**QUESTION: Explain how measurements of the angular power spectrum of the cosmic microwave background are used in the determination of cosmological parameters.**

This information comes from Ch. 30.2 of Carroll & Ostlie (2006), Ch. 8.6 of Schneider (2006), and of course Emberson (2012).

To first order, the CMB is a perfect blackbody peaked at 2.73 K, or 2 mm, a consequence of inflation (which generated nearly identical initial conditions) and the same evolution since (see below for why the blackbody is perfect). To higher order, this is not true.

Following inflation, the quantum (matter, not just baryon) density fluctuations were expanded to much larger than the particle horizon. As these fluctuations come into the particle horizon, they could react to each other, communicating their overdensities via acoustic waves (since photon propagation was stymied). The photons sloshed with the baryons, and therefore at the time of recombination there were characteristic overdensities of photons as well. Effects ("primary anisotropies") directly pertaining to inhomogeneities at the time of last scattering include:

- **The Sachs-Wolfe effect**: photons are redshifted due to passing out of an overdense dark matter region. They are also time dialated, and therefore do not cool as much as their surroundings. The combined effect, called the Sachs-Wolfe effect, is to make photons cooler in overdensities (compared to the mean) and hotter in the underdensities.
- **Peculiar velocities**: velocities of individual regions of the universe Doppler shift the photons trapped in them.
- **Enhanced baryon density**: in regions of dark matter overdensity baryons are also overdense. On scales larger than the horizon at recombination the distribution of baryons traces the distribution of dark matter. On smaller scales, the pressure of the baryon-photon fluid result in adiabatic heating when compressed.
- **Silk damping**: on small scales photons and baryons are not coupled as there is a finite mean free path of photons (set by time between Thompson scattering events). Temperature fluctuations is smeared out by photon diffusion below $\sim 5$'.

The first three effects are highly coupled to one another. On scales larger than the sound horizon at recombination the first two effects partly compensate for each other. On scales smaller, dark matter densities drive enhanced baryon densities, while pressure in the baryon-photon soup act as a restoring force, creating forced damped oscillations. These are known as baryon acoustic oscillations (BAO), and are qualitatively described below.

Secondary effects include Thompson (decreases anisotropy) and inverse Compton (Sunayev-Zel'dovich effect) scattering of CMB photons, gravitational lensing of CMB photons, and the "integrated Sachs-Wolfe effect". The last effect is due to photons falling into and out of a potential well. If the potential well did not change over time, the net effect is zero (possibly some lensing), but if the potential did change, the net effect is measureable.

The overall CMB anisotropy is written as $\delta T(\theta,\phi)/T = \frac{T(\theta,\phi)-T}{T}$ ($T$ is the mean temperature):

$$\frac{\delta T(\theta,\phi)}{T} = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} a_{l,m} Y_m^l(\theta,\phi), \tag{41}$$

where $Y_m^l(\theta,\phi)$ is a spherical harmonic. To remove the arbitrary choice of where $\phi = 0$, we average over all $m$ values. We then define:

$$C_l = \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{l,m}|^2. \tag{42}$$

Notice how $C_l$ is positive even if $a_{l,m}$ is negative - this means temperature peaks and troughs all contribute. The angular power spectrum is then written out as $\frac{\Delta_T^2}{T} = l(l+1)C_l/2\pi$ or $\Delta_T = T\sqrt{l(l+1)C_l/2\pi}$, as $\Delta_T$ tells us the contribution per logarithmic interval in $l$ to the total temperature fluctuation (Ryden 2003, pg. 200). The result of performing such an analysis using, say, WMAP data, gives us Fig. 18.

Each region of the power spectrum describes some aspect of cosmological evolution. Note that we can convert between $l$ and effective angular size simply through $l \sim 180^\circ/\theta$.

The region far to the left of the first peak represents scales so large as to not be in acoustic causal contact at recombination. The anisotropy at these scales therefore directly reflects the matter fluctuation spectrum $P(k)$. For a Harrison-Zel'dovich spectrum, we expect $\Delta_T^2$ to approximately be constant for $l < 50$.

