Most of this solution was from Ryden (2003).
Classical Big Bang theory (i.e. without inflation) has three significant problems

1. **The horizon problem**: the CMB we view is isotropic to one part in 10^5, and since we are just receiving these photons right now, the two "sides" of the universe could not have communicated with each other. In fact, the Hubble sphere at the time of last scattering has a diameter $2c/H(t_{\rm ls}) \approx 0.4$ Mpc, approximately 2 degrees on the sky. There should, therefore, have been no prior communciations between various patches of the sky before last scattering.

2. **The flatness problem**: we may rewrite Eqn. 4 as

$$1 - \Omega(t) = \frac{-\kappa c^2}{R^2 a^2 H^2} = -\frac{H_0^2(1-\Omega_0)}{H^2 a^2} \tag{46}$$

   From Sec. 1.11, we know that $|1-\Omega_0| \lesssim 0.02$, but Eqn. 46 requires, then, that for earlier times the universe be even flatter (in fact, $1 - \Omega(t) \leq 10^{-60}$ during the Planck era). Classical Big Bang theory does not explain this fine-tuning.

3. **The monopole problem**: various GUT theories predict that as space cooled to below 10^{12} TeV, spontaneous symmetry breaking between the electroweak and strong forces occured. This phase trasition created various topological defects, including pointlike defects (magnetic monopoles), linelike defects (cosmic strings) and so on. These objects would have been highly non-relativistic, and therefore would have begun to dominate the evolution of the universe at around $t = 10^{-16}$ s. This did not occur, and we do not observe monopoles or other topological defects today.

The solution to all three problems is to invoke a short period of exponential a increase very early in the history of the universe. As an example, suppose $H^2 = H_i^2 = \frac{\Lambda c^2}{3}$ during inflation (before and after, the universe is radiation dominated). Then:

$$a(t) = \begin{cases} a_i(t/t_i)^{1/2} & \text{if } t < t_i \\ a_i \exp\left(H_i(t-t_i)\right) & \text{if } t_i < t < t_f \\ a_i \exp\left(N\right)(t/t_f)^{1/2} & \text{if } t > t_f \end{cases} \tag{47}$$

This gives $a(t_f)/a(t_i) = e^N$, where $N = H_i(t_f - t_i)$. Let us also assume $t_f - t_i = 100/H_i$ (i.e. 100 Hubble times); this is a reasonably short time during the GUT Epoch, since $1/H_i \sim 10^{-36}$ s. This means $N = 100$. $e^{100} = 10^{43}$, so length scales in the universe increased by 43 orders of magnitude. This easily allows the last scattering surface to have once been in causal contact[5]. We may perform a more detailed calculation by noting that the particle horizon distance is the proper (co-moving times a) distance travelled by a photon from $t = 0$ to $t = t_i$ or t_f (before or after inflation), and this shows that the horizon increased from 10^{-27} m to 1 pc. The flatness problem is easily addressed as well, since $\frac{1-\Omega(t_f)}{1-\Omega(t_i)} = \frac{\dot{a}(t_i)^2}{\dot{a}(t_f)^2}$. Using Eqn. 47, we see that $\frac{1-\Omega(t_f)}{1-\Omega(t_i)} = e^{-2N}$, meaning that the universe was much, much flatter after inflation, whatever its initial curvature. Lastly, since a increased by 43 orders of magnitude, volume increased by 129 orders of magnitude. As it turns out, we would expect one magnetic monopole for every 10^{61} Mpc^3.

1.16.1. *What caused inflation?*

There is no general consensus as to what caused inflation. The energy density of dark energy today is ~ 0.004 TeV/m^3, while $\rho_\Lambda = \frac{3H_i^2}{8\pi G}$ (since $H^2 = \frac{\Lambda c^2}{3}$ during inflation) gives a ridiculous 10^{105} TeV/m^3.

Suppose, then, there was a scalar field $\phi(\vec{r}, t)$ known as an "inflaton field", associated with a scalar potential $V(\phi)$. From the derivation in (Ryden 2003, pg. 247 - 248), if V changes very slowly, then it is in what is known as a "metastable false vacuum", and as a result $P_\phi \approx -V_\phi$, which, like a cosmological constant, would drive inflation. When ϕ finally changes sufficiently to minimize V, inflation ends. We note that inflation also significantly cools down the universe ($T \propto 1/a$); minimization of V may release enough energy to reheat the universe.

1.16.2. *How does inflation affect the large scale structure of the universe?*

Aside from flattening the universe and homogenizing it, inflation also carried Planck-scale energy fluctuations (10^{-35} m) to macrophysical scales (in our example, 10^8 m). This could be what seeded the initial spectrum of density fluctuations in the universe.

