General Qualifying Exam Solutions: Galaxies

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Q1) Mass of the Milky Way

What is the total mass (in both dark matter and in stars) of the Milky Way galaxy? How does this compare to M31 and to the LMC? How is this mass determined?

Short Answer

Table 1: Galaxy Masses

Object	Total $[M_\odot]$	Stars $[M_\odot]$
Milky Way	$\approx 10^{12}$	$\approx 10^{10}$
Andromeda (M31)	$\approx 2 \times M_{MW}$	
LMC	$\approx 10^{10}$	$\approx 10^{19}$

There are variety of methods to determine the total masses of these objects. For more details, besides the included notes, see Jo Bovy's detailed Galaxy Dynamics notes.

Methods:

1. Rotation curves: $M(< r) = \frac{r v_c^2}{G} \propto r$
2. Virial theorem: $M = \frac{1}{G} \frac{\sum w_i |v_i|^2}{\sum w_i / r_i}$
3. Velocity distribution cutoff of stars
4. Via Globular Clusters:
   - Escape velocity: $v_{esc} = \sqrt{\frac{GM(<r)}{r}}$
   - Photometrically for baryonic mass + scaling relations for the DM
5. Local group timing argument / Spherical Jeans equation $M(< r) = -\frac{r\sigma_r^2}{G}\left(\frac{\mathrm{d}\ln(\nu\sigma_r^2)}{\mathrm{d}\ln r} + 2\beta\right)$
6. Wolf Mass Estimate: $M(< r_{1/2}) = 3G^{-1}\sigma_{los}^2 r_{1/2}$

The easiest method for the LMC is Spherical Jeans.
The Spherical Jeans is hard for the Milky Way because we do not (yet!) have large data samples with three-dimensional velocities at large distance. Determining $\sigma_r(r)$ and especially $\beta(r)$ is difficult. At $r \gg R_0$, the solar radius, $\sigma_r(r)$ is approximately equal to $\sigma_{los}(r)$, because to a good approximation the Sun is sitting at the center of the Galaxy. In this case $v_{los} \approx v_r$. For the same reason, $\beta(r)$ can only be measured using tangential velocities (i.e., proper motions).

Q1) Herman ExtraGal Q2

• List masses
• invoke virial theorem to relate M & v.
• get v from 21 cm line width (Doppler broadened), Tully-Fisher (max rot. v $\propto$ L profile of spiral)

EG 2

• Mass determination of MW Galaxy

Galaxy	Mass w/ DM	Mass w/out DM
MW	$\sim 10^{12} M_\odot$	$\sim 10^{10} M_\odot$
M31	$\sim 10^{12} M_\odot$	$\sim 10^{11} M_\odot$
LMC	$\sim 10^{10} M_\odot$	$\sim 10^{9} M_\odot$

• Assume galaxy is virialized (dynamically relaxed, in equilibrium) and determine rotation curve

$U = -2K$

$-\frac{GMm}{r} = -mv^2$

$v = \sqrt{\frac{GM}{r}} \rightarrow M = \frac{v^2 r}{G}$

- Measure velocities (or velocity dispersion) of galaxy components as function of radius
- Measure width of Doppler broadened 21 cm line from HI to get velocity dispersion
- could use Tully-Fisher relation to get max. rotational velocity based on luminosity profile of spiral, and Faber-Jackson relation to get velocity dispersion based on luminosity of ellipticals.

R - Max rotational velocity v. L
S - spirals
T - Tully-Fisher

D - velocity dispersion v. L
E - ellipticals
F - Faber-Jackson

• The mass of a galaxy is often expressed in terms of its M/L ratio.

QUESTION 2

What is the total mass (in both dark matter and in stars) of the Milky Way galaxy? How does this compare to M31 and to the LMC? How is this mass determined?

The masses of the MW and Andromeda galaxy (M31) have been measured by a variety of methods, but often with conflicting results that have led to a debate around which galaxy is more massive. Mass judging criteria based on observations of the surface brightness of the stellar halo, the number of globular clusters (which correlates with total mass albeit with scatter), and the amplitude of the inner gas rotation curve suggest that M31 is more massive. On the other hand, if the mass estimate is based on criteria such as the velocities of satellite galaxies, distant globular clusters, or tidal radii of nearby dwarf spheroidals, then the MW appears more massive. The current consensus, however, is that the two galaxies are roughly of the same mass ($\sim 10^{12} M_\odot$), with M31 probably the slightly more massive of the two, though this is based on the rather indirect mass estimates described above for M31 (Watkins et al. 2010).

