In[75]:= `Clear["Global`*"]`

Defining the variables

G : graviational constant

M_{gc} : GC mass

M_{BH} : BH mass

μ : M_{BH} / M_{gc}

r : GC - centric radius

a : Plummer scale length

x : r / a

In[76]:= $\$Assumptions = \{G > 0, M_{gc} > 0, \mu > 0, r > 0, a > 0, x > 0\}$

Out[76]= $\{G > 0, M_{gc} > 0, \mu > 0, r > 0, a > 0, x > 0\}$

GC (Plummer)

In[77]:= $\text{PhiPlummer}[x\_] := -\frac{G * M_{gc}}{a \sqrt{1 + x^2}};$

PhiPlummer[x]

Out[78]= $-\frac{G\, M_{gc}}{a \sqrt{1 + x^2}}$

In[79]:= $\text{PlummerDensity}[x\_] := \frac{3\, M_{gc}}{4\, \text{Pi}\, a^3} \left(1 + x^2\right)^{-5/2};$

PlummerDensity[x]

Out[80]= $\frac{3\, M_{gc}}{4\, a^3\, \pi \left(1 + x^2\right)^{5/2}}$

BH (Kepler)

The black hole is assumed to be at the center of the GC.
Therefore, its potential is given by:

In[81]:= $\text{PhiKepler}[x\_] := -\frac{G * \mu * M_{gc}}{a * x}$

PhiKepler[x]

Out[82]= $-\frac{G\, \mu\, M_{gc}}{a\, x}$

Velocity Dispersion

Create the potential

In[83]:=
```
Phi[x_] := PhiPlummer[x] + PhiKepler[x]
Phi[x]
```

Out[84]=
$$-\frac{G\,M_{gc}}{a\sqrt{1+x^2}}-\frac{G\,\mu\,M_{gc}}{a\,x}$$

In[85]:=
```
dPhi[x_] := D[Phi[x], x]
dPhi[x]
```

Out[86]=
$$\frac{G\,x\,M_{gc}}{a\left(1+x^2\right)^{3/2}}+\frac{G\,\mu\,M_{gc}}{a\,x^2}$$

The number density is given only be the Plummer potential (except directly at 0)

In[87]:=
```
ν[x_] := PlummerDensity[x] / Mgc
```

In[88]:=
```
ν[x]
```

Out[88]=
$$\frac{3}{4\,a^3\,\pi\left(1+x^2\right)^{5/2}}$$

Actually do it

In[89]:=
```
sig2 = 1/ν[x] Integrate[ν[xp] * dPhi[xp], {xp, x, Infinity}]
```

Out[89]=
$$-\frac{1}{6\,a\,x\left(1+x^2\right)}G\left(-6\,\mu-36\,x^2\,\mu-70\,x^4\,\mu-56\,x^6\,\mu-16\,x^8\,\mu+48\,x^3\sqrt{1+x^2}\,\mu+48\,x^5\sqrt{1+x^2}\,\mu+16\,x^7\sqrt{1+x^2}\,\mu+x\sqrt{1+x^2}\,(-1+16\,\mu)\right)M_{gc}$$