### 1.16.3. *Is inflation the only way to explain the three observations above?*

This information is from Wikipedia (2012a).

The horizon problem can be solved by increasing the speed of light. The monopole problem exists primarily because the GUT is well-accepted - since no topological defects have ever been produced in experiment, it may be that they do not exist. Penrose recently found that, from a purely statistical standpoint, there are many more possible initial conditions in the universe which do not have inflation and produce a flat universe, than there are which do have inflation and produce a flat universe, suggesting that using inflation to solve the flatness problem introduces a far larger fine-tuning problem than leaving it be.

## 1.17. *Question 16*

**QUESTION: Define and describe the 'fine tuning problem'. How do anthropic arguments attempt to resolve it?**

Fine-tuning refers to circumstances when the parameters of a model must be adjusted very precisely in order to agree with observations (Emberson 2012).

One major cosmological fine tuning problem is the $\Omega_\Lambda$ fine-tuning problem. WMAP 7-year results give $\Omega = 0.725$; if we were to backtrack this to the Planck Era, we would get a remarkably tiny number. As a first order estimate, assume the universe is radiation dominated; then $\Omega_\Lambda = \Omega_{\Lambda,0}\frac{H_0^2}{H^2} = \Omega_{\Lambda,0}\frac{T_0^4}{T^4}$, and at the Planck scale $T_0/T = 1.88 \times 10^{-32}$ $\Omega_\Lambda \approx 10^{-127}$ (Dodelson 2003, pg. 392).

This is related to the problem elucidated in Sec. **??**, that the vacuum energy density is about 100 orders of magnitude higher the observed $\rho_\Lambda$. If QFT is correct, then vacuum energy exists and should greatly accelerate the universe, and the fact that it does not indicates that the true cosmological constant $\Lambda_t$ is actually negative and cancels with the vacuum energy constant $\Lambda_v$ (to *60 - 120 decimal places of accuracy*!) to produce the effective $\Lambda_o$ ("observed $\Lambda$") we see (Shaw & Barrow 2011).

A related issue is the coincidence problem, which asks why the timescale $t_{\Lambda_o} \sim \Lambda_o^{-1/2}$ is approximately the age of the universe (or the nuclear timescale of stars), rather than much smaller (Shaw & Barrow 2011). In my view, this is a rewording of the problem, since saying that the $\Lambda_o$ timescale is of order the nuclear timescale is equivalent to asking why the observed $\Omega_\Lambda = 0.725$ and not nearly 1 at present, which then requires that $\Omega_\Lambda$ be tiny during the Planck Epoch.

The fine-tuning of $\Lambda_o$ to be nearly zero is problematic because we know of no known physical principle that constrains it (it is a fundamental constant of the universe) except for the vacuum energy density, which it is clearly not equal to. There are a number of possible solutions to this question (from Page (2011)):

1. $\Lambda_t - \Lambda_v = \Lambda_o$ simply by chance. This seems highly unsatisfactory: if we assume $\Lambda_o$ is completely randomly distributed (or perhaps distributed with a peak near $\Lambda_t$) it seems highly unlikely $\Lambda_o$ would be so close to zero. This claim is impossible to disprove, however.

2. $\Lambda_t - \Lambda_v = \Lambda_o$ (or is highly likely to be $\Lambda_o$) for a fundamental physical reason that we have yet to discover. For example, it may be required by the laws of physics that $\Lambda_t - \Lambda_v = 0$, and the nonzero $\Lambda_o$ comes from a separate physical principle (Shaw & Barrow 2011). While there is no way to disprove this claim either, it becomes more unattractive.

3. There is a multiverse, and $\Lambda_t - \Lambda_v$ equal various values in different universes. We inhabit a universe where $\Lambda_o$ is nearly zero because if it was even just a few orders of magnitude larger, atoms would disintegrate and life (that we know of) would not form.

4. The universe is fine-tuned so that life (that we know of) will form, and because of this $\Lambda_o$ is nearly zero.

The cosmological constant fine-tuning problem is one of several fine-tuning problems, as apparently changing any one of the fundamental constants in the universe (those constants that are not constrained by any physical theory, ex. the relative strength of gravity to the other four forces) may lead to wildly different-looking universes. The four options above may apply to any one of them.

The last two options are variations of the anthropic principle, which is a philosophical consideration that observations of the physical universe must be compatible with the conscious life that observes it (Wikipedia 2012a). The fact, therefore, that we observe ourselves living in a fine-tuned universe or an unlikely member of the multiverse is because if the universe were different, we would not exist to live in it.

Indeed, $\Omega_\kappa$ was once among the ranks of fine-tuned universal constants. A small variation in the curvature would be greatly amplified (the flatness problem in Sec. 1.16) so that either the universe would quickly become empty, preventing the formation of large scale structure, or collapse in on itself. This issue was solved by option 2 - inflation was developed as a physically plausible mechanism to create a very flat universe.

