Dynamic visualization of high-dimensional functions via low-dimension projections and sectioning across 2D and 3D display devices

A thesis submitted for the degree of

Doctor of Philosophy

by

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Contents

Acknowledgements

I would like to thank . . .

Declaration

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma in any university or equivalent institution, and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

Nicholas S Spyrison

Preface

The material in Chapter 1 has been submitted to *Something interesting jornal* for possible publication.

The contribution in Chapter 3 of this thesis was presented in the super awesome confrence held in Dublin, Ireland, in July 2015.

Abstract

This thesis is about ...

Chapter 1

Introduction

This is where you introduce the main ideas of your thesis, and an overview of the context and background.

In a PhD, Chapter 2 would normally contain a literature review. Typically, Chapters 3–5 would contain your own contributions. Think of each of these as potential papers to be submitted to journals. Finally, Chapter 6 provides some concluding remarks, discussion, ideas for future research, and so on. Appendixes can contain additional material that don't fit into any chapters, but that you want to put on record. For example, additional tables, output, etc.

Chapter 2

Literature review

2.1 Touring

2.1.1 Overview

In univariate datasets histograms, or smoothed density curves are employed to visualize data. In bivariate data scatterplots and contour plots (2-d density) can be employed. In three dimensions the two most common techniques are: 2-d scatter plot with the 3rd variable as an aesthetic (such as, color, size, height, *etc*.) or rendering the data in a 3-d volume using some perceptive cues giving information describing the seeming depth of the image [1]. When there are 4 variables: 3 variables as spatial-dimensions and a 4th as aesthetic, or a scatterplot matrix consisting of 4 histograms, and 6 unique combinations of bivariate scatterplots.

Let p be the number of numeric variables; how do we visualize data for even modest values of p (say 6 or 12)? It's far too common that visualizing in data-space is dropped altogether in favor of modeling parameter-space, model-space, or worse: long tables of statistics without visuals (Wickham, Cook, and Hofmann, 2015). Yet, we all know of the risks inherant in relying too heavily on parameters alone (Anscombe, 1973; Matejka and Fitzmaurice, 2017). So why do we move away from visualizing in data-space? Scalability,

[1]Graphs of data depicting 3 dimension are typically printed on paper, or rendered on a 2-d monitor, they are intrinsically 2-d images. They are sometimes referred to as 2.5-d, or more frequently erroneously referred to as 3-d, more on this later.

in a word, we are not familiar with methods that allow us to concisely depict and digest $p \geq 5$ or so dimensions. This is where dimensonality reduction comes in. Specifically, we will be focusing on a specific group called touring. In the interest of time I will not belabor the diversity of dimentionality reduction, (see [Grinstein, Trutschl, and Cvek (2002); Carreira-Perpinán (1997); heer_tour_2010] for a quick summary). Suffice it to say that touring has a couple of salient features: linear transfromations such that we can interpolate back to the oiginal variable space and does not discard dimensions, something that is common to other linear techniques. By emploring the bredth of tours we are able to preserve the visualization of data-space, and with it, the intrinsic understanding of structure and distribution of data that is more susinct or beyond the reach of statistic valules alone.

Touring is a linear dimensonality reduction technique that orthagonally projects p-space down to $d(\leq p)$ dimensions. Many such projections are interpolated, each making local rotations in p-space. These frames are then viewed in order to the effect of watching an animation of the lower dimensional embedding changing as p-space is manipulated. Shadow puppets offer a useful analogy to aid in conceptualizing touring. Imagine a fixed light source facing a wall. When a hand or puppet is introduced the 3-dimensional object projects a 2-dimensional shadow onto the wall. This is a physical representation of a simple projection, that from $p = 3$ down to $d = 2$. If the object rotates then the shadow correspondingly changes. Observers watching only the shadow are functionally watching a 2-dimensional tour as the 3-dimensional object is manipulated.

Terminology

n, p (sometimes called d by Wegman, or n), d (sometimes called k by wegman, or d in tourr)

2.1.2 History

Touring was first introduced by Asimov in 1985 with his purposed Grand Tour(Asimov, 1985) at Stanford University. In which, Asimov suggested three types of Grand Tours:

torus, at-random, and random-walk. The specifics of which will be discussed below in the Typology section.

TALK ABOUT maths Here::

Note that the the above methods have no input from the user aside from the starting basis. The bulk of touring development since has largely been around dynamic display, user interaction, geometric representation, and application.

This works well when the number of dimensions being toured is small (in the neighborhood of 5-10), yet the number of view, or 2-frames and we can produce from p-space suffers from the so called blessing/curse of dimensionality. In which the plethora of degrees of freedom either offer many (non-unique) solutions to a problem or something that becomes ever increasing unlikely,

2.1.3 Tour path

A fundamental aspect of touring is the path of rotation. Of which there are four primary distinctions(Buja et al., 2005): random choice, precomputed choice, data driven, and manual control.

