Lambda calculus

Functional models of computation

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History

- 1928 - Hilbert's *Entscheidungsproblem* [1]
  - Is there an *algorithm* for deciding whether a proposition in first-order logic is true or false?
- Replacement for set theory as foundation of mathematics
  - 1930 - Combinatory logic (*Curry, Schönfinkel*)
  - 1932 - λ-calculus (*Church*)
  - 1935 - Kleene-Rosser paradox
- Effective computability
  - 1935 - Untyped λ-calculus (*Church, Kleene, Rosser*)
  - 1936 - Turing machine
  - 1936 - Church-Turing thesis
- 1936 - Undecidability of first-order logic
  - Halting problem of Turing machine
  - Equivalence of λ-terms

[1] German for "decision problem"

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David Hilbert

- **Haskell Curry**
- Wilhelm Ackermann
- John von Neumann
- Ernst Zermelo
- …

Alonzo Church

- **Stephen Cole Kleene**
- **J. Barkley Rosser**
- Alan Turing
- Dana Scott
- Michael O. Rabin
- …

Syntax

$$term ::= \underbrace{var}_{\text{Variable}} \mid \underbrace{term\ term}_{\text{Application}} \mid \underbrace{\lambda var.\, term}_{\text{Abstraction}}$$

Examples

Conventions

Tree representation

α-conversion

Free and bound variables

Substitution

α-equivalence

β-conversion

β-reduction

β-abstraction

η-conversion

Convertibility

Normal order reduction

First Church-Rosser theorem

Second Church-Rosser theorem

Normal order reduction

Recursion

Fixed-point combinator

Curry's Y-combinator

$$Y = \lambda f.\,(\lambda x.\,f(xx))\ (\lambda x.\,f(xx))$$

Turing's Θ-combinator

$$\Theta = (\lambda xy.\,x(xxy))\ (\lambda xy.\,x(xxy))$$

Church-Turing thesis

Undecidability

Church numerals

Relation to folds

Algebraic data types

Predecessor

Q&A