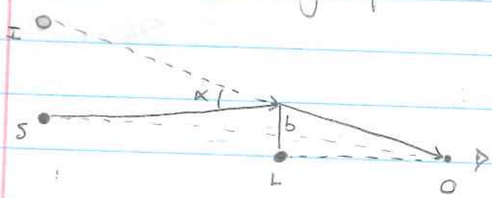
P1

- draw diagram
- write down eqn, explain its origins (geodesic + Schw. eqn)
- do dimensional analysis to justify eqn, potentially explain when its useful.

Gravitational Microlensing

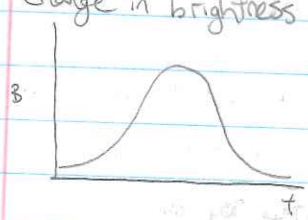
- Light passing by a point mass M is deflected by an angle $\boxed{\alpha = \frac{4GM}{bc^2}}$. This comes from general relativity, making use of the geodesic equation and the Schwarzschild metric.
- the deflection angle predicted by GR is twice that of Newtonian gravity.



- This deflection angle makes sense dimensionally. As the lense gets more massive, space becomes more warped and the light is deflected by a larger angle. As the light rays stray farther from the lens, space is less warped further out and so the deflection angle is smaller. The deflection is caused by gravity, so G has to be there, and the photons travel at c, so that's in there too.

- See EG6 and EG13 for applications to galaxy cluster detection and mass determination.

- Change in brightness of lensed source as lens moves across it:



- because lensing magnifies image

- determine baseline of telescope assuming diffraction limited
- explain that interferometers work by correlating signals, which are affected by phase lag and thus depend on baseline
- draw response function as sinc(x)

P2

Interferometer

- This is sort of similar to the single slit experiment in that we get an interference pattern that will depend on the size of the slit/baseline of the telescopes. So we need to figure out the angular size of the object we want to resolve, and then we can describe the response/intensity given that value: $\sin\theta = \lambda/D$

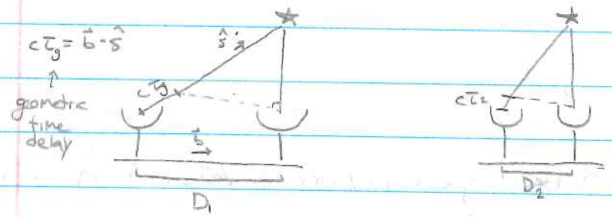
- use small angle approx. so $\sin\theta \sim \theta$

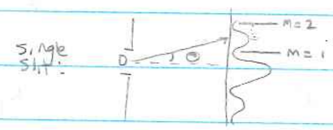
$\theta = \frac{1}{50}$ arcseconds $= \frac{1}{50}'' \cdot \frac{1^\circ}{3600''} \cdot \frac{1\text{ rad}}{57^\circ} = 10^{-7}$ rad

$\lambda = 2\ \mu m = 2 \times 10^{-6}$ m

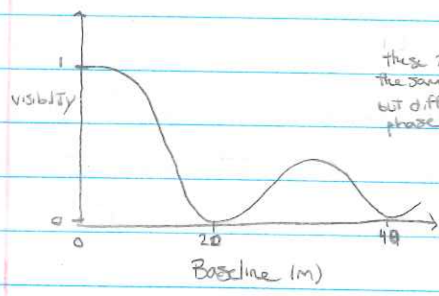
$\therefore D = \frac{\lambda}{\theta} = \frac{2 \times 10^{-6}\text{ m}}{10^{-7}\text{ rad}} = 20$ m

- this is the baseline of the interferometer. Changing the baseline will change the intensity you receive, since it'll adjust the delay time between the two telescopes:





- This will affect the interference pattern you observe. Recall that for the single slit experiment, the minima occur at integer multiples: $\sin\theta = \frac{m\lambda}{D}$



these 2 are the same V, but different phase lag.

- Dish 1 will measure a voltage of $V_1 = V\cos(\omega(\tau - \tau_g))$.
  Dish 2 will measure a voltage of $V_2 = V\cos(\omega\tau)$.
  You multiply these together and take the average, which gives you your final signal: $R = \frac{V^2}{2}\cos(\omega\tau_g)$.
  - correlated ≡ multiplied and time-averaged.

- See www.cv.nrao.edu/course/astr534/Interferometers1.html

P3

- Assume *Rschw., write down $F_t = \frac{dg}{dr} L$, get to $M \sim F_t^{-1/2}$ and $M \sim 10^{3-4} M_\odot$
- Explain that since $M \sim F_t^{-1/2}$, $\uparrow M$ = easier to survive
- Explain that shredding a star means $F_t > F_g$ and the Roche limit must be overcome. Assume $R_{roche} = R_{schw.}$ so $M \sim 10^8 M_\odot$, SMBH

- Black Hole Shredding and Tidal Disruption Flares

- Assuming we have a non-rotating BH, the location of the event horizon will correspond to the Schwarzschild radius:

$$r_S = \frac{2GM}{c^2}$$

- The tidal force you'd need to withstand is:

$$F_T = \frac{dg}{dr} L$$

- where L = your length and $g = \frac{-GM}{r^2}$

$$\therefore F_T = \frac{d}{dr}\left(-\frac{GM}{r^2}\right) L$$

$$= \frac{2GML}{r^3} \leftarrow r_s$$

$$= 2GML\left(\frac{2GM}{c^2}\right)^{-3}$$

$$= \frac{c^6 L}{4G^2M^2}$$

$$\therefore \boxed{M = \sqrt{\frac{c^6 L}{4G^2 F_T}}}$$

- The tidal force a typical human can withstand is ~10-100 g's (Most people black out at 5 g's, but wouldn't be ripped to shreds yet). Plugging in all the numbers gives a value of $\boxed{M \sim 10^{3-4} M_\odot}$
- Based on the equation $F_T = c^6L/4G^2M^2$, $\boxed{F_T \propto M^{-2}}$, so as $M \uparrow$, $F_T \downarrow$. So more massive BHs are more survivable, and v.v.

