Cosmology Qual Solutions

Last Edited: December 2018

Preface

This work is a collaborative effort by the cosmology class of fall 2018. It is written and compiled primarily by Bethany and Ajay.

It is based upon the works of:

Miranda Herman
https://github.com/mkherman/Quals2018/blob/master/1_Cosmology.pdf

Jeffery Emberson
http://www.astro.utoronto.ca/~cczhu/EmbersonQual.pdf

Charles Zhu
http://www.astro.utoronto.ca/~cczhu/ZhuQual.pdf

Ned Wright
http://www.astro.ucla.edu/~wright/cosmolog.htm

This is a collaborative version for the code adverse and may be altered at any time.
Anyone with access to the document is encouraged to make changes and add comments.

For my official notes please see my repository:
https://github.com/AstroLudwig/Qualifying-Exam (In Prep)

To convert equations into latex please download the Auto-Latex equation add on.
https://sites.google.com/site/autolatexequations/tutorial

Question 1 - Recombination

What is recombination? At what temperature did it occur? Explain why this does not match the ionization potential of hydrogen.

Relevant Equations

- $E = k_b T$ Thermal Energy
- $\frac{n_p n_e}{n_e} \propto T^{3/2} e^{-1/T}$ Saha Equation
- $x_e = \frac{n_e}{n_p + n_H}$ Free Electron Fraction

Solution

What is recombination?

- The epoch where a sea of electrons, atomic nuclei, and photons became neutral atoms for the first time.
- Temperature dropped due to expansion.Photoionization no longer occured often enough to maintain the ionization fraction of Hydrogen.

At what temperature did it occur?

- Assuming that the reaction happens fast enough to keep things in thermal equilibrium we can use the Saha equation to relate the ionization fraction to the temperature.
  - $\frac{n_p n_e}{n_e} \propto T^{3/2} e^{-1/T}$
- This can be further simplified if we require that $n_e = n_p$ and we use the free electron fraction.The temperature ends up being about~1/4 eV or
  - $13.6eV \frac{1}{8.6e - 5eV} K \approx 150000K$
  -

Explain why this does not match the ionization potential of hydrogen

- The ionization potential of hydrogen is 13.6 eV or
  - $13.6\text{eV} \frac{1}{8.6e - 5 \text{ eV}} K \approx 150000 \text{ kelvin}$
- The temperature of recombination is significantly lower than this because CMB photons aren't uniform.The cmb is very close to a blackbody spectrum which has an exponential tail known as the Wien's Tail.This tail contains a non negligible number of photons which are still able to ionize hydrogen. This is made worse by the fact that photons also outnumbered baryons $10^9 : 1$ because of baryogenesis and the resultant particle/antiparticle annihilation.

Follow Up

•The process was not instantaneous. As recombination progressed, the number of free electrons available for thomson scattering which kept photons and baryons in thermal equilibrium, decreased. This caused a drop in the opacity for the photons until the optical depth was low enough (τ= 1) that photons could stream freely through the universe: The CMB. This corresponds to the surface of last scattering where radiation and matter decoupled.

•If the photon:baryon ratio were higher there would be more ionization and recombination would happen later.

•If the ionization energy of Hydrogen was lower, it would take longer to cool off significantly past this temperature due to wien's tail so recombination would happen later.

Question 2 - Flat Universe

The universe is said to be "flat", or, close to flat. What are the properties of a flat universe and what evidence do we have for it?

Relevant Equations:

- $H(t)^2 = \frac{8\pi G}{3}\left[\rho(t) + \frac{\rho_{cr} - \rho_0}{a(t)^2}\right]$ Friedmann Equation
- $H(a)^2 = H_0^2\left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right]$ Friedmann Equation
- $\rho_c = \frac{3H_0^2}{8\pi G}$ Critical Density

Solution

What are the properties of a flat universe?

- Parallel Lines do not Converge or Diverge
- Sum of Angles in a triangle = 180 deg
- Density of the universe is the critical density

What evidence do we have for it?

- By measuring the density of the universe we can determine the curvature. We can probe the different components many ways (CL Spectrum of CMB, galaxy cluster distribution, etc.). We can also look at the angular scale of CMB fluctuations which depend on the universe's geometry.
- If the universe were positively curved, the peak of the CMB Cl power spectrum would occur at larger angular scales (lower multipoles)
- If it was negatively curved, the peaks would be at smaller angular scales (higher multipoles).
- Since the expansion rate evolves differently for different densities we can also use Type Ia SNR to map distance to z and infer Ω_0.
- From the Friedman Eq the density of the universe determines its geometry and tells us how the scale factor evolves over time.
- $\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$
- $\Omega_0 = \Omega_r + \Omega_m + \Omega_\Lambda, \;\; 1 - \Omega_0 = \Omega_k \rightarrow \text{ if } \Omega_0 = 1, \Omega_k = 0$

Follow Up

- The universe was likely much flatter in the early universe as even slight deviations in the density would've changed our expansion significantly. Inflation solves this flatness problem by flattening everything out through exponential expansion, so it doesn't matter how curved the universe was pre inflation.

Question 3 - CDM Power Spectrum

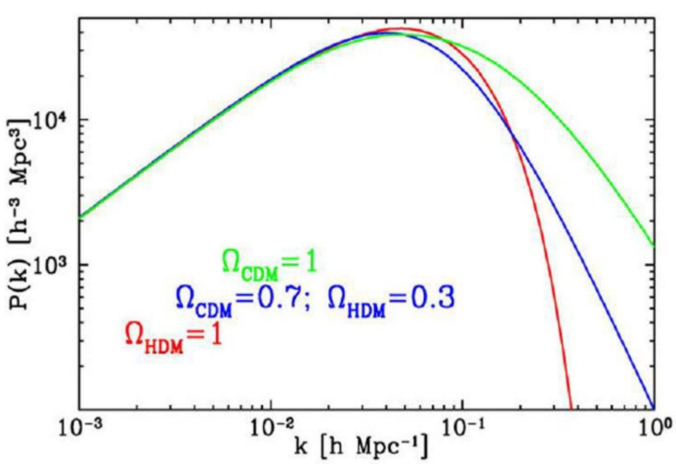
Outline the development of the Cold Dark Matter spectrum of density fluctuations from the early universe to the current epoch.
Rephrased: Describe how fluctuations in the matter density grow with time in different cosmological epochs and how this leads to the matter power spectrum we see.

Solution
Gravitational instability increases density fluctuations over time.

- On large scales, the universe is homogeneous.
    - But on small scales you see anisotropies with small amplitudes in the cmb that can be described with a gaussian random field.
    - These become bigger over time.
- To talk about this we define a fractional overdensity, or relative density contrast
    - $$\delta = \frac{p(\vec{r},t)}{\bar{p}(t)} - 1$$
- In over dense regions, Δp >0, δ >0, there is a stronger gravitational field than the mean.
- Expansion is related to gravity so these over dense regions expand more slowly, while conversely,underdense regions expand much quicker than the mean.•There's some positive feedback between density creating a gravitational field and the gravitational field making the region more dense.

The Matter-Power Spectrum

- The power spectrum is parameterized by a power law $P(k) = k^n$
- Harrison and Zeldovich argue that n has to equal 1 or else you're introducing a preferred scale for mass scale for fluctuations. $P(k) = k$
- A transfer function is introduced which is dependent on the cosmological model. This is where it matters if the universe is matter or radiation dominated. The assumption we make here is that we can use linear perturbation theory, so fluctuation amplitudes have to be small. Often this just sets the initial condition.
- For small k, $P(k) = k$.
- For large k, $P(k) = k^{-3}$,
- Peaks at $k_{peak} \sim 2 \times 10^{-2} hMpc^{-1}$ corresponding to $\lambda_{peak} \sim 350h^{-1}Mpc$



Question 4 - Big Bang

State and explain three key pieces of evidence for a Big Bang origin for the observable Universe.

Solution

- Hubble's Law
  - At large scales all galaxies are receding away from us ($v = H_0 d$) implying that the universe is expanding. If we extrapolate backward, there must have been a time where everything was much closer together. Hence, the big bang.
- CMB $\Rightarrow$ Cosmic Microwave Background
  - Thermal black body radiation from when baryons and photons were in thermal equilibrium. It's homogeneous and isotropic, implying that at some point the entirety must have been in causal contact and therefore the universe must have been much smaller and hotter in the past.
  - $T = T_0(1+z) = 2.7K(1+z)$, as Z goes to ∞ , really freaking hot.
- BBN $\Rightarrow$ Big Bang Nucleosynthesis
  - Subatomic particles combined to form the lightest elements first as the universe expanded and cooled enough for the photon energy to decrease past the atomic binding energies
  - By knowing or assuming some initial conditions for the universe and knowing relevant interaction cross section, you can calculate the expected primordial abundances of elements and compare the current measurements.

Question 5 - Big Bang Nucleosynthesis

Why are only very light elements (H, D, He, and traces of Li) synthesized in the first three minutes of the Big Bang?

(I haven't figured out how to convert latex tables into google doc yet)

Time after BB	Description	Chemical Reaction
$<$ 1 second	The neutron:proton ratio is maintained at thermal equilibrium	$p + e^{-1} \Longleftrightarrow n + \nu$
$\approx$ 1 second	Temperature cools, is slightly less than the neutron:proton mass difference, these weak reactions become slower than expansion and the ratio freezes out at about 1:6	$n + e^{+} \Longleftrightarrow p + \bar{v}$
$>$ 1 second	Only neutron decay changes number of neutrons. Half life of a neutron is 615 seconds, $\sim$ 10 minutes. **Without further reactions to preserve neutron the Universe would be pure Hydrogen.**	$n \rightarrow p + e^{-1} + \bar{\nu}$
$>$ 100 seconds	Deuterons preserve the neutron. The reaction releases 2.2 MeV but since photons are a billion times more numerous than protons the reaction doesn't proceed until T $<$ 0.1 MeV. The neutron proton ratio is about 1:7.	$p + n \Longleftrightarrow d + \gamma$
	Further reactions proceed to make helium nuclei including He_3, He_4, and radioactive Hydrogen H_3 (triatomic). Because the binding energy of helium is 28 MeV more bound than deuterons and the temperature has fallen to 0.1 MeV, the reactions go one way.	$d + n \rightarrow H_3 + \gamma$ $H_3 + p \rightarrow He_4 + \gamma$ $d + p \rightarrow He_3 + \gamma$ $He_3 + n \rightarrow He_4 + \gamma$
	Reactions that don't produce a photon occur and can happen even faster.	$d + d \rightarrow He_3 + n$ $d + d \rightarrow H_3 + p$ $H_3 + d \rightarrow He_4 + n$ $He_3 + d \rightarrow He_4 + p$
	H_3 has a 12 year half life and decays into He_3 so none survive to the present.	$H_3 \rightarrow He_3 + e^{-} + \bar{\nu}_e$
	Be_7 decays into Li_7 with a 53 day half life and does not survive.	$He_3 + He_4 \rightarrow Be_7 + \gamma$ $Be_7 + e^{-} \rightarrow Li_7$

Table 1: BBN Timeline

Deuteron is the nucleus of deuterium.
Deuterium is the heavy form of hydrogen H_2.
Deuteron is just a nucleus with a proton where hydrogen has only one proton and no neutron.
Deuterium peaks around 100 seconds and is rapidly swept up into helium nuclei.
Very few helium nuclei combine into heavier nuclei giving an abundance of lithium.

