) Centre of pressure h^* is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Where I_G = M.O.I of trapezoidal ABCD about its C.G.

Let

I_{G_1} = M.O.I. of rectangle FBCE about its C.G.

I_{G_2} = M.O.I. of two Δs ABF and ECD about its C.G.

$$I_{G_1} = \frac{bd^3}{12} = \frac{10 \times 6^3}{12} = 180 \, m^4$$

I_{G_1} is the M.O.I. of the rectangle about the axis passing through G_1

$\therefore$ M.O.I of the rectangle about the axis passing through the C.G of the trapezoidal $= I_{G_1} +$ Area of rectangle $\times x_1^2$

Where x_1 is distance between the C.G of rectangle and C.G. of trapezoidal

$= (3.0 - 2.769) = 0.231m$

$\therefore$ M.O.I of FBCE passing through C.G. of trapezoidal

$= 180 + 10 \times 6 \times (0.231)^2$

$= 183.20 m^4$

Now I_{G_2} = M.O.I of $\triangle$ ABD in ~~above~~ figure below (*) about $G_2 = \frac{bd^3}{36}$

$= \frac{(16-10) \times 6^3}{36} = 36 m^4$

The distance between the C.G. of triangle and C.G. of trapezoidal $= (2.769 - 2.0) = 0.769$

$\therefore$ M.O.I of the two Δs about an axis passing through C.G. of trapezoidal

$= I_{G_2} +$ Area of triangle $\times (0.769)^2$

$= 36.0 + \frac{6 \times 6}{2} \times (0.769)^2$

$= 36.0 + 10.64 = 46.64$

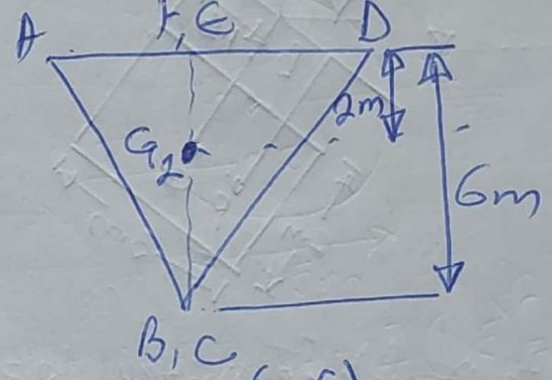


fig (*)

$\therefore I_G$ = M.O.I of trapezoidal about its C.G.

= M.O.I of rectangle about the C.G. of trapezoidal + M.O.I of triangles about the C.G. of the trapezoidal

$= 183.20 + 46.64$

$= 229.84 m^4$

$\therefore h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

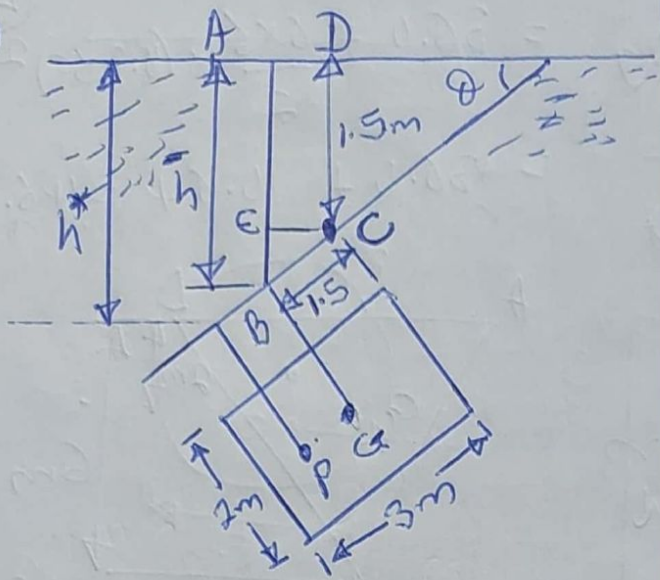
Where $A = 78$, $\bar{h} = 2.769$

$h^* = \frac{229.84}{78 \times 2.769} + 2.769$

$= 1.064 + 2.769$

$= 3.833 m$

⑥



Width of plane surface, $b = 2m$
Depth, $d = 3m$
Angle $\theta = 30^0$
Distance of upper edge from free water surface = 1.5m