- How does this relate to the mass range of BHs that have TDFs due to shredding MS stars? Let's consider a Solar-type star for simplicity, so $M = 1 M_\odot$ and $R = 1 R_\odot$. To pull apart a star, the tidal force must exceed the star's own gravitational force (ie the Roche limit must be overcome):

$$F_T = F_g$$

$$R_{roche} = R_* \left(\frac{2M_{BH}}{M_*}\right)^{1/3}$$ * see Wikipedia for derivation. Just uses point mass on star.

$\therefore$ if we assume the maximum radius/distance the star can get is r_s, we have:

$$M_{BH} = \frac{M_*}{2}\left(\frac{r_s}{R_*}\right)^3$$

$$= \frac{M_*}{2}\left(\frac{2GM_{BH}}{c^2 R_*}\right)^3$$

$$M_{BH} = \sqrt{\frac{c^6 R_*^3}{4 M_* G^3}}$$

$$\boxed{M_{BH} \sim 10^8 M_\odot}$$

- If you consider other M/R values for MS stars, the range is $\sim 10^{7-9} M_\odot$. So this is in the range of SMBHs. Of course assuming $R_{roche} = r_s$ isn't really right, so there's that... Anyway, the BH mass range for shredding a human is much ~~smaller~~ lower... than that for TDFs.

- Tidal disruption flares occur when a star wanders too close to a SMBH, where tidal forces rip it apart, producing a stream of debris that will then be accreted onto the BH. As the gas is heated it produces a luminous flare of radiation (usually x-ray), which we can observe.

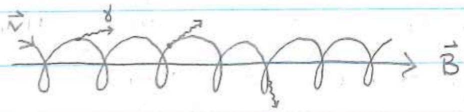
P4

- Sync. radiation is produced by relativistic e⁻'s in B-field, which are accelerated and emit dipole radiation beamed in the direction of $\vec{v}$
- Sync. rad. spectrum follows a power law, $F \sim \nu^{-\alpha}$
- Radio galaxies have power law continuum due to synch. rad. by AGN

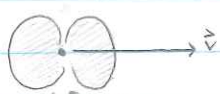
Synchrotron Radiation

- An accelerated electron will produce radiation. An electron moving through a B-field will experience a Lorentz force, $\vec{F}_B = q\vec{v} \times \vec{B}$, causing it to accelerate and radiate. The non-relativistic version of this is cyclotron radiation, but when the moving charged particles are relativistic, the Lorentz factor $\gamma = (1-(\frac{v}{c})^2)^{-1/2}$ must be included and you get synchrotron radiation.

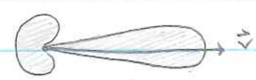
$$\nu_s = \gamma^2 \nu_{gyro} = \gamma^2 \frac{eB}{2\pi m_e}$$



- Synchrotron radiation is often called 'non thermal' because the e⁻'s kinetic energy can't be due to the thermal energy of the particles - the temperature would have to be much higher than what we observe ($\frac{1}{2}m_e v^2 = \frac{3}{2}k_B T_e$) to explain the velocities only based on thermal motion. Instead, the particles are moving relativistically, and as they're accelerated inward radially, they emit dipole radiation and we observe relativistic beaming, meaning the light is almost entirely emitted in the direction of the electron's velocity. When this elongated beam sweeps past the LOS of the observer, a pulse is observed, and the Fourier transform of this pulse gives you the spectrum of the radiation.

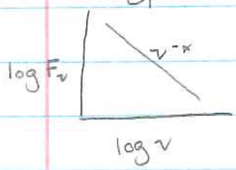


rest frame



observer frame

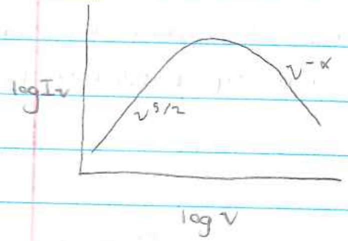
- Synchrotron radiation is generally linearly polarized, but can be circularly polarized too.
- The spectrum of synchrotron radiation looks like a powerlaw, $F \propto \nu^{-\alpha}$.



Generally there's less flux at higher energies / frequencies, probably because there are fewer e⁻'s with such high velocities / energies to produce that flux?

- A synchrotron radiation source can also exhibit self-absorption, since every emission process must have a balancing absorption process. At low

Frequencies the optical depth for absorption is large, so absorption becomes important so that synchrotron radiation can never exceed black body emission.



generally $\alpha \sim 0.7$ for extended radio components of galaxies
$\alpha \sim 0$ for compact core components.

- Radio galaxies are elliptical galaxies with active nuclei. Their continuum spectra can be described by a power law, the source of which is synchrotron radiation emitted mainly from the relativistic jet coming from their AGN. In our own galaxy, radio emission can come from synchrotron radiation in the ISM, which has a magnetic field.

- Forbidden lines are low prob. transitions due to oscillating electric quadrupole or magnetic dipole moments
- Important in diffuse gas where density is low, so collisional de-excitation doesn't compete w/ forbidden lines
- Important for cooling ISM b/c low prob. of absorption, so can escape freely

P5

- Forbidden Lines

- Atomic transitions take place sort of "only" when the initial and final levels have quantum numbers that satisfy the selection rules; for electric dipole radiation:

$\Delta l = \pm 1$ $\quad\quad$ $\Delta j = 0, \pm 1$

$\Delta m_l = 0, \pm 1$ $\quad\quad$ $\Delta S = 0$

- If the selection rules aren't met, an electric dipole will not radiate. However, the transition is still possible, but only by oscillating electric quadrupoles or magnetic dipoles. These result in much lower transition probabilities, which is why they're 'forbidden'; they're just uncommon ($\sim 10^{-8}$ times as likely).
  - For instance a jump from $2P_1$ to $2P_2$ is not ok ($\Delta l = 0$), but $2P$ to $2S$ is ok ($\Delta l = -1$). If only the last rule is broken, this is a semiforbidden transition.

- Because these transitions occur via spontaneous emission, there's a competing process (collisional de-excitation) which can suppress the photon emission. ie de-excite the e^- before it can emit - E transferred to colliding particle. Thus forbidden transitions, with their low transition probabilities, can be washed out in high density regions. Forbidden lines are thus important in tenuous gas like planetary nebulae and the ISM.

