The Hubble parameter is given by Eqn. 9, and so BAO can constrain the (present-day) density ratios Ω_m, Ω_k, Ω_r and Ω_Λ. Having $H(z)$ is extremely important for determining the behaviour of dark energy over time (i.e. it is not obvious if the equation of state for dark energy does not change over time). Moreover, having both $d_A(z)$ and $H(z)$ allows a much greater constraint on possible cosmologies, as the two values are related to one another. For example, having d_A alone cannot constrain dark energy evolution due to a degeneracy between the dark energy EOS and Ω_k/a^2, but having both d_A and $H(z)$ immediately allows us to determine Ω_k. Theoretically, BAO can also constrain the growth of large-scale structure through changes in amplitude of the power spectrum.

1.14.1. *Why is BAO often used in conjunction with CMB?*

This answer comes verbatim from Emberson (2012).

The complementary probes of the CMB and galaxy clustering observations can be combined to break each others' parameter degeneracies in order to better constrain cosmological parameters. For instance, for a fixed primordial spectrum, increasing DM density shifts the matter power spectrum up to the right while shifting the CMB peaks down to the left. On the other hand, the addition of baryons boosts the amplitude of odd-numbered peaks in the CMB spectrum, but suppresses the power spectrum rightward of its peak as well as increasing its oscillation amplitude. Finally, increasing the abundance of hot dark matter (i.e. neutrinos) suppresses galaxy clustering on small scales while having essentially no effect on the CMB.

1.14.2. *What is the BAO equivalent of higher-l CMB peaks?*

I have no idea.

1.15. *Question 14*

QUESTION: Explain how weak lensing measurements can be used in the determination of cosmological parameters.

This information comes from Schneider (2006), Ch. 6.5 and 8.4.

Weak lensing is when a gravitational lens only moderately distorts the image of a background object. Typically, these background objects are at larger angles to the lens than strongly lensed objects. The distortion, or shear, is sufficiently small that it cannot be distinguished in a single image (since we do not know the true shape of any background object), but since the shear is over a large number of background objects, it can be detected statistically.

The distortion is known as a shear because it reflects the contribution of the tidal forces to the local gravitational field of the lens. The shear results from the derivative of the deflection angle (since the background object is tiny!), and the deflection angle is an integral over the surface density Σ of the lens,

$$\theta(\vec{\xi}) = \frac{4G}{c^2} \int \Sigma(\vec{\xi}') \frac{\vec{\xi}' - \vec{\xi}}{|\vec{\xi}' - \vec{\xi}|^2} d^2\xi \tag{45}$$

where we have projected the geometry of the system onto the sky (i.e. flattening along the radial axis) and are representing the 2D vector position of objects on the sky as $\vec{\xi}$. $\vec{\xi}'$ is the impact parameter vector of the light being lensed. measuring weak lensing, then, gives us a parameter-free method of determining the surface mass density of galaxy clusters, dark matter included. This method can also be used to search for clusters alongside the Sunayev-Zel'dovich effect.

According to Nick Tacik's qualifier notes, microlensing also features "convergence", which is a slight magnification of the background object.

There are a number of uses of weak lensing:

- **Cosmic shear** is microlensing due to the large-scale structure of the universe. This effect is extremely subtle (1% on angular scales of a few arcminutes). Mapping cosmic shear gives us statistical properties about the density inhomogeneities of the universe, much like galaxy surveys do. Indeed, we can determine the two-point correlation function of ellipticities, and relate this to the matter power spectrum $P(k)$. The matter power spectrum is directly related to cosmological parameters (Fig. 24). Microlensing is advantageous because no assumptions need to be made about whether or not dark matter and baryons track each other.

  The most significant result from cosmic shear has been Ω_m combined with the normalization σ_8 of the power spectrum. The two values are almost completely degenerate, and for an assumed $\Omega_m = 0.3$ we can obtain $\sigma_8 \approx 0.8$.

- One obvious cosmological usage of this is to determine the **mass to light ratio of galaxy clusters**, which places constraints on the baryon-dark matter ratio, if reasonable theoretical models for mass-to-light of baryonic objects can be created.

