1.9.3. *Can SNe II be used as standard candles?*

Yes; in particular the recombination bump of SNe II-P can be used as a standardizable candle (Kasen & Woosley 2009). In these SNe, there is a tight relationship between luminosity and expansion velocity (as measured from $\sim$5000 ÅFe II absorption lines), explained by the simple behavior of hydrogen recombination in the supernova envelope (Kasen & Woosley 2009). There is sensitivity to progenitor metallicity and mass that could lead to systematic errors, and overall SNe II are dimmer than SNe Ia, however (Kasen & Woosley 2009).

1.10. *Question 9*

QUESTION: Rank the relative ages of the following universes, given an identical current-day Hubble constant for all of them: an accelerating universe, an open universe, a flat universe.

Most of this information comes from Emberson (2012) and Ryden (2003).

In general, the age of the universe can be determined (assuming a is normalized such that $a_0 = 1$) by solving the Friedman-Lemâtre equation, Eqn. 4. Equivalently, we note that $H(z(t)) = \dot{a}/a$, which can be rewritten as

$$dt = \frac{da}{aH} \tag{36}$$

There are, of course, time dependencies on both sides, but since a and z are simply functions of time (if a and z are not positive-definite, then we can parameterize both t and a or z as a function of some parameterization θ) and we can treat z as an independent variable to solve for t by rewriting this equation with Eqn. 9 to obtain:

$$t = \frac{1}{H_0}\int_0^\infty \frac{1}{1+z}\frac{1}{(\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + (1-\Omega_0)(1+z)^2 + \Omega_{\Lambda,0})^{1/2}}dz. \tag{37}$$

Needless to say, for non-trivial cosmologies this requires some kind of numerical simulation. (For true order-of-magnitude enthusiasts, however, the right side is ~ 1, giving us the Hubble time.) Let us consider a few more trivial cosmologies.

As noted in Sec. 1.1.4, in a Λ-dominated universe, $H = H_0$, and as a result $a = Ce^{H_0 t}$. Such a universe is eternally old, since a never goes to zero. In an empty, $\Lambda = 0$ universe (which naturally would be open), $H^2 = \frac{c^2}{R^2}\frac{1}{a^2}$, meaning $a = \frac{c}{R}t$, and $H = 1/t$. The age of the universe is then $1/H_0$. In a matter-dominated critical universe, $\kappa = 0$, which results in $a = H_0^{1/3}\left(\frac{3}{2}t\right)$, which gives $H_0 = 2/3t$, and therefore the age is $2/3H_0$. For a flat universe in general, we can assume the energy density goes like $a^{-(1+3w)}$, and we obtain $t = \frac{2}{3H_0(1+w)}$ - a critical radiation-dominated universe has an age $1/2H_0$. The shorter time is due to the fact that in all matter dominated, non-empty universes $H(t)$ decreases over time, and for the same H_0 today $H(t)$ must have been much larger in the past, leading to a shorter amount of time needed to expand the universe to its current size. Open universes will have ages that lie somewhere between $1/H_0$ and $2/3H_0$, while supercritical universes, which cannot easily be calculated (see below), will have ages shorter than $2/3H_0$ because the slowdown of $H(t)$ becomes even more extreme than for a critical universe. See Fig. 15.

Our answer then, is that a flat universe is younger than an open universe, which is younger than an accelerating universe, if we assume the open and flat universes are matter dominated, and the accelerating universe is Λ dominated. If H_0 is fixed, then, in general, if Ω_κ is kept fixed while $Omega_\Lambda$ is increased, the age of the universe increases. If Ω_Λ is kept fixed while increasing Ω_κ, the age decreases. (Ω_m must vary to compensate for fixing one Ω while moving the other.)

From concordance cosmology, the universe is 13.7 Gyr.

1.10.1. *What is the fate of the universe, given some set of Ωs?*

In a matter-dominated universe, where $\Omega_\kappa = 1 - \Omega_m$, we may rewrite the FL equation as

$$\frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3} + \frac{1-\Omega_m}{a^2}. \tag{38}$$

Without solving for anything, we can easily see that if $\Omega_m > 1$ there will be a maximum size to the universe when $H^2 = 0$. This problem actually can be solved analytically (pg. 106 of Ryden) to yield $a(\theta) = \frac{1}{2}\frac{\Omega_m}{\Omega_m - 1}(1-\cos(theta))$ and $t(\theta) = \frac{1}{2H_0}\frac{\Omega_m}{(\Omega_m-1)^{3/2}}(\theta - \sin\theta)$. The universe, therefore, begins and ends in finite time. Similarly, an analytical solution also exists for $\Omega_m < 1$, though from our previous discussion it is obvious that this universe expands forever. In matter-dominated universes, therefore, matter determines fate.