On the other hand, the masses of the two galaxies are reasonably well constrained within the first few kpc from knowledge of their gas rotation curves via 21-cm radio observations (Carroll & Ostlie 2007, pg. 914). Of course, this only samples the inner regions of the galaxies, and in order to probe further out into the vast dark matter halos it is necessary to resort to satellite kinematics. Unfortunately, the uncertainties in such techniques are plagued by low sample sizes as well as the fact that there is seldom knowledge of the proper motion of the satellites to complement their observed radial velocity and distance data. Because of the latter, assumptions must be made on the eccentricities of the satellite orbits thereby affecting the mass determination; see equation (170) below.

In order to convert satellite kinematics into mass estimates we begin by analyzing the virial theorem for a spherically symmetric collection of N test particles (e.g. planetary nebulae, stars, globular clusters, satellite galaxies) orbiting a point mass M. For this situation the virial theorem dictates that

$$GM = \frac{\langle v^2 \rangle}{\langle 1/r \rangle}, \tag{170}$$

where angular brackets denote average values. If the distribution of test particles is spherically symmetric then $\langle v^2 \rangle = 3\langle v_r^2 \rangle$ and $\langle 1/r \rangle = 2/\pi \langle 1/R \rangle$, where v_r is the observed radial velocity and R the projected separation. Substituting these identities into equation (170) for a collection of N test particles yields a mass estimate of the form

$$M = \frac{3\pi}{2G} \frac{\sum_i v_{r_i}^2}{\sum_i 1/R_i}. \tag{171}$$

Despite its easy appearance, the virial theorem does not provide accurate mass estimates. There are many problems associated with its use including its failure to converge as $N \to \infty$ (Bahcall & Tremaine 1981).

Instead of using the virial theorem, a more reliable mass estimate is based on the projected mass $q \equiv v_r^2 R/G$. The variable q has dimensions of mass and with a suitable multiplicative factor can be used as an estimator to the mass M. For a general distribution of test particles it turns out that the expectation value of q is

$$\langle q \rangle = \frac{\pi M}{32}(3 - 2\langle e^2 \rangle), \tag{172}$$

where $\langle e^2 \rangle$ is the expectation value of the square of the eccentricities of the particles orbits. Using this relation and the definition of q it is straightforward to arrive at a mass estimate of the form

$$M = \frac{C}{G}\frac{1}{N}\sum_i v_r^2 R_i, \tag{173}$$

where C is a constant of order unity depending on the test particles' eccentricities. Unlike the virial theorem method, the central value theorem applies to this method and guarantees that the sum will converge to the true mass M with an error proportional to $1/\sqrt{N}$ (Bahcall & Tremaine 1981).

Applying a modified form of equation (173) to 26 satellite galaxies of the Milky Way and 23 satellite galaxies of M31, Watkins et al. (2010) determine the masses of the two galaxies within 300 kpc from their centres to be $M_{\rm MW} \sim 3 \times 10^{12}\ M_\odot$ and $M_{\rm M31} \sim 1 \times 10^{12}\ M_\odot$. These values are rather volatile inasmuch as the exclusion of the satellite galaxies with ambiguous velocity and distance measures changes the mass estimates by a factor of roughly 2.

The value for $M_{\rm MW}$ is in good agreement with the study by Xue et al. (2008) in which 2401 blue horizontal branch (BHB) stars (which have high luminosities and nearly constant absolute magnitudes within a restricted colour range) from the SDSS are used to constrain the MW's circular velocity curve up to 60 kpc. From this the total mass within 60 kpc is determined and subsequently used to estimate the mass of the entire halo to be $M_{\rm MW} \sim 1 \times 10^{12}\ M_\odot$. Of course, Newton's theorem asserts that any mass outside of the limiting radius of 60 kpc will have no observational effect in a spherical or elliptical system and so estimating the halo mass in this way requires an initial assumption on the structure of the dark matter halo. Indeed, the estimate by Xue et al. (2008) is based on the assumption of an NFW halo profile.