(i) Total pressure $F = \rho g A \bar{h}$

where $\rho = 1000\ kg/m^3$

$A = b \times d = 3 \times 2 = 6m^2$

$\therefore \bar{h}$ = Depth of C.G. from free water surface = $1.5 + 1.5 \sin 30^0$

(ie $\bar{h} = AE + EB = 1.5 + 1.5 C \sin 30^0$)

$\bar{h} = 1.5 + 1.5 \times \frac{1}{2} = 2.25\ m$

$F = 1000 \times 9.81 \times 6 \times 2.25$
$= 132435 N.$

ii) Centre of pressure h^* is

$h^* = \frac{I_G \sin^2 \theta}{A \bar{h}} + \bar{h}$

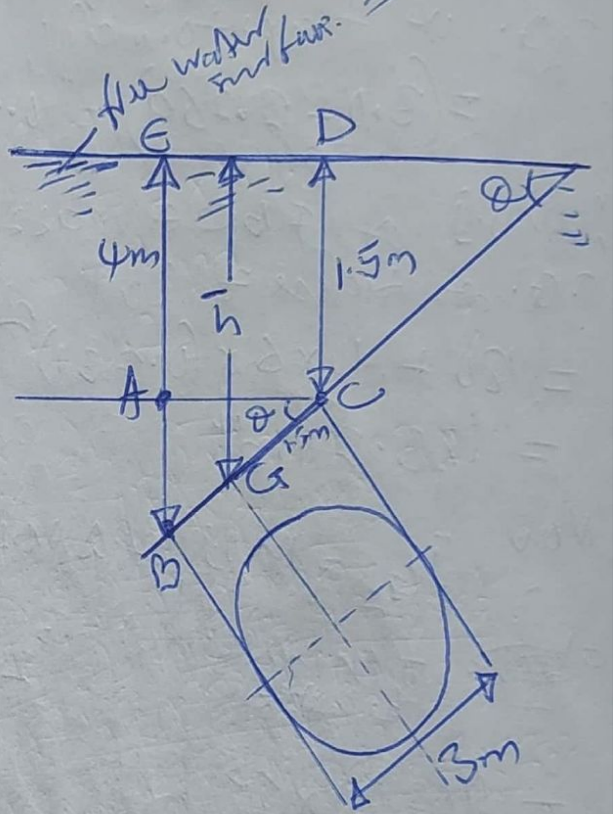
where $I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12}$
$= 4.5 m^4$

$\Rightarrow h^* = \frac{4.5 \times \sin^2 30^0}{6 \times 2.25} + 2.25$

$= 0.0833 + 2.25$

$= 2.3333m$

7)



Diameter of plate $d = 3.0m$

$\therefore$ Area $= A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (3.0)^2$

$A = 7.0685 m^2$

Distance $DC = 1.5m$,
$BG = 4m$

Distance of C.G. from free surface

$= \bar{h} = CD + GC \sin\theta$

$\Rightarrow \bar{h} = 1.5 + 1.5 \sin\theta$

But $\sin\theta = \frac{AB}{BC} = \frac{BE - AE}{BC}$

$= \frac{4.0 - DC}{3.0} = \frac{4.0 - 1.5}{3.0}$

$= \frac{2.5}{3.0} = 0.8333$

$\therefore \bar{h} = 1.5 + 1.5 \times 0.8333$

$= 1.5 + 1.249$

$= 2.749\,m$

i) Total pressure $F = \rho g A \bar{h}$

$F = 1000 \times 9.81 \times 7.0685 \times 2.749$

$= 190621\,N$

ii) Centre of pressure h^*,

$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

Where $I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64}(3)^4$

$= 3.976\,m^4$

$\Rightarrow h^* = \frac{3.976 \times (0.8333) \times 0.8333}{7.0685 \times 2.749} + 2.749$

$= 0.1420 + 2.749$

$= 2.891\,m$

8) Radius of gate = 2m

width of gate = 1m

Horizontal force F_x = force on the projected area of the curved surface on vertical plane