- Forbidden transitions in diffuse gas are important for <u>cooling</u>. Atoms are collisionally excited when a free e^- in the gas impacts an 'ion' in the ground level, exciting the ion due to the transfer of KE. The free e^- thus loses energy, and since collisional <u>de</u>-excitation is negligible for low-density environments, spontaneous emission will release a γ to drop the ion back down, so it loses some energy too. Since the transition probability is low, the probability of absorption is also low, so these lines can freely escape the nebulae and cool the gas.
  - Forbidden line cooling dominates at $T \sim 10^{4-5}$ K, and depends on the density as n^2
    ↳ cooling rate: $\Lambda = n_e n_{ion} q_{12} \Delta E_{12}$

P6

- Polytropic EoS is stellar model describing pressure profile as function of density only
- Plug this into HSE eqn, solve differential eqn, soln will describe density profile as function of r
- $n = 3$ describes radiative MS stars ($P \sim \rho T \sim T^4$), $n = 1.5$ describes WDs, degenerate relativistic e^- gas.

• Polytropic Equation of State

• Polytropes are hypothetical stellar models where the stellar pressure depends only on the density:

$P = K\rho(r)^{1+1/n}$ or $P = K\rho(r)^{\frac{n+1}{n}}$

• By considering that stars must be in hydrostatic equilibrium, you can insert the polytropic EoS into HSE equation, $\frac{dP}{dr} = -\frac{d\Phi}{dr}\rho$, make use of Poisson's equation, $\nabla\Phi = 4\pi G\rho$, and rework things to get the Lane-Emden equation. Solutions to this equation will depend on the polytropic index, n, and these solutions are polytropes $\Theta_n(\xi)$. These can then directly give you the density profile of the star, $\rho(r)$.

- so to reiterate: $P = K\rho(r)^{\frac{n+1}{n}}$ ← polytropic equation of state; pressure profile

  sort of like Temp. $\Theta_n(\xi)$ sort of like radius ← polytrope; sol'n to LE eqn

  $\rho(r)$ ← density profile

• We can only get analytic solutions to the LE equation for $n = 0, 1, 5$. Everything else requires numerical solutions with boundary conditions on $P_{central}$, $\rho_{central}$, R, etc. Two of the most important solutions are $n = 3/2$ and 3.

- $n = 3/2$ → degenerate, non-relativistic electron gas $P \sim \rho^{5/3}$
- $n = 3$ → degenerate, relativistic electron gas $P \sim \rho^{4/3}$

• As n increases, the density distribution is more heavily weighted toward the center of the body. Objects that are well approximated w/ polytropic EoS are:

- $n = 3$ → radiation dominated stars, like MS stars
  - → by assuming an ideal gas ($P \sim \rho T$) and constant temperature gradient $\nabla = 1/4$ ($T \sim P^{1/4}$) $P \sim T^4$, $P_{rad} \sim T^4$?, you get $P \sim \rho^{4/3} \rightarrow n = 3$
- $n = 1.5$ → white dwarfs
  - → we arrive at $P \sim \rho^{5/3}$ by considering kinetic theory of gas (??) such that P is related to the # density of the gas, particle momentum, Fermi energy etc.

- observed ^{e} neutrino flux 1/4 that predicted by solar models based on PP chain reactions producing ν_e's of various energies
- Solution is than ν's change flavors due to presence of e^-s in matter as they travel through Sun
- Detectors were only sensitive to ν_e's, not ν_τ or ν_μ

P7

- Solar Neutrino Problem

- The observed neutrino flux detected from the solar core was much lower (~1/4) than that predicted by models of solar nuclear fusion and our understanding of neutrinos themselves. The standard solar model describes the number of (electron) ν's we would expect to see based on the PP chain:

$p^+ + p^+ \rightarrow {}^2H + e^+ + \nu_e$ ← principle ν source, but harder to detect b/c lower E.

${}^2H + p^+ \rightarrow {}^3He + \gamma$

① ${}^3He + {}^3He \rightarrow {}^4He + 2p$

② ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$
${}^7Li + p^+ \rightarrow {}^4He + {}^4He$

③ ${}^7Be + p \rightarrow {}^8B + \gamma$
${}^8B \rightarrow {}^8Be + e^+ + \nu_e$
${}^8Be \rightarrow 2\,{}^4He$

- The solution to the neutrino deficit is that neutrinos can change flavor if they have small but finite rest masses, basically due to the presence of electrons in matter (MSW effect). So the electron neutrinos can transform into tau/muon neutrinos as they travel through the Sun. This explains why we detect fewer neutrinos than expected - our detectors were only sensitive to ν_e's, not ν_τ or ν_μ. However, SNO lab can detect interactions b/w ν's of all types with D_2O, and has confirmed that observed neutrino fluxes match that predicted by the Standard Solar Model plus the MSW effect.

P8

- Fusion is stable in MS stars due to stellar thermostat, maintaining HSE
- $\uparrow E \to \uparrow T \to \uparrow P \to \uparrow R \to$ work done, energy used, contracts, repeats
- Unstable in thin shells (He flashes / thermal pulses on AGB) and stars w/ degenerate cores where P indep. of T (WDs)

(Un)Stable Nuclear Fusion

- Nuclear fusion in a MS star is stable thanks to the 'stellar thermostat' and maintaining hydrostatic equilibrium. As nuclear fusion generates energy, the temperature increases. This increases the pressure (from the ideal gas law), which causes the star to expand. Work is done in this expansion, which results in a loss of energy and so the star relaxes back to its equilibrium.
- To see how HSE implies this stability, we can do some Math...

– Mass conservation: $\frac{dM}{dr} = 4\pi r^2 \rho \to \rho \sim M/R^3 \to \frac{\dot{\rho}}{\rho} = \frac{-3\dot{r}}{r}$

– HSE: $\frac{dP}{dr} = \frac{-GM}{r^2}\rho \to P \sim \frac{M}{R}\cdot\rho \to P \sim M^2/R^4 \to \frac{\dot{P}}{P} = \frac{-4\dot{r}}{r}$

– ideal gas: $P \sim \rho T \to T \sim P/\rho \to T \sim M/R \to \boxed{\frac{\dot{T}}{T} = \frac{-\dot{r}}{r}}$

↳ Thus as temperature increases (due to increased fusion), the radius must expand, which must decrease the temperature, and thus contract the star again, keeping it in HSE and thus stable.

