$\boxed{Q2}$

$\dot{x}_1 = x_2, \quad \dot{x}_2 = -3x_1 + x_2(2 - 3x_1^2 - x_2^2)$

$0 = x_2, \quad 0 = -3x_1 + x_2(2 - 3x_1^2 - x_2^2)$

$\Rightarrow x_1 = 0$

$(0,0)$ equilibrium pionts

$\boxed{\dfrac{\partial f_1}{\partial x_1} = 0}$

$0 = -3x_1 + 2x_2 - 3x_1^2 x_2 - x_2^3$

$\dfrac{\partial f_2}{\partial x_1} = -3 + 0 - 6x_1x_2 - 0$

$\dfrac{\partial f_1}{\partial x_2} = 1$

$0 = -3x_1 + 2x_2 - 3x_1^2 x_2 - x_2^3$

$0 = -3x_1 + 2x_2 - 3x_1^2 - 3x_2^2$

$\dfrac{\partial f}{\partial x_2} = 0 + 2 - 3x_1^2 - 3x_2^2$

$$J = \begin{bmatrix} 0 & 1 \\ -3 - 6x_1x_2 & 2 - 3x_1^2 - 3x_2^2 \end{bmatrix}_{(0,0)}$$

$$J = \begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix}$$

$$\left.\begin{aligned} \lambda_1 &= 1 + j1.4 \\ \lambda_2 &= 1 - j1.4 \end{aligned}\right\} \rightarrow \lambda_{1,2} = \alpha \pm j\beta$$

$\because a > 0 \ \therefore$ unstable focus

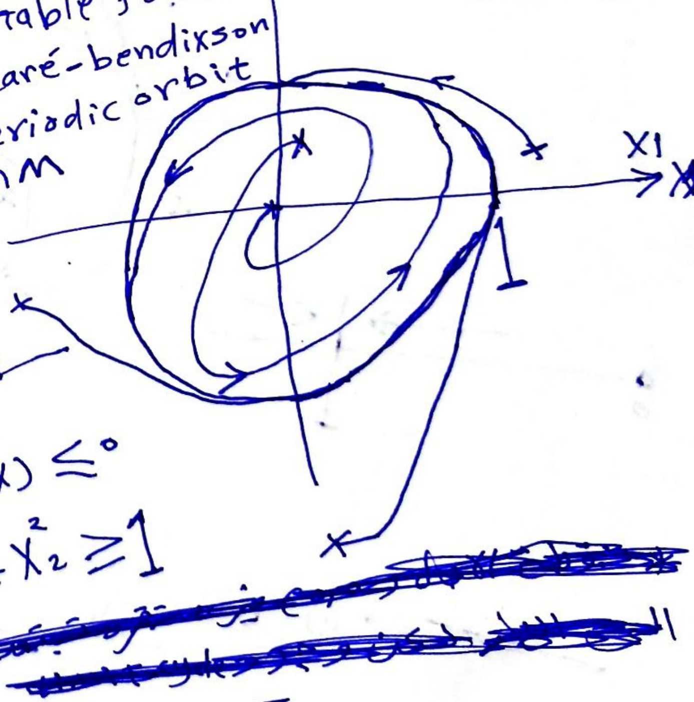
origin is unstable focus

* اثبات وجود limit cycle

$M = \{V(x) \leq 1\}$

Let $V(x) = 3x_1^2 + x_2^2$

by the Poincaré-bendixson there is a periodic orbit in M



$f(x) \cdot \nabla V(x) = 6x_1\dot{x}_1 + 2x_2\dot{x}_2$

$= 6x_1(x_2) + 2x_2(-3x_1 + x_2(2 - 3x_1^2 - x_2^2))$

$= 6\cancel{x_1x_2} - 6\cancel{x_1x_2} + 2x_2^2(2 - 3x_1^2 - x_2^2)$

$= 2x_2^2(2 - 3x_1^2 - x_2^2)$

$\leq 2x_2^2(2 - 6x_1^2 - 2x_2^2)$

$= 4x_2^2(1 - 3x_1^2 - x_2^2)$

$= 4x_2^2(1 - V)$

$f(x) \cdot \nabla V(x) \leq 0$

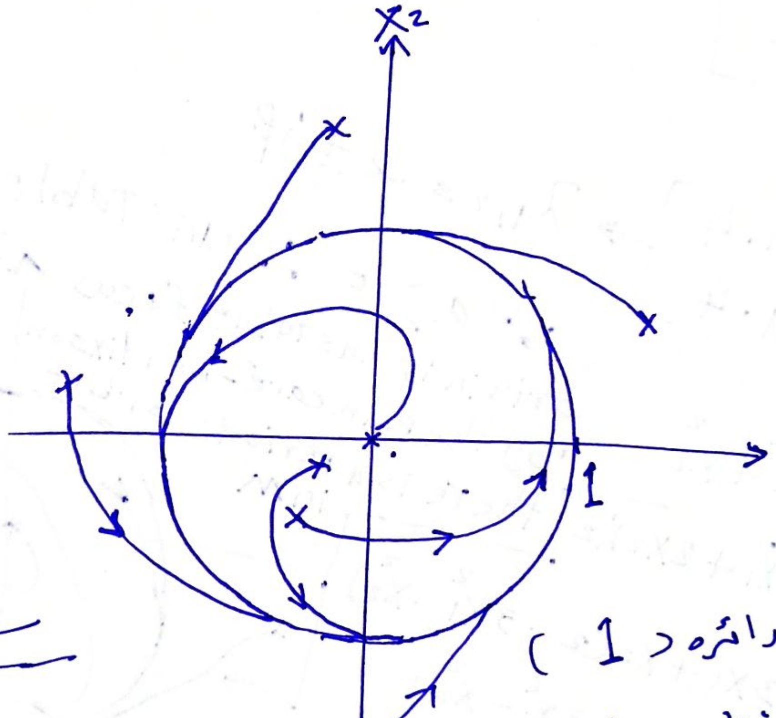
for $3x_1^2 + x_2^2 \geq 1$

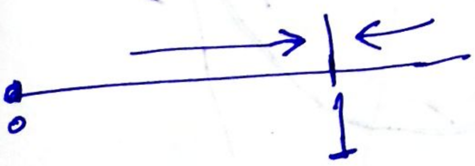
~~[illegible]~~

$0 \longrightarrow 1 \longleftarrow$

Limit cycle radius $= 1$

Limit cycle radius = 1





* المسارات داخل الدائره (1) التي تبتعد عن origin تلتحق مع Limit cycle

* المسارات الناشئه خارج الدائرة تجذب الى الدائره وتلتحق بالدائره (Limit cycle)