$=$ force on $BO = \rho g A \bar{h}$

Where A = Area of $BO = 2 \times 1 = 2\,m^2$,

$\bar{h} = \frac{1}{2} \times 2 = 1\,m$;

$F_x = 1000 \times 9.81 \times 2 \times 1$

$= 19620\,N$

This will act at a depth of $\frac{2}{3} \times 2 = \frac{4}{3}\,m$ from surface of liquid;

Vertical force, F_y

F_y = Weight of water (imagined) supported by AB

$F_y = \rho g \times$ Area of $AOB \times 1.0$

$= 1000 \times 9.81 \times \frac{\pi}{4}(2)^2 \times 1.0$

$= 30819\,N$.

This will act a distance of $\frac{4R}{3\pi} = \frac{4 \times 2.0}{3\pi} =$

0.848 m from OB

$\therefore$ Resultant force F is given by

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{19620^2 + 30819^2}$$

$$= 36534.4 N.$$

- The angle made by the resultant with horizontal is given by

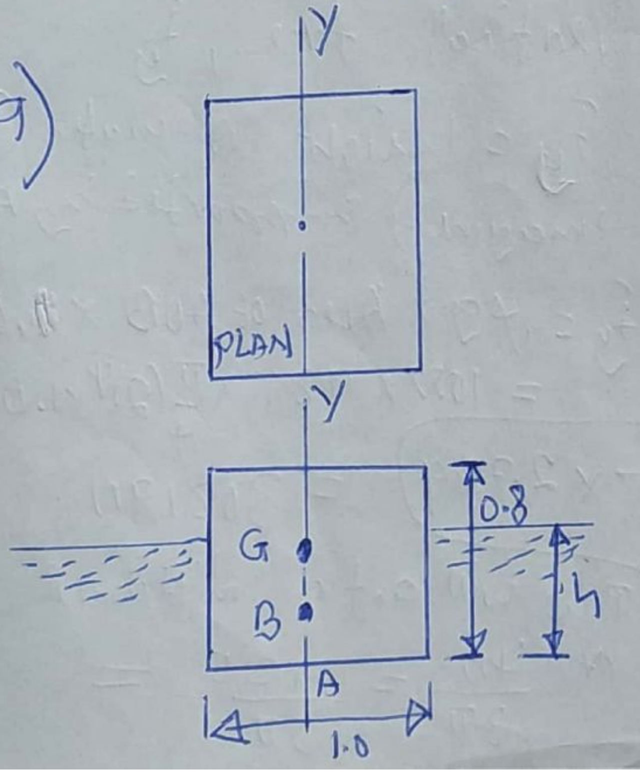
$$\tan\theta = \frac{F_y}{F_x} = \frac{30819}{19620}$$

$$= 1.5708$$

$$\theta = \tan^{-1}(1.5708)$$

$$= 57^{\circ} 31'$$

9)



Dimensions of block = 2×1×0.8

Let depth of immersion = h m

Sp. gr. of wood = 0.7

- Weight of wooden piece = Weight density of wood × volume

$= 0.7 \times 1000 \times 9.81 \times 2 \times 1 \times 0.8 N$

- Weight of water displaced = weight density of water × volume of the wood submerged in water $= 1000 \times 9.81 \times 2 \times 1 \times h N$

For equilibrium,

weight of wood piece = weight of water displaced

$\therefore 700 \times 9.81 \times 2 \times 1 \times 0.8 = 1000 \times 9.81 \times 2 \times 1 \times h$

$$\Rightarrow h = \frac{700 \times 9.81 \times 2 \times 1 \times 0.8}{1000 \times 9.81 \times 2 \times 1}$$

$= 0.7 \times 0.8 = 0.56 m$

$\therefore$ Distance of centre of Buoyancy from bottom

i.e.

