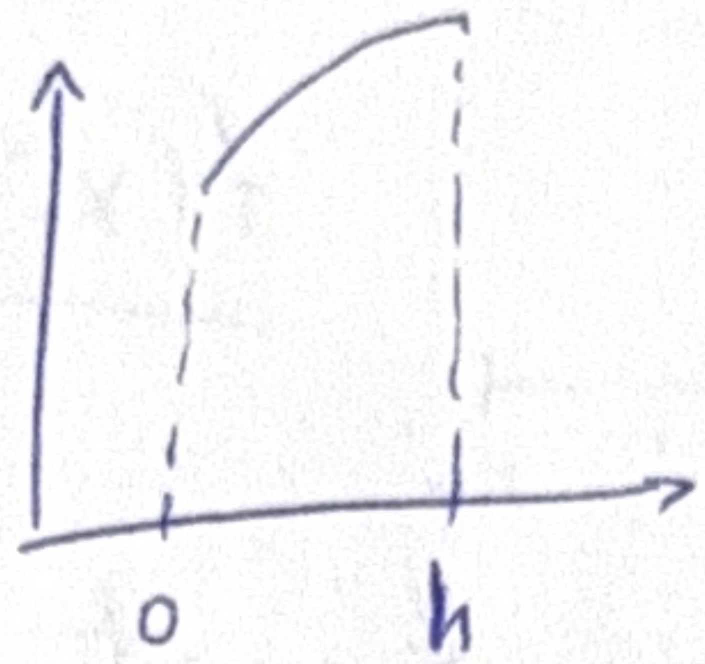
Выч. мат. Семинар

$$|J - S| = \left|\sum_{i=0}^{n-1} (J_i - S_i)\right| \le \sum_{i=0}^{n-1} |J_i - S_i|$$

$$J_i = \int_0^h f(x)\,dx$$



$$S_0 = f(0)\cdot h$$

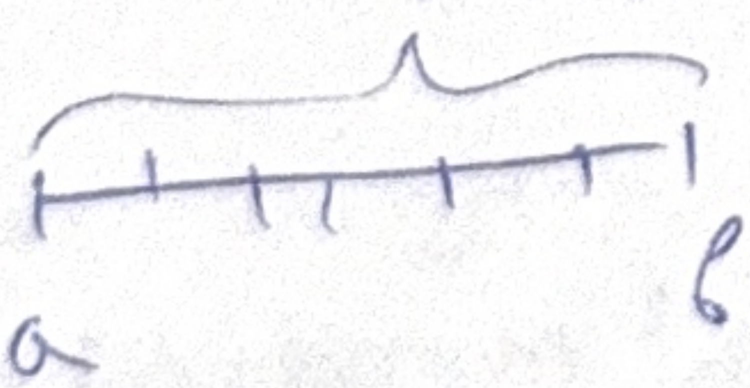
$$\int_0^h f(x)\,dx = \int_0^h \left[f(0) + \frac{f'(0)x}{1!} + O(x^2) \right] dx =$$

$$= f(0)$$

$$|J - S| = \ldots \le \sum_{i=0}^{n-1} |J_i - S_i| = \sum_{i=0}^{n-1} \left| f'(x_i)\frac{h^2}{2} + O(h^3)\right| \le$$

$$\le \sum_{i=0}^{n-1} \left|f'(x_i)\frac{h^2}{2}\right| + \sum_{i=0}^{n-1} |O(h^3)| \le \frac{h^2}{2} \max_i f'(x_i) \sum_{i=0}^{n-1} 1 +$$

