Most of this information comes from Bassett & Hlozek (2010).

Oscillations of the baryon-photon fluid occur, as noted in Sec. 1.13, because the fluid is attracted to matter-energy overdensities. Material streaming into these overdensities become adiabatically compressed, leading to their expansion. As this expansion is also adiabatic, the material cools as it expands, eventually losing radiation support. It then begins to re-collapse.

This picture describes a parcel of material being oscillated in the plasma. The *wave itself*, however, manifests as an outgoing overdensity of baryons, photons and neutrinos travelling at the sound speed $c_s = 0.577c$.

Consider an overdensity of photons, baryons and dark matter. The photon-baryon fluid has high overpressure and as a result photons and baryons (and neutrinos) launch a sound wave into the surrounding medium. The dark matter mainly stays in place, though is partly gravitationally affected by the outgoing wave (see Sec. 2.2 of Eisenstein et al. 2007 for details). The soundwave travels outward until recombination occurs, at which point the photons and baryons decouple. The photons become free to propagate outward, and become the first peak of the CMB, while the baryons, no longer affected by outward pressure, stall at the sound horizon at recombination, r_H. Over time, gravitational attraction to the central dark matter overdensity drives most baryons into the centre of the original overdensity. Since gravitational attraction works both ways, dark matter also clumps r_H, which then becomes permanently etched in the large scale structure of the universe.

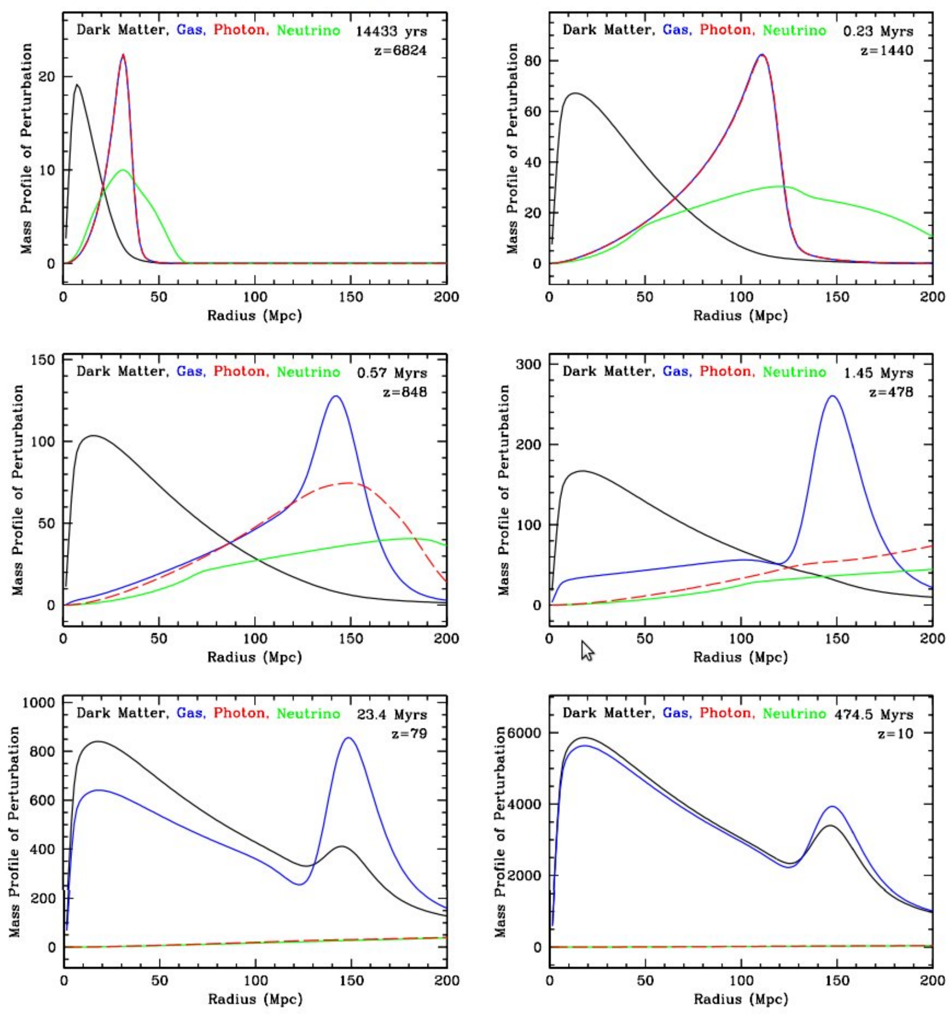

FIG. 21.— A series of snapshots of evolution of an overdensity of point-like initial overdensity which is present in baryons, dark matter, neutrinos and photons. On the y-axis is radial mass profile, and on the x-axis comoving radius. The overpressure of the baryon-photon fluid immediately drives an outgoing sound wave. After recombination the photons and neutrinos stream away, while the baryons, having lost outward pressure support, stall at a characteristic distance from the initial overdensity. Gravitationaly collapse now occurs, and most of the baryons stream back into the dark matter overdensity, but the characteristic bump at large distance is maintained. From Eisenstein et al. (2007), their Fig. 1.

