QUESTION 2

What is the total mass (in both dark matter and in stars) of the Milky Way galaxy? How does this compare to M31 and to the LMC? How is this mass determined?

The masses of the MW and Andromeda galaxy (M31) have been measured by a variety of methods, but often with conflicting results that have led to a debate around which galaxy is more massive. Mass judging criteria based on observations of the surface brightness of the stellar halo, the number of globular clusters (which correlates with total mass albeit with scatter), and the amplitude of the inner gas rotation curve suggest that M31 is more massive. On the other hand, if the mass estimate is based on criteria such as the velocities of satellite galaxies, distant globular clusters, or tidal radii of nearby dwarf spheroidals, then the MW appears more massive. The current consensus, however, is that the two galaxies are roughly of the same mass ($\sim 10^{12} M_\odot$), with M31 probably the slightly more massive of the two, though this is based on the rather indirect mass estimates described above for M31 (Watkins et al. 2010).

On the other hand, the masses of the two galaxies are reasonably well constrained within the first few kpc from knowledge of their gas rotation curves via 21-cm radio observations (Carroll & Ostlie 2007, pg. 914). Of course, this only samples the inner regions of the galaxies, and in order to probe further out into the vast dark matter halos it is necessary to resort to satellite kinematics. Unfortunately, the uncertainties in such techniques are plagued by low sample sizes as well as the fact that there is seldom knowledge of the proper motion of the satellites to complement their observed radial velocity and distance data. Because of the latter, assumptions must be made on the eccentricities of the satellite orbits thereby affecting the mass determination; see equation (170) below.

In order to convert satellite kinematics into mass estimates we begin by analyzing the virial theorem for a spherically symmetric collection of N test particles (e.g. planetary nebulae, stars, globular clusters, satellite galaxies) orbiting a point mass M. For this situation the virial theorem dictates that

$$GM = \frac{\langle v^2 \rangle}{\langle 1/r \rangle}, \tag{170}$$

where angular brackets denote average values. If the distribution of test particles is spherically symmetric then $\langle v^2 \rangle = 3\langle v_r^2 \rangle$ and $\langle 1/r \rangle = 2/\pi \langle 1/R \rangle$, where v_r is the observed radial velocity and R the projected separation. Substituting these identities into equation (170) for a collection of N test particles yields a mass estimate of the form

$$M = \frac{3\pi}{2G} \frac{\sum_i v_{r_i}^2}{\sum_i 1/R_i}. \tag{171}$$

Despite its easy appearance, the virial theorem does not provide accurate mass estimates. There are many problems associated with its use including its failure to converge as $N \to \infty$ (Bahcall & Tremaine 1981).

Instead of using the virial theorem, a more reliable mass estimate is based on the projected mass $q \equiv v_r^2 R/G$. The variable q has dimensions of mass and with a suitable multiplicative factor can be used as an estimator to the mass M. For a general distribution of test particles it turns out that the expectation value of q is

$$\langle q \rangle = \frac{\pi M}{32}(3 - 2\langle e^2 \rangle), \tag{172}$$

where $\langle e^2 \rangle$ is the expectation value of the square of the eccentricities of the particles orbits. Using this relation and the definition of q it is straightforward to arrive at a mass estimate of the form

$$M = \frac{C}{G} \frac{1}{N} \sum_i v_r^2 R_i, \tag{173}$$

where C is a constant of order unity depending on the test particles' eccentricities. Unlike the virial theorem method, the central value theorem applies to this method and guarantees that the sum will converge to the true mass M with an error proportional to $1/\sqrt{N}$ (Bahcall & Tremaine 1981).

Applying a modified form of equation (173) to 26 satellite galaxies of the Milky Way and 23 satellite galaxies of M31, Watkins et al. (2010) determine the masses of the two galaxies within 300 kpc from their centres to be $M_{\rm MW} \sim 3 \times 10^{12}\, M_\odot$ and $M_{\rm M31} \sim 1 \times 10^{12}\, M_\odot$. These values are rather volatile inasmuch as the exclusion of the satellite galaxies with ambiguous velocity and distance measures changes the mass estimates by a factor of roughly 2.