- *grand tour*, a constrained random choice p-space. Paths are constrained for changes in direction small enough to maintain continuity and allow for user comphrehension
  - torus-surface (Asimov, 1985)
  - Geodesic
  - at-random
  - random-walk
  - *local tour*, a sort of grand tour on leash, such that it goes to a nearby random projection before returning to the original position and iterating
- *guided tour*, data driven tour optimiazing some objective function via (stochastic) gradient descent (Hurley and Buja, 1990).
  - holes (Cook, Buja, and Cabrera, 1993) - iterates projections that add more white space to the center of the projection.

  - cmass (Cook, Buja, and Cabrera, 1993) - find the projection with the most density or mass in the center.
  - lda (Lee et al., 2005) - linear discrimin ant analysis, seeks a projection where 2 or more classes are most separated.
  - pda - pricipal component analysis finding where the data is most spread (1d only).
  - other user-defined objective function (Wickham et al., 2011).

- *planned tour*, Precomputed choice, In which the path has already been generated or defined.

  - *little tour* (McDonald, 1982), where every permutation of variables is stepped through in order, analogous to a brute-force or exhaustive search.
  - a saved path of any other tour

- *manual tour* - Manual control, a constrained rotation on selected manipulation variable and magnitude(Cook and Buja, 1997). Typically used to explore the local area after identifying an interesting feature from another tour.

- *dependance tour*, combination of n independent 1d tours. A vector describes the axis each variable will be displayed on. **ie** $c(1,1,2,2)$ is a 4 to 2d tour with the first 2 variables on on the first axis, and the remaining on the second.

  - *correlation tour* (Buja, Hurley, and McDonald, 1987), a special case of the dependance tour, analogous to canonical correlation analysis

2.1.4 Geometrics and display dimension

Up to this point we have been talking about 2d scatterplots, which offer the first and a simple case for viewing lower-dimensional embeddings of p-space. However, other geometrics (or geoms) offer perfectly valid orthonormal projections as well.

- 1d geoms

  - 1-d densities: such as histogram, average shifted histograms(**scott85**), and kernal density(**scott95**).
  - image: (**Wegman**)
  - time series: where multivariate values are independantly lagged to view peak and trough allignment. Currently no package implementation, but use case is discussed in (Cook and Buja, 1997).

- 2d geoms
  - 2-d density (**NS**)
  - scatterplot
  -

- 2.5d, 3d geoms {ADD FOOTNOTE ABOUT 2.5d vs 3d}
  - Anaglyphs, sometimes called stero, where (typically) red images are positioned for the left channel and cyan for the right, when viewed with corresponding filter glasses give the depth perception of the image.
  - Depth, which use some subset of depth cues, most commonly size and/or color of data points.

- *d*-dim geoms
  - Andrews crurves (Andrews, 1972), smoothed variant of parallel coordinate plots, discussed below.
  - Chernoff faces (Chernoff, 1973), variables linked to size of facial features for rapid cursory like-ness comparison of observations.
  - Parallel coordinate plots (Ocagne, 1885), where any number of variables are ploted in parallel with observations linked to their cooresponding variable value by polylines.
  - Scatterplot matrix (Becker and Cleveland, 1987), showing a triangle matrix of bivariate scatterplots with 1-d density on the diagonal.
  - Radial glyphs, radial variants of parallel coordinates including radar, spider, and star glyphs (Siegel et al., 1972).

2.1.5 Aplication

Below is a non-exhaustive list of software implementing touring in some degree, ordered by descending year:

- Spinifex (**spinifex**) – for Linux, Unix, and Windows.
- Tourr (Wickham et al., 2011) – for Linux, Unix, and Windows. R package.
- CyrstalVision (Wegman, 2003) – for Windows.
- GGobi (Swayne et al., 2003) – for Linux and Windows.
- DAVIS (Huh and Song, 2002) – Java based, with GUI.
- VRGobi (Nelson, Cook, and Cruz-Neira, 1998) – for use with the C2 in steroscopic 3d displays.
- ExplorN (Carr, Wegman, and Luo, 1996) – for SGI Unix.
- XGobi (Swayne, Cook, and Buja, 1991) – for Linux, Unix, and Windows (via emulation).
- XLispStat (Tierney, 1990) – for Unix, and Windows.
- Prim-9 (Asimov, 1985; Fisherkeller, Friedman, and Tukey, 1974) – on an internal operating system.

Support and maintenance of such implementations give them a particularly short life span, while conceptual abscraction and technically heavier implementations have hampered user growth. There have been notable efforts to deminish the barriers to entry and make touring more approachable as a data exploration tool [Huh and Song (2002); Swayne et al. (2003); Wegman (2003); Wickham et al. (2011); huang_tourrgui:_2012].

