QUESTION: What is recombination? At what temperature did it occur? How does this relate to the ionization potential of Hydrogen?

Most of this information comes from Ryden (2003), filtered through Emberson (2012). Subscript 0 will represent present-day values.

Recombination is when the universe cooled to the point at which protons combined with electrons to form hydrogen atoms. The resulting massive decrease in opacity caused the universe to become optically thin, and the photon field of the universe decoupled from its matter counterpart[2]. Recombination does not refer to a single event or process: the epoch of recombination is the time at which the baryonic component of the universe went from being ionized to being neutral (numerically, one might define it as the instant in time at which the number density of ions is equal to the number density of neutral atoms). The epoch of photon decoupling is the time at which the rate at which photons scatter from electrons becomes smaller than the Hubble parameter (at the time). When photons decouple, they cease to interact with the electrons, and the universe becomes transparent. Third, the epoch of last scattering is the time at which a typical CMB photon underwent its last scattering from an electron. The three processes are related because hydrogen opacity is the driver of all three.

For simplification, let us assume the universe is made completely of hydrogen atoms. ^{4}He has a higher first ionization energy significantly higher than that of H, and therefore helium recombination would have occured at an earlier time.

The degree of H ionization is determined by the Saha equation, which can be derived (especially for H, where it is easy) from the grand partition function Ξ (Carroll & Ostlie 2006),

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-\chi_i/k_B T} \tag{15}$$

where i indicates the degree of ionization, χ_i is the ionization energy from degree i to degree $i+1$. Suppose we ignore excited internal energy states (note that Ryden does this, but does not make it explicit); then $Z_{H^+} = Z_p = g_p = 2$ and $Z_H = g_H = 4$, for all possible spin states of the nucleus and electron. This gives us (multiplying the lefthand side of Eqn. 15 by V/V)

$$\frac{n_H}{n_p} = n_e\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-3/2} e^{\chi/k_B T} \tag{16}$$

where $\chi = 13.6$ eV and $n_e = n_p$. Using the fact that the number of photons is $0.243\left(\frac{k_B T}{\hbar c}\right)^3$ and $n_p = n_\gamma \eta$, with $\eta \approx 5.5\times10^{-10}$ (this does not change by much throughout the era of recombination), we may eliminate n_e and solve for when the left side is equal to one, i.e. when X, the ionization fraction, is equal to 1/2. This gives us $T = 3740$ K, $z = 1370$ (0.24 Myr for a matter dominated and flat universe). Past this temperature, photons became too cold to ionize H. The evolution of X with redshift is given in Fig. 3.

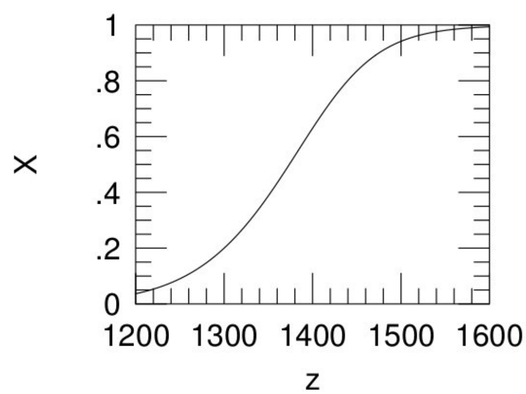

FIG. 3.— Change in ionized fraction X as a function of redshift. From Ryden (2003), her Fig. 9.4.

1.2.1. *Couldn't photons have decoupled from baryons before recombination?*

Photons are coupled to baryons through photoionization and recombination, though in the very hot universe the dominant interaction would have been Thomson scattering off free electrons, with a rate given by $\Gamma = n_e\sigma_e c$. Values can be calculated using $n_e = \frac{0.22\mathrm{m}^3}{a^3}$ (if $a = 1$ today) and $\sigma_e = 6.65\times10^{-29}$. Photons decouple (gradually) from baryons when Γ exceeds H, equivalent to saying the mean free path λ exceeds c/H. The critical point $\Gamma = H$ occured at $z \approx 42$, $T \approx 120$ K, long past recombination.

[2] Since atoms were always ionized before this, "recombination" is almost a misnomer!

If we perform the same calculation during the era of recombination, setting n_e using the analysis above and obtaining H from $H(t) = H_0\Omega_m \frac{a_0^3}{a^3} = H_0\Omega_m \frac{1}{(1+z)^3}$, we obtain $z \approx 1100$ and $T \approx 3000$ (exact answers are difficult without modelling, since during the final stages of recombination the system was no longer in LTE).

1.2.2. *What is the last scattering surface?*

The last scattering surface is the $\tau = 1$ surface for photons originally trapped in the optically thick early universe. The age $t_0 - t$ of this surface can be found using

$$\tau = 1 = \int_t^{t_0} \Gamma(t)dt \tag{17}$$

In practice this is difficult, and so we again estimate that $z \approx 1100$ for last scattering.