Question 6 - Type IA Supernova

Explain how and why Type Ia Supernovae are used in the measurements of cosmological parameters.

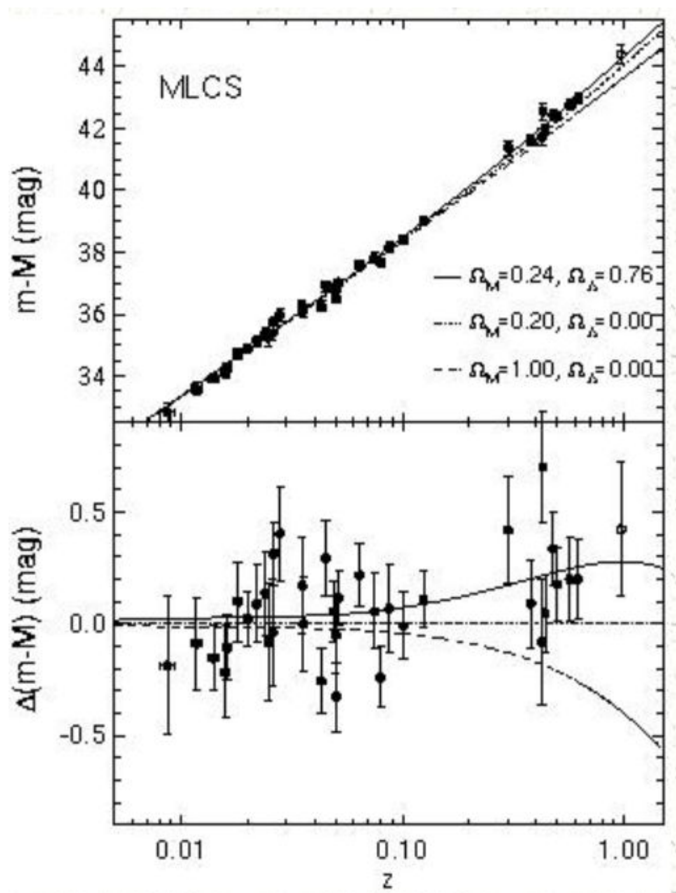
Relevant Equations:

- $m_{apparent} - M_{absolute} = 5\log_{10}(\frac{d}{10})$ Distance Modulus
- $v = cz$
- $v = H_0 d$
- $d = \frac{c}{H_0} z$
- $H = H_0\sqrt{\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda}$

For supernova, we can measure a) the apparent magnitude and b) the light curve.
From the light curve we can get the absolute magnitude.
Plugging the absolute and apparent magnitudes into the distance modulus gives us the distance to these supernova.

Follow Up
Why are type Ia SN standardizable?

- Type Ia SN occur when a companion binary star accretes mass onto a white dwarf until the Chandrasekhar limit is reached, in which case it explodes. The Chandrasekhar limit is pretty universal so you get a consistent progenitor mass.



Question 7 - Observational Cosmology

Describe two methods, other than Type Ia supernovae, by which the cosmological parameters can be determined by astronomical observations.

Baryon Acoustic Oscillations - BAO - Can be used as a standard ruler.
Relevant Equations:

- $\Delta\theta = \dfrac{\Delta\chi}{d_A(z)}$
- $\Delta\theta \rightarrow$subtended angle. $\Delta\chi \rightarrow$ length of ruler. $d_A(z) \rightarrow$ angular diameter distance.
- $d_A(z) \propto \displaystyle\int_0^z \frac{dz}{H(z)}$
- $c\Delta z = H(z)\Delta\chi \rightarrow$ the redshift interval can be measured from data thus determining the Hubble Parameter.

In the early universe, things are clumpy. Sound waves can propagate because things are also very dense. The initial sound wave mechanism is set off by high amounts of pressure in particular regions which compresses this hot photon-baryon fluid which then decompresses like a sound wave does.
The electrons follow the photons because they're charged. The protons follow the electrons because otherwise there would be this huge electrical field.
These ripples travel onward for about 150 Mpc until recombination where the expansion of the universe drops the temperature, the electrons and protons combine to form neutral hydrogen, which no longer has to follow along with the photons. This leaves a matter ripple/shell frozen in place.
We can predict where this should occur so SDSS went and looked to see if there was a bump in the correlation function between galaxies and how clustered they are. This correlation function just describes how far galaxies should be from each other if everything just got pushed homogeneously and the bump in it shows that there is actual structure to how the matter got pushed.
Since that BAO signal is confirmed, and we know its comoving distance and angular size, we can place constraints on cosmological parameters.

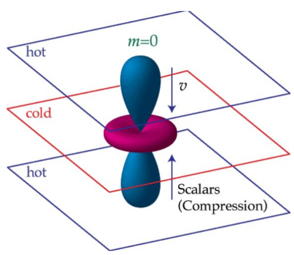
Angular Power Spectrum of CMB.
The CMB map is a superposition of BAO shells with the temperature anisotropies resulting from the level of compression and rarefaction of the baryon photon fluid. We can express the temperature fluctuations in terms of spherical harmonics and get the amplitude of the fluctuations at different scales. The first peak in the CMB power spectrum is from the BAO waves reaching maximum compression when the universe became transparent at recombination. It's position tells us about how curved the universe is. The position of the first peak would shift left or right depending on a positively or negatively curved universe, respectively. Comparing the heights of the first two peaks tells us about Ω_m and the heights of the second and third peaks tells us about dark matter since the dark matter pulls the baryon fluid in more during compression.

Other methods: Weak Gravitational Lensing, CMB foreground like SZ and ISW effects.

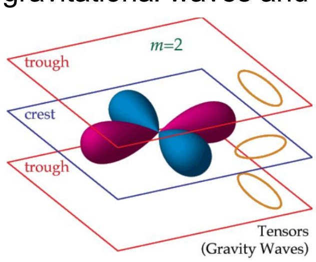
Question 8 - CMB Polarization

Why is the cosmic microwave background expected to be weakly polarized, and what is practically required to observe this signal?

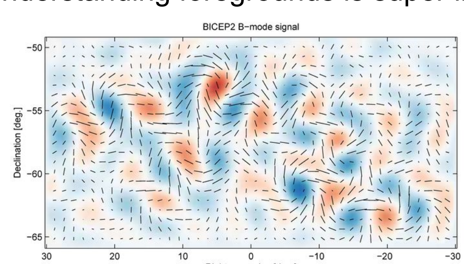
- The cosmic background radiation is blackbody radiation and should therefore be unpolarized. However, measurements have confirmed a finite polarization exists.
    - Since BB's absorb photons at all polarizations it should emit at all polarizations. No preferred direction means unpolarized.
- Temperature/density fluctuations produce a local quadrupole moment. This results in radiation coming toward an electron along a preferred axis.
- Thomson scattering produces linearly polarized light when the electron interacts with this radiation.
- The quadrupole can come about in two ways.
    - An electron falls into a gravitational potential well.

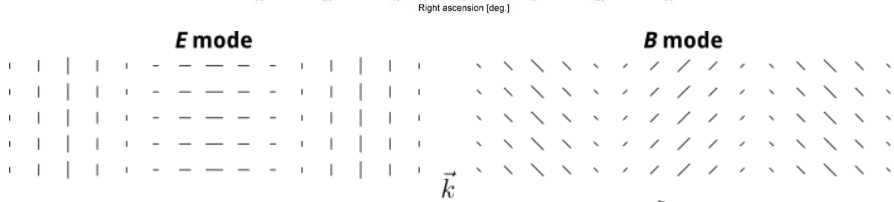


    - As primordial gravitational waves stretch and squeeze space they affect the density of the photon baryon fluid. This is cool because inflation predicts gravitational waves and finding a polarization in the cmb supports that.



- Measuring polarization requires a detector with polarized sensors to measure the E and B field components of the CMB. Bicep2 detected CMB B modes (but it turned out to only be dust -- this is why understanding foregrounds is super important)

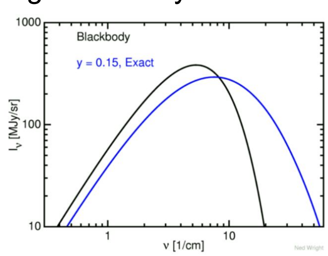




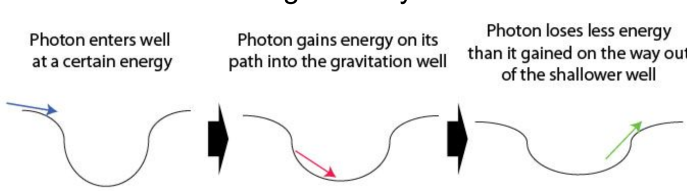
Question 9 - CMB Foregrounds

Our view of the cosmic microwave background is affected by what is along the line of sight. Give two examples of CMB foregrounds that also provide information about the cosmic parameters.

- It's a long journey from the CMB to us and a lot of opportunity for things to get in the way of CMB photons. Most of it is galactic stuff like like synchrotron radiation and dust but some can tell us about cosmological parameters.
- Thermal Sunyaev Zel'dovich $\Rightarrow$ **ICM scatters photons, shifts BB by optical depth $*$ thermal energy/ rest mass energy. Can measure distance and Ω_m**
    - Involves scattering of CMB photons by rapidly moving electrons in the hot gas in clusters of galaxies.
    - It is possible to use a combination of the SZ effect and the X-ray emission from the hot gas to derive a distance to the cluster.
    - This effect is proportional to (1) the number density of electrons, (2) the thickness of the cluster along our line of sight, and (3) the electron temperature. The parameter that combines these factors is called the Kompaneets y parameter, with $y = \tau(kT/mc^2)$. Tau (τ) is the optical depth or the fraction of photons scattered, while (kT/mc^2) is the electron temperature in units of the rest mass of the electron.
    - The SZ effect also amplifies the first peak in the power spectrum which can tell us about Ω_m
    - The usual order of magnitude for y is about 0.0001, which is very small.



