Д/з по выч мату.

1) $f(x) = \sin(x)$ $\quad x \in [0, 1]$ $\quad h = \frac{1}{2}$

а) $x_1 = 0$

$x_2 = \frac{1}{2}$

$x_3 = 1$

$-\cos(x)$

$$f(x) - L_3(x) = \frac{f^{(n)}(\xi)}{n!} \omega_n(x) \qquad \prod_{i=1}^{n}(x - x_i) =$$

$= (x-x_1)(x-x_2)\cdot(x-x_3) =$

$= x\left(x-\frac{1}{2}\right)(x-1)$

$=$

$M_3 = \max|f^{(3)}(x)|$ на $[0, 1]$

$= \max|-\cos(x)| = 1$

$\omega(x) = x\left(x - \frac{1}{2}\right)(x-1)$

$\omega' = 3x^2 - 3x + \frac{1}{2} = 0$

$x = \frac{3 \pm \sqrt{3}}{6}$

$\max|\omega(x)| = \frac{\sqrt{3}}{36}$

$$|f(x) - L_3(x)| \leqslant \frac{1}{\underbrace{3!}_{=6}} \cdot \frac{\sqrt{3}}{36} = \frac{\sqrt{3}}{216}$$

б) $h = \frac{1}{10}$ => 11 точек

$$f(x) - L_{11}(x) = \frac{f^{(11)}(\xi)}{11!} \prod_{i=1}^{11} (x - x_i) =$$

$$= -\cos(x)$$

$M_{11} = \max|-\cos(x)|$ на $[0,1] = 1$

$$|f(x) - L_{11}(x)| \leq \frac{1}{11!}\left(\frac{1}{10}\right)^{11} = 2,5 \cdot 10^{-19}$$

г) $f(x) = \exp(x)$

а) $$f(x) - L_3(x) = \frac{f^{(3)}(x)}{3!} \prod_{i=1}^{3} (x - x_i) =$$

$$= \frac{e^x}{6} \cdot x\left(x - \frac{1}{2}\right)(x-1)$$

e^x → max на $[0,1]$ в 1 $e^1 = e$

$\max = \frac{\sqrt{3}}{36}$

$$|f(x) - L_3(x)| \leq \frac{e}{6} \frac{\sqrt{3}}{36} = \frac{e\sqrt{3}}{216}$$

б) $$f(x) - L_{11}(x) = \frac{f^{(11)}(x)}{11!} \prod_{i=1}^{11} (x - x_i)$$

$$|f(x) - L_{11}(x)| \leq \frac{e}{11!}\left(\frac{1}{10}\right)^{11} = 6,81 \cdot 10^{-19}$$

$$L_4(x) = \sum_{i=1}^{4} f_i \prod_{\substack{j=1 \\ j\neq i}}^{4} \frac{x-x_j}{x_i-x_j} =$$

$x_1 = 1 \quad f_1 = 0$

$x_2 = 0 \quad f_2 = 7$

$x_3 = 2 \quad f_3 = -1$

$x_4 = 3 \quad f_4 = 3$

$$= 7\cdot\left(\frac{x-1}{0-1}\right)\left(\frac{x-2}{0-2}\right)\left(\frac{x-3}{0-3}\right) +$$

$$+ -1\left(\frac{x-1}{2-1}\right)\left(\frac{x-0}{2-0}\right)\left(\frac{x-3}{2-3}\right) +$$

$$+ 3\cdot\left(\frac{x-1}{3-1}\right)\left(\frac{x-0}{3-0}\right)\left(\frac{x-2}{3-2}\right) =$$

$$= -\frac{7}{6}(x-1)(x-2)(x-3) +$$

$$+ \frac{1}{2}(x-1)(x-3)x +$$

$$+ \frac{1}{2}(x-1)(x-2)x =$$

$$= \frac{1}{2}(x-1)x\,(\overbrace{x-3+x-2}^{2x-5}) - \frac{7}{6}(x-1)(x-2)(x-3) =$$

$$= \left(\frac{1}{2}x^2 - \frac{1}{2}x\right)(2x-5)\ \boxed{-\frac{7}{6}x^3} + 7x^2 - \frac{77x}{6} + 7 =$$

$$\boxed{x^3} - \frac{7}{2}x^2 + \frac{5}{2}x =$$

$$= -\frac{1}{6}x^3 + \frac{7}{2}x^2 - \frac{62}{6}x + 7$$