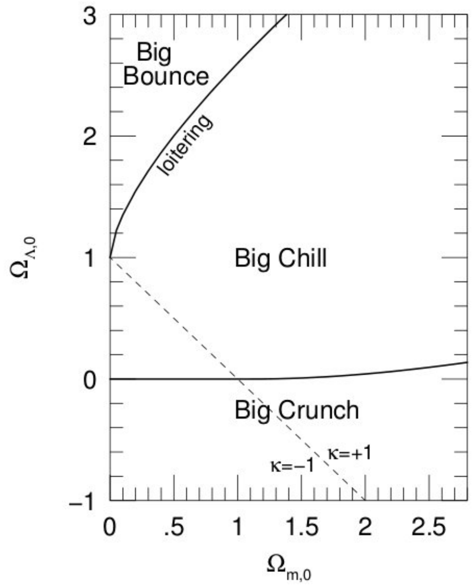

FIG. 16.— A parameter space study of possible fates of the universe given an Ω_m and an Ω_Λ. Curvature is assumed to be $1 - \Omega_m - \Omega_\Lambda$. From Ryden (2003), her Fig. 6.3.

not created by the Big Bang (or fused in stars before main sequence turnoff), is produced through the spallation of heavy elements due to galactic cosmic rays. Over time, the abundance of ^{9}Be increases, and therefore it serves as a clock. Observations of ^{9}Be content in GC MS turnoff stars suggest the MW is 13.6 ± 0.8 Gyr.

1.10.3. *Is Λ caused by vacuum energy?*

This information comes from Shaw & Barrow (2011).

The contribution of vacuum energy to the energy-momentum tensor is $-\rho_{\mathrm{vac}} g^{\mu\nu}$; i.e. vacuum energy density is fixed (so total vacuum energy scales with volume), and therefore $w = -1$, and Eqn. 8 gives a negative pressure. Formally, the value of ρ_{vac} is actually infinite, but if quantum field theory is only valid up to some energy scale E, then $\rho_{\mathrm{vac}} \propto E^4$. We know that QFT could be valid up to supersymmetry breaking (1000 GeV), the electroweak scale (100 GeV) or the Planck scale (10^{18} GeV). The vacuum energy density is therefore anywhere between 10^{12} to 10^{72} GeV^4. Actual measurements of ρ_Λ give 10^{-48} GeV^4, meaning that the energy density of ρ_Λ is approximately 60 to 120 orders of magnitude smaller than the vacuum energy density.

This problem gets worse; see Sec. 1.17.

WTF IS A LENGTH SCALE

1.11. *Question 10*

QUESTION: What are the currently accepted relative fractions of the various components of the matter-energy density of the universe? (i.e., what are the values of the various $Omega_{\mathrm{i}}$'s)

The relative fractions (with respect to the critical density $3H_0^2/8\pi G$) of Ω_b (baryon density), Ω_c (dark matter density), Ω_Λ (dark energy density), Ω_r(radiation density), Ω_ν (neutrino density) and Ω_κ (curvature) are:

$$
\begin{aligned}
\Omega_b &= 0.0458 \pm 0.0016 \\
\Omega_c &= 0.229 \pm 0.015 \\
\Omega_m &= 0.275 \pm 0.015 \\
\Omega_\Lambda &= 0.725 \pm 0.016 \\
\Omega_r &= 8.5 \times 10^{-5} \\
\Omega_\nu &< 0.0032 \\
\Omega &= 1.000 \pm 0.022 \\
\Omega_\kappa &= 0.000 \pm 0.022
\end{aligned}
$$

These values, except for Ω_r, come from Komatsu et al. (2011) and Jarosik et al. (2011) (for Ω_ν), or are calculated from them (in the case of Ω_m, Ω and Ω_κ). In both papers, WMAP 7-year data, alongside BAO and H_0, were used. Ω_r was calculated from $\Omega_m a_{rm}$, where a_{rm}. Additional parameters, such as SNe Ia measurements of luminosity distance, can help further constrain values such as Ω_κ.

These values are for the relative fractions of various components *today*, and so should all have subscript 0 attached to them (this has been ommitted for clarity). To determine what these values were at any time t, we evolve them in accordance with their equation of state (i.e. $a^{-(1+3w)}$) and divide by the critical density at the time $3H^2/8\pi G$. This is covered in depth in Sec. 1.1.3.