- Nuclear fusion is unstable in <u>thin shells</u>, which is what drives He (?) flashes and thermal pulses on the AGB. Its also unstable for stars that develop degenerate cores, where the ideal gas law no longer applies and P is independent of T. In this case an increase in temperature doesn't change the pressure and so no cooling occurs, and you get a nuclear runaway, which is what destroys WDs as Type Ia SNe. See S7.

P9

- NS are supported by n° deg. P where all states are occupied up to the Fermi energy
- NSs can be much more compact than WDs b/c n°s don't have to overcome repulsion
- n°s don't decay b/c the resulting e^- would need an energy above the Fermi energy, which exceeds the max possible by β decay

- Neutron Decay / Neutron Degeneracy Pressure

- A neutron star is supported against gravity by neutron degeneracy pressure. n° degeneracy pressure is analogous to e^- P_{deg}. Basically, above the Chandrasekhar limit the Fermi energy of the e^-s increases to a point where it's energetically favorable to combine e^- and p^+ to form n°. In e^- P_{deg}, as density increases the e^-s progressively fill the lower energy states, and additional e^- are forced to occupy higher energy states even at low T, since the Pauli exclusion principle for fermions prevents electrons from occupying the same state. Neutrons are also fermions and obey this rule, but they can be packed much more tightly because the neutrons themselves are only affected by the strong nuclear force. Thus neutron stars can be much more compact than WDs.

- So the reason neutrons in a NS don't decay into e^-s and p^+s is because this would be energetically unfavorable. Neutron decay takes the form $n^\circ \rightarrow p^+ + e^- + \bar{\nu}_e$ or $n^\circ + \nu_e \rightarrow p^+ + e^-$, so in order for this to happen the electron would have to be given an energy above the Fermi energy (the Fermi energy is the energy of the highest occupied state (minus the energy of the ground state)). The Fermi energy will exceed the maximum possible energy of the beta-decay electron, so the neutron simply can't decay.

- Technically a neutron star does have some beta decay, but it is balanced by electron capture. This balance requires a ratio of $8n^\circ : 1p^+ : 1e^-$ to maintain it.

P10

- Assume virial thm, $E = \frac{1}{2}U$, $L = E/t = 4\pi r^2 \sigma T^4$, get eqn for T
- Get λ from Wien's Law
- Make approximations for λ, T based on M, R: star = IR, WD = UV, NS+ = Xray

- Accretion Temperature

- To determine the typical T of matter accreting onto various objects, we can start by assuming the system is virialized, that the accreting matter is in a disk and that it's a steady state (ie the mass of the disk doesn't change, so dM/dt = constant).

- virial theorem:

$$2K = -U$$

$$E = K + U = -\frac{1}{2}U + U = \frac{1}{2}U = \frac{GMm}{2r}$$ (I ignored the negative sign b/c why not)

$$L = E/t = \frac{GMm}{2rt} = \frac{GM\dot{m}}{2r}$$

- assume radiation is Blackbody:

$$L = 4\pi r^2 \cdot \sigma T^4$$

$$\therefore 4\pi r^2 \sigma T^4 = \frac{GM\dot{m}}{2r}$$

- assume matter accretes onto surface of star, $r = R$:

$$\therefore \boxed{T = \left(\frac{GM\dot{m}}{8\pi\sigma R^3}\right)^{1/4}}$$

- I think you can get $\dot{m}$ from $\pi R^2 \rho v$, but IDK.
- Note that turbulence in the disk reduces the temperature, as it encourages momentum transport which means energy doesn't have to be radiated away.

- To get the wavelength, you can use Wien's displacement law:

$$\lambda = \frac{500\,nm \cdot 5800\,K}{T}$$

	Star	WD	NS	BH	SMBH
$M/M_\odot$	3	0.8	1.4	5	10^8
$R/R_\odot$	2	10^{-2}	10^{-5}	10^{-5}	400
$\dot{m}/M_\odot$	$10^{-9}\ yr^{-1}$	$10^{-9}\ yr^{-1}$	$10^{-9}\ yr^{-1}$	$10^{-9}\ yr^{-1}$	$3\ yr^{-1}$
T	1600 K	65000 K	9×10^6 K	10^7 K	5×10^5 K
λ	1.8 μm; IR	45 nm; UV	0.3 nm; Xray	0.2 nm; Xray	5.7 nm; Xray

- BB peaks at λ you're trying to observe at, producing thermal noise
- CCDs are semiconducting material, and thermal energy is enough to free e^-s and collect in potential wells, reducing SNR
- Short λ are made of different material w/ higher energy sensitivity (UV),
- radio antennae don't count photons but measure E-field of radio wave

P11

Detector Cooling

- Long wavelength detectors must be cooled so that their black body radiation peak doesn't contribute to the radiation being observed. That is, most room-temperature objects radiate in the IR, and this produces thermal noise or 'dark current' on a detector like a CCD.

- CCDs are basically composed of semiconducting material where e^-s are easily freed by incoming γs, which then collect in the potential wells (or 'light buckets') of the CCD and are read out as a voltage. If the detector is warm enough, the e^-s have enough energy to just escape on their own, resulting in thermal noise or dark current. Cooling the CCD reduces this effect from $\sim 10^3\ e^-/px/s$ to $\sim 1\ e^-/px/s$, drastically improving the SNR of your observations.

- This isn't necessary for short wavelength detectors because they're not sensitive to IR wavelengths. That is, the material the detector is made of is different, and more energy will be required to free e^-s than the thermal (bad word choice) temperature could provide.

- Radio antennae generally don't require cooling, but their receivers can be cooled to minimize interference from thermal noise. Radio telescopes don't use CCDs (which count photons) but rather measure radio waves and their amplitude (usually the E-field component).

P12

- Assume # γs $/m^2/s$ is $F/h\nu$ = constant, n
- Signal / # total photons depends on A, t : $N = n\pi(D/2)^2 \Delta t$
- error/noise is Poisson, so $\sqrt{N}$
- $SNR_A > SNR_B$ due to larger diameter.