There were many such perturbations in the photon-baryon fluid before recombination, and so we expect many such outgoing waves. Since at high redshift perturbations can adequately be described by linear theory, we can simply add all these waves up to form a large series of outgoing waves overlapping each other. Statistically, then, there would still be a preferred length, r_H (comoving) at which galaxies prefer to be separated.

The preferred length scale can be measured using the two-point correlation function $\xi(r)$. The correlation function and the power spectrum $P(k)$ are related by the radial Fourier transform $P(k) = \int_{-\infty}^{-\infty} \xi(r) exp(-ikr) r^2 dr$. A δ-function in $\xi(r)$ due to the preferred length scale therefore results in a series of oscillations in $P(k)$. These are what are commonly called the baryon acoustic oscillations, or BAO (see Fig. 23).

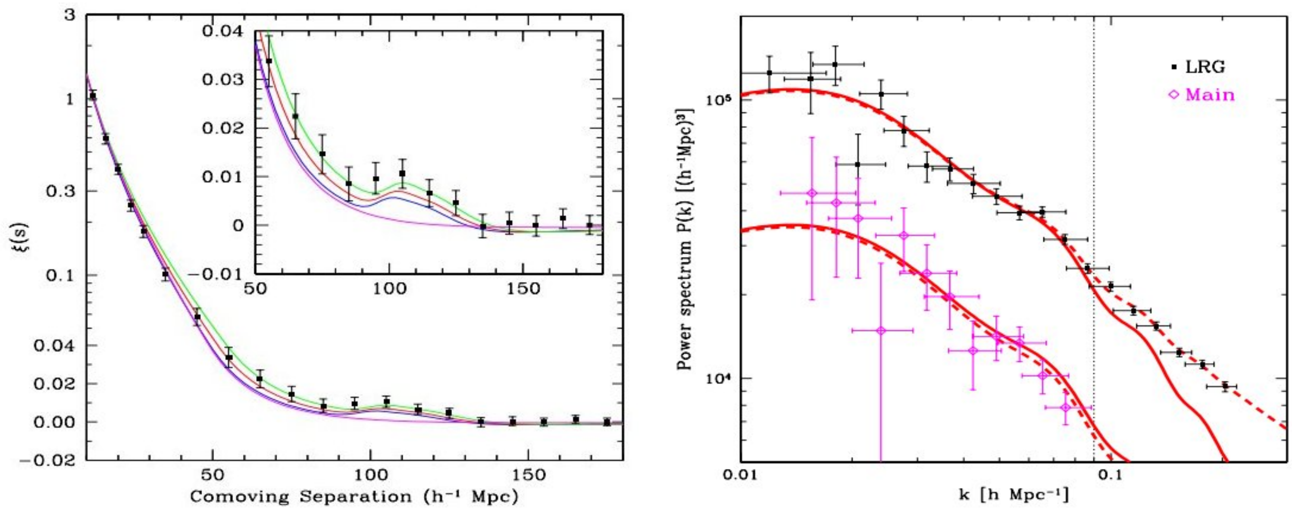

FIG. 22.— Left: the baryon acoustic peak in the two-point correlation spectrum ξ. Points are from the SDSS LRG sample, and solid lines represent models with $\Omega_m h^2 =$ (from top to bottom) 0.12, 0.13, 0.14 and 0 ($Omega_b h^2 = 0.024$ for all). Right: baryon acoustic oscillations in the SDSS power spectrum. Magneta points are the main SDSS sample, and black points are the LRG sample. Lines represent ΛCDM fits to the WMAP3 data, while the dashed lines indicate non-linear corrections. From Bassett & Hlozek (2010), their Figs. 1.1 and 1.2.

Suppose we had the results of galaxy clustering (in Fourier space using spherical coordinates) from a large galaxy survey. If we decompose the results into transverse (θ, ϕ) and radial (r) modes, we can inverse Fourier-transform these modes back into physical space to determine the preferred radial and transverse lengths galaxies cluster at (see Bassett & Hlozek 2010, pg. 12, for complications), represented by Δz and $\Delta\theta$, respectively. As seen in Fig. 23, we can measure

$$H(z) = \frac{c\Delta z}{s_\parallel} \tag{43}$$

and[4]

$$d_A(z) = \frac{s_\perp}{(1+z)\Delta\theta} \tag{44}$$

because we know the co-moving distance $s_\parallel = s_\perp$ from the r_H at last scattering and $z_{\rm recomb} \approx 1100$ from CMB measurements. We can then perform a simultaneous measurement of the Hubble parameter and the angular distance at a given redshift.