1.11.1. *What are the consequences of these numbers on the nature of the universe?*

The fact that Ω_0 is 1 within error (i.e. $\Omega_\kappa = 0$) indicates that the universe is flat, and therefore adheres to standard Euclidean geometry, and is likely spatially infinite. The current Ω_m and Ω_Λ indicate that the universe was once matter dominated and in the future will be much more Λ-dominated ($\frac{\Omega_m}{\Omega_\Lambda} = \Omega_{m,0}\Omega_{\Lambda,0}\frac{1}{a^3}$). This means that the expansion of the universe was once $\propto t^{2/3}$ and in the far future will approach $\propto e^{Ht}$. From a similar argument, we can find that in the very distant past (since $\Omega_m/\Omega_r = 1/a$) the universe was dominated by radiation. See Fig. 2 for a schematic.

Using the concordance model, we can determine that the co-moving distance to the furthest galaxy within our particle horizon is 16 Gpc (46 Gly). Due to the Hubble sphere receding, this horizon is larger than one would expect if galaxies receding with $v_{\rm pec} > c$ could never be seen (see Sec. 1.7)

1.11.2. *How do we determine Ω_r from the CMB?*

This information comes from Dodelson (2003), pg. 40, but integrating the Planck function is better explained in Hickson (2010), pg. 52.

The current photon energy density from the CMB can be calculated from the energy density of a blackbody:

$$U = \frac{4\pi}{c}\int_0^\infty I_\nu d\nu \tag{40}$$

where I_ν is the Planck Function, $\frac{2h\nu^3}{c^2(e^{h\nu/k_BT}-1)}$. This integration gives $U = \frac{8\pi^5 k_B^4}{15h^3c^3}T^4 = aT^4$. If we input the COBE FIRAS CMB temperature of 2.725 ± 0.002 K and divide by the current critical density of the universe $3H_0/8\pi G$, we obtain $\Omega_r \approx 4.7 \times 10^5$ today.

The present-day galaxy luminosity is 2×10^8 L$_\odot$/Mpc^3. From this the energy density of radiation from starlight can be estimated, and it turns out to be about $\sim 3\%$ of the CMB radiation. A more detailed estimate gives $\sim 10\%$. A miniscule amount of Ω_b has been turned into Ω_r.

1.11.3. *How are other values empirically determined?*

There are multiple techniques to determine the various Ω_i values empirically. These techniques are complementary, and generally a method of fitting is used to constrain a universe with χ (usually 6) numbers of unknown variables.

Techniques include (Dodelson 2003, Ch. 2.4):

1. There are four established ways of measuring Ω_b. The simplest way is to observe the luminosity of galaxies, and the X-ray luminosity of the hot gas between clusters (which house the majority of baryons in the universe), and convert to a mass. Another is to look at the column density of H from absorption in quasar spectra. The third is to use CMB anisotropy (see Sec. 1.13). Lastly, the matter density (or matter/radiation density ratio, since Ω_r is easily measured) helps determine the outcome of BBN.

2. The most obvious way of determining Ω_m is to use observations sensitive to Ω_b/Ω_m and then use the apparent value of Ω_b to determine Ω_m. For example, the temperature of virialized IGM in a cluster is sensitive to the total mass of the cluster, and can be determined using X-ray observations or the Sunayev-Zel'dovich effect (Sec. 2.18). Counting large clusters and matching the number to the expected number of large halos from N-body simulations also gives an estimate (Schneider 2006, pg. 314). Ω_m can be also be determined through baryon acoustic oscillations (Sec. 1.14) and the cosmic peculiar velocity field (peculiar velocities of galaxies, discernable by redshift surveys, are related to the rate at which overdensities grow, which depends on Ω_m).

3. The cosmic neutrino background remains unobserved. We know that Ω_ν acts much like a radiation field (since neutrinos are relativistic), but they decoupled from baryons just before BBN, when temperatures were still high enough to generate electron-positron pairs. At the end of pair production, the radiation field inherited some of the annihilation energy, while the neutrinos did not. Nevertheless, a detailed calculation (Dodelson 2003, pg. 44 - 46) can be used to show $\Omega_\nu \approx 0.68\Omega_r$ (number from Ryden). The component is also constrained when other Ω values, and the curvature of the universe, are known, hence why WMAP results, combined with HST H_0 and BAO, can place upper limits on it.