Signal to Noise Ratios

- We have 2 telescopes observing the same source. To determine the SNR of each observation, we start by assuming the source has some flux F, so the # of photons $/m^2/s$ we will see is $n = F/h\nu$.
- To determine the signal, we want to know how many photons we'll collect. This depends on our telescope size and exposure time:

$$N = n \cdot A \cdot \Delta t$$
$$= n\pi\left(\tfrac{D}{2}\right)^2 \Delta t$$

- To determine the noise, we can assume photon noise is Poisson distributed, so the error is $\sqrt{N}$. Realistically we should also include the sky background, the dark current/thermal noise, and the read out noise, but roughly we can just say:

$$SNR = N/\sqrt{N}$$
$$= \sqrt{N}$$
$$= \sqrt{n\pi(D/2)^2 \Delta t}$$

- Comparing our 2 telescopes:

$$\frac{SNR_A}{SNR_B} = \frac{\sqrt{n\pi(10m/2)^2(1\,min)}}{\sqrt{n\pi(1m/2)^2(10\,min)}}$$
$$= \frac{\sqrt{(10m)^2(1\,min)}}{\sqrt{(1m)^2(10\,min)}}$$
$$= \boxed{\sqrt{10}}$$

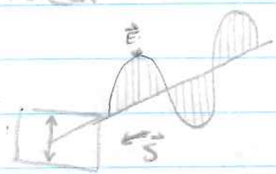
$\therefore$ The SNR of A is greater than B by more than a factor of 3. The diameter of the telescope is more important than the exposure time for achieving high SNR observations.

P13

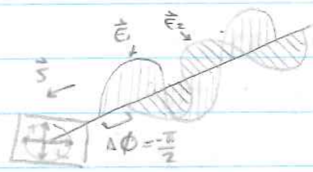
- If light travels in a single plane = linear, if its in 2 ⊥ planes = circular
- Mathematically, the phase diff b/t/w the E-field components of 2 plane waves will be 0 = linear, 90 = circular
- Linear polarization in CMB (Thomson scattering), scattered starlight by elongated dust particles, circular from sync. in AGN.

- Linear and Circular Polarization

- Light is a transverse EM wave, and for unpolarized light, all planes of propagation are equally probable. If light travels in a single plane, it is linearly polarized, and if it travels in 2 perpendicular planes w/ a 90° difference in phase but equal amplitude, the light is circularly polarized.



Linear



Circular

- Mathematically, let's consider the E-field of the light, which can be written in terms of the x and y components, so our light is really 2 perpendicular plane waves 1 & 2:

$E_x = E_1 \cos(\omega t + \phi_1)$

$E_y = E_2 \cos(\omega t + \phi_2)$

- If $\phi_1 = \phi_2$, you'll get linearly polarized light (ie no phase difference)

- If $\phi_2 - \phi_1 = \pm\frac{\pi}{2}$, and $E_1 = E_2$ (ie same amplitude), you get circularly polarized light. Right circular polarization shows the total E vector moving CW, so $\Delta\phi = \phi_2 - \phi_1 = -\frac{\pi}{2}$. Left circular polarization shows the E vector moving CCW, so $\Delta\phi = \frac{\pi}{2}$.
- If $E_1 \neq E_2$, you get elliptical polarization.

- Linear polarization is seen in the CMB as a result of Thomson scattering, and in starlight scattered by elongated dust particles in the ISM.
- Synchrotron radiation can be seen as either linear or circular polarization depending on your viewing angle, and this is relevant in AGN with synchrotron radiation emitted from strong radio jets.

P14

- $FOV = \text{plate scale} \times \text{CCD size} = \frac{1\,\text{rad}}{\text{focal length}} \times \#\text{pixels} \times \text{pixel size} = \frac{1}{F\# D} \times \#px \times px\,size$
- Etendue is property of light characterizing how spread out it is. Will be conserved for solid angle and area: $D_1^2\, FOV_1^2 = D_2^2\, FOV_2^2 = \mathcal{E} = \text{const.}$
- 5m telescope = 103" × 103", 10 m = ½ that.

- Field of View

- The field of view of telescope/CCD can be determined by considering the plate scale (a term originating from the use of photographic plates in the olden days) and the size of your CCD:

$$FOV = \text{plate scale} \times \text{CCD size}$$

- the plate scale depends on the focal length of your telescope, which depends on your telescope diameter and focal ratio. The size of your CCD is just the number of pixels times the size of each pixel:

$$FOV = \frac{1\,\text{rad}}{\text{focal length}} \times \#\text{pixels} \times \text{pixel size}$$

$$= \frac{206265''}{f^{\#} \cdot D} \times N \times \Delta x$$

- this $f^{\#}$ is the number in your focal ratio. So if you have $f/16$, $f^{\#} = 16$.

$$\therefore FOV = \frac{206265''}{(16)(5m)} \times 2000 \times 20\,\mu m$$

$$= 103''$$

$$\therefore FOV^2 = \boxed{103'' \times 103''}$$

- Étendue is a property of light that characterizes how 'spread out' the the light is in terms of area and solid angle (french for 'expanse'?). For any optics system, this is conserved:

$$\mathcal{E} = D^2 \cdot FOV^2 = \text{constant}$$

- thus if you use a 10 m telescope (2× the diameter), the FOV will decrease by a factor of 2:

$$D_1^2 \cdot FOV_1^2 = \mathcal{E} = D_2^2 \cdot FOV_2^2$$

$$D_1 \cdot FOV_1 = D_2 \cdot FOV_2$$

$$FOV_2 = \frac{D_1}{D_2} FOV_1$$

$$= \frac{5m}{10m}(103'')$$

$$\therefore FOV_2^2 = \boxed{52'' \times 52''}$$

- Gaussian = continuous random variable, $f = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$, Doppler broadening, CMB spatial anisotropies
- Poisson = discrete random variable, $f = \frac{\nu^n}{n!} e^{-\nu}$, photon counting, SNe occurrence
- Log Normal = $x = e^y$ where y = gaussian, $f = \frac{1}{\sqrt{2\pi}\,\sigma} \frac{1}{x} \exp\left(\frac{-(\ln x - \mu)^2}{2\sigma^2}\right)$, chabrier IMF $< 1 M_\odot$, Sun's FUV continuum

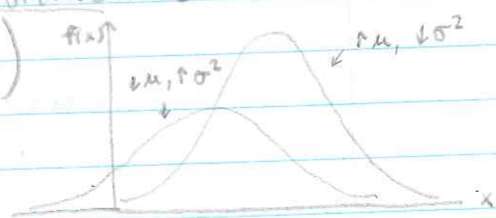
P15

Gaussian, Poisson, and Log-Normal Distributions

Gaussian

- The probability distribution function of a continuous random variable x is:

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$



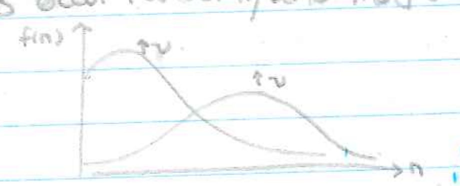
- μ = mean
  σ^2 = variance (σ = stdev)
- the Gaussian PDF is called a normal PDF when $\mu = 0$ and $\sigma = 1$.
- the mean comes from $\int_{-\infty}^{\infty} x\, f(x)\, dx = \mu$ and the variance comes from $\int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$ for continuous variables.
- the Gaussian PDF is important b/c of its relation to the Central Limit theorem, which states that for a sufficiently large number of independent continuous random variables x_i (ie $n \to \infty$), with μ_i and σ_i^2, the mean will be ~ Gaussian distributed with $\mu = \sum_{i=1}^{n} \mu_i$ and $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$.

- Two examples where Gaussian pdfs apply are in the thermal (Doppler) broadening of spectral lines and CMB spatial anisotropies (also PSF and the random noise in radio signals).

Poisson

- The Poisson pdf describes the distribution for a discrete random variable n, which can only be an integer. It's basically the binomial distribution in the limit of a large number of events with a low probability of success, but
$N \to \infty$ & $Np = \lambda$ → where the expectation value of the # of successes stays finite. This makes the Poisson pdf useful for counting events with some average occurrence rate but where individual events occur randomly (?) and independently.

$$f(n; \nu) = \frac{\nu^n}{n!} e^{-\nu}$$



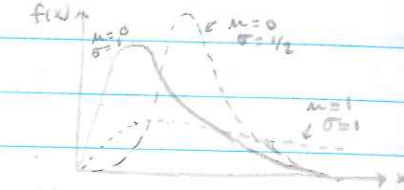
- ν = mean = variance
- if the mean value ν is large, a Poisson variable can be treated as a continuous variable following a Gaussian distribution (CLT)

- Examples where Poisson pdfs apply are photon counting, the # of SNe that will occur in a galaxy over some time, and the monthly mean # of sun spots.

Log-Normal

- If a continuous random variable y is Gaussian (with μ and σ^2), then $X = e^y$ follows a log normal distribution:

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot \frac{1}{x} \cdot \exp\left(\frac{-(\ln x - \mu)^2}{2\sigma^2}\right)$$



- the mean and variance are functions of μ and σ^2
- Basically, if X is a continuous random variable, $\ln(x)$ is normally distributed. Note that x must be positive, otherwise $\ln(x)$ is undefined.
- Apparently this is useful for modeling phenomena whose growth rate is independent of size

- Examples where log-normal distributions apply are the Chabrier IMF for stars w/ $M < 1M_\odot$, the intensity of the Sun's FUV continuum, the x-ray flux variations in some Seyfert I galaxies, and the number of words written in sentences by George Bernard Shaw.

P16

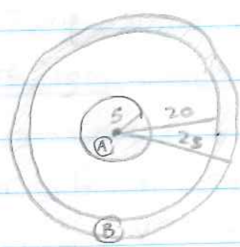
- Frac. precision = $1/SNR$, $SNR = N_* / \sqrt{N_* + (1 + \frac{n}{n_{bkg}}) N_{sky}}$; $N_{sky} = \frac{n}{n_{bkg}} \cdot N_{annulus}$
- Error prop: $\delta R = \sqrt{(\frac{\partial R}{\partial x}\delta x)^2 + \ldots}$ for $R(x, \ldots)$
- Assume measurements aren't correlated, dark current unimportant

- Aperture Photometry, Fractional Precision, and Uncertainty

- The fractional precision is really just asking for how high your noise is relative to your signal. So this is just an SNR calculation:

$$\frac{S}{N} = \frac{N_*}{\sqrt{N_* + \left(1 + \frac{n}{n_{bkg}}\right)\left(N_{sky} + N_{Dark} + N_{read}^2\right)}}$$

- where N_* is the e^- count of the source/star
  - n is the # of pixels in your central aperture
  - n_{bkg} is the # of pixels in your annulus used to estimate background
  - N_{sky} is the e^- count of the sky in your central aperture * NOT PER PIXEL.
- we will ignore N_{Dark} and N_{read}, assuming they're negligible.



$N_A = N_* + N_{sky} = 10^4 e^-$

$n_A = n = \pi(5px)^2 = \pi \cdot 25$ px

$N_B = N_{sky\ in\ bkg} = 8100 e^-$

$n_B = n_{bkg} = \pi(25\,px)^2 - \pi(20\,px)^2 = \pi \cdot 225$ px.

$$N_{sky} = \frac{n}{n_{bkg}} \cdot N_B = \frac{\pi \cdot 25}{\pi \cdot 225} \cdot 8100 e^- = 900 e^-$$

$$N_* = N_A - N_{sky} = 10^4 e^- - 900 e^- = 9100 e^-$$

$$\therefore \frac{S}{N} = \frac{(9100 e^-)}{\sqrt{(9100 e^-) + \left(1 + \frac{\pi \cdot 25}{\pi \cdot 225}\right)(900 e^-)}} \approx 90$$

$$\therefore \frac{N}{S} = \text{fractional precision} = \frac{1}{90} \approx \boxed{1\%}$$

- The fact that our flux calibration is good to 1% means that all of our N's will have a ±1% error associated with them. Our SNR calculation would then vary by either $\sqrt{1.01}$ or $\sqrt{0.99}$ which doesn't change our fractional precision by much. True error propagation involves nasty derivatives and stuff. To do this we'd have to assume our measurements aren't correlated.

$R = R(X, Y, \ldots)$

$$\delta R = \sqrt{\left(\frac{\partial R}{\partial x}\delta x\right)^2 + \left(\frac{\partial R}{\partial y}\delta y\right)^2 + \ldots}$$

P.17

- mean, median, mode, stdev $= \sqrt{\frac{1}{N}\sum(x-\bar{x})^2}$
- outliers = Z-score, # stdevs you're willing to accept, $z_i = \frac{|x_i - \bar{x}|}{\sigma}$, $z_i \approx 3$
- remove outliers: mode + median unchanged, mean shift a little, stdev ↓

- Mean, Median, Mode, Standard Deviation, and Outliers.