Schommer et al. (1992) measure the velocities of individual stars in 83 star clusters in the LMC to arrive at a mass estimate for the galaxy. Using equation (173) they find the mass of the LMC to be $M_{\rm LMC} \sim 2 \times 10^{10}\ M_\odot$, roughly $1/100$ that of the Milky Way. They compare this to an estimate based on a rotation curve constructed from their cluster rotation data in addition to earlier

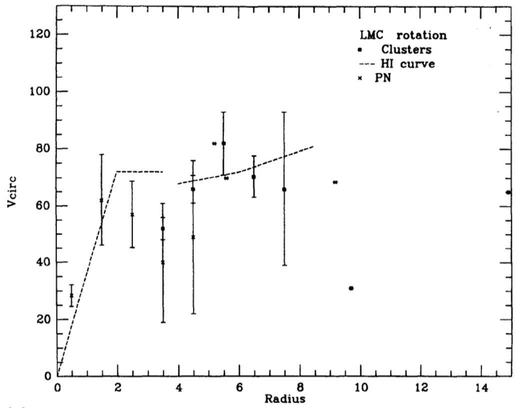

FIG. 29.— The rotation curve of the LMC built from rotational data of star clusters, H I, and PN. The dashed line shows the H I rotation curve whereas the points represent data from the clusters and PN. The error bars are the standard deviations of the mean of the velocities in the given bin while points with no error bars denote single objects. The bottom axis is in units of degrees and can be converted to physical lengths by noting that the LMC is 48 kpc from the MW. Image taken from Schommer et al. (1992).

data on the circular velocities of H I and planetary nebulae (PN) in the LMC. This is shown in Figure 29. To estimate the mass of the LMC from the rotation curve, one needs to compare the the circular velocity of the MW and the LMC at a distance of 8.5 kpc, the distance from the Sun to the centre of the MW. For the MW this value is roughly 200 km s^{-1} while for the LMC it is roughly 20 km s^{-1} since the circular velocity is roughly 20% larger than the rotation velocity (Weinberg 2000). Then since $M \propto Rv^2$ we have that $M_{\rm LMC} \sim 1 \times 10^{10}\, M_\odot$, in good agreement with the value above. This estimate would obviously be improved with more measurements of mass tracers at large radii. In principle, this can be achieved by H I though at such large radii its kinematics may be disturbed by hydrodynamical processes from tidal interactions (i.e. the Magellanic Stream).

Another quick check on the mass of the LMC is to investigate its tidal interactions with the MW. From the point of view of the MW, the LMC is an oversize globular cluster. Its tidal radius is measurable and depends both on the MW rotation curve and the LMC mass (and also weakly on the LMC mass profile). The tidal radius of the LMC can be estimated by observing the extent of its stellar halo. From this its mass is estimated via

$$M_{\rm LMC} = 2\left(\frac{r_t}{R_{\rm LMC}}\right)^3 M_{\rm MW}, \tag{174}$$

where r_t is the tidal radius and $R_{\rm LMC}$ is the distance to the LMC. This analysis is considered by Weinberg (2000) in which a tidal radius of 10.8 kpc is used to constrain the mass of the LMC at $M_{\rm LMC} \sim 2 \times 10^{10}\, M_\odot$.

Q1) Campbell ExtraGal Q2

1.2 Question 2

What is the total mass (in both dark matter and in stars) of the Milky Way galaxy? How does this compare to M31 and to the LMC? How is this mass determined?

1.2.1 Short answer

The MWG has a total mass of about $10^{12}\,\mathrm{M}_\odot$. This is about the same as M31 (Andromeda), and larger than the LMC which is roughly $10^{10}\,\mathrm{M}_\odot$. Assuming virial equlibrium, the mass can be determined from the flat part of the rotation curve via

$$V_0 = \sqrt{\frac{GM(<R)}{R_0}}\ [\mathrm{m\,s^{-1}}].$$

If spiral galaxies are being observed, the Tully-Fisher relation $L \propto v_{\mathrm{max}}^4$ can be used to determine the maximum orbital velocity in replacement of the rotation curve. Similarly, if elliptical galaxies are being observed, the Faber-Jackson relation $L \propto \sigma_v^4$ can be used instead.

1.2.2 Additional context

The components of the Milky Way Galaxy (MWG) have total masses as follows: a disk mass of $4.5 \times 10^{10}\,\mathrm{M}_\odot$, bulge mass of $4.5 \times 10^9\,\mathrm{M}_\odot$, dark halo mass of $2 \times 10^{12}\,\mathrm{M}_\odot$, and BH mass of $4 \times 10^6\,\mathrm{M}_\odot$.
The Galactic disk rotates, with rotational velocity $V(R)$ depending on the distance R from the center. We can estimate the mass of the Galaxy from the distribution of the stellar light and the mean mass-to-light ratio of the stellar population, since gas and dust represent less than $\sim 10\%$ of the mass of the stars. From this mass estimate we can predict the rotational velocity as a function of radius simply from Newtonian mechanics. However, the observed rotational velocity of the Sun around the Galactic center is significantly higher than would be expected from the observed mass distribution. If $M(< R_0)$ is the mass inside a sphere around the Galactic center with radius $R_0 = 8\,\mathrm{kpc}$, then the rotational velocity from Newtonian mechanics is