$$+ |O(h^3)| \sum_{i=0}^{n-1} 1 = \frac{h^2}{2} M_1 n + nO(h^3) =$$

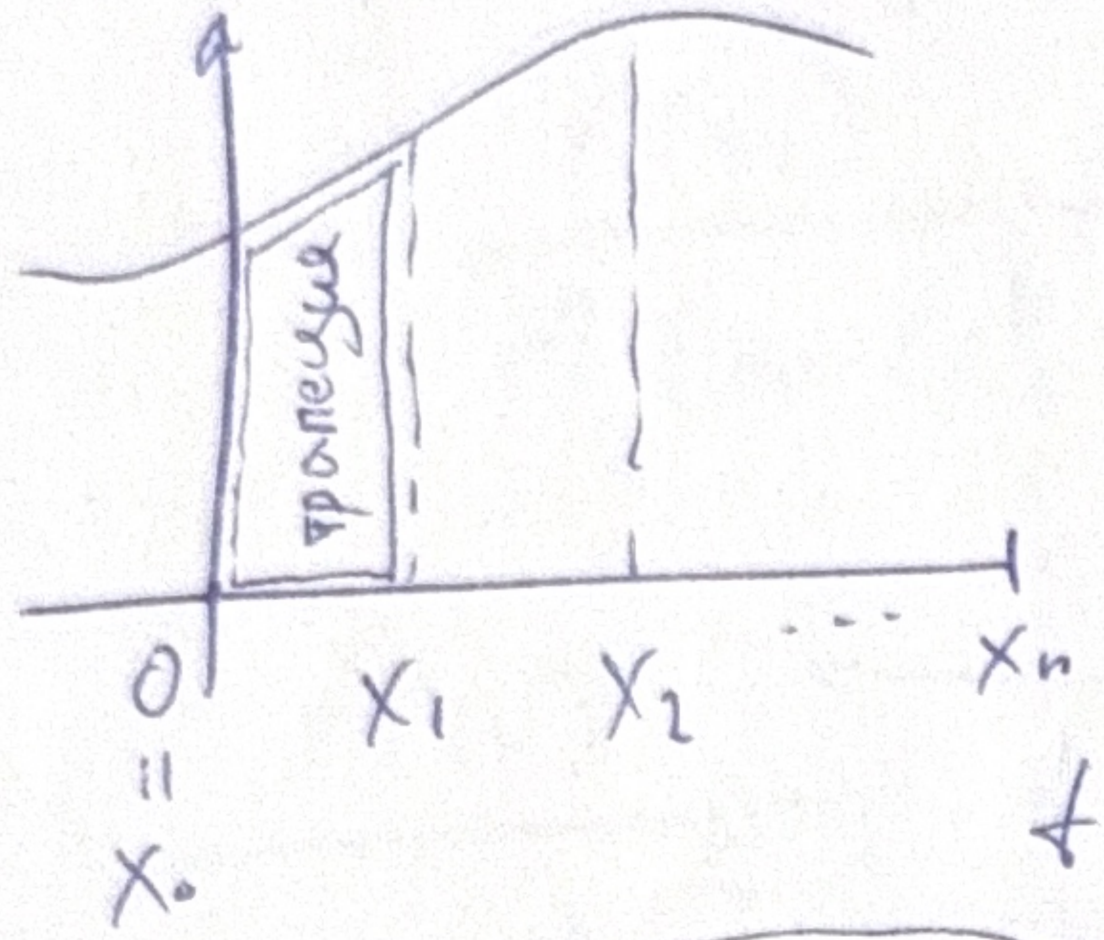


$$h = \frac{b-a}{n}$$

$$= \boxed{\frac{h^2}{2} M_1 \frac{b-a}{h}} + \frac{b-a}{\cancel{h}} O(h^{\cancel{3}\,2})$$