**THERE IS A REGION THAT IS IN GRAVITATIONAL CONTACT BUT NOT ACOUSTIC CONTACT - WHAT HAPPENS TO THOSE?**

The first peak is determined by the largest structure that could have been in acoustic causal contact at last scattering. The soundspeed of a relativistic gas, from $\sqrt{P/\rho}$ is $c_s = c/\sqrt{3}$, where $c$ is the speed of light. We estimate the size of

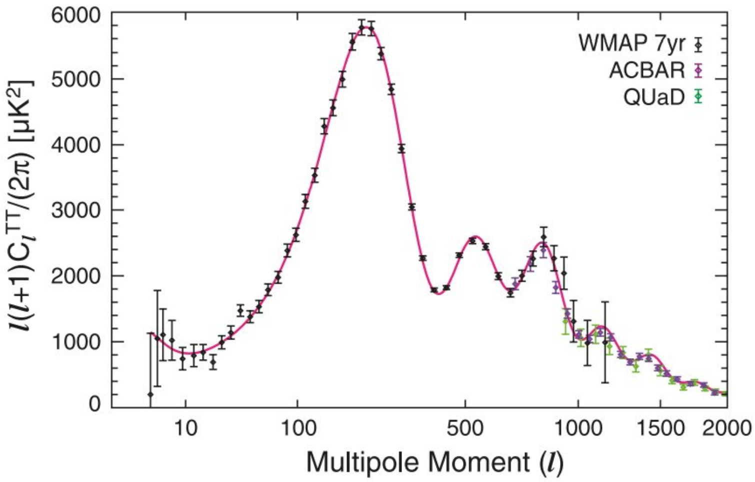


FIG. 18.— WMAP 7 year data CMB power spectrum, alongside data from ground-based ACBAR and QUaD. Note that this is the square of the CMB SED. The pink line is a best fit 6-parameter ΛCDM model. From Komatsu et al. (2011).

the horizon at about 1.8° (see below), which means the angular size of the first peak should be $\theta \sim 1.04°$, or $l \sim 175$. Since the system must react to the data it receives from the horizon, it is sensible that $l = 200$ is the true value (a more detailed calculation can show that $l = 200/\sqrt{\Omega_0}$.

We can model, *a la* pg. 1265 - 1266 of Carroll & Ostlie (2006) baryon-photon soup oscillations as a vertical piston with two regions of material. The upper and lower regions of the piston both start out with the same density, but because gravity pulls down, the lower region is compressed. The difference in densities between the two regions at maximum compression is much greater than the difference in densities between the two regions at maximum rarefactions, since the equilibrium position of the piston is one in which the upper region is less dense than the lower region.

Likewise, in the CMB, the initial distribution of matter over/underdensities was centred around the average density of the universe, not the equilibrium density configuration. As a result, a region that has just come into causal contact will first adiabatically contract. Since the baryon-photon fluid is essentially a trapped glob of photons, $P = \frac{1}{3}U/V$, meaning that radiation pressure will increase dramatically, acting as a restoring force, and the gas eventually expands again. This oscillatory motion changes the temperature of the fluid - hottest at maximum compression, coldest at maximum expansion (modulated by the Doppler effect, which also plays a role in shaping emission from these regions). Since the equilibrium position of the system is a (less extreme) compression, the compression will be stronger than the rarefaction. From all this, we conclude that the first peak in the CMB power spectrum is due to the maximum compression, on the largest possible scale at last scattering, of baryon-photon fluid. When photons decoupled from baryons at last scattering, the CMB retained the imprint of excess temperature at this scale.

The first trough is produced by an area somewhat smaller than the horizon. Its oscillation speed was faster than the first peak oscillation, and therefore it reached $\delta T = 0$ at time of last scattering.