When Λ is added to the mix, we solve for

$$\frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3} + \frac{1-\Omega_m-\Omega_\Lambda}{a^2} + \Omega_\Lambda. \tag{39}$$

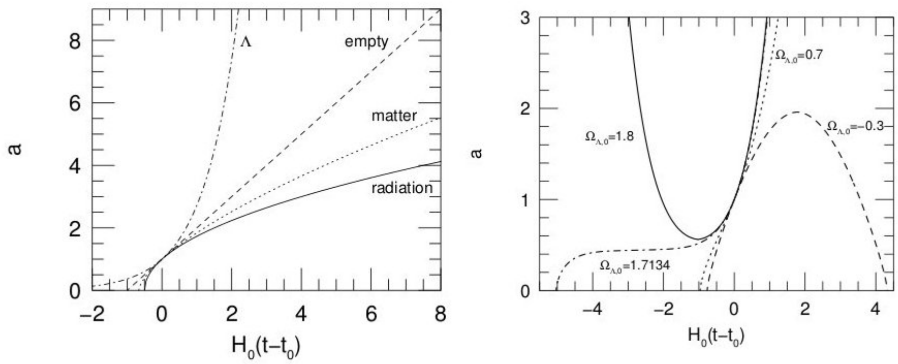

FIG. 14.— Left: a plot of scale factor a as a function of time for a Λ-dominated, empty $\Lambda = 0$, critical matter-dominated and critical radiation-dominated universe. Curves have been scaled so that they all correspond to H_0 today. The point at which $a = 0$ is the age of each universe. Right: the scale factor a as a function of time for universes with $\Omega_m = 0.3$ and varying Ω_Λ. From Ryden (2003), her Figs. 5.2 and 6.4.

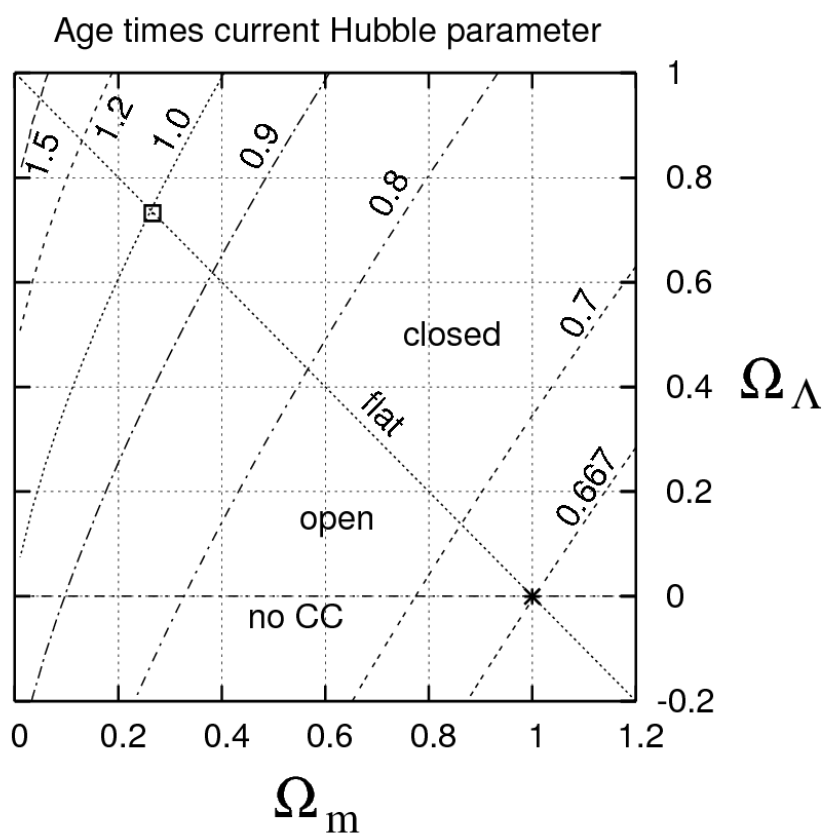

FIG. 15.— Age of the universe, scaled to the concordance cosmology value of 13.7 Gyr.