- Integrated Sachs-Wolfe Effect $\Rightarrow$ **a particle falls into well and expansion makes it easier to get out. Particle gets to keep some energy. Tells us about expansion**
    - Light travelling through a supercluster picks up energy in the form of speed and heat like a particle rolling down a valley.
    - Normally it would give all that energy back when rolling back up the valley, but dark energy changes the shape of the valley as the particles rolls through it to make it shallower and the particle gets to keep some of that heat.
    - It's a gravitational redshift that occurs between the surface of last scattering and the earth but happens when the universe is still dominated by radiation. If the universe were matter dominated than large scale gravitational potential energy wells and hills don't evolve significantly.



Gravitational well of galaxy supercluster - the depth shrinks as the universe (and cluster) expands

Question 10 - Inflation

Describe cosmological inflation. List at least three important observations it is intended to explain.

Solution

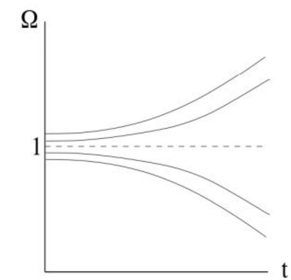
Inflation occured 10^{-36} to 10^{-34} seconds after the big bang when quantum fluctuations allowed a small region of space to enter a true vacuum state in a universe otherwise filled with false vacuum. The conceptual part of this is tricky. A simple approach is that the universe was temporarily dominated by a positive cosmological constant Λ_i with $w = -1$.

Relevant Equations

- $a(t) = e^{H_i t}$
- $w = p/\rho$, $\rho \propto a^{-3(1+w)}$ with $w = -1$

Flatness Problem $\Rightarrow$ The Universe is nearly flat today, and even flatter in the past. Seems fine tuned.

- We can write the friedmann equation as
  $$1 - \Omega(t) = \frac{H_0^2(1-\Omega_0)}{H(t)^2 a(t)^2}$$
- $\Omega(t) = 1$ is unstable equilibrium. Any deviation from this would quickly make things less flat.
- Inflation reduces the dependence of the current flatness on the initial conditions.



Horizon Problem $\Rightarrow$ How is the Universe so homogeneous if not all particles are in causal contact?

- Distance to last scatter $d_p(t_0) = c\int_{t_{ls}}^{t_0} \frac{dt}{a(t)}$, but $t_{ls} << t_0$ only slightly smaller than current horizon distance.
- If you look at two points separated by 180 degrees on the sky, their current proper distance is $\approx 2d_{hor}(t_0)$, hence they are causally disconnected.
- CMB measurements shows that those particles are the same temperature to within one part is 10^5. How are they in thermal equilibrium?
- Universe is matter dominated at last scattering, $a = (3/2H_0 t)^{2/3}$, you can get the angular diameter distance $d_a = \frac{c}{1+z}\int \frac{dz}{H(z)}$, which shows you that anything not in a 2 degrees radius is not in causal contact and $d_{hor}(t_{ls}) \approx 0.4 \text{ Mpc}$.
    - If you factor in inflation and allow 100 e-foldings $d_{hor}(t_{ls}) \approx 10^{43} \text{ Mpc}$ allowing the last scattering surface to be in causal contact.

Monopole Problem

- It would be beautiful if all of the forces were originally unified in the beginning of the universe. We know that the EM and weak forces become the same force at super high temperatures after all. However one of the predictions of this is that at one point the universe would have been dominated by magnetic monopoles since they occur at every defect in the quantum field and we can't find them.
- Inflation dilutes the number of magnetic monopoles practically out of existence.

Question 11 - Fine Tuning

Define and describe a ‘fine tuning problem'. How do anthropic arguments attempt to resolve it?

Fine Tuning Problem

- Generally, a fine tuning problem refers to circumstances when the parameters of a model must be adjusted very precisely in order to agree with observations.
- Imagine walking into your boss’ office and finding a pencil balanced on its point. You continue to find this pencil balanced every day for the next year until you finally inquire about its origins. Your boss acknowledges that it is quite remarkable to find the pencil this way, but claims that there is no need to explain its origins since he found it this way when he moved into his office. You would probably not find this answer satisfactory, but, on the other hand, if the pencil had simply been lying there, you probably wouldn’t have asked about it in the first place.
- In the case of the flatness problem, the pencil standing up on one end is the fact that $\Omega_0 = 1$.
- To prove that if it's flat now, it must have been flatter in the past:
  $H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{a^2}$, plug in H^2 from $\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G}{3H^2}\rho$ $\Rightarrow$ $\frac{1-\Omega}{\Omega} = \frac{-3\kappa c^2}{8\pi G \rho a^2}$

  The very early universe was in the radiation dominated era where $\rho \propto a^{-4}$ and $t^{1/2}$

  $$\frac{1-\Omega}{\Omega} \propto \frac{t}{t_0}\frac{1-\Omega_0}{\Omega_0}$$

  Current measurements from WMAP produce $\Omega_0 = 1.002$, if we look at the time when the universe was 1 second year old, $\frac{1-\Omega}{\Omega} \sim 10^{-20}$

  The reason this is surprising is that it isn't a natural state to be in, in fact it’s an unstable equilibrium point similar to the pencil.

Anthropic Principle

In the example of the flat universe problem, if you consider other universes that aren’t flat, you quickly find that they aren’t universe’s with life in it.

- If the universe is open $\Omega_0 < 1$, the density is insufficient to halt the expansion of the universe resulting from the Big Bang. After only a short period of time the universe would be expanding too quickly for gravitationally bound systems to form.
- If the universe is closed $\Omega_0 > 1$, the density of the universe will halt the expansion of the universe and cause it to collapse at some finite time.
- Since the human population clearly exists, then it must be that the universe is flat.

It is unremarkable that the universe has fundamental constants that happen to fall within the narrow range thought to be compatible with life.

Life happens to exist because of the constants and not the other way around.

Question 12 - Correlation Function

Define the two-point correlation function. How is it related to the power spectrum? How is the C_l spectrum of the CMB related to low redshift galaxy clustering?

Define the two-point correlation function.
Describes the likelihood of finding a galaxy at point X given that another galaxy exists with some characteristic separation.Given some distance what is the probability of finding 2 galaxies separated by that distance?Takes into account the fact that galaxies are not randomly distributed in space but are found in groups and clusters.The correlation function is obtained observationally by averaging over the density products for a large number of pairs of galaxies with given separation r.
$\xi(r) = (r/r_0)^{-\gamma}$, $\gamma \approx 1.7$
If galaxies are more concentrated $\xi(r) > 0$ and vice versa.

How is it related to the power spectrum?
The correlation is related to the power spectrum via fourier transform:

$$P(k) = 2\pi \int_0^\infty dx x^2 \frac{sin(kx)}{kx} \xi(x)$$

We can measure the correlation separation between galaxies and determine the functional form of the power spectrum.

How is the Cl spectrum of the CMB related to low redshift galaxy clustering?

The relevance of low Z galaxy clustering to the Cl spectrum of the CMB is through secondary anisotropies.Low z galaxies can produce changes to the spectrum via the ISW effect, the SZ effect, and gravitational lensing

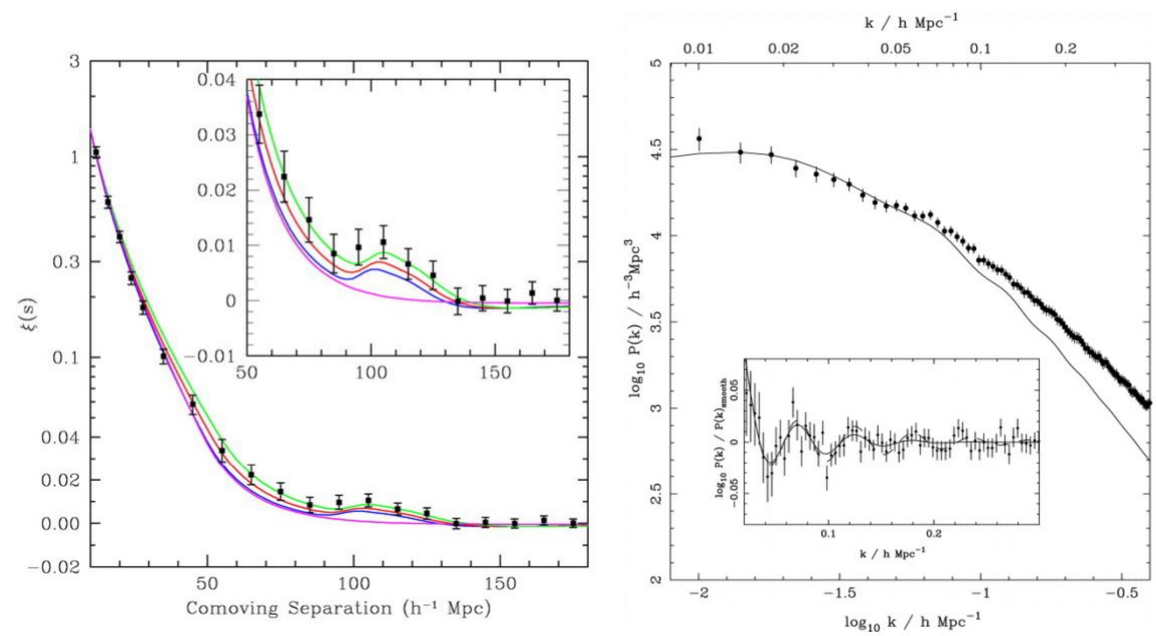


FIG. 24.— (left) The correlation function computed from SDSS galaxies. The peak at $100/h$ Mpc separation denotes the horizon scale at recombination. The solid lines show models with $\Omega_{mat}h^2 = 0.12, 0.13, 0.14$ (green, red, blue respectively), all with $\Omega_{bary}h^2 = 0.024$. The magenta lines shows a pure CDM model ($\Omega_{mat}h^2 = 0.105$) which lacks the acoustic peak. Image taken from Eisenstein et al. (2005). (right) The matter power spectrum as computed from analysis of galaxies identified by the SDSS. The inset shows the baryon wiggles caused by the baryonic acoustic oscillations (BAO). Image taken from (Percival et al. 2007).

Question 13 - Cosmological Constant

Consider a cosmological model including a positive cosmological constant. Show that, in such a model, the expansion factor eventually expands at an exponential rate. Sketch the time dependence of the expansion factor in the currently favoured cosmological model.