$$AB = \frac{h}{2} = \frac{0.56}{2} = 0.28 m$$

$\therefore AG = 0.8/2.0 = 0.4 m$

$BG = AG - AB = 0.4 - 0.28$

$= 0.12 m$

- The meta-centric height

is given by

$$GM = \frac{I}{V} - BG$$

where $I = \frac{1}{12} \times 2 \times 1^3 = \frac{1}{6} m^4$

V = volume of wood in water

$V = 2 \times 1 \times h = 2 \times 1 \times 0.56$

$V = 1.12 m^3$

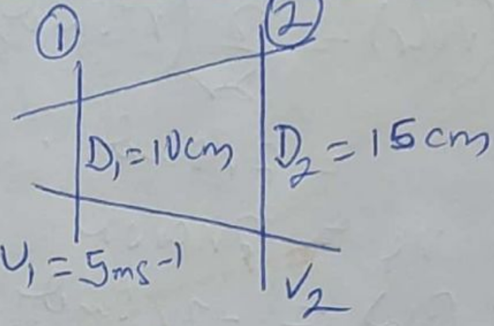
$$GM = \frac{1}{6} \times \frac{1}{1.12} - 0.12$$

$= 0.1488 - 0.12$

$= 0.0288 m$

Tutorials sheet 3

1)



$A_1 = \frac{\pi}{4} D_1^2 = 0.007854 m^2$

$V_1 = 5 ms^{-1}$

$A_2 = \frac{\pi}{4} D_2^2 = 0.01767 m^2$

i) Discharge through pipe is given by

$Q = A_1 \times V_1 = 0.03927 m^3 s^{-1}$

ii) we know that

$A_1 V_1 = A_2 V_2$

$\Rightarrow V_2 = \frac{A_1 V_1}{A_2}$

$V_2 = 2.22 ms^{-1}$

2) Given

$u_r = r \sin\theta$, $u_\theta = 2r\cos\theta$

from physically possible flow, the continuity equation $\frac{\partial}{\partial r}(r u_r) + \frac{\partial}{\partial \theta}(u_\theta) = 0$, should be satisfied.

Now $u_r = r \sin\theta$

multiplying the above equation by r, we get

$r u_r = r^2 \sin\theta$

Differentiating the

preceding equation w.r.t r, we get :

$$\frac{\partial}{\partial r}(r u_r) = \frac{\partial}{\partial r}(r^2 \sin\theta)$$

$$= 2r\sin\theta .$$

Now $u_\theta = 2r\cos\theta$

$$\Rightarrow \frac{\partial}{\partial \theta}(u_\theta) = -2r\sin\theta$$

$$\therefore \frac{\partial}{\partial r}(r u_r) + \frac{\partial}{\partial \theta} u_\theta =$$

$$2r\sin\theta - 2r\sin\theta = 0.$$

Hence the continuity equation is satisfied. Hence the given velocity components represent a physically possible flow.

3) The velocity components u, v and w are $u = 4x^3$, $v = -10x^2 y$, $w = 2t$

- for the point $(2,1,3)$, we have $x = 2$, $y = 1$ and $z = 3$ at time $t = 1$.

Hence velocity components at $(2,1,3)$ are

$u = 4 \times (2)^3 = 32$ units

$v = -10(2)^2(1) = -40$ units

$w = 2 \times 1 = 2$ units

$\therefore$ velocity vector V at $(2,1,3) = 32i - 40j + 2k$

or Resultant velocity

$$|\vec{V}| = \sqrt{u^2 + v^2 + w^2}$$

$$= 51.26 \text{ units}$$

- Acceleration is given by :

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Substituting the acceleration components at $(2,1,3)$ at time $t = 1$ are

$a_x = 1536$ units

$a_y = 320$ units

$a_z = 2.0$ units

$\therefore$ Acceleration is

$$\vec{A} = a_x i + a_y j + a_z k$$

$$\vec{A} = 1536 i + 320 j + 2k$$

$$|\vec{A}| = 1568.9 \text{ units}$$

4) The continuity equation for incompressible fluid is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

i) $u = x^2 + y^2 + z^2 \quad \therefore \frac{\partial u}{\partial x} = 2x$

$v = xy^2 - yz^2 + xy \quad \therefore \boxed{\frac{\partial v}{\partial y} = 2xy - z^2 + x}$

- Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation.

