The University of Texas at Austin
Department of Electrical and Computer Engineering

EE381V: Learning-Based Optimal Control, Fall 2022

HOMEWORK 1

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1. *LQR with slew rate penalty.* Consider a discrete time, linear time-invariant system. Assume the stage-wise cost is quadratic in the state and input. In addition, consider a quadratic cost penalizing the slew rate (change in control input). The task is to regulate the system to the zero point.

   Formulate the finite-horizon unconstrained optimal problem above and derive the recursive DP equations.

2. *LQR with bilinear cost and affine dynamics.* Consider a system with stage-wise cost

$$c(x_k, u_k) = \frac{1}{2}x_k^T Q_k x_k + \frac{1}{2}u_k^T R_k u_k + u_k^T H_k x_k + q_k^T x_k + r_k^T u_k + h_k,$$

   terminal cost

$$c_N(x_k) = \frac{1}{2}x_k^T Q_N x_k + q_N^T x_k + h_N,$$

   and dynamics

$$x_{k+1} = A_k x_k + B_k u_k + d_k.$$

   Derive the discrete time LQR controller for the above system.

   Hint: Assume the cost-to-go is of the following form

$$J_k(x_k) = \frac{1}{2}x_k^T V_k x_k + v_k^T x_k + w_k.$$

3. *Coding iLQR.* Code the iLQR algorithm in Python. Use the system below to test your implementation.

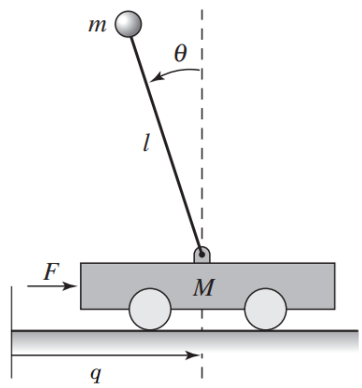

Figure 1: A schematic the cart-pole system.

The differential equations governing the dynamics of the balance system, pictured above, are given by

$$M_t\ddot{q} - ml\ddot{\theta}\cos(\theta) + c\dot{q} + ml\sin(\theta)\dot{\theta}^2 = F$$
$$-ml\cos(\theta)\ddot{q} + J_t\ddot{\theta} + \gamma\dot{\theta} - mgl\sin(\theta) = 0$$

where $M_t = M + m$ and $J_t = J + ml^2$. c and γ are viscous damping coefficients. Set $M = 10kg$, $m = 80kg$, $c = 0.1Ns/m$, $J = 100kgm^2/s^2$, $l = 1m$, $\gamma = 0.01Nms$ and $g = 9.8m/s^2$.

Plot and compare the optimal control sequence, state values and cost obtained using iLQR and LQR (model obtained by linearising around $\theta = 0$ and $q = 0$) for a reasonable finite horizon duration. Use two different initial conditions, $\theta_0 = 10°$ and $\theta_0 = 30°$. Consider the usual quadratic cost and choose the same cost matrices for both the controllers.