This information is cribbed from Emberson (2012).

The tired light hypothesis was an attempt to reconcile the steady state theory of the universe with redshift. It posited that as light travels across cosmological distances, it loses energy at a rate

$$\frac{d(h\nu)}{dl} = -H_0 h\nu, \tag{27}$$

i.e. galaxies are not receding from us, but rather light itself "grows tired". The CMB, and to a lesser extent the nature of galaxies at high redshift, are both strong evidence against the static universe the tired light hypothesis is in support of.

A more direct argument against this hypothesis is that the flux received by an observer from a source of luminosity L should, in a tired light steady state universe, be $f = \frac{L}{4\pi d_c^2(1+z)}$, while in an expanding (curved) universe it is $f = \frac{L}{4\pi S_\kappa^2(1+z)^2}$; the second $1+z$ is due to the stretching out of a shell of radiation with thickness dr. This can be tested using standardizable candles, such as Cepheids and SNe Ia. Another direct argument is that in a tired light universe the angular size of an object should scale with the square of the co-moving distance, $d\Omega = dA/r^2$. In an expanding universe, however, this value should be $d\Omega = dA(1+z)^2/S_\kappa^2$ (which also means that surface brightness $dF/d\Omega \propto (1+z)^{-4}$ instead of remaining constant as in a static universe). Observations of elliptical galaxies at $z \approx 0.85$ have show that the angular size of objects $d\Omega$ is consistent with an expanding universe.

The steady state universe assumes the "perfect" cosmological principle, that the universe is spatially homogeneous and isotropic, as well as homogeneous in time. It does assume that the universe is expanding, though to keep the universe temporally homogeneous this would mean $\dot{a}/a = H_0$, i.e. the universe expands exponentially. To maintain the same density over time, matter is thought to be created *ex nihilo* throughout the universe, and the exponential expansion of the universe would allow distant radiation and stellar material to be removed from the observable universe.

A number of observational evidence speak against this model of the universe as well. Galaxies evolve with redshift - $z > 0.3$ galaxies in rich clusters tend to be bluer than their counterparts at low redshift, indicating that the universe has had a variable star formation rate throughout its history. Even more series is the CMB. It is conceivable that the CMB is the result of either the radiation field, or the emission of stars downscattered to microwaves by intergalactic dust. The first is difficult to believe because the sum contribution of emission from the creation field at all redshifts would have to precisely equal a blackbody. The second suffers from the same problem, as well as the issue that, because the universe is isotropic, this dust would have to exist locally. The densities required to produce a CMB would also require $\tau = 1$ by $z = 2$, meaning that we would not be able to see radio galaxies with $z > 2$. This is not the case, further invalidating the steady state universe.

1.7. *Question 6*

QUESTION: Sketch a graph of recession speed vs. distance for galaxies out to and beyond the Hubble distance.

This answer is cribbed from Emberson (2012), with a helpful dallop of Davis & Lineweaver (2004).

The expansion of the universe imparts a wavelength shift on the spectra of galaxies, denoted $z = \Delta\lambda/\lambda$. In the low-$z$ limit the relationship between z and proper distance is $c(1+z) = Hd$. This shift was originally attributed to a "recession velocity" (i.e. the galaxies are moving away from us at a velocity v) and called Hubble's Law:

$$v = H_0 d \tag{28}$$

From the RW metric, at any given moment in time an object has proper distance (from us) $d_p(t) = a(t)d_c$, giving us

$$\frac{d(d_p)}{dt} = \dot{a}d_c = H(t)d_p(t). \tag{29}$$

Therefore Hubble's Law is *universal*, in that given some time t the universe undergoes homologous expansion as given by $H(t)$. If we sketched a graph of recession speed vs. comoving distance (i.e. proper distance today), it would be a straight line!

We, however, cannot measure the co-moving distance, so a more reasonable relationship to draw would be the relationship between recessional velocity and redshift z. We can calculate this by noting that $v(t) = H(t)d_p(t) = \frac{\dot{a}}{a}ad_c = \dot{a}c\int_{t_{\rm em}}^{t_0}\frac{1}{a}dt'$. t is separate from both the emission and reception times of the photon, and this is because there is no unique recessional velocity (it changes over time!) that we could point to. Converting this expression to an integration over redshift ($da = -1/(1+z)^2dz = -dz/a^2$) and assuming $a_0 = 1$, we obtain

$$v(t,z) = \dot{a}\int_0^z \frac{c}{H}dz' \tag{30}$$