Solution

- **The scale factor eventually increases at an exponential rate if the cosmological constant is positive.**

The Friedmann equation is written as:

$$(\frac{\dot{a}}{a})^2 = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$$

For large a, only Ω_Λ is left

$$(\frac{\dot{a}}{a})^2 = \Omega_\Lambda$$

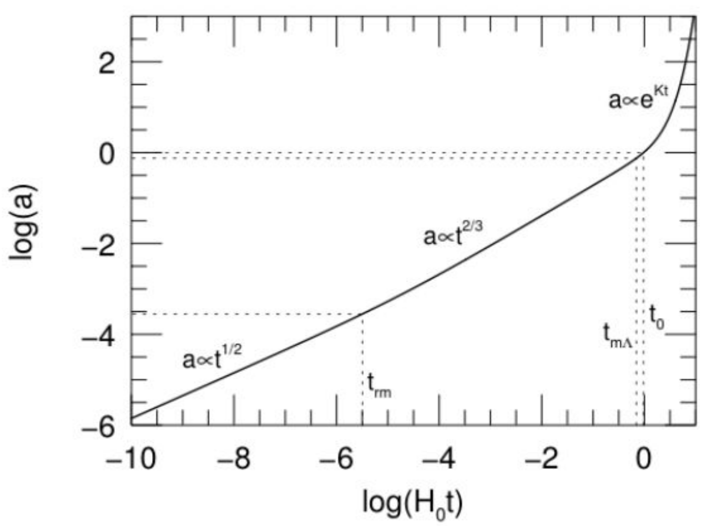
$$\frac{1}{a}da = \sqrt{\Omega_\Lambda}dt$$

$$ln(a) = \sqrt{\Omega_\Lambda}t$$

$$a = e^{\sqrt{\Omega_\Lambda}t}$$

$$a \propto e^t$$

- **Show the time dependence of the expansion factor in the currently favoured cosmological model.**

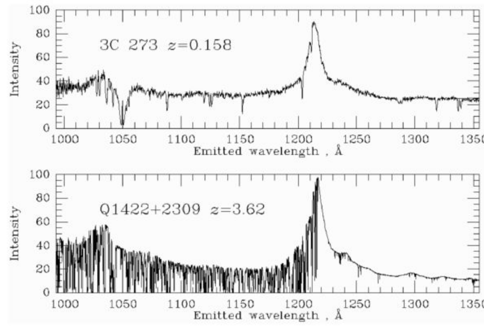


Question 14 - Reionization

Define and describe the epoch of reionization. What are the observational constraints on it?

Solution

The reionization of the universe was due almost exclusively to photo-ionization.Collisional ionization is ruled out since the ICM wasn't hot enough for efficient collisional ionization.Reionization took place from z~12−6

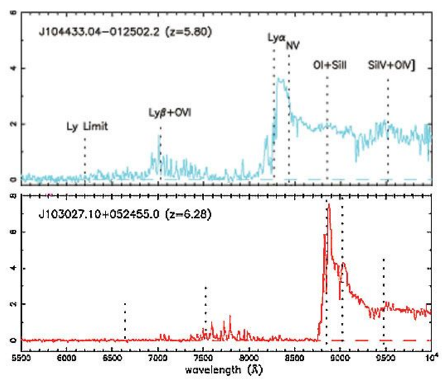


Lyman Alpha Forest

The presence of neutral hydrogen absorbs much more of the lyman alpha series. Direct observational evidence we have of the existence and properties of the general IGM.

Gunn-Peterson Trough of Quasars

Can constrain the end of reionization. The Gunn Peterson trough is a feature of the spectra of quasars due to the presence of neutral hydrogen in the Intergalactic Medium (IGM). The trough is characterized by suppression of electromagnetic emission from the quasar at wavelengths less than that of the Lyman-alpha line at the redshift of the emitted light. The discovery of the trough in a z = 6.28 quasar, and the absence of the trough in quasars detected at redshifts just below z = 6 presented strong evidence for the hydrogen in the universe having undergone a transition from neutral to ionized around z = 6.



The top is a close quasar, the bottom is very distant. Note the height of the spectral lines on the left side of the spectrum.

CMB Polarization

Can constrain the beginning of reionization. The degree of polarization from thomson scattering is related to the optical depth and therefore the abundance of free electrons.

Kinetic SZ Effect

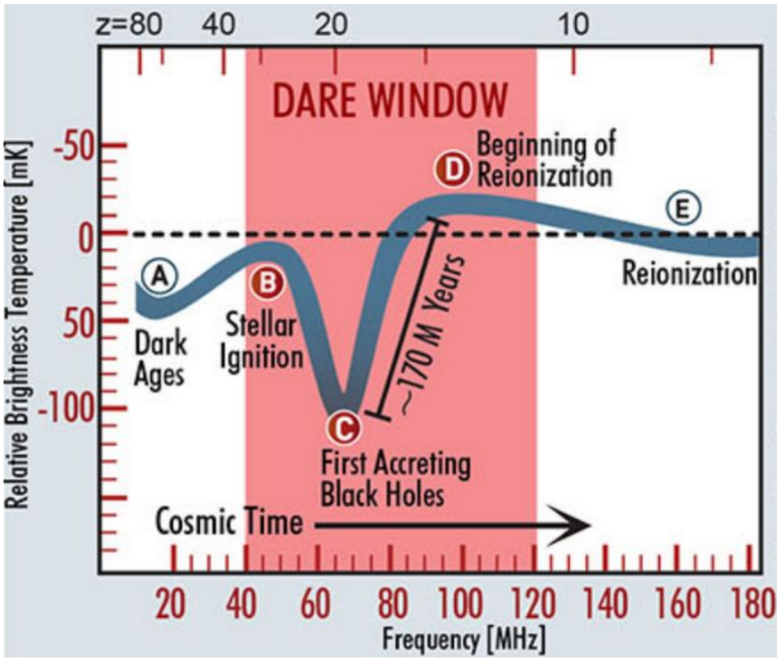
Can tell us how long reionization lasted. The kinetic SZ effect looks at the temperature fluctuations in the CMB. The peculiar velocities of the ionized bubbles produce a doppler shift of CMB photons and the strength of this effect scales with the number of ionized bubbles present.

21 cm Spin Flip

Can trace the ionized fraction of hydrogen. This is the only way to probe the dark ages from recombination to reionization. It can provide a picture of the matter power spectrum after recombination as well as providing the picture for how the universe was reionized. In the late (z <9) reionization epoch the 21 cm line is proportional to ionized fraction of hydrogen.

Question 15 - 21 cm Line

The 21 cm line of hydrogen is expected to show up in absorption against the cosmic microwave background at some redshifts, and in emission at other redshifts. What physical processes lead to this behaviour?



21 cm spin flip can be a spontaneous emission or induced through collisions.

0. Prior to Dark Ages, Redshift z~200 - Post Recombination
    - The intergalactic medium is still dense enough to couple photons to baryons via collisions so we don't see any significant absorption or deviation from the CMB temperature. No 21 cm signal.

A. Dark Ages, z~80, -
    - As the universe expands, collisions can't keep baryons and photons coupled anymore and the temperature drops so neutral hydrogen with electrons in a parallel configuration will seek the lower energy state with electrons in anti parallel configurations. When one of the electrons flips its spin it emits a 21 cm line which can be detected even when everything else around it is dark.

B. Stellar Ignition, z~30
    - The first stars and dwarf galaxies form in the universe and their collective luminosity puts an end to the cosmological dark ages.
    - They excite hydrogen to produce the lyman series photons

C. First Accreting Black Holes, z~20
    - Galaxies begin to emit X-rays which heat gas the 21 cm line counteracts the lyman gun peterson trough.

D. Beginning of Reionization, z~15
    - Stars produce UV radiation which ionizes the ISM. No 21 cm emission is seen in bubbles around these stars.

E. Reionization, z~6
    - No more 21 cm line

Question 16 - Perturbation Modes

What is the difference between scalar and tensor modes of perturbation in the early universe, and how can you detect their presence?

Types of fluctuations: scalar and tensor
Observables: the temperature, E-mode, B-mode, and temperature cross polarization power spectra.

You can decompose your metric perturbations into different “types” that have different mathematical properties.

- Scalar:
    - Density perturbations, they don’t have directional information.
    - Scalar perturbations source fluctuations in temperature, as well as E-mode polarization of the CMB.
    - We can detect their presence in the CMB by looking at the polarization power spectrum and C_l power spectrum for temperature fluctuations.

- Tensor
    - Distortion of space-time metric in the form of gravitational waves.
    - They have some handedness to them.
    - These source temperature fluctuations and E-mode polarization, but also source B-mode polarization(!).
    - Many models of inflation predict that tensor modes were indeed present in the early universe, so that’s why observing B-modes is a “smoking gun” for inflation.
    - Primordial gravitational waves haven’t yet been discovered, thought to be very faint.
- Vector
    - Have a magnitude and a single direction
    - Easily damped by the expansion of the universe since velocities are difficult to maintain.
    - We don’t observe these and don’t expect to.

To detect such modes we can observe the CMB and determine its linear polarization. That polarization map can be decomposed into E and B modes. BICEP2 for instance had linearly polarized antenna and just observed everywhere on the sky.

Systematics

- E modes can be sheared into B modes through weak lensing but only on small scales

Question 17 - Cosmic Neutrino Background

What are the similarities and differences between the cosmic neutrino background and the cosmic microwave background?

The CNB

- Produced when neutrinos decoupled from the photon-baryon fluid at t ~ 1 second.
- The expansion of the universe exceeded the reaction rate of neutrino interactions.
- The temperature dependent cross section became too small as the universe cooled
- Neutrinos then streamed freely through the universe.
- Peak temperature of 1.9 K
- Very difficult to observe neutrinos from our own sun, much less 14 billion light years away.
- We look for indirect evidence of it.
    - Has an effect on the small scale part of the C_l spectrum.
        - Neutrinos diffusing outward damps the structure at small angles, just like Silk damping.
    - Element abundance.

	CNB	CMB
Particle	Neutrinos	Photons
Decoupling of	Neutrinos and Photon-Baryon Fluid	Photons and Baryons
Time	1 second	380,000 years
Temp Then	10^10 K	3000 K
Temp Now	1.9 K	2.73 K

If the universe cools uniformly for all its constituents why is $T_{cnb} < T_{cmb}$?

- Electron-Positron annihilation.
    - This event produced an in flux of photons with more energy, giving the overall CMB progenitor photon distribution a slightly higher temperature than the neutrinos.
    - This annihilation period occurred around t = 6s when it became more energetically favorable to produce photons rather than electrons.

