This information is cribbed from Emberson (2012).

The tired light hypothesis was an attempt to reconcile the steady state theory of the universe with redshift. It posited that as light travels across cosmological distances, it loses energy at a rate

$$\frac{d(h\nu)}{dl} = -H_0 h\nu, \tag{27}$$

i.e. galaxies are not receding from us, but rather light itself "grows tired". The CMB, and to a lesser extent the nature of galaxies at high redshift, are both strong evidence against the static universe the tired light hypothesis is in support of.

A more direct argument against this hypothesis is that the flux received by an observer from a source of luminosity L should, in a tired light steady state universe, be $f = \frac{L}{4\pi d_c^2(1+z)}$, while in an expanding (curved) universe it is $f = \frac{L}{4\pi S_\kappa^2(1+z)^2}$; the second $1+z$ is due to the stretching out of a shell of radiation with thickness dr. This can be tested using standardizable candles, such as Cepheids and SNe Ia. Another direct argument is that in a tired light universe the angular size of an object should scale with the square of the co-moving distance, $d\Omega = dA/r^2$. In an expanding universe, however, this value should be $d\Omega = dA(1+z)^2/S_\kappa^2$ (which also means that surface brightness $dF/d\Omega \propto (1+z)^{-4}$ instead of remaining constant as in a static universe). Observations of elliptical galaxies at $z \approx 0.85$ have show that the angular size of objects $d\Omega$ is consistent with an expanding universe.

The steady state universe assumes the "perfect" cosmological principle, that the universe is spatially homogeneous and isotropic, as well as homogeneous in time. It does assume that the universe is expanding, though to keep the universe temporally homogeneous this would mean $\dot{a}/a = H_0$, i.e. the universe expands exponentially. To maintain the same density over time, matter is thought to be created *ex nihilo* throughout the universe, and the exponential expansion of the universe would allow distant radiation and stellar material to be removed from the observable universe.

A number of observational evidence speak against this model of the universe as well. Galaxies evolve with redshift - $z > 0.3$ galaxies in rich clusters tend to be bluer than their counterparts at low redshift, indicating that the universe has had a variable star formation rate throughout its history. Even more series is the CMB. It is conceivable that the CMB is the result of either the radiation field, or the emission of stars downscattered to microwaves by intergalactic dust. The first is difficult to believe because the sum contribution of emission from the creation field at all redshifts would have to precisely equal a blackbody. The second suffers from the same problem, as well as the issue that, because the universe is isotropic, this dust would have to exist locally. The densities required to produce a CMB would also require $\tau = 1$ by $z = 2$, meaning that we would not be able to see radio galaxies with $z > 2$. This is not the case, further invalidating the steady state universe.

1.7. *Question 6*

QUESTION: Sketch a graph of recession speed vs. distance for galaxies out to and beyond the Hubble distance.

This answer is cribbed from Emberson (2012), with a helpful dallop of Davis & Lineweaver (2004).

The expansion of the universe imparts a wavelength shift on the spectra of galaxies, denoted $z = \Delta\lambda/\lambda$. In the low-$z$ limit the relationship between z and proper distance is $c(1+z) = Hd$. This shift was originally attributed to a "recession velocity" (i.e. the galaxies are moving away from us at a velocity v) and called Hubble's Law:

$$v = H_0 d \tag{28}$$

From the RW metric, at any given moment in time an object has proper distance (from us) $d_p(t) = a(t)d_c$, giving us

$$\frac{d(d_p)}{dt} = \dot{a}d_c = H(t)d_p(t). \tag{29}$$

Therefore Hubble's Law is *universal*, in that given some time t the universe undergoes homologous expansion as given by $H(t)$. If we sketched a graph of recession speed vs. comoving distance (i.e. proper distance today), it would be a straight line!

We, however, cannot measure the co-moving distance, so a more reasonable relationship to draw would be the relationship between recessional velocity and redshift z. We can calculate this by noting that $v(t) = H(t)d_p(t) = \frac{\dot{a}}{a}ad_c = \dot{a}c\int_{t_{\mathrm{em}}}^{t_0}\frac{1}{a}dt'$. t is separate from both the emission and reception times of the photon, and this is because there is no unique recessional velocity (it changes over time!) that we could point to. Converting this expression to an integration over redshift ($da = -1/(1+z)^2dz = -dz/a^2$) and assuming $a_0 = 1$, we obtain

$$v(t,z) = \dot{a}\int_0^z \frac{c}{H}dz' \tag{30}$$

and if we assume $t = t_0$ (we want the *current*) recessional velocity, then $v(t_0, z) = \frac{1}{H_0}\int_0^z \frac{dz'}{H}$. This is plotted against z in Fig. 9.