The value for $M_{\rm MW}$ is in good agreement with the study by Xue et al. (2008) in which 2401 blue horizontal branch (BHB) stars (which have high luminosities and nearly constant absolute magnitudes within a restricted colour range) from the SDSS are used to constrain the MW's circular velocity curve up to 60 kpc. From this the total mass within 60 kpc is determined and subsequently used to estimate the mass of the entire halo to be $M_{\rm MW} \sim 1 \times 10^{12}\, M_\odot$. Of course, Newton's theorem asserts that any mass outside of the limiting radius of 60 kpc will have no observational effect in a spherical or elliptical system and so estimating the halo mass in this way requires an initial assumption on the structure of the dark matter halo. Indeed, the estimate by Xue et al. (2008) is based on the assumption of an NFW halo profile.

Schommer et al. (1992) measure the velocities of individual stars in 83 star clusters in the LMC to arrive at a mass estimate for the galaxy. Using equation (173) they find the mass of the LMC to be $M_{\rm LMC} \sim 2 \times 10^{10}\, M_\odot$, roughly 1/100 that of the Milky Way. They compare this to an estimate based on a rotation curve constructed from their cluster rotation data in addition to earlier

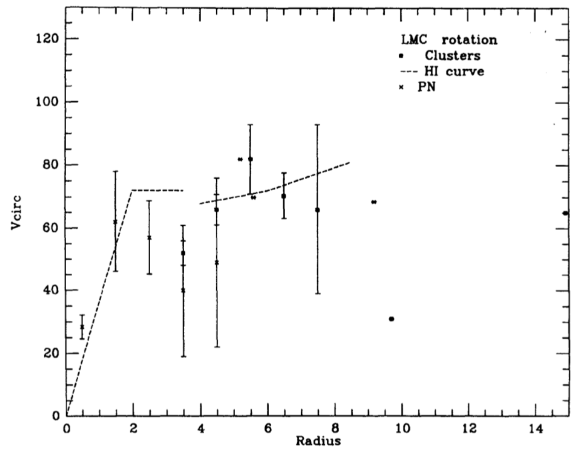

FIG. 29.— The rotation curve of the LMC built from rotational data of star clusters, H I, and PN. The dashed line shows the H I rotation curve whereas the points represent data from the clusters and PN. The error bars are the standard deviations of the mean of the velocities in the given bin while points with no error bars denote single objects. The bottom axis is in units of degrees and can be converted to physical lengths by noting that the LMC is 48 kpc from the MW. Image taken from Schommer et al. (1992).

data on the circular velocities of H I and planetary nebulae (PN) in the LMC. This is shown in Figure 29. To estimate the mass of the LMC from the rotation curve, one needs to compare the the circular velocity of the MW and the LMC at a distance of 8.5 kpc, the distance from the Sun to the centre of the MW. For the MW this value is roughly 200 km s^{-1} while for the LMC it is roughly 20 km s^{-1} since the circular velocity is roughly 20% larger than the rotation velocity (Weinberg 2000). Then since $M \propto Rv^2$ we have that $M_{\rm LMC} \sim 1 \times 10^{10}\, M_{\odot}$, in good agreement with the value above. This estimate would obviously be improved with more measurements of mass tracers at large radii. In principle, this can be achieved by H I though at such large radii its kinematics may be disturbed by hydrodynamical processes from tidal interactions (i.e. the Magellanic Stream).

Another quick check on the mass of the LMC is to investigate its tidal interactions with the MW. From the point of view of the MW, the LMC is an oversize globular cluster. Its tidal radius is measurable and depends both on the MW rotation curve and the LMC mass (and also weakly on the LMC mass profile). The tidal radius of the LMC can be estimated by observing the extent of its stellar halo. From this its mass is estimated via

$$M_{\rm LMC} = 2\left(\frac{r_t}{R_{\rm LMC}}\right)^3 M_{\rm MW}, \tag{174}$$

where r_t is the tidal radius and $R_{\rm LMC}$ is the distance to the LMC. This analysis is considered by Weinberg (2000) in which a tidal radius of 10.8 kpc is used to constrain the mass of the LMC at $M_{\rm LMC} \sim 2 \times 10^{10}\, M_{\odot}$.