2.2 Virtual reality

Chapter 3

spinifex: extending *tourr* with manual tours and graphic display

3.1 Abstract

Touring teqniques offer a great opertunity for data-space vizualization of ($p > 3$) multivariate data sets. This paper discusses the *R* package *spinifex*, which adds support for the manual tour, which is particularly usefuly for exploring the local structure after identifying a feature of interest, perhaps via guided tour. Additionally, *spinifex* extends graphic outputs to *plotly* and *gganimation*. This work extends the functionality of and is compatible with *tourr*.

Keywords: grand tour, projection pursuit, manual tour, tourr, touring, high dimenson visualization, hihg dim vis, dimensionality reduction, visualization, statistical graphics, data-space.

3.2 Introduction

Both classical and contemorary visualizations of data are presented in two dimensions, that of a computer monitor or in print. How is it that we come to view and share data that exsists in $p > 3$ dimensions? In an appeal to brievity we shall ignore model and parameter summarization due to there shortcomings (Anscombe, 1973; Matejka and Fitzmaurice, 2017). Within the realm of data-space visualization we are left with projecting higher volumes and embedding them within lower dimensional spaces that we can vizualize.

This is not a new phenomena, such linear projections have been in use for quite some time. (Pearson, 1901; Fisher, 1936) and the myriad of single value deconcomposistion (SVD) techniques from numerous disciplines use such embeddings. Previous application look at data in one (or few) static orientations, after some objective optimization. For instance in PCA, we reorient p-dimensions such that we have a reference to the ordered components that describe a descending ammount of variation held within the data. Yet we still have p components remaining to visualize. Where does the dimension reduction come in? From plotting only the first two or three and potenially another dimension tied to data point asthetic. This is maximizes the ammount of variation that can be display in an emedding, but regually discards a large proportion of the variation held within the data.

More recently non-linear dimensionality techniques have become popular, such as t-distributed stchocastic neighbor embedding (t-SNE) (Maaten and Hinton, 2008), building off of sammon mappings (Sammon, 1969). Such non-linear methods make for astounding distinction when in low d embeddings, but contain inhearant shortcomings. Namely: that the non-linear tranformations break inter-operability back to the original data-space, and that they can suffer from overfitting. If there is no inherent clustering with the data, it's posible that noise within the variables may become the proment feature and be displayed erroneously as group clustering.

(Asimov, 1985; Buja and Asimov, 1986) suggested grand tours in which random walks in p-space can be interpolated and embedded in d dimensions which are then viewed in sequence. The broader scope of touring has some beneficial features, namely: touring keeps the original dimensionality in tact unlike tradition static linear-projections, and maintains inter-operabilty in the orginal dimensions, a primary drawback of non-linear dimensionality reduction.

TODO: clean up above, talk about what

3.3 Terminology and demystifying projection:

TODO: clean up the terminology section

basis, data, n, p, d,

Suppose that we have tri-variate data, $\mathbf{X}_{[8,\ 3]}$, the corners points of a rectanguloid. We can describe the relative orientation by defining

For every p-dimensional space can be described by the direction and magnitude of axes in a square matrix that we call a basis. Imgine 3 axes of an XYZ Caresian volume (*ie.* a basis $\in \mathbb{R}^p$). In matimatical form we would write this as a diagonal identity matrix of demension 3.

This basis has some nice properties that are mathimatically nice to preserve, namely, that each axis as is at a right angle to the other (*orthagonal*), and are unit *normal* (length or norm equal to one). If matrix meets both of these criteria we call it *orthonormal*.

3.4 Manual tour

Let's explore the process behind the manual tour

Given:

$\mathbf{X}_{[n,\ p]}$ A dataset contaning n observations of p numeric variables.

$\mathbf{B}_{[p,\ d]}$ An orthonormal [1] basis describing the current orientation projecting p down to d dimension.

$$\mathbf{X}_{[n,\ p]} = \begin{bmatrix} X_{1,\ 1} & \dots & X_{1,\ p} \\ X_{2,\ 1} & \dots & X_{2,\ p} \\ \vdots & \ddots & \vdots \\ X_{n,\ 1} & \dots & X_{n,\ p} \end{bmatrix}$$

[1] Where each variable is both: orthagonal, at right angles (dot product is 0) to the other variabels, and unit vectors, a norm = 1

$$\mathbf{B}_{[p,\ d]} = \begin{bmatrix} B_{1,\ 1} & \dots & B_{1,\ d} \\ B_{2,\ 1} & \dots & B_{2,\ d} \\ \vdots & \ddots & \vdots \\ B_{p,\ 1} & \dots & B_{p,\ d} \end{bmatrix}$$

TODO: move this down to a usage example.

For ease of computation we will be wroking mostly with the basis and not the data, once basis manipulation is done postmultiply the data by the basis to get back to data-space.

