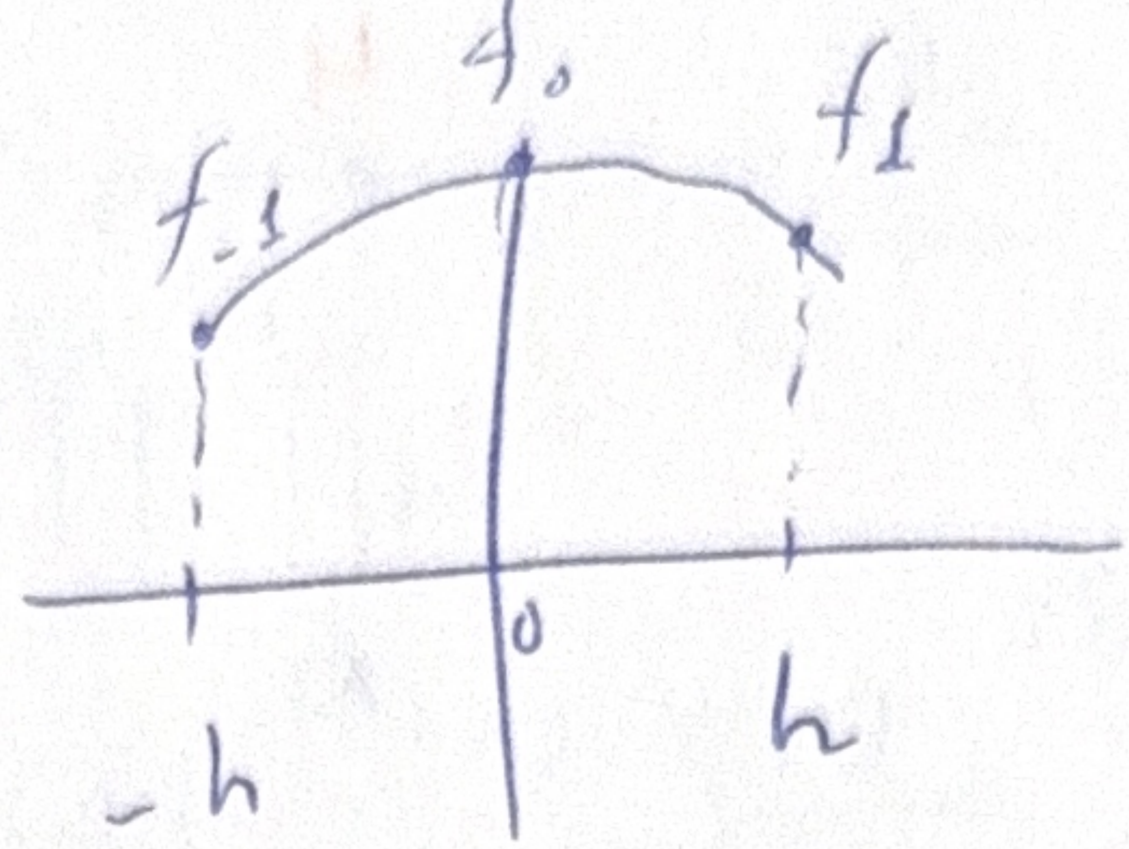
Выч. мат Семинар

$$y = ax^2 + bx + c$$

$$y(0) = f_0$$

$$c = f_0$$



$$a = \frac{f_1 + f_{-1} - 2f_0}{2h^2}$$

$$\left.\begin{aligned} ah^2 + bh + f_0 &= f_1 \\ ah^2 - bh + f_0 &= f_{-1} \end{aligned}\right\} \Rightarrow 2ah^2 + 2f_0 = f_1 + f_{-1} \Rightarrow$$

$$\int_{-h}^{h} y\,dx = \int_{-h}^{h} (ax^2 + bx + c)\,dx = \left(\frac{a}{3}x^3 + \frac{b}{2}x^2 + cx\right)\Bigg|_{-h}^{h} =$$

$$= \frac{2a}{3}h^3 + 2ch = \frac{f_1 + f_{-1} \cancel{+} - 2f_0}{3}h + 2f_0 h =$$