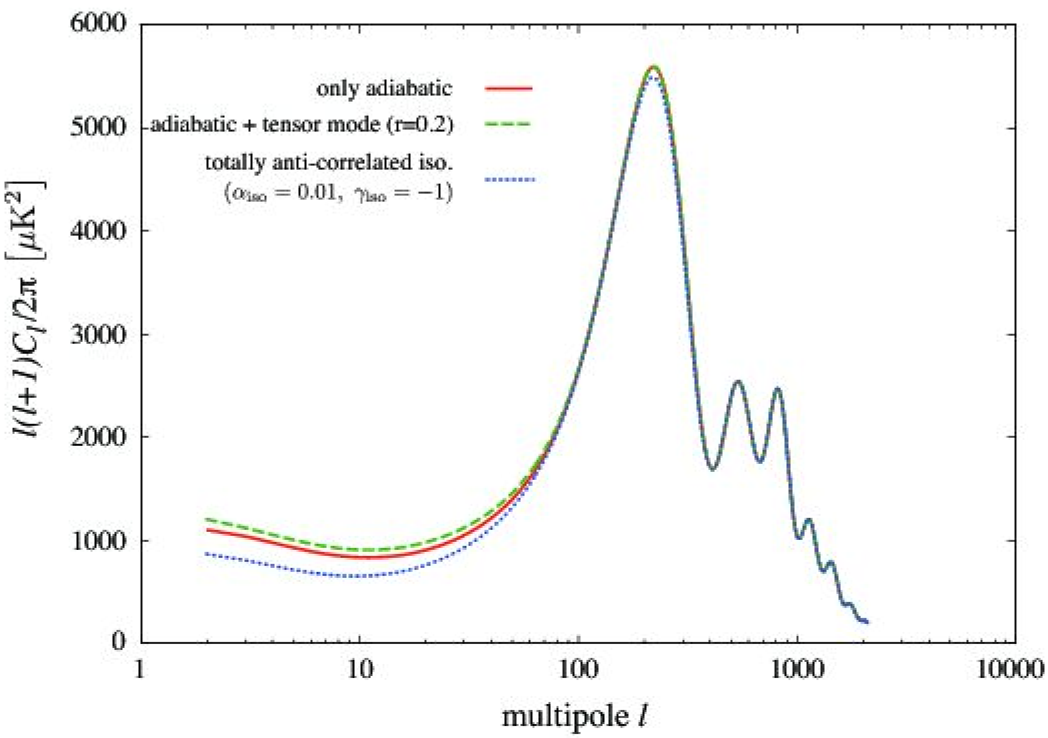
Question 18 - Isocurvature/Adiabatic

What is the difference between an isocurvature mode and an adiabatic mode, in terms of the initial density perturbations in the early universe? How do we know that the initial conditions are mostly adiabatic?

- Isocurvature
    - Density Fluctuations are all balanced
    - No *net* over or under densities, if the density of one component increases the other components decrease.
    - No effect on the local curvature
- Adiabatic
    - Density Fluctuations are proportional to each other.
    - If the density of baryons goes up in a certain region so does the density of everything else.
    - Local curvature in spacetime is changed

We know the initial conditions of the universe must have been in adiabatic modes based on the shape of the C_l power spectrum of the CMB.

If we had more isocurvature modes the power spectrum would be smeared out as overdensities in photons and baryons wouldn't necessarily follow each other. The resulting temperature fluctuations would have different amplitudes



Question 19 - Freeze Out

What is freeze out? Compute the time at which was at a temperature of 1 MeV. Why is this an important time/temperature in cosmology?

What is freeze out?

Freeze out is when your reaction rate between particles becomes smaller than the expansion rate of the universe, leaving us with some relic abundance of the particle that we can calculate.

Compute the time at which the temperature was at 1 MeV

We assume the Universe is radiation dominated, so $a \sim t^{1/2}$.

We assume $T = \frac{T_0}{a}$.

When you plug is T = 1 MeV and T0 = 2.75 K, you get t = 1 second.

Why is this an important time/temperature of cosmology?

This time is important because this allowed the density of neutrons and protons to stay ~ constant. Neutrinos interact with baryons via the weak force, however at T = 1 MeV, the expansion rate H of the Universe exceeded the interaction rate Γ of the neutrinos, effectively decoupling them from the baryons. Then, neutrons could not decay to make neutrinos at the output anymore, hence the neutrons froze out.

```
T = 1*u.MeV
To = 2.75*u.K * k_B.to(u.eV/u.K)
from astropy.constants import k_B

t0 = 13.8*u.Gyr
a = To.to(u.MeV) / T
t = t0.to(u.s) * (a**2.)
print(t)
```

```
0.024456478572147 65 s
```

Note: the calculation above is wrong, but can’t think of anything better.

Question 20 - S.H.0

Discuss the simple harmonic oscillator picture of the CMB and how the matter density,the baryon density, the radiation density and the curvature affect the CMB power spectrum.

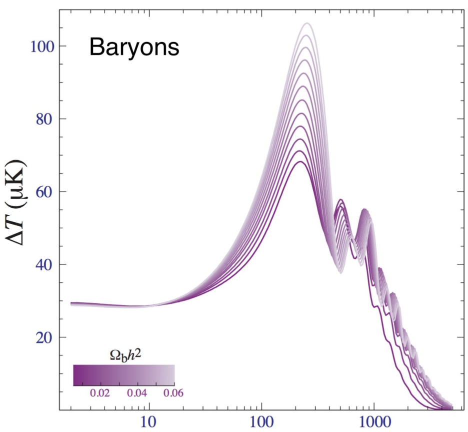
Baryons and photons are being stretched and compressed in sound waves during the early universe before baryon-photon decoupling. Since the process is periodic, we can approximate it using the S.H.O.

The speed of the wave is given by the sound speed $c_s = \sqrt{\frac{\dot{p}}{\dot{\rho}}}$

The restoring force is the pressure gradient.

The S.H.O is given by $\ddot{\theta} + c_s^2 k^2 \theta = 0$

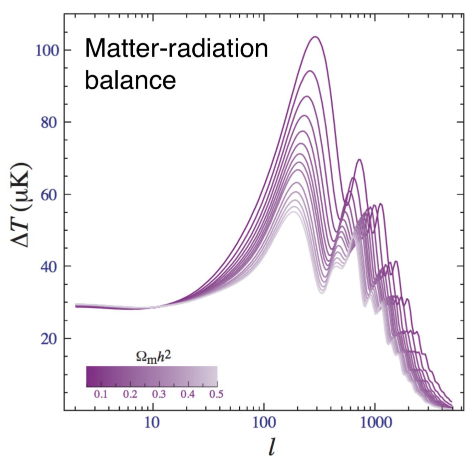
These Temperature oscillations represent heating & cooling of the fluid that is compressed or rarefied by an acoustic standing wave.



As we increase baryons, the first peak gets larger because there is more pressure from the baryon-photon fluid.

The troughs don’t get much deeper however because we’re not adding dark matter to create the potential wells.

If we were to add dark matter the peaks would go down and the troughs would deepen.

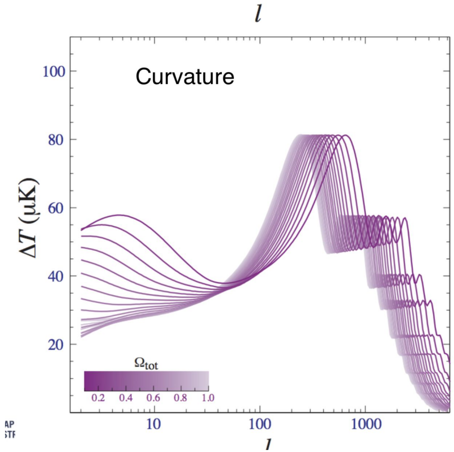


Adding more matter or decreasing radiation decreases the peak size because radiation is the dominant source of the pressure.

Raising the dark matter density reduces the overall amplitude of the peaks.

Lowering the dark matter density eliminates the baryon loading effect so that a high third peak is an indication of dark matter.

Radiation drives peaks more than matter.



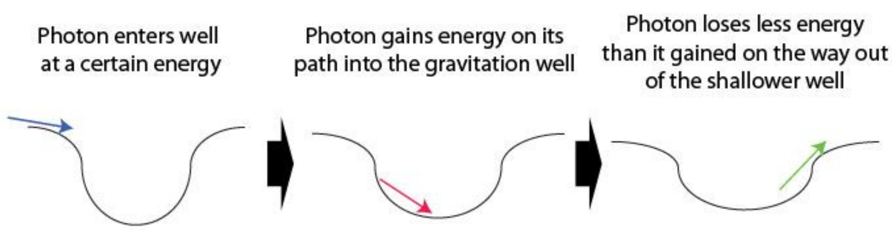
The first peak in the CMB power spectrum is from the BAO waves reaching maximum compression when the universe became transparent at recombination. It's position tells us about how curved the universe is. The position of the first peak would shift left or right depending on a positively or negatively curved universe, respectively.

Question 21 - ISW

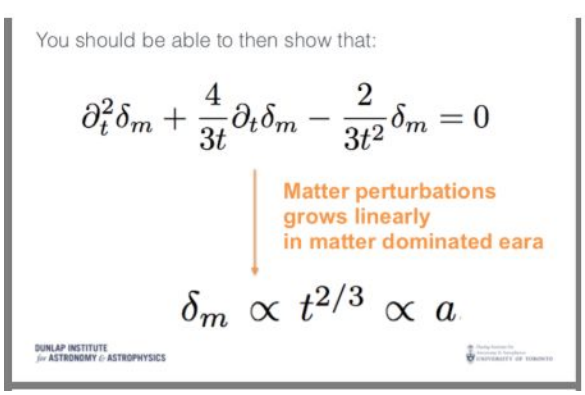
What is the integrated Sachs Wolfe effect, and how does it constrain the dark energy density?

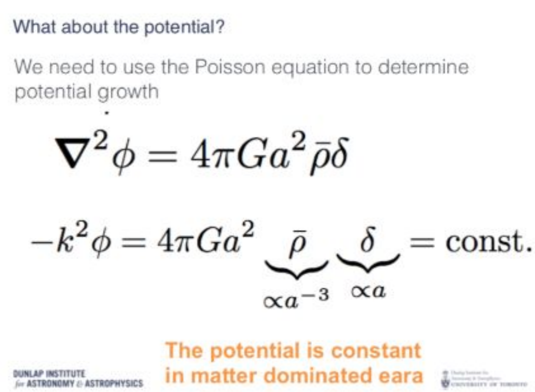
Integrated Sachs-Wolfe Effect ⇒ **a particle falls into well and expansion makes it easier to get out. Particle gets to keep some energy. Tells us about expansion. ISWE creates T fluctuations by variations in the gravitational potential of a cluster. These T fluctuations are seen on large angular scales (theta ~ 1 degree) rather than smaller scales (where behaviour of photons and baryons matters).**

- Light travelling through a supercluster picks up energy in the form of speed and heat like a particle rolling down a valley.
- Normally it would give all that energy back when rolling back up the valley, but dark energy changes the shape of the valley as the particles rolls through it to make it shallower and the particle gets to keep some of that heat.
- It's a gravitational redshift that occurs between the surface of last scattering and the earth but happens when the universe is still dominated by radiation. If the universe were matter dominated than large scale gravitational potential energy wells and hills don't evolve significantly.
- In a matter dominated era, the potential remains constant, even when the overdensities scale as a (see Figures below). **This is because the scale factor cancels out in the Poisson equation** $\delta_m \propto t^{2/3} \propto a$ **and** $\rho \propto a^{-3}$**.**
  You get an a left over in radiation/dark-energy dominated eras.
- ISW can constrain dark energy density because if there is more DE, the potential wells become smaller at the output and vice versa if you have less DE.