The second peak is generated by rarefaction (recall that $C_l$ included an absolute value!). As discussed earlier, rarefactions are weaker than compression because the equilibrium state is closer to compression, and so the first peak is greater in magnitude than the second peak, and the relative suppression of the second peak increases with $\Omega_b$. This is because increasing $\Omega_b$ moves the equilibrium position to a greater compression, "loading down" the plasma (Hu & Dodelson 2002). **WAIT AT SOME POINT WE'D EXPECT BARYONS TO OUTNUMBER DARK MATTER, AND THIS WOULDN'T BE TRUE!** Indeed, all odd peaks are compressions, and all even peaks rarefactions, which accounts for, in our universe, the fact that odd peaks are slightly taller than even peaks.

The third peak, due to compression, is sensitive to the density of dark matter $\Omega_c$. This is because when photons contribute substantially to the universe's overdensities, their dilution during maximum compression due to the expansion of the universe, helps to amplify acoustic oscillations (Hu & Dodelson 2002). Since the low $l$ modes were launched when radiation dominated the most, they are the most amplified by this effect, explaining why the first peak of the CMB is taller than all those that follow it. When $\Omega_c$ is increased, the position of matter-radiation equality is shifted, lowering most of the peaks. Increasing $\Omega_c$ also increases baryon compression, however, which allows the odd peaks to retain some of their height. The relative height of the third to the second peak, then, helps pin down $\Omega_c$.

For $l > 1000$, Silk damping becomes significant, and the peaks die off.

Fig. 20 shows detailed theoretical calculations of the power spectrum as functions of various cosmological parameters. $\Omega_0$ is related to curvature, which serves to shift the peaks of the CMB (by changing the time of last scattering), and disturb the spectrum on very large scales (the latter is a function of the integrated Sachs-Wolfe effect, which strengthens with curvature). Changing $\Omega_\Lambda$ serves to distort very large scales (because expansion rate modifies the integrated Sachs-Wolfe effect) and shift the peaks of the CMB (because it changes, slightly, the time of last scattering). Increasing $\Omega_b$ increases the odd-numbered peaks, and decreases the strength of Silk damping (since it decreases the photon mean free path). Increasing $\Omega_m$ dramatically decreases peak amplitudes for reasons described earlier, and slightly increases the $l$ on peaks (from changing the time of last scattering).