-

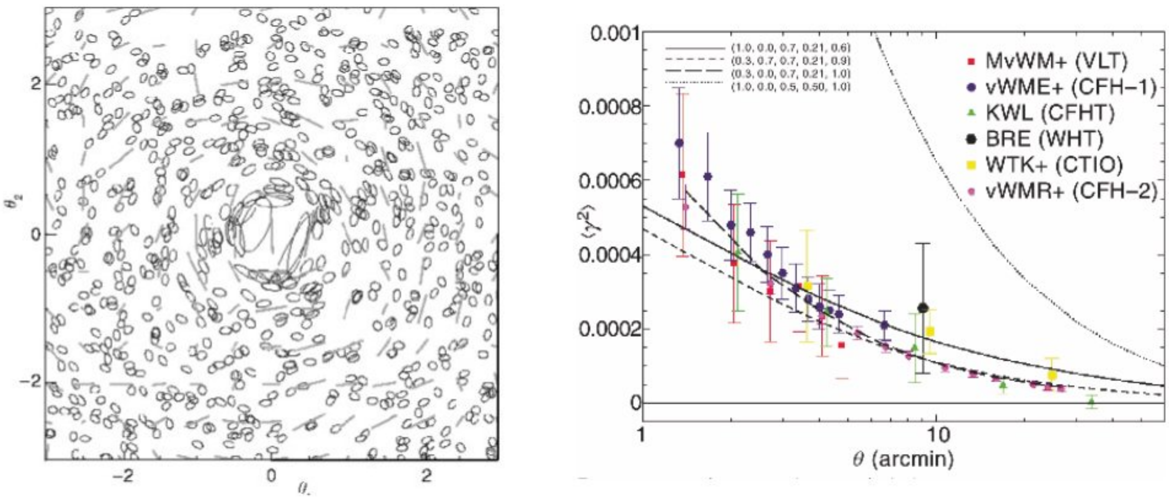

FIG. 24.— Left: computer simulation of gravitational microlensing of distant galaxies. Right: measurement of cosmic shear dispersion as a function of angular scale from multiple teams, overplotted on shear dispersion curves of universes with varying properties (labelled on the upper left; numbers mean Ω_m, Ω_Λ, h, shape parameter Γ and power spectrum normalization σ_8). From Schneider (2006), his Fig. 6.35 and 8.15.

1.15.1. *How is weak lensing measured?*

This information comes from Schneider (2006), Ch. 6.5.

To measure weak lensing we require a large number of well-resolved background objects, meaning we need a deep and wide image. Systematic observations of weak lensing have only become feasible in recent years due to the development of wide-field cameras, improvement of the dome seeing at many telescopes and development of dedicated analysis software.

Because measurement of cosmic shear requires high precision, one major source of error is actually insufficient knowledge of the redshift distribution of background galaxies needed for microlensing. High-redshift galaxy surveys are therefore needed to help reduce errors.

1.15.2. *Can strong lensing be used to determine cosmological parameters?*

Strong lensing tends to generate multiple images (culminating in an Einstein Ring if the source is directly behind the lens), and the photons from each image have different travel times (due to both having to move through a gravitational potential well and due to the geometric differences between different paths). The differences in travel time Δt can be measured because luminosity variations of the source are observed at different times in different images. Δt linearly scales with the current size of the universe, and therefore scales with H_0^{-1} due to the Hubble law. See Fig. 25.

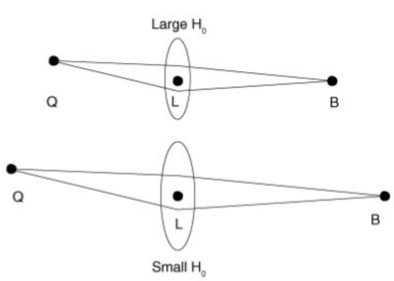

FIG. 25.— Schematic of how Δt can constrain H_0. Above is a large H_0 universe, while below is a small H_0 universe (larger because $v = H_0 d$), with all other observables kept constant. The Δt is larger for the small H_0 universe, and hence is $\propto H_0^{-1}$. From Schneider (2006), his Fig. 3.44.

1.16. *Question 15*

QUESTION: Describe cosmological inflation. List at least three important observations which it is intended to explain.