Gravitational well of galaxy supercluster - the depth shrinks as the universe (and cluster) expands





Question 22 - Boltzmann

How is the Boltzmann equation used to compute the relative number densities in the universe? Describe the procedure you would use to set up a set of Boltzmann equations

Boltzmann statistics are a good way of describing large number densities of particles. In cosmology we use these statistics to describe reaction rates and freeze out.
To do so you would integrate the boltzmann equation from before to after freeze out when
$\frac{dn}{dt} = 0$

3.3.1 Boltzmann Equation

In the absence of interactions, the number density of a particle species i evolves as

$$\frac{dn_i}{dt} + 3\frac{\dot{a}}{a}n_i = 0\,. \tag{3.3.80}$$

This is simply a reflection of the fact that the number of particles in a fixed physical volume ($V \propto a^3$) is conserved, so that the density dilutes with the expanding volume, $n_i \propto a^{-3}$, cf. eq. (1.3.86). To include the effects of interactions we add a collision term to the r.h.s. of (3.3.80),

$$\frac{1}{a^3}\frac{d(n_i a^3)}{dt} = C_i[\{n_j\}]\,. \tag{3.3.81}$$

This is the *Boltzmann equation*. The form of the collision term depends on the specific interactions under consideration. Interactions between three or more particles are very unlikely, so we can limit ourselves to single-particle decays and two-particle scatterings / annihilations. For concreteness, let us consider the following process

$$1 + 2 \rightleftarrows 3 + 4\,, \tag{3.3.82}$$

i.e. particle 1 can annihilate with particle 2 to produce particles 3 and 4, or the inverse process can produce 1 and 2. This reaction will capture all processes studied in this chapter. Suppose we are interested in tracking the number density n_1 of species 1. Obviously, the rate of change in the abundance of species 1 is given by the difference between the rates for producing and eliminating the species. The Boltzmann equation simply formalises this statement,

$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = -\alpha\, n_1 n_2 + \beta\, n_3 n_4\,. \tag{3.3.83}$$

We understand the r.h.s. as follows: The first term, $-\alpha n_1 n_2$, describes the destruction of particles 1, while that second term, $+\beta\, n_3 n_4$. Notice that the first term is proportional to n_1 and n_2 and the second term is proportional to n_3 and n_4. The parameter $\alpha = \langle \sigma v \rangle$ is the *thermally averaged cross section.*[17] The second parameter β can be related to α by noting that the collision term has to vanish in (chemical) equilibrium

$$\beta = \left(\frac{n_1 n_2}{n_3 n_4}\right)_{\rm eq} \alpha\,, \tag{3.3.84}$$

where $n_i^{\rm eq}$ are the equilibrium number densities we calculated above. We therefore find

$$\boxed{\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = -\langle \sigma v \rangle \left[n_1 n_2 - \left(\frac{n_1 n_2}{n_3 n_4}\right)_{\rm eq} n_3 n_4 \right]}\,. \tag{3.3.85}$$

[17] You will learn in the *QFT* and *Standard Model* courses how to compute *cross sections* σ for elementary processes. In this course, we will simply use dimensional analysis to estimate the few cross sections that we will need. The cross section may depend on the *relative velocity* v of particles 1 and 2. The angle brackets in $\alpha = \langle \sigma v \rangle$ denote an average over v.

Question 23 - Linear Perturbation Theory

Describe linear perturbation theory and its relevance to studying the evolution of structure in the universe.

The Poisson Equation

- Specifies the relationship between matter density and gravitational potential
- Is linear so homogeneous matter distribution and density fluctuations can be considered separately.
- Assume matter is pressureless (dust) with density and velocity fields

The Continuity Equation

- Describes the matter fluid
- Matter is conserved: the density decreases if the fluid has a diverging velocity field (thus, if particles are moving away from each other)

The Euler Equation

- Describes the conservation of momentum and the behaviour of the fluid under the influence of forces.
- Since we are considering pressureless dust $\Delta P = 0$

This answer to this is based on the lecture slides titled “Fields, clustering, and dark matter” in the class notes.

- Talk about continuity, Poisson, Euler equations and how they lead to the equations that describe evolution of perturbations over time.
- Talk about Jeans scale. Perturbations with wavelengths larger than Jeans wavelength (small k) are **unstable**, whereas wavelengths smaller the Jeans wavelength (large k) **supports oscillations**.
- Matter perturbations grow **linearly** in matter dominated era, $\delta_m \propto t \propto a$.
- Potential is **constant** in matter dominated era
- Radiation is effectively unclustered, so it **‘damps’** growth of matter perturbations. In radiation dominated era, $\delta_c = \text{constant}$ or $\delta_c = \ln t$.
- Growth **suppresses** during dark energy dominated era. In DE dominated era, $\delta_m \propto e^{-2t\sqrt{\Lambda/3}} \propto a^{-2}$.

Question 24 - Growth equation

Write down the general growth equation, and discuss what assumptions we employ to solve for the growth of density perturbations in a matter dominated universe. How do structures grow in a radiation dominated universe?

General growth equation

$$\frac{\partial^2 \delta}{\partial t^2} + 2\,\frac{\dot{a}}{a}\,\frac{\partial \delta}{\partial t} = 4\pi\, G \bar{\rho}\, \delta + \frac{c_s^2}{a^2}\,\nabla^2 \delta$$

Growth of perturbations for CDM and BM respectively,

$$\frac{\partial^2 \delta_c}{\partial t^2} + 2\,\frac{\dot{a}}{a}\,\frac{\partial \delta_c}{\partial t} = 4\pi\, G \bar{\rho}\, \left(\frac{\Omega_c}{\Omega_m}\,\delta_c + \frac{\Omega_b}{\Omega_m}\,\delta_b\right)\,,$$
$$\frac{\partial^2 \delta_b}{\partial t^2} + 2\,\frac{\dot{a}}{a}\,\frac{\partial \delta_b}{\partial t} = 4\pi\, G \bar{\rho}\, \left(\frac{\Omega_c}{\Omega_m}\,\delta_c + \frac{\Omega_b}{\Omega_m}\,\delta_b\right) + \frac{c_s^2}{a^2}\,\nabla^2 \delta_b\,.$$

In the matter dominated (EdS) Universe,

In the Einstein–de Sitter model we can solve the perturbation evolution equations analytically. The Einstein–de Sitter model is the flat, matter-only model $\Omega_m = 1$ at all times. Because observations are consistent with a flat Universe, the Einstein-de Sitter model is in fact a good description of the Universe between the end of radiation domination a $z \approx 3300$ and the start of the dark-energy era at $z \lesssim 1$.

The background evolution in the Einstein–de Sitter Universe is given by

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}} \tag{15.65}$$
$$H(t) = \frac{2}{3}\,t^{-1}\,. \tag{15.66}$$

The decaying mode is therefore $\delta_-(t) \propto t^{-1}$. The growing mode is found from Equation (15.63) and is

$$\delta_+(t) \propto t^{2/3} \propto a(t)\,. \tag{15.67}$$

Density perturbations in the Einstein–de Sitter Universe therefore grow as the scale factor.

In the radiation dominated era, the growth equation becomes:

Since density contrast evolves over cosmological time

$$\partial_t^2 \delta_c \sim H^2 \delta_c \gg 4\pi G \bar{\rho}_c \delta_c$$

which means we ignore

$$\partial_t^2 \delta_c + \frac{1}{t} \partial_t \delta_c - \cancel{4\pi G \bar{\rho}_c \delta_c} = 0$$

which leads to solutions

$$\delta_c = \text{const.} \quad \delta_c \propto \ln t.$$

Radiation is effectively unclustered, so 'damps' growth of matter perturbations

Question 25 - Bias

Discuss bias in relation to constraining the growth of structure.

You can determine the matter power spectrum by calculating the two-point correlation function and applying a fourier transform.
The caveat is that number density does not necessarily equal matter density.
A bias is a way to get from number density to

matter density

$$\frac{\Delta n_{gal}}{n_{gal}} = b\frac{\Delta\rho}{\rho}$$

This bias parameter is a complicated function of scale, environment, and galactic properties. The difficulty in estimating this parameter dominates systematic uncertainties in determining the matter power spectrum P(k).

