If we perform the same calculation during the era of recombination, setting n_e using the analysis above and obtaining H from $H(t) = H_0\Omega_m \frac{a_0^3}{a^3} = H_0\Omega_m \frac{1}{(1+z)^3}$, we obtain $z \approx 1100$ and $T \approx 3000$ (exact answers are difficult without modelling, since during the final stages of recombination the system was no longer in LTE).

1.2.2. *What is the last scattering surface?*

The last scattering surface is the $\tau = 1$ surface for photons originally trapped in the optically thick early universe. The age $t_0 - t$ of this surface can be found using

$$\tau = 1 = \int_t^{t_0} \Gamma(t)dt \tag{17}$$

In practice this is difficult, and so we again estimate that $z \approx 1100$ for last scattering.

1.3. *Question 2*

QUESTION: The universe is said to be "flat", or, close to flat. What are the properties of a flat universe and what evidence do we have for it?

This information comes mainly from Emberson (2012), with supplement from Carroll & Ostlie (2006).

As noted in Sec. **?**, the FLRW universe may only have three types of curvature. When $\kappa = 1$, the universe is positively curved, since $R\sin(r/R) < r$ (i.e. the actual size of the object would be smaller than its physical size, consistent with the fact that two straight lines intersecting on a circle will eventually meet again) and when $\kappa = -1$ the universe is negatively curved, since $R\sinh(r/R) > r$.

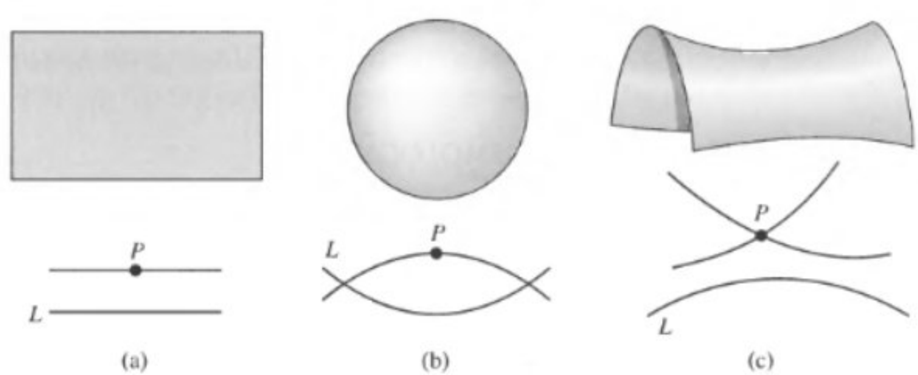

FIG. 4.— Schematic of two dimensional analogues to possible curvature in an FLRW universe. The behaviour of lines parallel at a point P within each space is also drawn. A Euclidian plane has no curvature, a sphere has positive curvature and a saddle has negative curvature. From Carroll & Ostlie (2006), their Fig. 29.15.

Fig. 4 shows the two primary geometric features of a flat, closed and open universe. In open and closed universes, parallel lines tend to diverge (open) or converge (closed), while for a flat universe two parallel lines remain parallel indefinitely. Open and flat universes are infinite, while a closed universe may have a finite extent, since it "curves back" on itself. In $\Lambda = 0$ universes, the geometry of the universe is intimately related to the matter-energy density of the universe.

Measurement of the curvature of the universe is difficult. In a $\Lambda = 0$ universe it actually is greatly simplified, since curvature and expansion history are linked, and the age of the universe, combined with H_0, can be used to determine the curvature (or H_0 and q_0). In a universe with a non-zero cosmological constant, however, the age of the universe is decoupled from the curvature. Instead, we use a standard ruler: the first peak of the CMB power spectrum. This peak, due to the length of the sound horizon at decoupling, is

$$r_s(z_{rec}) = c\int_{z_{rec}}^{\infty} \frac{c_s}{H(z)} dz \tag{18}$$

where $c_s = (3(1 + 3\rho_{bary}/\rho_{ph}))^{-1/2}$ (Vardanyan et al. 2009). Detailed measurements of higher order peaks and their spacings in the CMB allow us to constrain both $H(z)$ and c_s, and obtain a preferred length scale (Eisenstein 2005). This is our standard ruler, and if we measure its current angular size θ, we can use Eqn. 14 alongside Eqn. 3 to determine

$$\frac{\theta}{1+z} = \frac{l}{S_\kappa} \tag{19}$$

Note that l is known, but S_κ depends on the co-moving distance between us and the CMB. This requires some knowledge of the subsequent expansion history of the universe, or else there is a degeneracy between Ω_m, Ω_Λ and Ω_κ (Komatsu et al. 2009). An additional constraint, such as a measurement of H_0, or the series of luminosity distance measurements using high-z SNe, allows us to constrain Ω_κ (Komatsu et al. 2009). See Fig. 5.