$$2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \frac{\partial w}{\partial z} = -3x - 2xy + z^2$$

$$\Rightarrow \partial w = (-3x - 2xy + z^2)\partial z$$

- Integrating both sides gives

$$\int dw = \int (-3x - 2xy + z^2)\,dz$$

$$w = \left(-3xz - 2xyz + \frac{z^3}{3}\right) + \boxed{\text{constant}}$$

Where constant of integration cannot be a function of z. But it can be a function of x and y that is $f(x, y)$

$$w = \left(-3xz - 2xyz + \frac{z^3}{3}\right) + f(x, y)$$

ii) $v = 2y^2 \quad \therefore \frac{\partial v}{\partial y} = 4y$

7

$w = 2xyz \quad \therefore \frac{\partial w}{\partial z} = 2xy$

- Substituting the values of $\frac{\partial v}{\partial y}$ and $\frac{\partial w}{\partial z}$ in continuity equation, we get

$$\frac{\partial u}{\partial x} + 4y + 2xy = 0$$

$$\Rightarrow u = -4xy - 2y\frac{x^2}{2} + f(y, z)$$

$$u = -4xy - x^2 y + f(y, z).$$

5) for the given fluid flow field:

$u = x^2 y \quad \therefore \frac{\partial u}{\partial x} = 2xy$

$v = y^2 z \quad \therefore \frac{\partial v}{\partial y} = 2yz$

$w = -2xyz - yz^2 \quad \therefore \frac{\partial w}{\partial z} = -2xy - 2yz$

for a case of possible steady incompressible fluid flow, the continuity equation should be satisfied

i.e. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$

$$\Rightarrow 2xy + 2yz - 2xy - 2yz = 0$$

Hence the velocity field

$\vec{V} = x^2yi + y^2zj - (2xyz + yz^2)k$ is a possible case of fluid flow.

- velocity at $(2,1,3)$ given

$\vec{V} = 4i + 3j - 21k$.

$|\vec{V}| = 21.587$ units.

Acceleration at $(2,1,3)$

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$

$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$

Acceleration $= a_x i + a_y j + a_z k$

$= 28i - 3j + 123k$

$|\vec{A}| = 126.18$ units

6) Given $V = 8x^3 i - 10x^2 y j$

$u = 8x^3$, $\frac{\partial u}{\partial x} = 24x^2$,

$\frac{\partial u}{\partial y} = 0$

$v = -10x^2y$, $\frac{\partial v}{\partial x} = -20xy$

$\frac{\partial v}{\partial y} = -10x^2$

i) shear strain rate is given by

$\frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$

$= \frac{1}{2}(-20xy + 0)$

$= -10xy$.

ii) Rotation in $x-y$ plane is given by:

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$= \frac{1}{2}(-20xy - 0)$

$= -10xy$.

As rotation $\omega_z \neq 0$, Hence flow is rotational.

7) $u = \frac{y^3}{3} + 2x - x^2y$

$\frac{\partial u}{\partial x} = 2 - 2xy$

$\frac{\partial u}{\partial y} = \frac{3y^2}{3} - x^2 = y^2 - x^2$

Also $v = xy^2 - 2y - \frac{x^3}{3}$

$$\frac{\partial v}{\partial y} = 2xy - 2$$

$$\frac{\partial v}{\partial x} = y^2 - \frac{3x^2}{3} = y^2 - x^2$$

i) for a 2-dimensional flow, continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2xy + 2xy - 2 = 0.$$

$\therefore$ It is a possible case of fluid flow.

ii) Rotation, w_z is given by:

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left\{(y^2 - x^2) - (y^2 - x^2)\right\}$$

$$= 0.$$

$\therefore$ Rotation is zero, which means it is a case of irrotational flow.

Tutorial sheet 4

1) Diameter of pipe $= 5\text{ cm} = 0.05$,

$P = 29.43\text{ N/cm}^2 = 29.43 \times 10^4\text{ N/m}^2$

$V = 2.0\text{ m/s}$

Datum head, $z = 5\text{m}$

Total head = pressure head + kinetic head + datum head

Pressure head $= \frac{P}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81}$

$= 30\text{m}$

Kinetic head $= \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.81}$

$= 0.204\text{ m}$

Total head $= \frac{P}{\rho g} + \frac{V^2}{2g} + z$

$= 30 + 0.204 + 5$

$= 35.204\text{ m}$

2)