Since there is only one normalization, a survey of galaxy cluster total masses at different redshifts (using hot gas and a mass-to-light conversion, gravitational microlensing, etc) can be used to determine cosmological parameters. This is because the minimum overdensity for collapse $\delta_{\rm min}$ is dependent on both the growth rate of overdensities and the expansion of the universe. Increasing Ω_m, for example, decreases $n(M,z)/n(M,0)$, since massive halo growth is more extreme the higher Ω_m is. A large Ω_Λ dampens massive halo growth.

1.5. *Question 4*

QUESTION: State and explain three key pieces of evidence for the Big Bang theory of the origin of the Universe.

This information is cribbed from Emberson (2012).

The Big Bang theory is the theory that the universe started off in an extremely hot, dense state, which then rapidly expanded, cooled, and became more tenuous over time. The Big Bang theory requires that at some point in the past a). the universe was born, b). the universe was extremely hot and c). objects were much closer together. The three key pieces of evidence are:

1. **Hubble's Law**: galaxies isotropically recede from our position with the relationship

$$\vec{v} = H_0\vec{r} \tag{26}$$

known as Hubble's Law. As it turns out, moving into the frame of another galaxy ($\vec{r}' = \vec{r} - \vec{k}$, $\vec{v}' = \vec{v} - H_0\vec{k} = H_0(\vec{r} - \vec{k}) = H_0\vec{r}'$) does not change any observations. At larger distances, Hubble's Law breaks down (see Sec. 1.7), but the rate of expansion only increases with distance. Because of this isotropic radial motion outward, we can back-calculate a time when all the galaxies ought to be together at one point. This time is $t_0 = r/v = 1/H_0 \approx 14$ Gyr, the Hubble Time. This gives an age to the universe, and indicates that in the distant past everything was closer together.

2. **The Cosmic Microwave Background**: the cosmic microwave background (CMB) is a near perfect isotropic blackbody with a (current) $T_0 \approx 2.73$ K. For a blackbody, $\lambda_{\rm peak} = 0.0029\,{\rm mK}/T$, $U = aT^4$ and $n = \beta T^3$, which gives us $n \approx 400$ cm^{-3}, $\epsilon \approx 0.25$ eV cm^{-3}, and $\lambda \approx 2$ mm. In Big Bang cosmology, this microwave background is the redshifted ($T \propto a^{-1}$) vestige of the surface of last scattering, when $T \approx 3000$ K and the universe became neutral enough for photons to travel unimpeded. This is evidence that the universe used to be hot.

3. **Big Bang Nucleosynthesis**: in the Big Bang theory, the lightest elements were created out of subatomic particles when the temperature dropped enough that the average photon was significantly below the binding energy of light elements. A detailed calculation of nucleosynthetic rates of H, D, He and Li during the first few minutes of the universe is consistent with the current abundances of light elements in the universe. See Sec. 1.8.

Additionally, no object has been found to be older than the currently accepted age of the universe, 13.7 Gyr. As we look back in time, we notice that the average galaxy in the universe looked considerably different - this evolution is consistent with ΛCDM cosmology, which has small, dense cores of dark matter forming due to gravitational instability, and then merging to form larger cores.

1.5.1. *What is Olbers's Paradox?*

Olbers's paradox is the apparent contradiction one has when an infinitely old, infinitely large universe with a fixed stellar density is assumed. In such a universe every single line of sight would eventually reach a star's photosphere. Since a typical photospheric temperature is ~ 5000 K and surface brightness is independent of distance, we would expect the entire sky to be at ~ 5000 K, roasting the Earth. Setting a finite age to the universe is one solution to the paradox; another would be that stars only formed in the last several billion years, and light from more distant stars have yet to reach us.

1.5.2. *Are there Big Bang-less cosmologies?*

It is impossible to generate a matter dominated universe for which there is no Big Bang. It is possible, however, for a Λ-dominated universe to be infinitely old, since an exponential (see Sec 1.1.4) never goes to zero. This is consistent with the steady state theory (Sec. 1.6).

1.6. *Question 5*

QUESTION: Define and describe the "tired light hypothesis" and the "steady state universe" as alternatives to the Big Bang. How have they been disproved observationally?