Select a manipulation variable, k. Initialize a zero vector e, and set the k-th element set to 1. Use the Gram-Schmidt process to orthornormalize the concatenation of the basis and e yielding the manipulation space.

$$\begin{aligned} \mathbf{M}_{[p,\ d+1]} &= Orthnormalize_{GS}(\mathbf{B}_{[p,\ d]}|\mathbf{e}_{k\ [p,\ 1]}) \\ &= Orthnormalize_{GS}\left(\begin{bmatrix} B_{1,\ 1} & \dots & B_{1,\ d} \\ B_{2,\ 1} & \dots & B_{2,\ d} \\ \vdots & \ddots & \vdots \\ B_{k,\ 1} & \dots & B_{k,\ d} \\ \vdots & \ddots & \vdots \\ B_{p,\ 1} & \dots & B_{p,\ d} \end{bmatrix} \quad | \quad \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \right) \end{aligned}$$

TODO: clean up the langue of phi, tie to manip var.

Select a vector ϕ_i, the angle of out-of plane rotation, orthagonal to the projection plane (relative to ϕ_1, the transformation ϕ_i - ϕ_1 proved to be helpful to discuss ϕ relative to the Z axis).

For i **in 1 to n_slides:**

For each ϕ_i, postmultiply the manipulation space by a rotation matrix, producing as many basis-projections.

$$\mathbf{P}_{b[p,\, d+1,\, i]} = \mathbf{M}_{[p,\, d+1]} * \mathbf{R}_{[d+1,\, d+1]}$$
$$= \begin{bmatrix} M_{1,\,1} & M_{1,\,2} & M_{1,\,3} \\ M_{2,\,1} & M_{2,\,2} & M_{2,\,3} \\ \vdots & \vdots & \vdots \\ M_{p,\,1} & M_{p,\,2} & M_{p,\,3} \end{bmatrix}_{[p,\, d+1]} * \begin{bmatrix} c_\theta^2 c_\phi s_\theta^2 & -c_\theta s_\theta(1-c_\phi) & -c_\theta s_\phi \\ -c_\theta s_\theta(1-c_\phi) & s_\theta^2 c_\phi + c_\theta^2 & -s_\theta s_\phi \\ c_\theta s_\phi & s_\theta s_\phi & c_\phi \end{bmatrix}_{[3,\, 3]}$$

For the $d = 2$

Where:

θ is the angle that lies on the projection plane (*ie*. on the XY plane)

ϕ is the angle orthagonal to the projection plane (*ie*. in the Z direction)

c_θ is the cosine of θ

c_ϕ is the cosine of ϕ

s_θ is the sine of θ

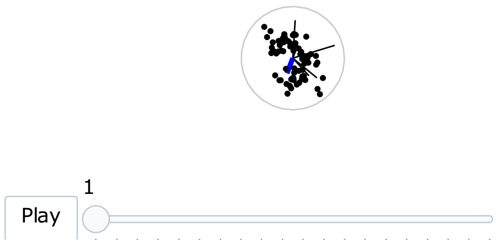
s_ϕ is the sine of ϕ

- To get back to data-space post multply each projection basis by the data, $\mathbf{X}_{[nxp]}$, for $\mathbf{P}_{d[nxd+1]}$.

$$\mathbf{P}_{d[n,\, d+1]} = \mathbf{X}_{[n,\, p]} * \mathbf{P}_{b[p,\, d+1]} = \begin{bmatrix} X_{1,\,1} & \dots & X_{1,\,p} \\ X_{2,\,1} & \dots & X_{2,\,p} \\ \vdots & \vdots & \vdots \\ X_{n,\,1} & \dots & X_{n,\,p} \end{bmatrix} * \begin{bmatrix} P_{b:1,\,1} & P_{b:1,\,2} & P_{b:1,\,3} \\ P_{b:2,\,1} & P_{b:2,\,2} & P_{b:2,\,3} \\ \vdots & \vdots & \vdots \\ P_{b:p,\,1} & P_{b:p,\,2} & P_{b:p,\,3} \end{bmatrix} \tag{3.1}$$

- View the first two variables from each projection in sequence for an XY scatterplot. The remaining variable is sometimes utilized to produce depth cues used in conjunction with the XY scatterplot.

```
## Warning: Only one ‘frame‘ variable is allowed
```



TODO: PDF output is a static image w/ play slider.

Chapter 4

Display dimensionality

4.1 My work

4.1.1 XGobbi vs the C2

Chapter 5

Human-computer interaction of 3d projections

5.1 Tour in 3D

5.1.1 ImAxes / IATK

Appendix A

Additional stuff

You might put some computer output here, or maybe additional tables.

Note that line 5 must appear before your first appendix. But other appendices can just start like any other chapter.

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