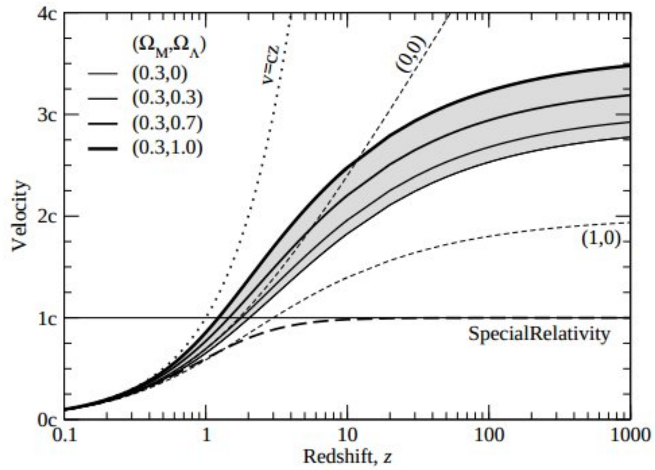

FIG. 9.— A plot of the recession velocity vs. redshift for several different FLRW cosmologies. How peculiar velocity would scale with redshift is also plotted. From Davis & Lineweaver (2004), their Fig. 2.

1.7.1. *How can we possibly see galaxies that are moving away from us at superluminal speeds?*

That nothing may move faster than the speed of light applies only to movement through space. Because it is space itself that expands, there is no restriction on maximum "speed" (more specifically $d_c da/dt$). Locally, galaxies (and light!) move in accordance with special relativity, but globally galaxies (and light!) can move away from us at superluminal speeds.

That being said, we can still see light being emitted by these objects from the distant past, so long as H is not constant! Suppose at $t_{\rm em}$ a photon is emitted by a galaxy at a critical distance $d_{p,\rm crit} = c/H$. The photon, while moving toward us in comoving space (and slowing down in comoving space) will initially have a fixed distance from us d_p, and will remain so as long as H is constant. If H decreases, however, which is the case for any universe accelerating less than exponentially, $d_{p,\rm crit}$ will increase, and d_p will be within the critical distance. The photon can then begin its journey toward us. In Fig. 10, notice how the light cone extends into the region where $d = c/H$.

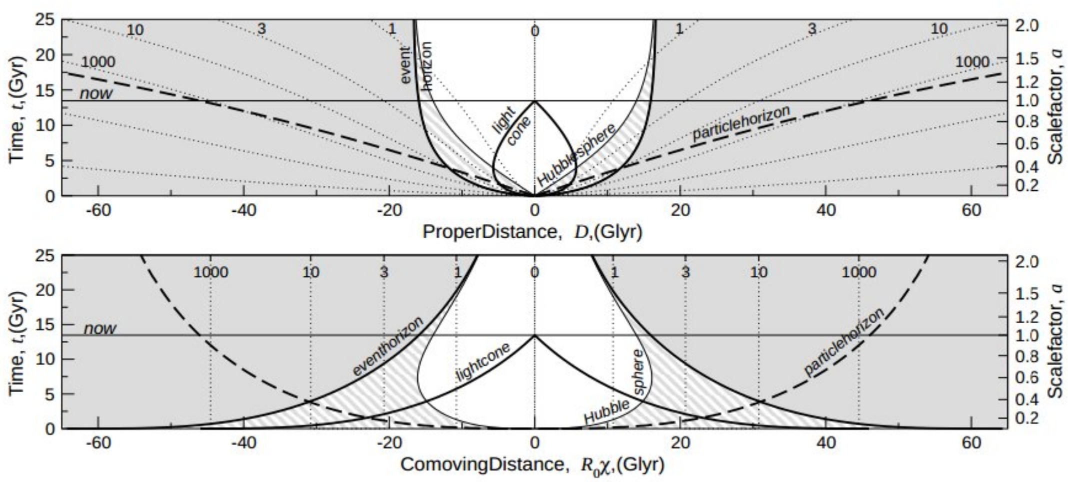

FIG. 10.— A space-time diagram of ΛCDM concordance cosmology. Above, time is plotted against proper distance, and below, time is plotted against comoving distance. "Light cone" represents the cone within which light could have reached us by now, and "event horizon" represents the furthest distance from which we will ever receive information on events at time t. The "particle horizon" represents the furthest distance objects that have ever been in in causal contact with us has gone. The "Hubble sphere" is the distance at which $d = c/H$, and the hatched region between the Hubble sphere and the event horizon represents events travelling superluminally away from us that we will one day be able to see. From Davis & Lineweaver (2004), their Fig. 1.