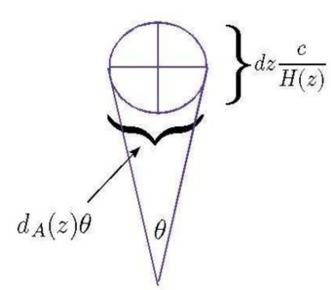

FIG. 23.— A schematic of what cosmological parameters can be measured from a spherical object of known radius. From Bassett & Hlozek (2010), their Fig. 1.6.

[4] Bassett & Hlozek (2010) write $s_\parallel(z)$ instead of $s_\parallel$. I am not sure why.

The Hubble parameter is given by Eqn. 9, and so BAO can constrain the (present-day) density ratios Ω_m, Ω_k, Ω_r and Ω_Λ. Having $H(z)$ is extremely important for determining the behaviour of dark energy over time (i.e. it is not obvious if the equation of state for dark energy does not change over time). Moreover, having both $d_A(z)$ and $H(z)$ allows a much greater constraint on possible cosmologies, as the two values are related to one another. For example, having d_A alone cannot constrain dark energy evolution due to a degeneracy between the dark energy EOS and Ω_k/a^2, but having both d_A and $H(z)$ immediately allows us to determine Ω_k. Theoretically, BAO can also constrain the growth of large-scale structure through changes in amplitude of the power spectrum.

1.14.1. *Why is BAO often used in conjunction with CMB?*

This answer comes verbatim from Emberson (2012).

The complementary probes of the CMB and galaxy clustering observations can be combined to break each others' parameter degeneracies in order to better constrain cosmological parameters. For instance, for a fixed primordial spectrum, increasing DM density shifts the matter power spectrum up to the right while shifting the CMB peaks down to the left. On the other hand, the addition of baryons boosts the amplitude of odd-numbered peaks in the CMB spectrum, but suppresses the power spectrum rightward of its peak as well as increasing its oscillation amplitude. Finally, increasing the abundance of hot dark matter (i.e. neutrinos) suppresses galaxy clustering on small scales while having essentially no effect on the CMB.

1.14.2. *What is the BAO equivalent of higher-l CMB peaks?*

I have no idea.

1.15. *Question 14*

QUESTION: Explain how weak lensing measurements can be used in the determination of cosmological parameters.

This information comes from Schneider (2006), Ch. 6.5 and 8.4.

Weak lensing is when a gravitational lens only moderately distorts the image of a background object. Typically, these background objects are at larger angles to the lens than strongly lensed objects. The distortion, or shear, is sufficiently small that it cannot be distinguished in a single image (since we do not know the true shape of any background object), but since the shear is over a large number of background objects, it can be detected statistically.

The distortion is known as a shear because it reflects the contribution of the tidal forces to the local gravitational field of the lens. The shear results from the derivative of the deflection angle (since the background object is tiny!), and the deflection angle is an integral over the surface density Σ of the lens,

$$\theta(\vec{\xi}) = \frac{4G}{c^2}\int \Sigma(\vec{\xi}')\frac{\vec{\xi}' - \vec{\xi}}{|\vec{\xi}' - \vec{\xi}|^2}d^2\xi \tag{45}$$

where we have projected the geometry of the system onto the sky (i.e. flattening along the radial axis) and are representing the 2D vector position of objects on the sky as $\vec{\xi}$. $\vec{\xi}'$ is the impact parameter vector of the light being lensed. measuring weak lensing, then, gives us a parameter-free method of determining the surface mass density of galaxy clusters, dark matter included. This method can also be used to search for clusters alongside the Sunayev-Zel'dovich effect.

According to Nick Tacik's qualifier notes, microlensing also features "convergence", which is a slight magnification of the background object.

There are a number of uses of weak lensing:

- **Cosmic shear** is microlensing due to the large-scale structure of the universe. This effect is extremely subtle (1% on angular scales of a few arcminutes). Mapping cosmic shear gives us statistical properties about the density inhomogeneities of the universe, much like galaxy surveys do. Indeed, we can determine the two-point correlation function of ellipticities, and relate this to the matter power spectrum $P(k)$. The matter power spectrum is directly related to cosmological parameters (Fig. 24). Microlensing is advantageous because no assumptions need to be made about whether or not dark matter and baryons track each other.

  The most significant result from cosmic shear has been Ω_m combined with the normalization σ_8 of the power spectrum. The two values are almost completely degenerate, and for an assumed $\Omega_m = 0.3$ we can obtain $\sigma_8 \approx 0.8$.

- One obvious cosmological usage of this is to determine the **mass to light ratio of galaxy clusters**, which places constraints on the baryon-dark matter ratio, if reasonable theoretical models for mass-to-light of baryonic objects can be created.

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