1.17.1. *Is the anthropic principle a scientifically or logically valid argument?*

Page claims that one variant of the anthropic principle, that cosmological principles are fine-tuned to maximize the amount/chance of life coming into being, is in principle testable (at least theoretically through modelling different universes).

My issue with the anthropic principle is that it mistakes cause for effect. The fact that we are here to observe the universe requires that $\Lambda_t - \Lambda_v = \Lambda_o$, not the other way around. The reverse would be true only if the existence of the universe requires observers that think like us, which in itself requires a corroborating physical principle (ex. collapse of the "existence" wave function requires a specific "observer" operator that corresponds to physical "intelligence"). If this were the case, the anthropic principle would simply be a stepping stone to a physical principle that sets the coefficients of the universe, and not a final explanation.

1.18. *Question 17*

QUESTION: Define the two-point correlation function. How is it related to the power spectrum? How is the C_l spectrum of the CMB related to low redshift galaxy clustering?

Most of this information comes from Schneider (2006).

Suppose we chose a patch of space dV centred at a point in space $\vec{x}$; we wish to determine the probability that we will find a galaxy in dV. We cannot (due to chaos) actually describe the density structure of the universe except on a statistical level, which in this case means the probability of finding a galaxy in dV centred on $\vec{x}$, averaged over all possible universes with the same statistical properties (this washes out random inhomogeneities; see **?**, pg. 282). If the universe were statistically homogeneous, then this probability would be $P_1 = \bar{n}dV$ (i.e. without gravity, all inhomogeneities are random), where $\bar{n}$ is the average number density of galaxies in the universe. We now consider the probability that a galaxy will be found in dV centred around $\vec{x}$ *and* a galaxy will be found in dV centred around another point $\vec{y}$ (again, in the statistical sense). If galaxy scattering were completely uncorrelated, then we would obtain

$$P = \bar{n}^2 dV^2 \tag{48}$$

i.e. uncorrelated probabilities simply multiply. Since galaxies gravitationally cluster, however, this cannot be the case. We therefore modify the above equation into

$$P = \bar{n}^2 dV^2 (1 + \xi(\vec{x}, \vec{y})) \tag{49}$$

The term $\xi(\vec{x}, \vec{y})$ is known as the two-point correlation function. Correspondingly, the correlation function for matter density $\langle \rho(\vec{x})\rho(\vec{y}) \rangle = \bar{\rho}(1 + \xi(\vec{x}, \vec{y}))$. If the universe obeyed the cosmological principle, then $\xi(\vec{x}, \vec{y}) = \xi(r)$, where $r = |\vec{x} - \vec{y}|$.

$\xi(r)$ is related to the power spectrum $P(k)$ by a spherical Fourier transform, as noted in Sec. 1.4:

$$P(k) = \int_{-\infty}^{-\infty} \xi(r) exp(-ikr) r^2 dr \tag{50}$$

(this is equivalent to $P(k) = 2\pi \int_{-\infty}^{-\infty} \xi(r) \frac{\sin(kr)}{kr} r^2 dr$). We recall that this is the method by which we can determine the matter power spectrum of the universe (if dark matter and baryons are coupled). Fig. **??** shows the relationship between the two functions.

We can now see the primary connection between the two-point correlation function and the CMB power spectrum, since there is an intimate connection between the matter power spectrum $P(k)$ and the CMB power spectrum Δ_T. Perhaps most importantly, the first peak of the CMB anisotropy spectrum corresponds to the sound horizon at last scattering. The co-moving preferred length of galaxy clustering as measured from the two-point correlation function, ∼140 Mpc, is a relic of the stalled baryon oscillation at the sound horizon at last scattering. In general, how the initial Harrison-Zel'dovich spectrum shifts over time to the current matter power spectrum depends on the expansion history of the universe and the fraction of baryons to dark matter. The expansion history depends on Ω_m, Ω_k and Ω_r, which all affect the CMB. Ω_b/Ω_c changes the amplitude of the BAO, as well as the height of the first peak of the CMB.

The CMB power spectrum has also been modified by secondary anisotropies, many of which are related to galaxy clustering. The integrated Sachs-Wolfe effect, for example, is due to the growth of dark matter halos, which also is what seeds the formation of galaxy clusters. At small angular scales, the image of the CMB is warped by microlensing from particuarly concentrated halos. Hot gas in the centres of galaxy clusters inverse Compton-scatter CMB photons (the Sunayev-Zel'dovich effect), which changes the overall thermal structure of the CMB along the line-of-sight to these clusters.