$$V_0 = \sqrt{\frac{GM(<R)}{R_0}}\ [\mathrm{m\,s^{-1}}].$$

From the visible matter in stars we would expect a rotational velocity of 160 km s, but we observe $V_0 = 220\,\mathrm{km\,s}$ (see Figure 3). This discrepancy, and the shape of the rotation curve $V(R)$ for larger distances R from the Galactic center, indicates that our Galaxy contains significantly more mass than is visible in the form of stars. This additional mass is called dark matter. Its physical nature is still unknown. The main candidates are weakly interacting elementary particles like those postulated by some elementary particle theories, but they have yet not been detected in the laboratory. Macroscopic objects (i.e., celestial bodies) are also in principle viable candidates if they emit very little light.

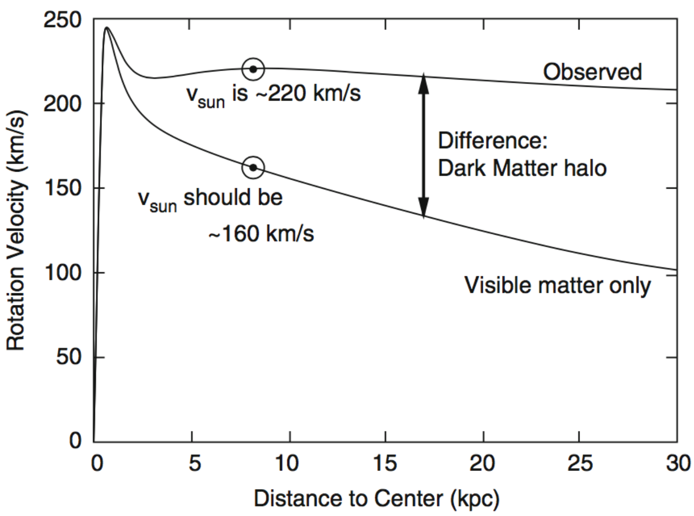

Figure 3: The upper curve is the observed rotation curve $V(R)$ of our Galaxy, i.e., the rotational velocity of stars and gas around the Galactic center as a function of their galactocentric distance. The lower curve is the rotation curve that we would predict based solely on the observed stellar mass of the Galaxy. The difference between these two curves is ascribed to the presence of dark matter, in which the Milky Way disk is embedded. This image is adapted from Nick Strobel's webpage at www.astronomynotes.com. Image taken from Schneider (2006).

Q1) Campbell ExtraGal Q2

1.2.3 Follow-up Questions

- If rotation curve/lensing measurements were instead due to modified gravity, how could we tell?
- What are Ω_m and Ω_b estimated to be? Why does the ratio of the two differ from the star/total mass ratio you have here? Where is all the extra baryonic mass?
- When you say we estimate stellar mass by "counting stars", what does that mean?

Q2) Nuclear Black Holes

What evidence is there that most galaxies contain nuclear black holes? How do those black holes interact with their host galaxies?

Q3) AGN

What are AGN? Describe different observational classes of them and how they may relate to each other.

Q4) Quasars

Draw a spectrum of a high-redshift quasar. What do quasar emission lines typically look like? Explain what we see in the spectrum at rest wavelengths bluer than 1216 Angstroms.

Q5) Methods of Galaxy Mass Determination

Describe three different methods used in the determination of the mass of a galaxy cluster.

Q6) SEDs of Single-Burst Galaxies

Draw the spectral energy distribution (SED) of a galaxy formed by a single burst of star formation at the ages of 10 Myrs, 2 Gyrs, and 10 Gyr. Please highlight the change over time in the 4000 Angstrom break.

Q7) Galactic Spiral Structure

What is galactic spiral structure and why is it thought to occur?

Q8) Stellar Initial Mass Funciton (IMF)

What is a stellar Initial Mass Function (IMF)? Explain how it is determined and how it is used.

Q9) Stellar Populations in the Galaxy

Characterize the stellar populations in the following regions: i) the Galactic bulge ii) the Galactic disk, outside of star clusters iii) open star clusters iv) globular clusters v) the Galactic halo

Q10) G-Dwarf Problem in the Solar Neighbourhood

What is the G-dwarf problem in the solar neighbourhood?

Q11) Stellar Orbits in Potential

Describe the orbits of stars in a galactic disk and in galactic spheroid.

Q12) Dynamical Relaxation

What is dynamical relaxation? Explain why this operates in star clusters but not in an elliptical galaxy.

Q13) Dynamical Friction

What is dynamical friction? Explain how this operates in the merger of a small galaxy into a large one.