It is then possible for a closed universe to expand forever. This is because Λ has a constant negative energy density, and the negative pressure caused by the cosmological constant only increases with time. If Ω_Λ is taken to extreme values, it may be impossible for a to drop below a certain value (as H^2/H_0^2 becomes negative) - such a universe must start and end with $a \to \infty$, the "Big Bounce". On the precipice of creating a Big Bounce, a universe can loiter at a fixed a (as curvature attempts to "fight" expansion from Ω_Λ) for long periods of time. Fig. 16 summarizes this discussion.

Concordance ΛCDM suggests the universe will continue to expand forever, approaching exponential expansion.

1.10.2. *How do we determine, observationally, the age of the universe?*

Lower limits on the age of the universe can be determined by looking at the ages of its contents. Globular cluster main sequence turnoff, for example, can be used to date GCs. These generally give $\sim 10 - 13$ Gyr. ^{9}Be, an isotope

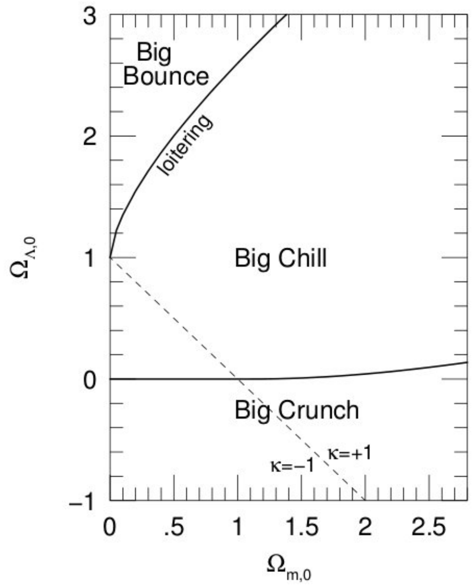

FIG. 16.— A parameter space study of possible fates of the universe given an Ω_m and an Ω_Λ. Curvature is assumed to be $1 - \Omega_m - \Omega_\Lambda$. From Ryden (2003), her Fig. 6.3.

not created by the Big Bang (or fused in stars before main sequence turnoff), is produced through the spallation of heavy elements due to galactic cosmic rays. Over time, the abundance of ^{9}Be increases, and therefore it serves as a clock. Observations of ^{9}Be content in GC MS turnoff stars suggest the MW is 13.6 ± 0.8 Gyr.

1.10.3. *Is Λ caused by vacuum energy?*

This information comes from Shaw & Barrow (2011).

The contribution of vacuum energy to the energy-momentum tensor is $-\rho_{\rm vac} g^{\mu\nu}$; i.e. vacuum energy density is fixed (so total vacuum energy scales with volume), and therefore $w = -1$, and Eqn. 8 gives a negative pressure. Formally, the value of $\rho_{\rm vac}$ is actually infinite, but if quantum field theory is only valid up to some energy scale E, then $\rho_{\rm vac} \propto E^4$. We know that QFT could be valid up to supersymmetry breaking (1000 GeV), the electroweak scale (100 GeV) or the Planck scale (10^{18} GeV). The vacuum energy density is therefore anywhere between 10^{12} to 10^{72} GeV^4. Actual measurements of ρ_Λ give 10^{-48} GeV^4, meaning that the energy density of ρ_Λ is approximately 60 to 120 orders of magnitude smaller than the vacuum energy density.

This problem gets worse; see Sec. 1.17.

WTF IS A LENGTH SCALE

1.11. *Question 10*

QUESTION: What are the currently accepted relative fractions of the various components of the matter-energy density of the universe? (i.e., what are the values of the various $Omega_{\rm i}$'s)

The relative fractions (with respect to the critical density $3H_0^2/8\pi G$) of Ω_b (baryon density), Ω_c (dark matter density), Ω_Λ (dark energy density), Ω_r(radiation density), Ω_ν (neutrino density) and Ω_κ (curvature) are:

$$
\begin{aligned}
\Omega_b &= 0.0458 \pm 0.0016 \\
\Omega_c &= 0.229 \pm 0.015 \\
\Omega_m &= 0.275 \pm 0.015 \\
\Omega_\Lambda &= 0.725 \pm 0.016 \\
\Omega_r &= 8.5 \times 10^{-5} \\
\Omega_\nu &< 0.0032 \\
\Omega &= 1.000 \pm 0.022 \\
\Omega_\kappa &= 0.000 \pm 0.022
\end{aligned}
$$

These values, except for Ω_r, come from Komatsu et al. (2011) and Jarosik et al. (2011) (for Ω_ν), or are calculated from them (in the case of Ω_m, Ω and Ω_κ). In both papers, WMAP 7-year data, alongside BAO and H_0, were used. Ω_r was