1.7.2. *Why can't we explain the Hubble flow through the physical motion of galaxies through space?*

If space were not expanding, but galaxies are moving away from us isotropically, then

$$v = c\frac{(1+z)^2 - 1}{(1+z)^2 + 1}. \tag{31}$$

If we assume $v = H_0 d_c$ applies to find the co-moving distance (in SR we have no way of accommodating further redshifting after photon emission, so we assume the galaxy still has the same velocity today, and follows the Hubble flow), we can use Eqn. 13 to determine the luminosity distance. We also use Eqns. 11 and 13 to determine the luminosity distance in GR. We compare this to the calculated luminosity distance using SNe Ia (any standardizable candle allows one to properly calculate the luminosity distance). The result is plotted in Fig. 11, and shows a clear bias against the special relativistic model. The reason why in the figure SR does even worse than Newtonian is simply because as $v \to c$, $d_c \to c/H_0$, resulting in a linear relationship between luminosity distance and redshift. This is not the case in either Newtonian or GR.

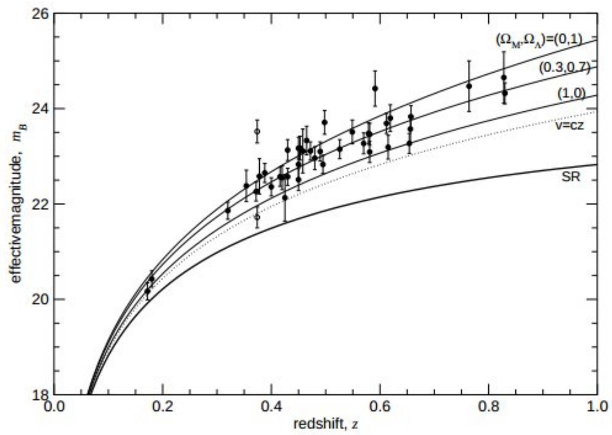

FIG. 11.— A plot of the magnitude-redshift relation, with a comparison between SR, Newtonian ($v = cz$) and several ΛCDM universes. Magnitude is calculated from luminosity distance. From Davis & Lineweaver (2004), their Fig. 5.

1.7.3. *Can galaxies with recession velocities $v > c$ slow down until $v < c$?*

Certainly! $v = H(t)d_p(t) = \dot{a}(t)d_c$, and therefore $v/c = \dot{a}(t)d_c/c$. For a matter-dominated universe $\dot{a}(t) \propto t^{-1/3}$ and therefore over time objects "slow down" (we cannot observe this, of course; light from these objects has yet to reach us!). This can be seen by the Hubble sphere expanding in Fig. 10.

1.8. *Question 7*

QUESTION: What happened in the first 3 minutes after the Big Bang? Why is only He (and tiny traces of Li) synthesized in the Big Bang?

A whole bunch of things happened in the first few minutes after the Big Bang, including inflation, CP symmetry breaking, neutrino decoupling. These features are summarized in Sec. 1.12. This question speaks mainly, however, of Big Bang nucleosynthesis (BBN).

The energy scale of BBN is set by the binding energy of nuclei - deuterium binding is about 10^5 times greater than the ionization energy of a hydrogen atom, and as a result BBN occured when $T \approx 4 \times 10^8$ K. The universe grew too cold to maintain such temperatures when it was only several minutes old.

The basic building blocks of matter are protons and neutrons. A free neutron has 1.29 MeV more energy than a proton, and 0.78 MeV more than a proton and electron. $n \to p + e^- + \bar{nu}_e$, then, is energetically (and entropically) highly favourable, and the half-life of a neutron is about 890 seconds.

At age $t = 0.1$ s, $T \approx 3 \times 10^{10}$ K, and the mean energy per photon was about $E \approx 10$ MeV, high enough to easily begin pair production. Neutrons and protons will be at equilibrium with each other via $n + \nu_e \leftrightharpoons p + e^-$ and $n + e^+ \leftrightharpoons p + \bar{nu}_e$, and given LTE, their densities will be given by the Maxwell-Boltzmann equation,

$$n = g\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{E}{k_BT}\right), \tag{32}$$

where the energy scale we consider is the rest mass of a proton vs. a neutron. The relative balance of neutrons and protons, then, is given by

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left(-\frac{(m_n - m_p)c^2}{k_B T}\right) \approx \exp\left(-\frac{Q}{k_B T}\right), \tag{33}$$

where Q = 1.29 MeV, which corresponds to $\sim 1.5{\times}10^{10}$ K. This shows a high preference for protons at low temperatures. In truth, however, $n+\nu_e \leftrightharpoons p+e^-$ is a weak reaction and the cross-sectional dependence of a weak reaction is $\sigma_w \propto T^2$. Since in a radiation-dominated universe $T \propto t^{-1/2}$, $\omega_w \propto t^{-1}$, and the neutron density is greater than $\propto t^{-3/2}$ (from $\rho \propto a^{-3}$ and the fact that neutron numbers are decreasing with temperature). As a result, Γ_w falls dramatically. When $\Gamma \approx H$, the neutrinos decouple from the neutrons and protons. This occurs (empirically) at about 0.8 MeV, or $T_{\rm freeze} = 9 \times 10^9$ K. Using Eqn. 33, we obtain 1 neutron for 5 protons.