- Mean: $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$
- Median: $x_{\frac{N+1}{2}}$ if N is odd — after sorting $\{x_i\}$ from small to large

  $\frac{1}{2}\left[x_{\frac{N}{2}} + x_{\frac{N+1}{2}}\right]$ if N is even

- Mode: most frequently occurring x_i; the value of x for which the pdf is at its maximum
- Standard deviation: $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$

- To check for outliers, I normally just check if $|x_i - \bar{x}| > 3\sigma$ and throw out values that satisfy this. This is essentially the same as the Z-score test, where $z_i = \frac{x_i - \bar{x}}{\sigma}$, and a value of $z_i \approx 3$ is usually taken.

- with 3σ this means x_i lies w/in 99.7% of the normal distribution, given that your data is Gaussian.

- If you remove some points, the mode shouldn't be affected, the median might be shifted by a point or so, the mean will change because it's less robust than the median (ie less resilient to change), and the stdev will go down.

- our results = 40% binary fraction
- To determine consistency, assume binomial dist w/ $\mu = Np$, $\sigma = \sqrt{Np(1-p)}$ (25, 3.5)
- Use Z-score to evaluate consistency. Acceptable w/in 1.5 σ of expected value

Binary Fraction

- If we observe 50 stars and find companions around 20 of them, we infer a binary fraction of $\boxed{0.4}$ - that is, 40% of all star systems contain binary stars.

- To determine the likelihood that this result matches a previous study, we can consider the Binomial distribution, which applies when:
- the # of observations is fixed
- each observation is independent
- the outcome of each observation is either T or F
- the probability of a T outcome is the same for each observation.

- The binomial distribution is:

$$f(n; N, p) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

where N = # observations total
n = # successes
p = probability of a success

- We want to test the hypothesis that our results come from a different distribution than the previous sample - essentially, is our measurement an outlier? To do this, we assume the probability of a binary system is the 50% given by the other sample. The Binomial pdf has the following properties we'll make use of:

$$\mu = Np$$

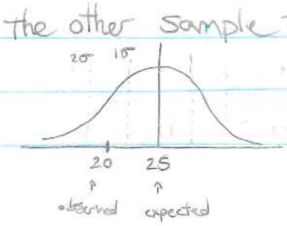
$$\sigma = \sqrt{\sigma^2} = \sqrt{Np(1-p)}$$

- Using $p = \frac{1}{2}$, $N = 50$, we get $\mu = 25$, $\sigma \approx 3.5$
- Let's set our confidence level to 3σ (that is, how confident are we that our value is not an outlier?). Then we want our Z-score (see P17) to be < 3:

$$\therefore \; z_i = \frac{|x_i - \bar{x}|}{\sigma} = \frac{|\text{observed} - \overset{\mu}{\text{expected}}|}{\sigma} = \frac{|20-25|}{3.5} \approx \underline{1.4}$$

or if z = 3, then x_i would have to be 14.5 ~ 15. our actual value is closer to the expected value than that, so it's w/in 3σ.

- thus our measurement is consistent with the other sample to within 1.5 sigma, which is acceptable.



P19

- $1-10^4$ Hz – ground based laser interferometry, collapse of massive stars, coalescing NSs + BHs
- $10^{-4}-1$ Hz – space based laser interferometry, WDs/NS/BHs spiraling into massive BHs, coalescence into SMBHs
- $10^{-7}-10^{-9}$ Hz – MS pulsar timing, early universe phase transitions, cosmic strings, Big Bang
- Detections inform us about cosm. params, test GR; non-detections constrain structure formation

- Gravitational Waves

- see www.slac.stanford.edu/econf/c940808/pdf/ssi94-002.pdf b/c it's beautiful

- Gravitational waves are oscillations of the fabric of spacetime itself due to the coherent bulk motion of huge amounts of mass-energy. Unlike EM waves, grav. waves are comparable to / larger than their sources, so we can't make pictures from them. The source of a grav. wave can't be much smaller than its grav. radius, r_s $\left(\frac{2GM}{c^2}\right)$, and can't emit strongly at periods smaller than the light travel time around this radius:

$$t = \frac{2\pi r_s}{c} = 4\pi GM/c^3$$

$$\therefore \nu = \frac{c^3}{4\pi GM} \sim 10^4 \text{ Hz} \cdot \frac{M_\odot}{M}$$

- to achieve a size on the order r_s and emit near ν_{max}, the object would need to be heavier than $M_{chandra}$ ($\sim 1 M_\odot$) $\rightarrow$ $\nu_{max} \sim 10^4$ Hz. More massive objects will have even lower frequencies. Note that most EM waves have $\nu > 10^7$ Hz.

- High Frequency Band: $1-10^4$ Hz
- upper limit set by max ν described above, lower limit set by noise limitations on Earth, like the grav. pull of atmospheric inhomogeneities that move overhead with the wind, and seismic vibrations.
- techniques include ground-based laser interferometers (LIGO) and resonant mass antennae. The former uses a beam splitter to send a laser beam down 2 axes, and then you measure the time delay for the return signal between the two to see if the distances are being compressed or expanded.
- sources include the collapse of stars into NSs or BHs, the coalescence of NS and stellar mass BH binaries in distant galaxies ($M < 1000 M_\odot$ though), and maybe stochastic background sources like vibrating cosmic strings/loops or the Big Bang itself.

- Low-Frequency Band: $10^{-4} - 1$ Hz
- techniques include space-based laser-interferometry, microwave-frequency Doppler tracking of spacecraft (via microwave signals sent from/back to Earth),

and optical tracking of spacecraft by each other.
- the lower ν limit is set by the expected difficulties of isolating spacecraft from the turbulence of fluctuating solar radiation pressure, solar wind, and cosmic rays.
- sources include short-period binary stars in our own galaxy, WDs/NSs/small BHs spiraling into massive BHs ($\sim 10^{5-7} M_\odot$), and coalescence of SMBH binaries ($10^{2-8} M_\odot$).