- Bias parameter corrects for the fact that that light does not trace mass perfectly
- Galaxy bias depends on galaxy mass, color, and physical scale
- So, there is a degeneracy between galaxy bias and matter power spectrum

But galaxies and matter are not the same thing

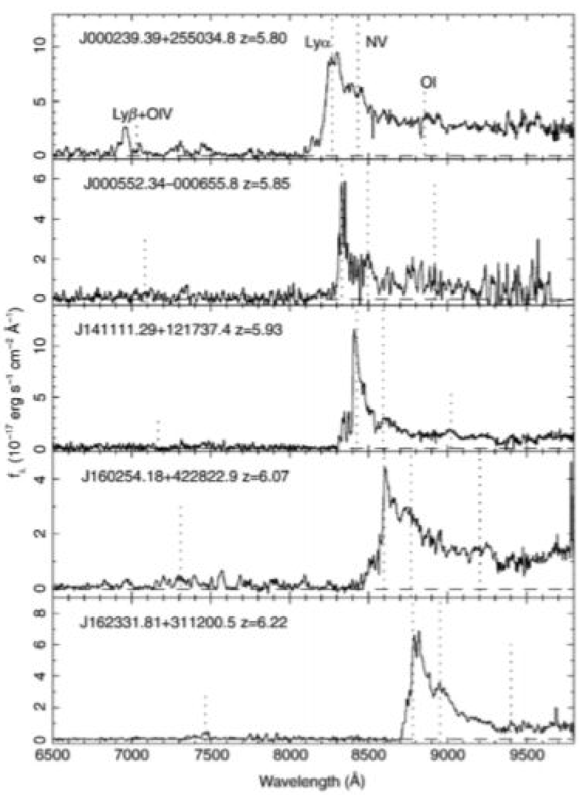
Cosmological theory most easily predicts the clustering of matter (dominated by dark matter, simple gravity)
We see galaxy positions. Galaxies live in dark matter halos.
Galaxy bias: $\delta_g = b\ \delta$

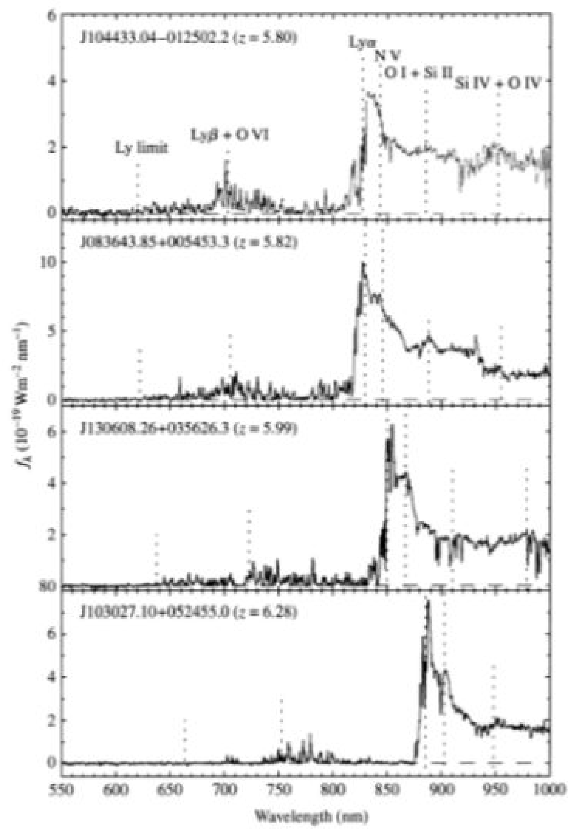
- valid on large scales: “linear bias”
- On those scales, $\xi_{gg}(r) = b^2\ \xi(r)$

Question 26 - Reionization Probes

Compare different reionization probes: Lyman alpha absorption from quasars, neutral hydrogen and the CMB polarization

Lyman alpha absorption





Lyman alpha: Light absorbed by neutral hydrogen in the Lyman alpha transition (n = 1 to 2). Many absorption lines in QSO seen (z < 6 after reionization). Gunn-Peterson trough (or flat spectrum in Lyman alpha wavelengths) is at z = 6, which is at the very end of reionization. Reionization happens from 15 < z < 6.

CMB Polarization

Can constrain the beginning of reionization. The degree of polarization from thomson scattering is related to the optical depth and therefore the abundance of free electrons.

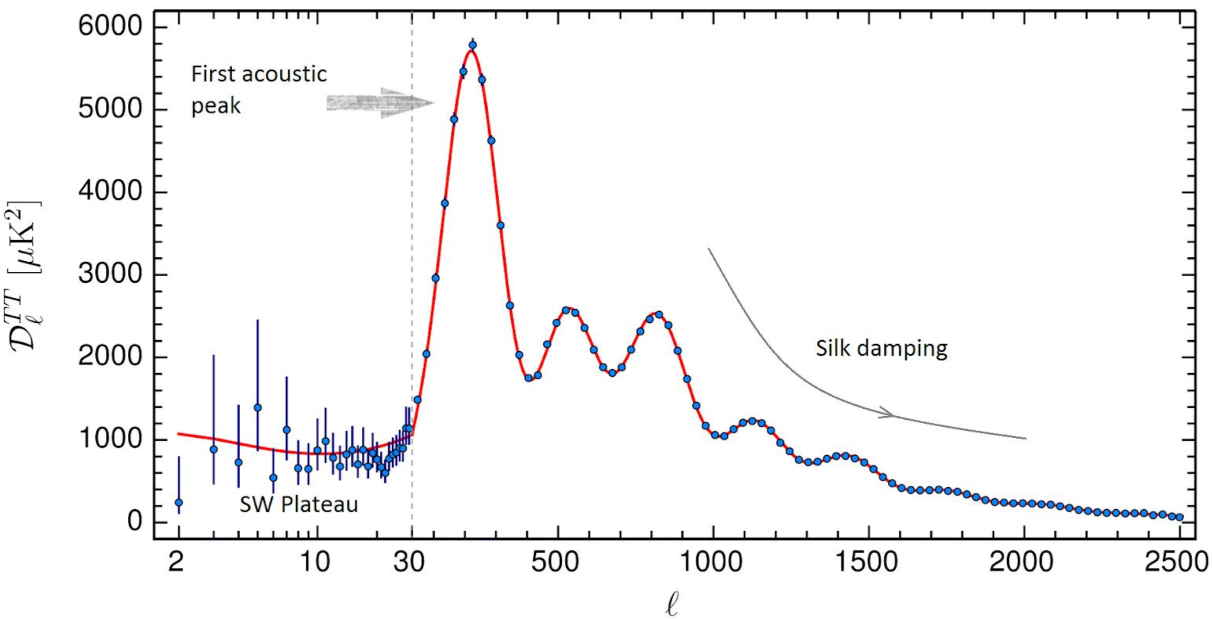
Neutral hydrogen

21 cm H I line emission and absorption is in late z <~ 9 in the reionization era is simply proportional to ionized fraction electrons.

Question 27 - Silk Damping

Describe Silk damping and what we can learn about cosmological parameters from the CMB tail.

Silk Damping



The coupling of baryons and photons is not perfect since, owing to the finite mean free path of photons, the two components are decoupled on small spatial scales.
This implies that on small length-scales, the temperature fluctuations can be smeared out by the diffusion of photons. This makes the CMB anisotropies more uniform on scales ~ mean free path. On these small scales, the T fluctuations are smeared or washed out by the diffusion of photons.

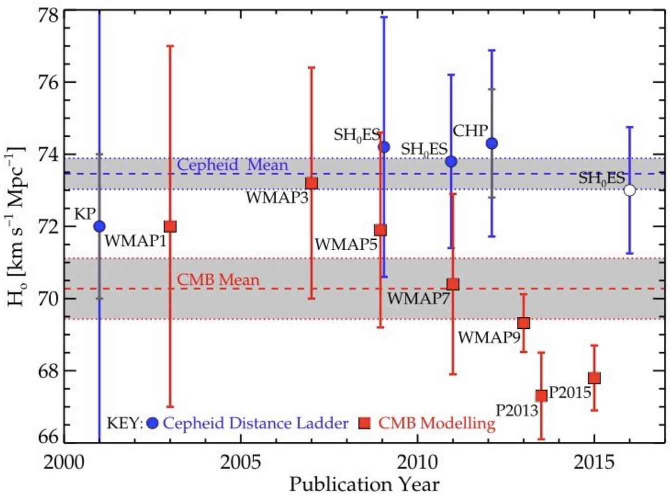
Structure can't exist on scales smaller than the mean free path of photons because on scales smaller than this photons and electrons aren't as coupled. The consequence of this is that variations decay exponentially which you see at the end of the power spectrum.

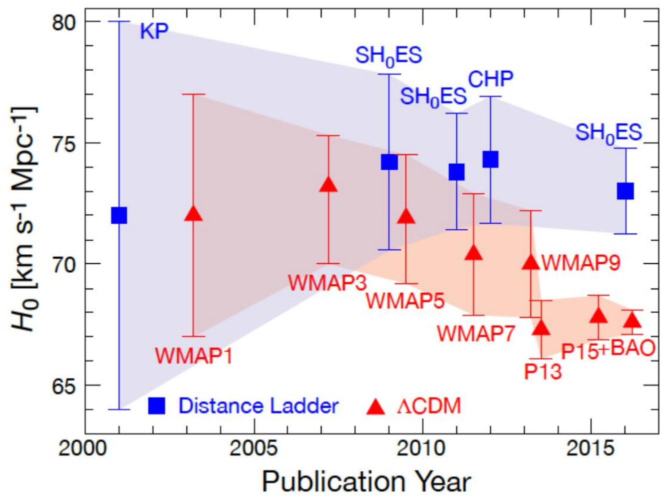
Cosmological parameters from CMB tail:

- Constrain curvature, baryonic density, and matter density
- Mean free path of photons depends on baryonic density, which constrains cosmo models
- Important to know the Silk diffusion damping model for your CMB observations to disentangle degeneracy with T fluctuations.
-

Question 28 - H0

Discuss three ways of constraining the Hubble constant, and possible degeneracies between the methods.

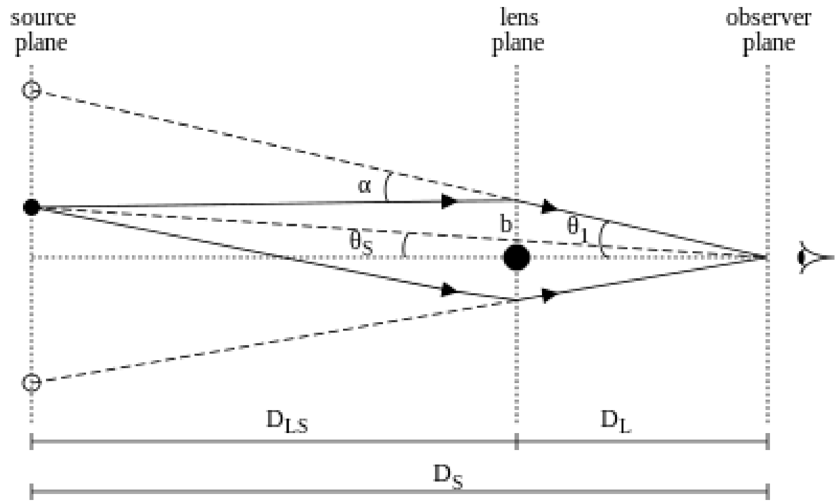




- CMB (BAO and other cosmological parameters)
  - Seeing what fits the power spectrum, assuming LambdaCDM. So if LambdaCDM is wrong, then H0 is wrong
- Supernovae Distance ladder
  - Standard candles - Type Ia (2 companion stars, where 1 is white dwarf)
  - Intrinsic luminosity is slightly different
  - Intrinsic luminosity depends on how fast light curve falls off in time
  - Distance modulus and velocity => H0
  - How well do we understand local astrophysics? Dust in front of SN can emit more IR that adds to IR of SN
- Gravity Waves:
  - Standard sirens
  - Know the absolute magnitude of the wave emission
  - Observed wave amplitude
  - Get distance from distance modulus
  - Get the redshift from difference in what frequency it should be merging
  - So have speed and distance
- Recessional velocity measurements in local Universe (Cepheid variables)
  - Edwin Hubble v = H0 d

Question 29 - Einstein Ring

Derive the equation for an Einstein ring for a point mass and discuss how things change when the lens is Extended.