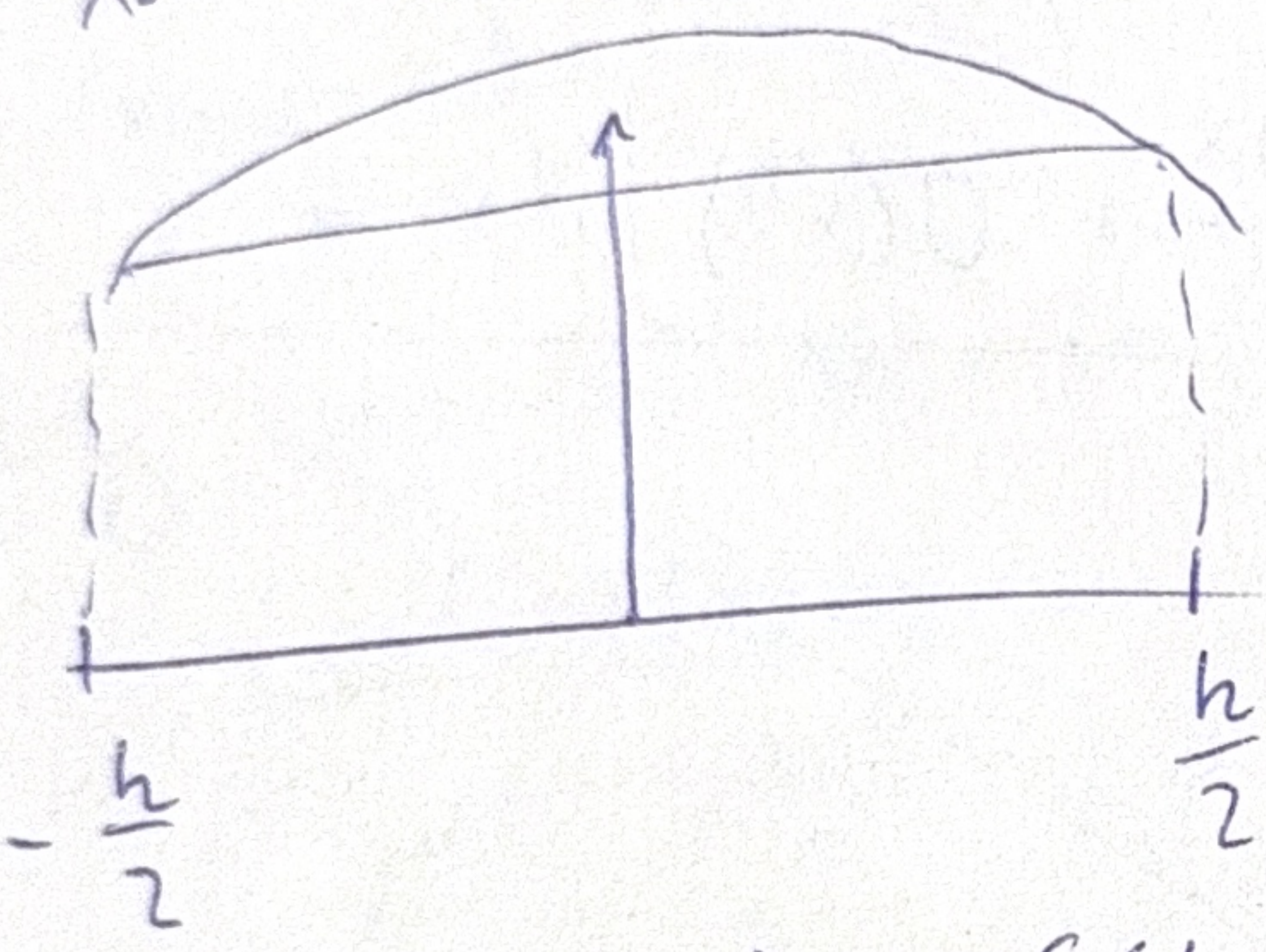
можно убрать т.к. для $b-a$ это незн.

Ф-ла трапеции



$$S = \sum_{i=0}^{n-1} \frac{f(x_i)+f(x_{i+1})}{2}h$$

$$\frac{f(x_0)+f(x_1)}{2}h + \frac{f(x_1)+f(x_2)}{2}h + \dots + \frac{f(x_{n-1})+f(x_n)}{2}h$$



$$S_i = \frac{f(-\frac{h}{2})+f(\frac{h}{2})}{2}h = f(0)h + \frac{f''(0)\frac{h^3}{4}}{2} + O(h^5)$$

$$I_i = \int_{-\frac{h}{2}}^{\frac{h}{2}} f(x)\,dx$$

$$|I-S| \le \sum_{i=0}^{n-1} |I_i - S_i| = \sum_{i=0}^{n-1} \left| f(0)h + \frac{f''(0)(h^3)}{24} + O(h^5) - \left(\frac{f(0)h}{2} + \frac{f''(0)\frac{h^3}{4}}{4} + O(h^5)\right)\right| =$$

$$= \sum_{i=0}^{n-1} \left| \frac{f(0)h}{2} - \frac{f(0)h^3}{48} \right|$$

$f(x)$ - в ряд Тейлора!

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + O(x^4)$$

$$\int f(x)\,dx = f(0)h + \frac{f''(0)(h^3)}{24} + O(h^5)$$

$$f\left(\pm\frac{h}{2}\right) = f(0) \pm \frac{f'(0)\frac{h}{2}}{1!} + \frac{f''(0)\frac{h^2}{4}}{2!} \pm \frac{f'''(0)\frac{h^3}{8}}{3!} + O(h^4)$$

Л/р в телефоне + $\int_0^{\pi} \sin x\,dx$, $h=6$ Посчитать всеми способами