#### 1.17.1. *Is the anthropic principle a scientifically or logically valid argument?*

Page claims that one variant of the anthropic principle, that cosmological principles are fine-tuned to maximize the amount/chance of life coming into being, is in principle testable (at least theoretically through modelling different universes).

My issue with the anthropic principle is that it mistakes cause for effect. The fact that we are here to observe the universe requires that $\Lambda_t - \Lambda_v = \Lambda_o$, not the other way around. The reverse would be true only if the existence of the universe requires observers that think like us, which in itself requires a corroborating physical principle (ex. collapse of the "existence" wave function requires a specific "observer" operator that corresponds to physical "intelligence"). If this were the case, the anthropic principle would simply be a stepping stone to a physical principle that sets the coefficients of the universe, and not a final explanation.

### 1.18. *Question 17*

**QUESTION: Define the two-point correlation function. How is it related to the power spectrum? How is the C$_l$ spectrum of the CMB related to low redshift galaxy clustering?**

Most of this information comes from Schneider (2006).

Suppose we chose a patch of space $dV$ centred at a point in space $\vec{x}$; we wish to determine the probability that we will find a galaxy in $dV$. We cannot (due to chaos) actually describe the density structure of the universe except on a statistical level, which in this case means the probability of finding a galaxy in $dV$ centred on $\vec{x}$, averaged over all possible universes with the same statistical properties (this washes out random inhomogeneities; see **?**, pg. 282). If the universe were statistically homogeneous, then this probability would be $P_1 = \bar{n}dV$ (i.e. without gravity, all inhomogeneities are random), where $\bar{n}$ is the average number density of galaxies in the universe. We now consider the probability that a galaxy will be found in $dV$ centred around $\vec{x}$ *and* a galaxy will be found in $dV$ centred around another point $\vec{y}$ (again, in the statistical sense). If galaxy scattering were completely uncorrelated, then we would obtain

$$P = \bar{n}^2 dV^2 \tag{48}$$

i.e. uncorrelated probabilities simply multiply. Since galaxies gravitationally cluster, however, this cannot be the case. We therefore modify the above equation into

$$P = \bar{n}^2 dV^2 (1 + \xi(\vec{x}, \vec{y})) \tag{49}$$

The term $\xi(\vec{x}, \vec{y})$ is known as the two-point correlation function. Correspondingly, the correlation function for matter density $\langle \rho(\vec{x})\rho(\vec{y})\rangle = \bar{\rho}(1 + \xi(\vec{x}, \vec{y}))$. If the universe obeyed the cosmological principle, then $\xi(\vec{x}, \vec{y}) = \xi(r)$, where $r = |\vec{x} - \vec{y}|$.

$\xi(r)$ is related to the power spectrum $P(k)$ by a spherical Fourier transform, as noted in Sec. 1.4:

$$P(k) = \int_{-\infty}^{-\infty} \xi(r) exp(-ikr) r^2 dr \tag{50}$$

(this is equivalent to $P(k) = 2\pi \int_{-\infty}^{-\infty} \xi(r) \frac{\sin(kr)}{kr} r^2 dr$). We recall that this is the method by which we can determine the matter power spectrum of the universe (if dark matter and baryons are coupled). Fig. **??** shows the relationship between the two functions.

We can now see the primary connection between the two-point correlation function and the CMB power spectrum, since there is an intimate connection between the matter power spectrum $P(k)$ and the CMB power spectrum $\Delta_T$. Perhaps most importantly, the first peak of the CMB anisotropy spectrum corresponds to the sound horizon at last scattering. The co-moving preferred length of galaxy clustering as measured from the two-point correlation function, $\sim$140 Mpc, is a relic of the stalled baryon oscillation at the sound horizon at last scattering. In general, how the initial Harrison-Zel'dovich spectrum shifts over time to the current matter power spectrum depends on the expansion history of the universe and the fraction of baryons to dark matter. The expansion history depends on $\Omega_m$, $\Omega_k$ and $\Omega_r$, which all affect the CMB. $\Omega_b/\Omega_c$ changes the amplitude of the BAO, as well as the height of the first peak of the CMB.

The CMB power spectrum has also been modified by secondary anisotropies, many of which are related to galaxy clustering. The integrated Sachs-Wolfe effect, for example, is due to the growth of dark matter halos, which also is what seeds the formation of galaxy clusters. At small angular scales, the image of the CMB is warped by microlensing from particularly concentrated halos. Hot gas in the centres of galaxy clusters inverse Compton-scatter CMB photons (the Sunayev-Zel'dovich effect), which changes the overall thermal structure of the CMB along the line-of-sight to these clusters.

#### 1.18.1. *Describe galaxy surveys.*

This information is from Schneider (2006), Ch. 8.1.