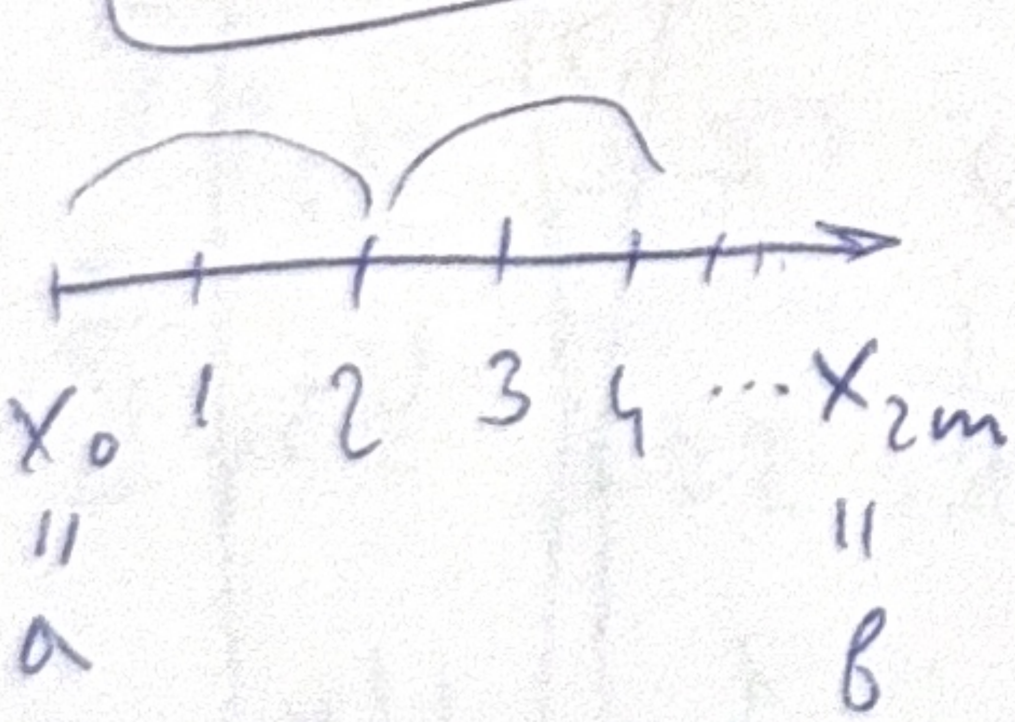
$$= \frac{h}{3}(f_1 + f_{-1} - 2f_0 + 6f_0) = \frac{h}{3}(f_1 + f_{-1} + 4f_0)$$

сместили систему отсчета в 0

$$S = \frac{h}{3}(f_2 + f_0 + 4f_1 + f_4 + f_2 + 4f_3 + \ldots) =$$

$\boxed{n = 2m}$

$$= \frac{h}{3}(f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \ldots f_{2n}) =$$



$$= \frac{h}{3}\left(f_0 + 4\sum_{i=0}^{m-1} f_{2i+1} + 2\sum_{i=1}^{m-1} f_{2i} + f_{2m}\right)$$

Определяем точность для ф-лы Симпсона:

$$f(x)\Big|_0 = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$$

$$f_1 = f(h)\Big|_0 = f(0) + \cancel{f'(0)h} + \frac{f''(0)h^2}{2!} + \cancel{\frac{f'''(0)h^3}{3!}} + \ldots$$

$$f_{-1} = f(-h)\Big|_0 = f(0) - \cancel{f'(0)h} + \frac{f''(0)h^2}{2!} - \cancel{\frac{f'''(0)h^3}{3!}} + \ldots$$