1.7.2. *Why can't we explain the Hubble flow through the physical motion of galaxies through space?*

If space were not expanding, but galaxies are moving away from us isotropically, then

$$v = c\frac{(1+z)^2 - 1}{(1+z)^2 + 1}. \tag{31}$$

If we assume $v = H_0 d_c$ applies to find the co-moving distance (in SR we have no way of accommodating further redshifting after photon emission, so we assume the galaxy still has the same velocity today, and follows the Hubble flow), we can use Eqn. 13 to determine the luminosity distance. We also use Eqns. 11 and 13 to determine the luminosity distance in GR. We compare this to the calculated luminosity distance using SNe Ia (any standardizable candle allows one to properly calculate the luminosity distance). The result is plotted in Fig. 11, and shows a clear bias against the special relativistic model. The reason why in the figure SR does even worse than Newtonian is simply because as $v \to c$, $d_c \to c/H_0$, resulting in a linear relationship between luminosity distance and redshift. This is not the case in either Newtonian or GR.

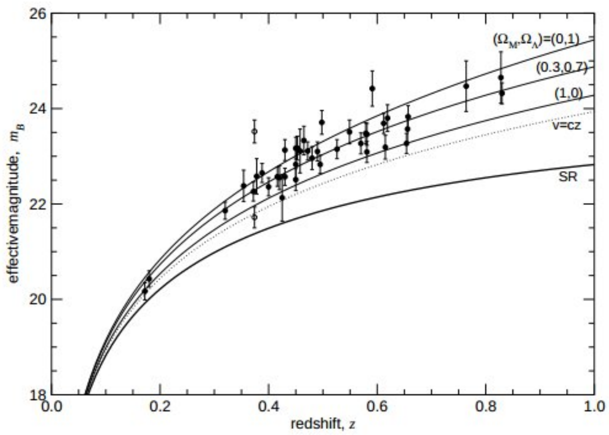

FIG. 11.— A plot of the magnitude-redshift relation, with a comparison between SR, Newtonian ($v = cz$) and several ΛCDM universes. Magnitude is calculated from luminosity distance. From Davis & Lineweaver (2004), their Fig. 5.

1.7.3. *Can galaxies with recession velocities $v > c$ slow down until $v < c$?*

Certainly! $v = H(t)d_p(t) = \dot{a}(t)d_c$, and therefore $v/c = \dot{a}(t)d_c/c$. For a matter-dominated universe $\dot{a}(t) \propto t^{-1/3}$ and therefore over time objects "slow down" (we cannot observe this, of course; light from these objects has yet to reach us!). This can be seen by the Hubble sphere expanding in Fig. 10.

1.8. *Question 7*

QUESTION: What happened in the first 3 minutes after the Big Bang? Why is only He (and tiny traces of Li) synthesized in the Big Bang?

A whole bunch of things happened in the first few minutes after the Big Bang, including inflation, CP symmetry breaking, neutrino decoupling. These features are summarized in Sec. 1.12. This question speaks mainly, however, of Big Bang nucleosynthesis (BBN).

The energy scale of BBN is set by the binding energy of nuclei - deuterium binding is about 10^5 times greater than the ionization energy of a hydrogen atom, and as a result BBN occured when $T \approx 4 \times 10^8$ K. The universe grew too cold to maintain such temperatures when it was only several minutes old.

The basic building blocks of matter are protons and neutrons. A free neutron has 1.29 MeV more energy than a proton, and 0.78 MeV more than a proton and electron. $n \to p + e^- + \bar{nu}_e$, then, is energetically (and entropically) highly favourable, and the half-life of a neutron is about 890 seconds.

At age $t = 0.1$ s, $T \approx 3 \times 10^{10}$ K, and the mean energy per photon was about $E \approx 10$ MeV, high enough to easily begin pair production. Neutrons and protons will be at equilibrium with each other via $n + \nu_e \leftrightharpoons p + e^-$ and $n + e^+ \leftrightharpoons p + \bar{nu}_e$, and given LTE, their densities will be given by the Maxwell-Boltzmann equation,

$$n = g\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{E}{k_B T}\right), \tag{32}$$