4. Ω_Λ can be constrained by observing the expansion history of the universe. Luminosity distances from SNe Ia can rule out to high precision that $\Omega_\Lambda > 0$. A combination of results from very different epochs (ex. CMB and SNe Ia out to $z = 1$) is necessary to properly remove the $\Omega_\kappa/\Omega_\Lambda$ ambiguity (Sec. 1.3).

1.11.4. *What are the six numbers that need to be specified to uniquely identify a ΛCDM universe?*

The six numbers are the (current) baryon fraction, the dark matter fraction, the dark energy fraction (ΛCDM assumes that $w = -1$ for dark energy), the curvature fluctuation amplitude Δ_R^2, the scalar spectral index n_s and the reionization optical depth τ. From these values, all others can be derived.

1.11.5. *Why is h often included in cosmological variables?*

h is the dimensionless Hubble parameter, defined as $H_0/(100\ \mathrm{km/(s\ Mpc)})$. It is often included in measurements of cosmological parameters that depend on H_0. Any error in the measurement of the current Hubble parameter can then be removed from the variable in question by including h in the expression (ex. $\Omega_c h^2$ for dark matter density), and relegating errors on H_0 to h.

Current estimates give $h = 0.72$.

1.12. *Question 11*

QUESTION: Outline the history of the Universe. Include the following events: reionization, baryogenesis, formation of the Solar system, nucleosynthesis, star formation, galaxy formation, and recombination.

This short chronology of the universe comes from Wikipedia (2011d), checked with Emberson (2012).

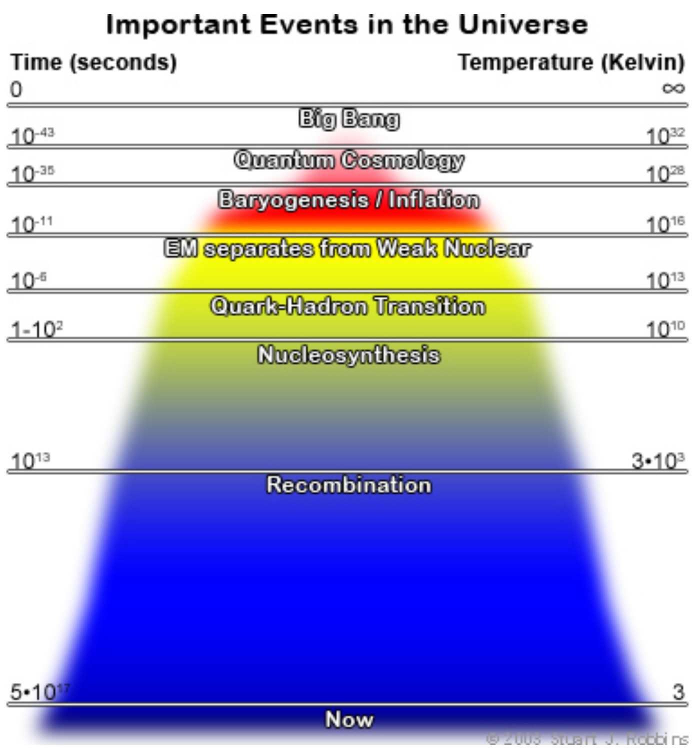

FIG. 17.— The history of the universe, with different cosmological eras, and their corresponding temperature and age ranges, labelled. From Robbins & McDonald (2006).

1. **Planck Epoch** (0 - 10^{-43} s?) - the Planck time is the time it takes light to travel one Planck length, or $t_P = l_P/c = \sqrt{\hbar G/c^5}$, and gives the length of time over which quantum gravity effects are significant. A physical theory unifying gravity and quantum mechanics is needed to describe this era. The four fundamental forces are merged.

2. **Grand Unification Epoch** (10^{-43} - 10^{-36} s?) - gravity separates from electronuclear force, and the universe can now be described by some flavour of grand unified theory (GUT). Separation of the strong nuclear force from the electroweak force at the end of the GUT Epoch produced magnetic monopoles in great quantity.

3. **Inflation** (10^{-36}–10^{-34} s?) - at 10^{-36} (?) seconds (and 10^{15} GeV) cosmological inflation (Sec. 1.16) is triggered. While the true microphysical cause of inflation is not known, Carroll & Ostlie (2006) describes inflation as a bubble of true vacuum (created via quantum fluctuation) surrounded by a false vacuum with a large energy