$$\frac{h}{3}\left(f_1 + f_{-1} + 4f_0\right) = \int_{-h}^{h} y\,dx = S$$

$$\parallel$$

$$\frac{h}{3}\left(6f(0) + f''(0)h^2 + \frac{f^{(4)}(0)h^4}{12} + \frac{f^{(6)}(0)h^6}{360} + O(h^7)\right)$$

$$Y = \int_{-h}^{h} f(x)\,dx = \int_{-h}^{h}\left(f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f''''(0)x^4}{4!} + O(x^5)\right)dx =$$

$$= f(0)2h + \frac{f''(0)h^3}{3} + \frac{f^{(4)}(0)h^5}{60} + O(h^7)$$

$$\left|Y_{2i+1} - S_{2i+1}\right| = \left|\cancel{\frac{2f''(0)h^3}{3}}\ \frac{f^{(4)}(0)h^5}{12}\,\frac{2}{15} + O(h^7)\right| \#$$

$$\sum_{i=0}^{m-1}\left|\frac{f^{(4)}(x_{2i+1})\cdot 2h^5}{4!}\cdot\frac{2}{15}+O(h^7)\right| \le$$

это →

из пред задания ответ

$$\le \sum_{i=0}^{m-1}\left|\frac{f^{(4)}(x_{2i+1})\,2h^5}{4!}\cdot\frac{2}{15}\right| + \left|\sum_{i=0}^{m-1}O(h^7)\right| \le$$

$$\le \frac{h^5}{3!\cdot 15}M_4\cdot m + O(h^7)\cdot m =$$

$$n = \frac{b-a}{h}$$

$$= \frac{h^{\cancel{5}\,4}}{3!\cdot 15}M_4\frac{(b-a)}{2\cancel{h}} + O(h^{\cancel{7}\,6})\frac{(b-a)}{2h} = \qquad n = 2m$$

$$= \frac{M_4 h^4 (b-a)}{3!\cdot 30} + \frac{(b-a)O(h^6)}{2} = \qquad m = \frac{b-a}{2h}$$

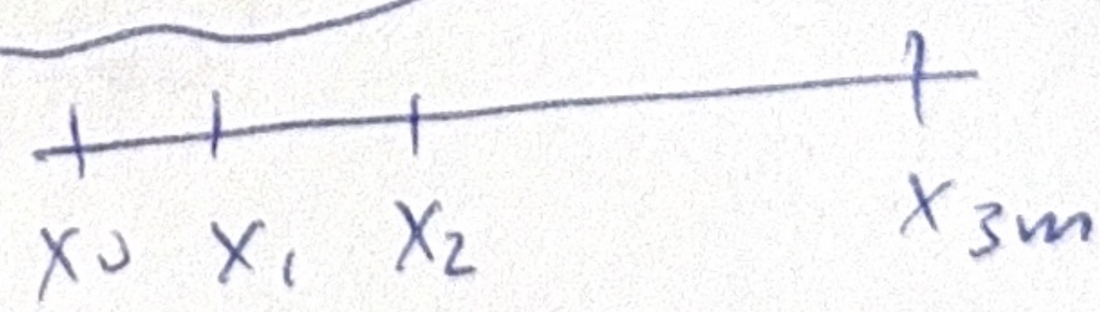
можно убрать

$$= \frac{(b-a)}{2}\left(\frac{M_4 h^4}{3!\cdot 15} + O(h^6)\right) =$$

$$= \boxed{\frac{M_4 h^4 (b-a)}{3!\cdot 30}} + O(h^6)$$

гл. член ошибки

Д/З обязательно



делим $(b-a)$ на интервалы длины $\qquad n = 3m = \frac{b-a}{h}$

3m

На каждом интервале полином(ы) строим $P_3(x)$

$$\int_{x_{3i}}^{x_{3i+3}} f(x)\,dx \approx \int_{x_{3i}}^{x_{3i+3}} P_3(x)\,dx$$

Выписать ф-лу числ инт., вычислить гл. член ошибки