Stellar Stream Track Reconstruction, with Errors

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Introduction

Stellar streams are sensitive probes of the Galactic potential. The likelihood of a model given stream data can only be assessed using simulations. However, comparison to simulation is challenging in a noisy 6D phase space in which even the stream paths are hard to quantify. Here we present a novel application of Self-Organizing Maps and first-order Kalman Filters to reconstruct the stream path, propagating measurement errors and data sparsity into the stream path uncertainty. The technique is Galactic-model independent, non-parametric, and works on phase-wrapped streams. We can uniformly analyze and compare data with simulation.

Data Ordering

Using Self-Organizing Maps (SOM) pre-trained on locally linearizing reference frames, we discover the 1D structure of a stellar stream and a path-distance-minimizing data ordering.

Self-Organizing Maps (SOM)

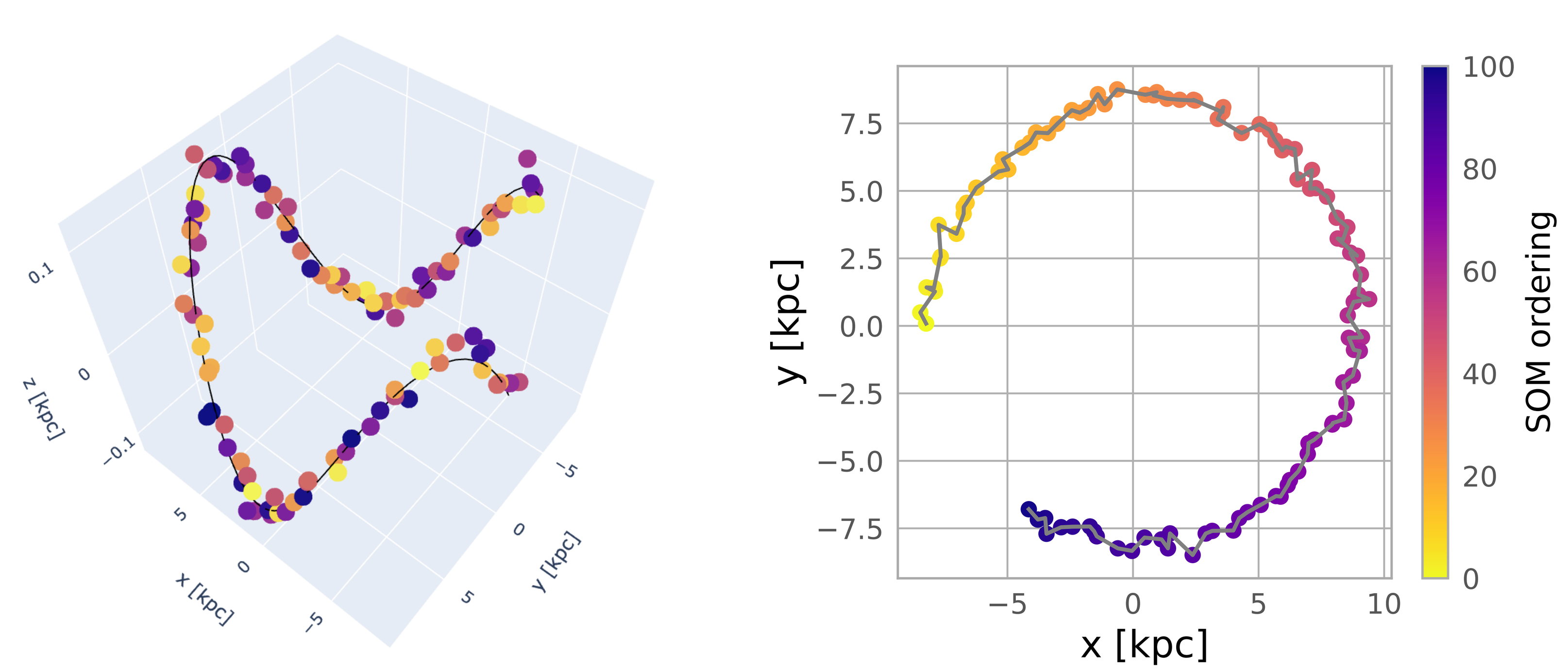
SOM are a neural network for low-dimensional, discrete representation of data. Using linked prototype vectors, SOM approximate the data by iteratively updating the topology of the prototypes to approximate the data distribution. In final form, each prototype maps nearby high-dimensional data to a lower-dimensional lattice [2] -- see Fig. 1.