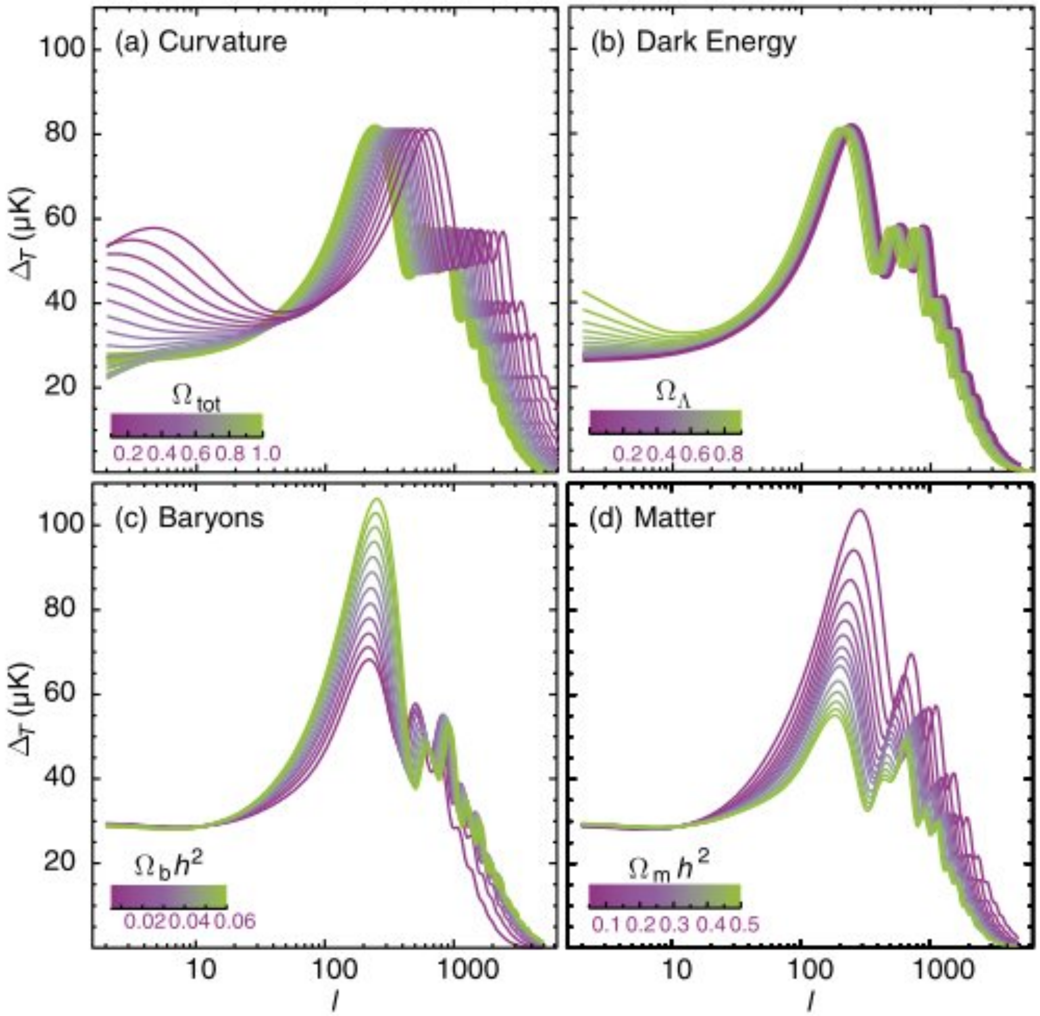


FIG. 19.— CMB power spectrum as a function of cosmological parameters $\Omega_0$, $\Omega_\Lambda$, $\Omega_b$ and $\Omega_m$. From Schneider (2006), his Fig. 8.24.

### 1.13.1. *How does the CMB power spectrum support the inflation picture?*

In general, a collection of random density perturbations would not be in phase, and we would not see regular peaks in the CMB power spectrum. Because of inflation, the entire universe received the same initial perturbation structure (i.e. statistically homogeneous initial conditions), and oscillations began at a certain scale when the sound horizon reached that scale.

### 1.13.2. *Derive the horizon size at recombination.*

See pg 170 - 171 of Schneider (2006).

The proper distance of the particle horizon is $r_H = a(t)d_c$. At large $z$, if we assume matter domination, then $r_H \approx 2\frac{c}{H_0}\frac{1}{\sqrt{(1+z)^3\Omega_m}}$. If we assume radiation domination, then $r_H \approx \frac{c}{H_0\sqrt{\Omega_r}}\frac{1}{(1+z)^2}$. Note that in our universe, matter domination and flatness necessary gives $r_H \approx 2\frac{c}{H_0}\frac{1}{(1+z)^{3/2}}$, which, since $t = \frac{2}{3H_0}a^{3/2}$, gives $r_H = 3ct$.

The angular size of the horizon is given by $r_H/d_A$, and Eqn. 14 can be modified to read $d_a \approx \frac{c}{H_0}\frac{2}{\Omega_m z}$ for high redshift and matter domination. This gives us $\theta_H \approx \sqrt{\Omega_m}1.8°$, so the size of the horizon is about 1° across the present-day sky. This derivation is modified if $\Omega_\Lambda$ is involved - instead of $\Omega_m$, $\Omega_m + \Omega_\Lambda$ is used - , which gives $\sim 1.8°$ for the horizon size.

### 1.13.3. *Why is the CMB a perfect blackbody?*

If we assume the universe is homogeneous, then there are no spatial temperature gradients. There is a temporal temperature gradient, which could produce absorption lines (Sec. 4.3), but conveniently $T \propto 1/a \propto 1 + z$ and $\lambda \propto 1 + z$ (and a redshifted blackbody looks like a blackbody), and therefore radiation produced during the entirety of recombination has the same blackbody spectrum.

### 1.13.4. *How is the CMB measured?*

The CMB was first observed by Arno Penzias and Robert Wilson at Bell Laboratories, using a horn-reflector radio antenna. Modern day observations are done using millimetre observatories situated in very dry regions (water absorption is strong at about 70 GHz). Examples include high altitude balloons such as Boomerang, ground-based observatories such as the South Pole Telescope, and spaceborne platforms such as COBE and WMAP.