Assume that all mass is concentrated at the center so that the lensing object is a point source.
For small angles the total deflection is given by

$$\alpha_1 = \frac{4G}{c^2}\frac{M}{b_1}$$

b_1 is the impact parameter, how close the light beam gets to the center of mass.

We can plug in $b_1 = \theta_1 D_L$

$$\alpha_1 = \frac{4G}{c^2}\frac{M}{\theta_1 D_L} \qquad \text{Eq 1}$$

θ_s is the angle at which one would see the source without the lens.
θ_1 is the observed angle of the image of the source with respect to the lens.

The lens equation is then:

$$\theta_1 D_S = \theta_S D_S + \alpha_1 D_{LS}$$

This can be rearranged

$$\alpha_1(\theta_1) = \frac{D_S}{D_{LS}}(\theta_1 - \theta_s)$$

Plugging this into equation 1 and rearranging

$$\theta_1 - \theta_s = \frac{4G}{c^2}\frac{M}{\theta_1}\frac{D_{LS}}{D_S D_L}$$

Making this dimensionless

$$\theta_1 = \theta_s + \frac{\theta_E^2}{\theta_1}$$

If the lens is not a point source but is extended, the Einstein radii of the Einstein ring does not change much so this means the Einstein radii is not very dependent on the mass distribution within the lens.

However other parameters of the lens are changed such as the image position and relative brightness.

Question 30 - Weak Lensing

What are some of the systematics in measurements of weak lensing? If they can be controlled, how are they accounted for?

1) The most important systematic arises in the image to catalog phase which is the **PSF** systematic, the idea is as follows your telescope has a response to light which is its point spread function and it's not a perfect delta function. For real telescopes you have to characterize that yourself and you do that by looking at non-lensed nearby stars. But obviously you won't get a star per pixel so you will have to Interpolate between stars and that process has very large error bars as a function of distance away from nearest star. So as a result the **psf uncertainty** can give you false info about the shape shear. A crude example to think of is the following, imagine your mirror had a giant bump in it everywhere you look it'll look like there is a black hole where that bump is because it'll distort everything!
2) PSF changes due to atmosphere and on a telescope basis
3) Another important systematic is inability to correct for certain electronic effects that are not multiplicative bias. In other words electronic effects that won't be removed by simply deconvolving the image from some fit. Instrumental noise.
4) The last important systematic in the image to catalog phase is the **selection bias**, that's basically a combination of things that account for you selecting a biased sample, the simplest bias arises from brightness you can only select objects of **certain levels of brightness**. Galaxy selection bias also present to color, as red galaxies are elliptical, so their intrinsic ellipticity is different from the blue sample. To measure the shear, you need to measure the intrinsic ellipticity of your galaxy.

In the catalog to science pipeline the main thing you need to know is that the shape measurement has both a multiplicative and an additive bias that takes the form

Shear_measured = (1+m) Shear_true + c

https://arxiv.org/abs/1210.7690
^^ Traditional source for weak lensing systematics discussion

Question 31 - Supernova Systematics

Discuss some of the systematics in supernova measurements. If they can be controlled, how are they accounted for?

- **K-correction**: The cosmological redshift affects the measurement of an object's spectrum because these observations are usually made within a specific wavelength region. For example, observations made with the V-band at 550 nm can be affected as the cosmological redshift brings shorter-wavelength radiation into the V band. This effect can be corrected by adding a compensating term called the K-correction to the distance modulus if the spectrum, Iλ, of the object is known.
  $$\mu = m - M = 5\log(\frac{d_L}{10pc}) + K$$
- **UV Spread**: Type Ia have a higher spread in the UV than in the optical or IR, which becomes problematic when cosmological expansion shifts the U band into the B band.
- **Reddening**: Intrinsic reddening of the SNe themselves and reddening due to dust should be handled separately, but in practice they are hard to deconvolve. Intrinsically fainter SNe Ia are redder than brighter ones, but the same effect occurs with dust extinction.
- **Galactic Evolution**: It is now well known that fainter SNe Ia tend to be embedded in older stellar populations; this translates to a $\sim 12\%$ brightness increase for $z = 1$SNe Ia due to increased star formation.
- **Curve Widths**: The width of the light curve is larger for Type Ia SNe at higher redshift than it is for local objects. This arises from the cosmological expansion which delays light signals by a factor of (1 + z).

Question 32 - Fisher

How are Fisher matrices used to forecast constraints in cosmology? (Part 1) Describe the difference between Fisher matrix techniques and Markov Chain Monte Carlo methods. (Part 2)

Fisher matrix

- Fisher information matrix is a way of measuring the amount of info that an observable X carries about an unknown parameter, p, that models X.
- The inverse of the Fisher matrix gives us the **covariance matrix**

  $$[F]^{-1} = [C] = \begin{matrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_{y^2} \end{matrix}$$

  σ_x and σ_y correspond to the 1 sigma errors on the parameters.

  $\sigma_{xy} = p\sigma_x\sigma_y$ where p is a correlation coefficient that varies from 0 (independent) to 1 (completely correlated)
- **Confidence Ellipses** can be plotted to show the degeneracy of solutions in parameter space.

$$a^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} + \sqrt{\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2}$$

$$b^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} - \sqrt{\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2}$$

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}$$

- These are multiplied by α depending on the confidence level we are interested in.
- For 68.3% CL (1-σ), $\Delta\chi^2 \approx 2.3$

$\alpha = \sqrt{\chi^2} \approx 1.52$

- The Fisher matrix can be manipulated
  - **Marginalization**: To find acceptable ranges for a parameter you can remove that parameter from the covariance matrix, take the inverse, and calculate a new fisher matrix.
  - **Priors**: If you already know one value to within some error you can include that. You would just add $1/\sigma^2$to the on-diagonal element corresponding to that variable.
  - **Combining different experiments**: just add the fisher matrices together
- The Fisher matrix can be used to find the **constraints** on the parameters
  - If we are given a Fisher matrix in terms of variables $p = (x, y, z)$ but we are interested in constraints on related variables $p' = (x', y', z')$. We can obtain a new Fisher matrix as follows:

$$F'_{mn} = \sum_{ij} \frac{\partial p_i}{\partial p'_m} \frac{\partial p_j}{\partial p'_n} F_{ij}$$

This can be evaluated in matrix form using a transformation.

$[F'] = [M]^T[F][M]$ where M is a matrix of partial derivatives.

- Types of observables: BAO, CMB power spectrum, CMB polarization
- Types of parameters: Omega_m, Omega_lambda, H_o, etc
- To get the **probability** that specific values are correct for measurements x and y:

$$\chi^2 = \frac{\left(\frac{\Delta x}{\sigma_x}\right)^2 + \left(\frac{\Delta y}{\sigma_y}\right)^2 - 2\rho\left(\frac{\Delta x}{\sigma_x}\right)\left(\frac{\Delta y}{\sigma_y}\right)}{1-\rho^2}$$

$$P(x, y) = \exp\left(-\frac{\chi^2}{2}\right)$$

In the case that the correlation coefficient, p, is equal to 0, we get back the familiar chi squared confidence interval. Fisher matrices are basically chi squared test on crack.

Question 32 - MCMC

MCMC

- Bayes Theorem
    - the posterior probability of the parameter set θ given the data D and the model M is given by:

$$P(\theta \mid D, \mathcal{M}) = \frac{\mathcal{L}(D \mid \theta, \mathcal{M})\, \pi(\theta \mid \mathcal{M})}{P(D, \mathcal{M})},$$

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- The MCMC method
    - Shifts the problem of calculating the unknown posterior probability distribution in the entire space, which can be extremely expensive for models with large number of parameters, to the problem of sampling from the posterior distribution. This is possible, for example, by growing a Markov chain with new states generated by the Metropolis sampler.

Difference b/w Fisher matrix and MCMC methods

- Fisher matrices can only tell you about models around some fiducial model that you choose (i.e., you have to choose a place where you take the derivatives). MCMC explores the entire parameter space.

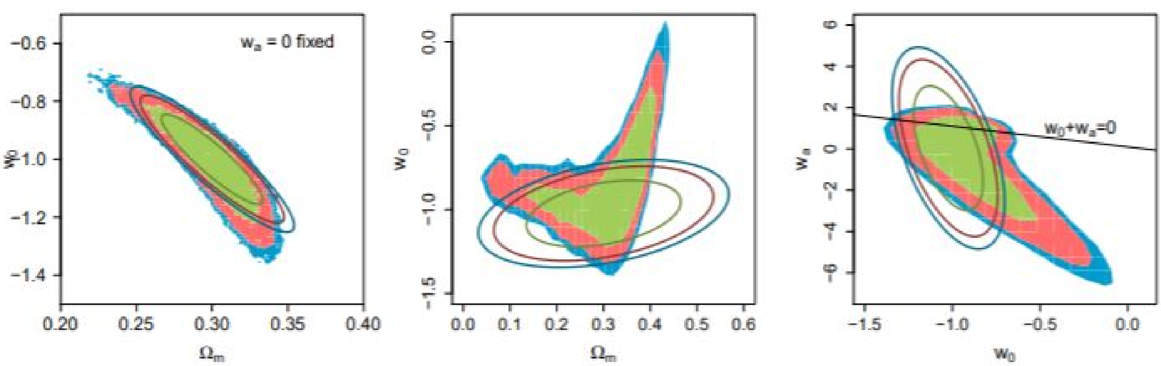

Figure 1. 68%, 90% and 95% confidence regions for a supernova survey. Filled contours correspond to the full posterior sampled with MCMC, while the solid lines represent the Fisher matrix results. The parameter spaces are $\{\Omega_m, w_0, M_{int}\}$ with fixed $w_a = 0$ (*left panel*), and $\{\Omega_m, w_0, w_a, M_{int}\}$ (*middle and right panels*). The parameters which are not shown have been marginalised in all panels.

Future Work

- Here's a paper on the difference between fisher and MCMC to read later: https://arxiv.org/pdf/1205.3984.pdf
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Question 33 - Degeneracy Direction

What is a degeneracy direction? Given an observable f(x,y), how would you compute the degeneracy direction between parameters x and y?

The long part of the confidence interval??

The degeneracy is the covariance of 2 parameters. Rho_12. The angular

Some cool confidence contour plots showing the degeneracy of parameters is can be seen here:
https://ned.ipac.caltech.edu/level5/March08/Linder/Linder3.html