1.18.1. *Describe galaxy surveys.*

This information is from Schneider (2006), Ch. 8.1.

With photometric sky surveys, the two-dimensional distribution of galaxies on the sphere can be mapped. To also determine the third spatial coordinate, it is necessary to measure the redshift of the galaxies using spectroscopy, deriving the distance from the Hubble law (or some model of $a(t)$). Actually performing such (spectroscopic!) surveys is daunting, and was not practical until the advent of CCDs and high-multiplexity spectrographs, which could take spectra of several thousand objects simultaneously. Modern surveys are defined by two parameters: the angular size of the sky covered, and the brightness cutoff of objects being observed.

Major galaxy surveys include the two-degree Field Galaxy Redshift Survey (2dFGRS, or 2dF for short), which covered approximately 1500 square degrees and objects with $B \lesssim 19.5$, and the Sloan Digital Sky Survey (SDSS), which covered a quarter of the sky, with a limiting surface brightness of about 23 magnitudes/arcsec^2 (SDSS 2012).

1.18.2. *What about three or higher point correlation functions?*

This information is from Schneider (2006), pg. 282.

Higher point correlation functions can be defined in the same manner that we have defined the two point correlation function. It can be shown that the statistical properties of a random field are fully specified by the set of all n-point correlations. Observationally, these functions are significantly harder to map, though.

1.19. *Question 18*

QUESTION: Consider a cosmological model including a positive cosmological constant. Show that, in such a model, the expansion factor eventually expands at an exponential rate. Sketch the time dependence of the expansion factor in the currently favoured cosmological model.

This question as been entirely answered in Secs. 1.1.4 and 1.10. Followups can be found there as well.

1.20. *Question 19*

QUESTION: Define and describe the epoch of reionization. What are the observational constraints on it?

This information is from a collection of sources (mostly Schneider (2006)); see my other document for details.

After recombination (3.8×10^5 yrs after the Big Bang) the vast majority of matter in the universe was neutral hydrogen. The epoch of reionization is the period in the universe's history over which the matter in the universe became ionized again. An understanding of reionization is important because of the role reionization plays in large-scale structure formation. The nature of reionization is directly linked to the nature of the reionizing sources, the first stars and active galactic nuclei in the universe (studies could shed light into everything from the early stages of metal enrichment in the uinverse and the clumpiness of the IGM, to the formation of the first supermassive black holes). Moreover, IGM ionization and temperature regulate galaxy formation and evolution.

It is currently believed that the first several generations of stars were the primary produces of photoionizing radiation during the epoch of reionization. The pre-population III star universe was a metal-free environment, meaning that the only methods of cooling involved H and He. All collapsing dark matter halos in the early universe had small masses, corresponding to low virial temperatures on the order of 10^3 K. Only H_2 emission cools baryons at $T_{\rm vir} \approx 10^3$ K efficiently; as a result, the baryon clouds formed $\gtrsim 100$ $M_\odot$ stars. When the baryon clouds corresponding to these small CDM halos collapse to form population III stars, the immediate regions surrounding these halos is ionized by the stars' radiation. Moreover, the radiation dissociates most of the H_2 in the universe (the dissociation energy is 11.3 eV, below the 13.6 eV minimum absorption energy of neutral ground-state H, so H_2-dissociating photons can travel without impedement) and prevent further star formation. After they die, population III stars seed the IGM with metals, which provide cooling for gas at much higher virial temperatures. Therefore baryons associated with larger halos can now collapse to form stars. The greater volume of ionizing photons from these stars begins ionizing more and more of the IGM. Eventually different expanding patches of ionized IGM merge, greatly accelerating reionization (since photos can now freely travel between bubbles). Eventually the last regions of the universe become fully ionized. This process can be seen in cartoon form in Fig. 26.

Because the recombination time for the IGM becomes longer than a Hubble time at z $\sim$ 8, very complicated reionization histories are allowed, and it is unclear exactly how longer reionization took. We know that reionization must occur over a time period of longer than $\Delta z > 0.15$, but it is very likely much longer.

Despite the fact that the universe is now fully ionized, the average density of HII in the universe is too low for the universe to again become opaque; assuming the universe somehow remained ionized *ad infinitum*, it would still have become transparent after 20 Myr.

Observational constraints on reionization include:

- **Ly-α observations** constrain the redshift at which recombination must have ended. Ly-α absorption lines (and, at high column densities, continuum absorption due to the Lyman limit at 912 Å) are created in quasar spectra by