The lack of neutrons prevented BBN from fusing to nickel. Proton-proton fusion is difficult due to Coulombic repulsion, and in the Sun the pp-chain has a timescale of several Gyr. This means that in the several minutes when the temperature of the universe was sufficiently high for nuclear fusion to occur, $p + n \leftrightharpoons D + \gamma$ dominated (neutron-neutron fusion has a very small cross-section). if every neutron binded to a proton, and the only nucleosynthetic product was ^{4}He, the fraction of ^{4}He/H would be (2 neutrons + 2 protons)/(6 free protons) = 1/3.

This fusion happened in several stages. The time of deuterium fusion (when $n_D/n_n = 1$) occured at $T \approx 7.6 \times 10^8$ K, or $t \approx 200$ s - this can be derived from the Saha equation ($g_D = 3$). Deterium can then be fused into tritium (^{3}H, half-life 18 years) or ^{3}He, and from there quickly fused into ^{4}He. ^{4}He is very tightly bound (hence α-decay), and there are no stable nuclei with atomic weight 5 or 8. Small amounts of ^{6}Li and ^{7}Li can be made via $^4\mathrm{He} + D \rightleftharpoons^6 \mathrm{Li} + \gamma$ and $^4\mathrm{He} +^3 \mathrm{H} \rightleftharpoons^7 \mathrm{Li} + \gamma$, which are fairly slow reactions. By the time the temperature has dropped to $T \approx 4 \times 10^8$ K at $t = 10$ min, BBN is over, and neutrons are locked up in ^{4}He and a small amount of Li.

1.8.1. *How does nucleosythesis scale with cosmological parameters?*

Nucleosynthesis depends critically on η, the baryon-to-photon ratio. A high ratio increases the temperature at which deuterium synthesis occurs, and hence gives an earlier start to BBN, allowing a greater conversion of D to ^{4}He. ^{7}Li is produced both by fusing ^{4}He and ^{3}He (decreases with increased baryon fraction) and by electron capture of ^{7}Be (increases). Fig. 12 shows the nucleosynthetic products of BBN as a function of baryon density.

1.8.2. *How do we determine primordial densities if D is easily destroyed in stars?*

One way is to look at Ly-α transitions in the neutral, high-redshift ISM. The greater mass of the D nucleus shifts slightly downward (i.e. more negative energy) the energy levels of the electron, creating a slightly shorter Ly-α transition.

1.8.3. *Why is there more matter than antimatter?*

When the temperature of the universe was greater than 150 MeV, quarks would roam free, and photons could pair-produce quarks. The various flavours of quarks were in LTE with each other, and very nearly equal. CP violation, however, produced a $\sim 10^{-9}$ bias in favour of quarks, and when the temperature cooled enough that quark pair production was no longer favourable, the quarks and antiquarks annihilated, producing an enormous photon to baryon ratio, and leaving only quarks. A similar situation occured for leptons.

1.8.4. *What are WIMPs?*

WIMPs (weakly interacting massive particles) are dark matter particle candidates that, due to their small cross-sections, would have stopped interacting with baryons at about the same time as neutrino decoupling. If WIMPs have masses < 1 MeV, they would be ultrarelativistic today and would have the same number density as neutrinos. This gives $\Omega_{\rm WIMP} h^2 \approx \frac{m_{\rm WIMP}}{91.5\,\rm eV}$ (this also applies to neutrinos). The mass of an individual WIMP, then must be < 100 eV. If instead the WIMP is massive, then it is not relativistic, and, then $\Omega_{\rm WIMP} h^2 \approx \left(\frac{m_{\rm WIMP}}{1\,\rm TeV}\right)^2$.

1.9. *Question 8*

QUESTION: Explain how Supernovae (SNe of Type Ia in particular) are used in the measurements of cosmological parameters.

This is adopted from my own qual notes.

Suppose a very bright standard candle exists throughout the history of the universe; since the luminosity of the candle is known, we would be able to use it to measure the luminosity distance (Eqn. 13, using $R = \frac{c}{H_0}\sqrt{|\Omega_\kappa|}$ and sinn to represent sin, sinh, etc.):

$$d_L = (1+z)\frac{c}{H_0}\frac{1}{\sqrt{|\Omega_\kappa|}}\mathrm{sinn}\left(\sqrt{|\Omega_\kappa|}H_0 \int_0^z \frac{dz}{H}\right) \tag{34}$$