Linearizing Reference Frames

Streams orbit the Galactic center of mass. The orbit path for the majority of streams, distant and less sensitive to Galactic structure, approximately describe a great circle.

Therefore, for a fixed origin point (α^*, δ), and rotation θ the transformation to a great circle frame defines a locally-linearizing set of sky coordinates (ϕ_1, ϕ_2) [1]. By ordering data along ϕ_1 we discover the approximate data ordering and circumvent the SOM network burn-in phase.

Figure: Mock Stellar Stream at Solar Circle.
Data coloring is index order. Errors are $\sim 2\times$ standard for cold streams.



(a) Mock Stream, randomly ordered. (b) Projection, reindexing by SOM

Reconstructing Stream Paths

A first-order Newtonian-dynamics, hidden-variable Kalman Filter is used to construct the stream path, propagating both measurement and sampling-sparsity-induced uncertainty into the path. The technique is injective and produces its own affine parameterization.

Figure: Reconstructed Stream Path & Uncertainties (blue).

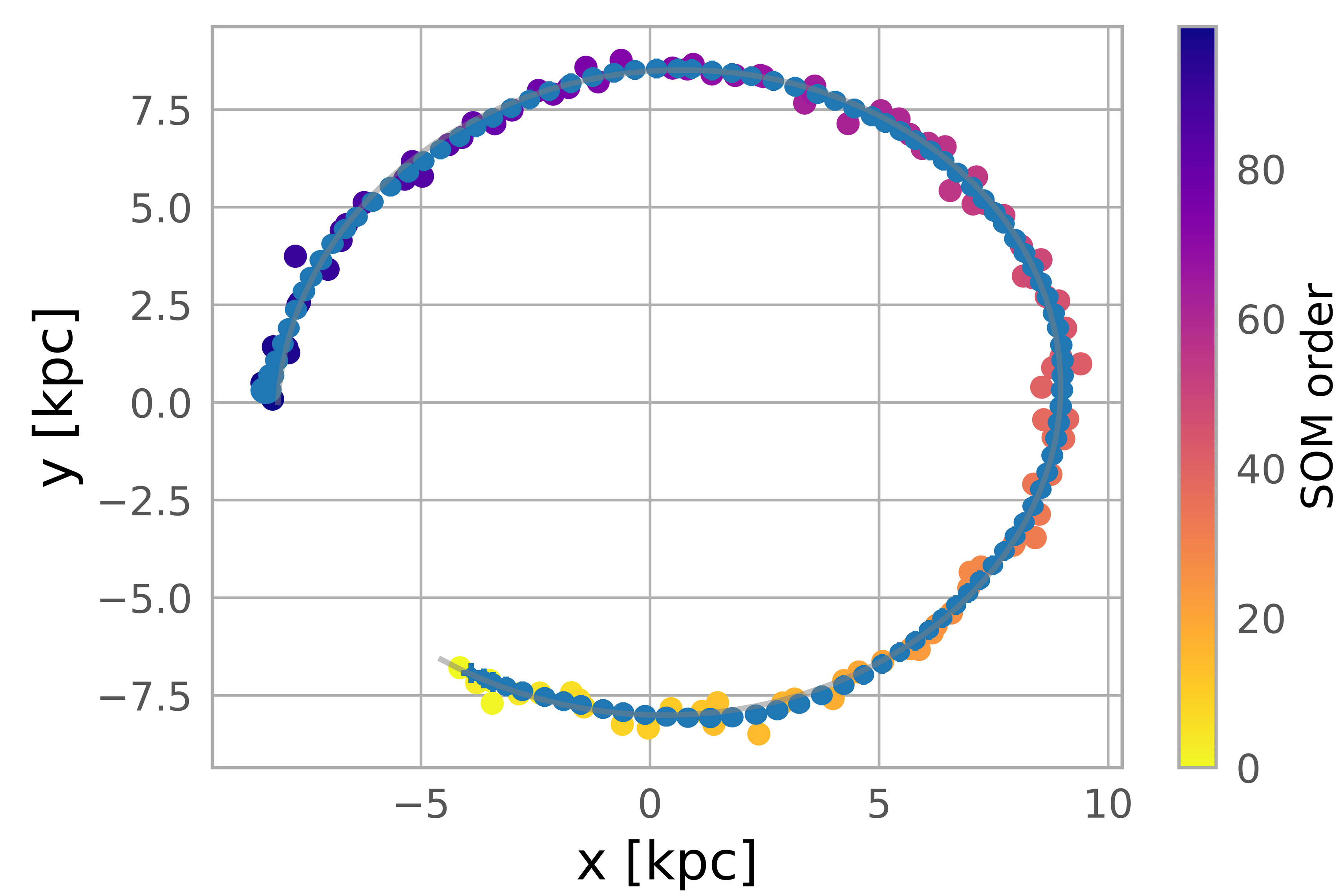
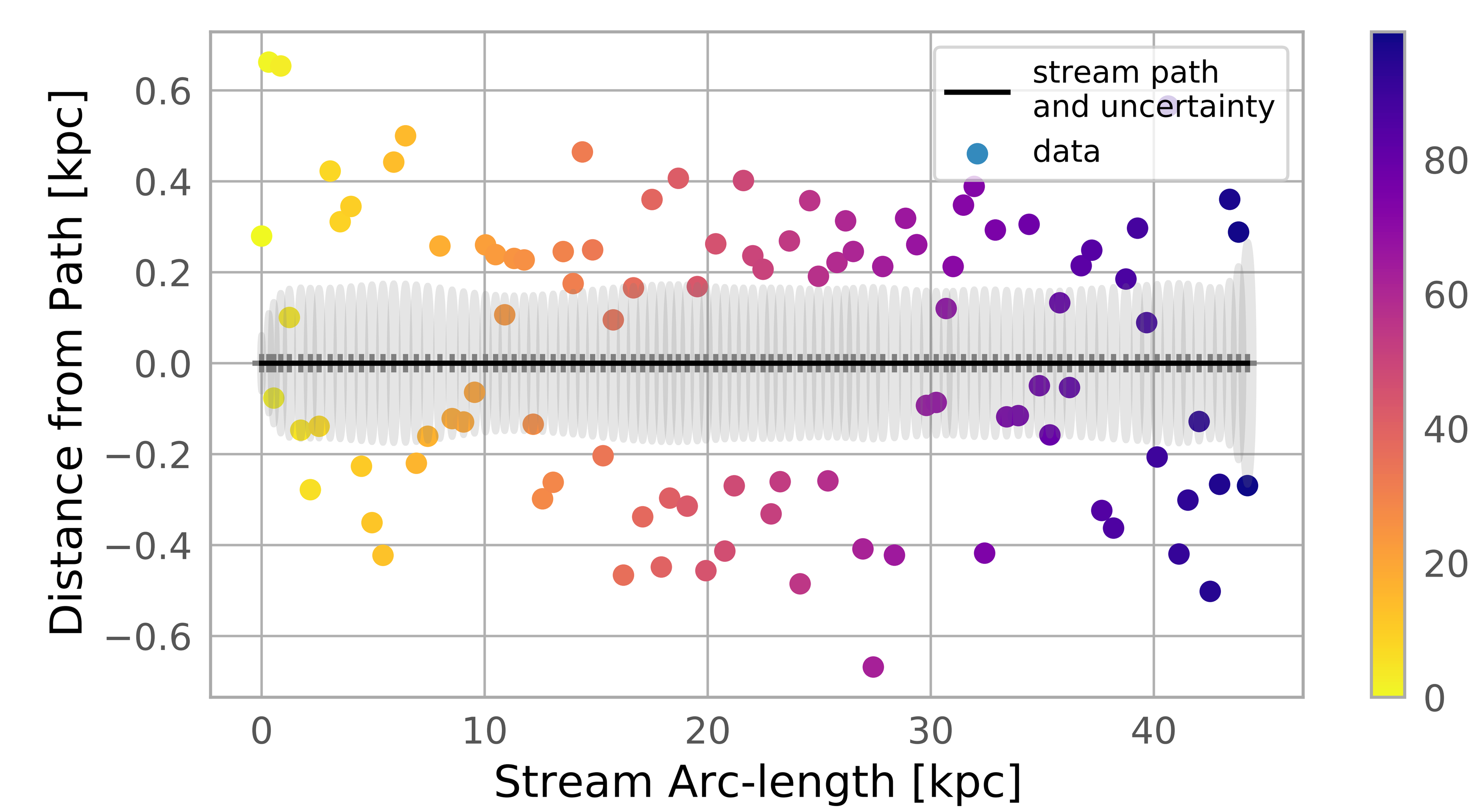