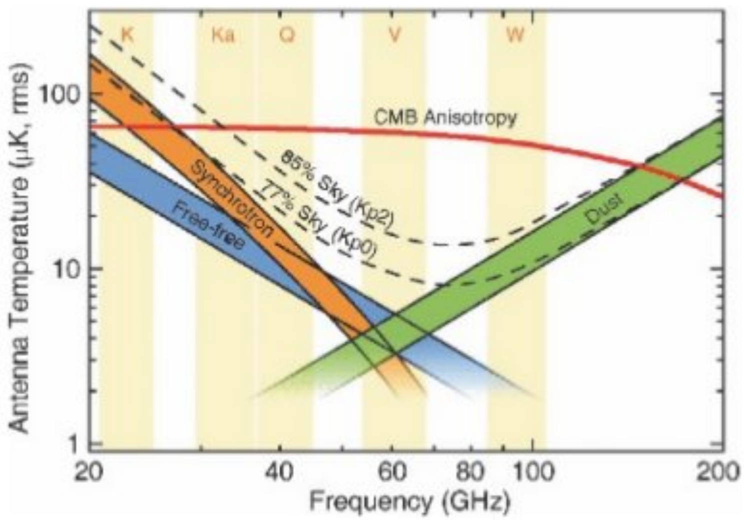


FIG. 20.— The CMB power spectrum (written in "antenna temperature", $\propto I_\nu \nu^{-2}$), with sources of noise superimposed. The five frequency bands of WMAP are also labelled. "Sky" is the summation of free-free, synchrotron and dust emission averaged over a certain fraction of the sky. From Schneider (2006), his Fig. 8.26.

When performing observations of the CMB, foreground sources from our Galaxy, namely synchrotron from relativistic electrons, bremsstrahlung from hot gas, and thermal emission from dust, (Fig. 20) must be removed. There are two ways to do this. The spectral index of synchrotron is -0.8 ($I_\nu \propto \nu^{-0.8}$, for bremsstrahlung it is 0, and for dust it is 3.5. For the CMB it is 2 (Rayleigh-Jeans tail). With this knowledge, a large number of spectral measurements could be made in the microwave, and the spectral indicies of foreground sources (highly different than CMB) could be fit for and eliminated. The other option is to use known tracers of each source of emission at other wavelengths where they dominate (radio for synchrotron, H-$\alpha$ for bremsstrahlung and FIR for dust), and use those measurements (and a model of emission) to remove them from the CMB map.

### 1.13.5. *Why did people use to think CMB anisotropy would be much larger than it is currently known to be?*

In Sec. **??** we discuss the growth of matter overdensities, and noted that for an Einstein-de Sitter (flat, critical) universe the density $D \propto a$, i.e. density grows with redshift. The average supercluster has $\delta \sim 1$, meaning that their collapse histories are reasonably well-described by $D \propto a \propto 1/(1+z)$. This means that at $z \approx 1100$, $\delta \approx 10^{-3}$. CMB overdensities are of order $10^{-5}$, which does not match. The solution is to invoke the fact that baryons, because they have yet to decouple with photons, would have not had as strong overdensities as dark matter, which did have overdensities of $10^{-3}$.

### 1.13.6. *What is the use of CMB polarization?*

Polarization occurs due to Thompson scattering of CMB photons, and is a 1% effect. Since Thompson scattering cannot produce polarization, it is the CMB anisotropy scattering off the surface of last scattering that must have produced it. This polarization will eventually be modified by reionization, and therefore measuring CMB polarization gives information at the redshift that reionization occurred (10.5 according to WMAP).

Electron motion causes different patterns to appear in CMB polarization. E-modes are caused by bulk movement of electrons during baryon-photon fluid oscillation, while B-modes are caused by gravitational radiation compression a ring of electrons. To date, B modes have not been observed.

## 1.14. *Question 13*

**QUESTION: Explain how measurements of bayron-acoustic oscillations can be used in the determination of cosmological parameters.**