[5] One might wonder why all points were not in causal contact at the Big Bang in any universe. This is because $H = \infty$ when $t = 0$ for universes with a Big Bang.

1.16.3. *Is inflation the only way to explain the three observations above?*

This information is from Wikipedia (2012a).

The horizon problem can be solved by increasing the speed of light. The monopole problem exists primarily because the GUT is well-accepted - since no topological defects have ever been produced in experiment, it may be that they do not exist. Penrose recently found that, from a purely statistical standpoint, there are many more possible initial conditions in the universe which do not have inflation and produce a flat universe, than there are which do have inflation and produce a flat universe, suggesting that using inflation to solve the flatness problem introduces a far larger fine-tuning problem than leaving it be.

1.17. *Question 16*

QUESTION: Define and describe the 'fine tuning problem'. How do anthropic arguments attempt to resolve it?

Fine-tuning refers to circumstances when the parameters of a model must be adjusted very precisely in order to agree with observations (Emberson 2012).

One major cosmological fine tuning problem is the Ω_Λ fine-tuning problem. WMAP 7-year results give $\Omega = 0.725$; if we were to backtrack this to the Planck Era, we would get a remarkably tiny number. As a first order estimate, assume the universe is radiation dominated; then $\Omega_\Lambda = \Omega_{\Lambda,0}\frac{H_0^2}{H^2} = \Omega_{\Lambda,0}\frac{T_0^4}{T^4}$, and at the Planck scale $T_0/T = 1.88 \times 10^{-32}$ $\Omega_\Lambda \approx 10^{-127}$ (Dodelson 2003, pg. 392).

This is related to the problem elucidated in Sec. **??**, that the vacuum energy density is about 100 orders of magnitude higher the observed ρ_Λ. If QFT is correct, then vacuum energy exists and should greatly accelerate the universe, and the fact that it does not indicates that the true cosmological constant Λ_t is actually negative and cancels with the vacuum energy constant Λ_v (to *60 - 120 decimal places of accuracy*!) to produce the effective Λ_o ("observed Λ") we see (Shaw & Barrow 2011).

A related issue is the coincidence problem, which asks why the timescale $t_{\Lambda_o} \sim \Lambda_o^{-1/2}$ is approximately the age of the universe (or the nuclear timescale of stars), rather than much smaller (Shaw & Barrow 2011). In my view, this is a rewording of the problem, since saying that the Λ_o timescale is of order the nuclear timescale is equivalent to asking why the observed $\Omega_\Lambda = 0.725$ and not nearly 1 at present, which then requires that Ω_Λ be tiny during the Planck Epoch.

The fine-tuning of Λ_o to be nearly zero is problematic because we know of no known physical principle that constrains it (it is a fundamental constant of the universe) except for the vacuum energy density, which it is clearly not equal to. There are a number of possible solutions to this question (from Page (2011)):

1. $\Lambda_t - \Lambda_v = \Lambda_o$ simply by chance. This seems highly unsatisfactory: if we assume Λ_o is completely randomly distributed (or perhaps distributed with a peak near Λ_t) it seems highly unlikely Λ_o would be so close to zero. This claim is impossible to disprove, however.

2. $\Lambda_t - \Lambda_v = \Lambda_o$ (or is highly likely to be Λ_o) for a fundamental physical reason that we have yet to discover. For example, it may be required by the laws of physics that $\Lambda_t - \Lambda_v = 0$, and the nonzero Λ_o comes from a separate physical principle (Shaw & Barrow 2011). While there is no way to disprove this claim either, it becomes more unattractive.

3. There is a multiverse, and $\Lambda_t - \Lambda_v$ equal various values in different universes. We inhabit a universe where Λ_o is nearly zero because if it was even just a few orders of magnitude larger, atoms would disintegrate and life (that we know of) would not form.

4. The universe is fine-tuned so that life (that we know of) will form, and because of this Λ_o is nearly zero.

The cosmological constant fine-tuning problem is one of several fine-tuning problems, as apparently changing any one of the fundamental constants in the universe (those constants that are not constrained by any physical theory, ex. the relative strength of gravity to the other four forces) may lead to wildly different-looking universes. The four options above may apply to any one of them.

The last two options are variations of the anthropic principle, which is a philosophical consideration that observations of the physical universe must be compatible with the conscious life that observes it (Wikipedia 2012a). The fact, therefore, that we observe ourselves living in a fine-tuned universe or an unlikely member of the multiverse is because if the universe were different, we would not exist to live in it.

Indeed, Ω_κ was once among the ranks of fine-tuned universal constants. A small variation in the curvature would be greatly amplified (the flatness problem in Sec. 1.16) so that either the universe would quickly become empty, preventing the formation of large scale structure, or collapse in on itself. This issue was solved by option 2 - inflation was developed as a physically plausible mechanism to create a very flat universe.