Figure: Path Residual: uncertainties reflect data.



Kalman Filter Math

Kalman filters estimate a joint probability distribution over the variables in a timeseries. For a hidden-velocity filter the state vector $\hat{\mathbf{x}}$ encodes the position and pseudo-velocity, and $\mathbf{P}$ the error therein. The Newtonian-dynamics transition matrix $\mathbf{F}$ gives the (prior) dynamics between states and $\mathbf{Q}$ the associated uncertainty, like intrinsic dispersion (see Fig. 3). $\mathbf{H}$ is the observation function, bringing states into measurement space. $\mathbf{z}, \mathbf{R}$ are the measurement mean and noise covariance [3].

$$\hat{\mathbf{x}} = [x\ v_x\ y\ v_y\ z\ v_z]^T \qquad \mathbf{F} = \mathrm{diag}_3\left(\begin{bmatrix} 1 & \Delta t \\ & 1 \end{bmatrix}\right) \qquad \mathbf{H} = \begin{bmatrix} 1\ 0 & & \\ & 1\ 0 & \\ & & 1\ 0 \end{bmatrix} \tag{1}$$

The Kalman Filter operates by updating at each time step (Δt); however, the times are not known. As a proxy, before each predict-update iteration we tune the time-step in the state transition matrix $\mathbf{F}$ by the smoothed point-to-point distance.

For each data point the Kalman filter process is two steps [2, 3]:

Predict

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k \quad \textit{a priori state estimate} \tag{2}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k \quad \textit{a priori estimate covariance} \tag{3}$$

Update

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \quad \text{pre-fit residual} \tag{4}$$

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k \quad \text{pre-fit residual covariance} \tag{5}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1} \quad \text{optimal Kalman gain} \tag{6}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \quad \textit{a posteriori estimate} \tag{7}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \quad \textit{a posteriori estimate covariance} \tag{8}$$

Subsequent steps are updated by previous steps, but not vice versa. To back-propagate information, the process is run in reverse by Rauch–Tung–Striebel (RTS) smoothing [4].

Conclusions

We reconstruct the mean path of stellar streams (eg Fig. 2) from sparse and noisy data, respecting both as sources of error. By using linearizing reference-frame transformations in conjunction with Self-Organizing Maps, we treat stellar-stream data as a pseudo time-series, to which first-order Kalman Filters can be applied. The path reconstruction properly propagates measurement errors and data sparsity into a path error (Fig. 3), allowing for equal treatment and more precise comparison of data and simulation.

References

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