- Very Low Frequency Band: $10^{-7} - 10^{-9}$ Hz
- techniques include millisecond pulsar timing. A grav. wave will perturb the arrival times of pulsar pulses, so if we see fluctuations in the timing of many pulsars and see the same time evolution, it could be due to grav. waves bathing Earth.
- possible sources include only a stochastic background from early universe processes like cosmic strings, phase transitions, and the BB. An object would need to be $\sim 10^{11} M_\odot$ to produce grav. waves, and we don't see compact objects this massive.

- Extremely Low Frequency Band: $10^{-15} - 10^{-18}$ Hz
- Techniques include measurement of the anisotropies in the CMB (quadrupolar anisotropy)
- Sources include the early universe processes described above, which would squeeze all of space inside our cosmological horizon in one direction and stretch it in another, producing a quadrupolar anisotropy.

- What can we learn from detections?
- compact binary inspiralling can tell us about H_0, the deceleration parameter, and the cosmological constant.
- small compact objects spiraling into massive BHs can tell us about the spacetime geometry around the latter, which can be used to test GR

- the lack of detections of fluctuations in ms pulsar timing suggests that galaxy formation likely wasn't seeded by the grav. pull of cosmic strings, as we don't see enough vibrating loops to support that.

P20

- self similarity = things look the same on all scales and their properties follow a simple power law relation
- Galaxy clusters, if scaled by mass. M_{gas}, T, L_{xray} all follow power laws
- Isothermal sphere collapse; $t_{ff} \sim 1/\sqrt{G\rho}$ so $\uparrow \rho$ = faster collapse, and since T = const, $\rho \sim 1/R^2$.

Self-Similarity

If something is self-similar, that means it looks roughly the same on any scale (think fractals or coastline). The parameters of a self similar object will then follow a power law scaling relation. This is what makes self-similarity so useful - it allows us to determine parameter values very easily, since there is a simple scaling relation between parameters. Obviously this doesn't work if your object doesn't look the same on the scales you're considering. IDK man.

One example is <u>galaxy clusters</u>. These are self-similar because galaxy clusters/groups of all masses are essentially identical objects <u>if</u> you scale them by their mass. Since they're self-similar, their properties follow simple scaling relations, like $M_{gas} \sim T^{3/2}$ and $L_{xray} \sim T^2$. However, the latter actually deviates (observationally it's closer to 3, not 2, in slope) and deviates more for galaxy groups. These relations are not expected to evolve with redshift. IDK why.

IGNORE THIS ONE!

Another example of self-similarity is the mapping between the <u>present-day prestellar core mass function and the stellar IMF</u>. The central portions of both functions have roughly log-normal shapes, but the CMF is shifted to higher mass by a factor of ~4. This suggests the shape of the IMF is directly inherited from the CMF, and there is self-similar mapping from the CMF onto the IMF. Basically this means the distribution of the <u>relative</u> masses spawned by a single core must also have a log-normal shape. For instance, the probability of forming a star with 0.4 - 0.44 $M_\odot$ from a 1-1.1 $M_\odot$ core is the same as the probability of forming a 4 - 4.4 $M_\odot$ star from a 10-11 $M_\odot$ core.

A simpler example is the <u>collapse of an isothermal sphere</u>. The free-fall time scale of collapse is $t_{ff} \sim 1/\sqrt{G\rho}$, so higher-density regions collapse faster, and the density distribution goes as $\rho \sim R^{-2}$ ($\rho \sim M/R^3$, $T = \text{const} \sim M/R \rightarrow M \sim R \rightarrow \rho \sim R^{-2}$). ← see P8! This solution is the same at almost any radius scale, making it self-similar.

See also Taylor & Sedov or Blanford & Payne or Shu (1977)

P20

P21

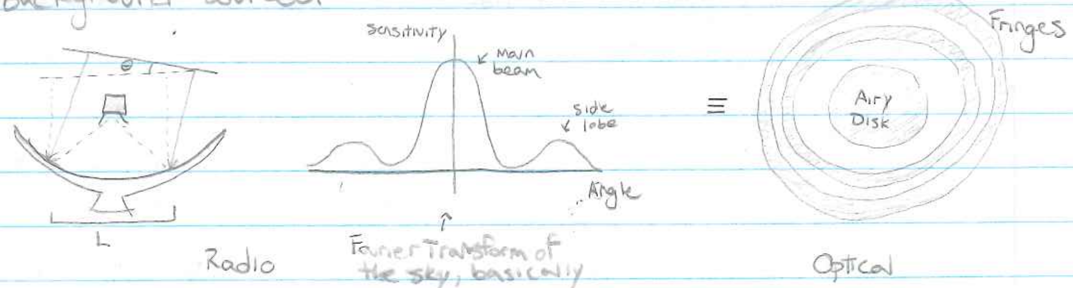
- Side lobes are due to constructive interference. When your source is slightly off axis, there is a phase diff b/w rays of light, which interfere.
- In radio you can add a feed horn to taper the antenna, and in optical you can apodize the mirror, smoothing out the sensitivity on the edges.

- Side Lobes and Diffraction - Limited Detectors

- Diffraction limited telescopes are ones whose angular resolution only depends on the telescope's optics and the observed wavelength. The refraction of light by the atmosphere is negligible if you're diffraction limited, ie the atmosphere doesn't reduce your resolution, which is really only true for radio observations, I think.

- The reason you get sidelobes is due to partial! constructive interference. If you have a source that's slightly offset from your optical axis, there'll be a phase difference b/w the 2 incoming rays from your plane wave of light. This means the rays can add constructively, so the telescope has a small increase in sensitivity at larger off-axis angles, so you'll pick up background sources.

$\theta = \frac{\lambda}{L}$



- To suppress radio side lobes, you can 'taper the antenna' by adding a feed horn. These are made specific sizes so they're only sensitive to radiation reflecting off a small part of the dish, which reduces the effects of side lobes (since they're at larger angles, but also reduces the sensitivity of your main beam.
- You can also build the telescope with an offset paraboloid dish (instead of a circular one) like the GBT, which also reduces side lobes.
- I have no idea about reducing sidelobes/fringe patterns in optical observations that are diffraction limited. If the seeing is bad then adaptive optics would improve your angular resolution, but that doesn't seem to apply if you're already diffraction limited.
- JK apodizing the mirror helps, so you reduce sensitivity of the primary mirror near its outer edges. This reduces airy rings but also reduces collecting area.

No more sticky tabs! ☺