Algorithms

Spring-2023 Midterm

This exam is designed to be done in the class time (1 hour 25 mins). Keeping into consideration the hard times and other midterms, the time for this test is 4 hours for you to finish and upload your documents to the eLearn. Put all files, pictures and your results in one pdf file and name it as LASTNAME_MidTerm .pdf and submit it through eLearn. **Any kind of plagiarism (help from your friends or getting help from internet) is prohibited**. Be sure that you submit your work before the time ends. The system will close after 8 PM and you will not be able to turn it in. So, make sure you submit by the due time.

1. **(5 Points)** Show that $(n^3+3n^2+3n+1) / (n+1)$ is $O(n^2)$. Use the definition and proof of big-O notation.

2. **(15 Points)** Prove using the definition of Omega notation that either 8^n is $\Omega(5^n)$ or not.

3. **(15 Points)** Find the running time function of the following algorithms as we learned in the class and then the tightest big-O notation.

a)
```
AlgorithmX(n)
1. x←1
2. for i←1 to n^2
3.     x←max(x,i)
4. for j←1 to n^2
5.     x←max(x,j)
6. return x
```

b)
```
AlgorithmY(n)
1. x←1
2. for i←1 to n^3
3.     j←1
4.     while j<n
5.         x←max(x, i*j)
6.         j←2*j
7. return x
```

c)
```
AlgorithmZ(n)
1. x←1
2. for i←1 to n^2
3.     j←1
4.     while j<n
5.         x←max(x, i*j)
6.         j←j+1
7. return x
```

4. **(15 Points)** Let T(n) be a running time function defined as $T(n) = 3n^2 + 2n + 5$, is this $\Theta(n^2)$? Explain prove your answer using the definitions of big-o and omega notations.

5. **(15 Points)** Write a pseudocode for the given recurrence relation below to calculate a_n, where n can be any positive integer.

   $a_1 = a_2 = 1$

   $a_n = 2a_{n-1} - 3a_{n-2}$

6. **(20 Points)** Solve the following recurrence relations using Master theorem.
   a. $T(n) = 3T\left(\frac{n}{3}\right) + n$
   b. $T(n) = 5T\left(\frac{n}{2}\right) + 2n^2$

7. **(15 Points)** Given the hash function as h(key) = key mod m where m=3, create an hash table for these entities given in order below: (e.g. 213, John is the first entity to be entered in your hash table):

213, John
122, Mark
345, Jane
234, Steph
340, Hong
423, Linda
143, Mary
467, Tonia
388, Jim
229, Hey

a. Using the hast table mentioned above, how many operations are needed to find whether the given entity 90, Michael is in the table or not.
b. What is the worst case scenerio for hash tables? Explain it in detail and given an example.