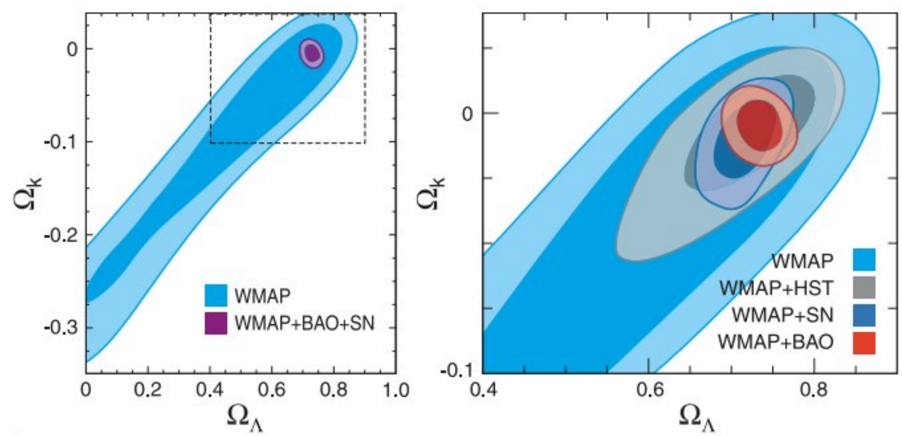

FIG. 5.— Joint two-dimensional marginalized constraint on the dark energy density Ω_Λ, and the spatial curvature parameter, Ω_κ. The contours show the 68% and 95% confidence levels. Additional data is needed to constrain Ω_κ: HST means H_0 from Hubble measurements, SN means luminosity distances from high-z SN, and BAO means baryon acoustic oscillation measurements from galaxy surveys. From Komatsu et al. (2009), their Fig. 6.

1.4. *Question 3*

QUESTION: Outline the development of the Cold Dark Matter spectrum of density fluctuations from the early universe to the current epoch.

Most of this information is from Schneider (2006), Ch. 7.3 - 7.5.

The growth of a single perturbation (described as one of the follow-up questions) in a matter-dominated universe can be described in the following way. We define the relative density contrast $\delta(\vec{r},t) = (\rho(\vec{r},t) - \bar{\rho})/\bar{\rho}$; from this $\delta(\vec{r},t) \leq -1$. At $z \sim 1000$ $|\delta(\vec{r},t)| << 1$. The mean density of the universe $\bar{\rho}(t) = (1+z^3)\bar{\rho}_0 = \bar{\rho}_0/a(t)^3$ from Hubble flow. Like in the classic Newtonian stability argument of an infinite static volume of equally space stars, any overdense region will experience runaway collapse (and any underdense region will become more and more underdense). In the linear perturbative regime, the early stages of this collapse simply make it so that the the expansion of the universe is delayed, so $\delta(\vec{r},t)$ increases. As it turns out, $\delta(\vec{r},t)$ can be written as $D_+(t)\delta_0(\vec{x})$ in the linear growth regime. $D_+(t)$ is normalized to be unity today, and $\delta_0(\vec{x})$ is the linearly-extrapolated (i.e. no non-linear evolution taken into account) density field today.

The two-point correlation function $\xi(r)$ (Sec. 1.18) describes the over-probability of, given a galaxy at $r = 0$, there will be another galaxy at r (or x, here). It describes the clustering of galaxies, and is key to understanding the large-scale structure of the universe. We define the matter power spectrum (often shortened to just "the power spectrum") as

$$P(k) = \int_{-\infty}^{-\infty} \xi(r) exp(-ikr) r^2 dr \tag{20}$$

Instead of describing the spatial distribution of clustering, the power spectrum decomposes clustering into characteristic lengths $L \approx 2\pi/k$, and describes to what degree each characteristic contributes to the total overprobability.

Since the two-point correlation function depends on the square of density, if we switch to co-moving coordinates and stay in the linear regime,

$$\xi(x,t) = D_+^2(t)\xi_0(x,t_0). \tag{21}$$

Likewise,

$$P(k,t) = D_+^2 P(k,t_0) \equiv D_+^2 P_0(k), \tag{22}$$

i.e. everything simply scales with time. This the evolution of the power spectrum is reasonably easily described.

The initial power spectrum $P_0(k)$ was generated by the quantum fluctuations of inflation. It can be argued (pg. 285 of Schneider (2006)) that the primordial power spectrum should be $P(k) = Ak^{n_s}$, where A is a normalization factor