Unit 5
Hypothesis Testing

Hypothesis Tests

A **hypothesis test** is a process that uses sample statistics to test a claim about the value of a population parameter.

If a manufacturer of rechargeable batteries claims that the batteries they produce are good for an average of at least 1,000 charges, a sample would be taken to test this claim.

A verbal statement, or claim, about a population parameter is called a **statistical hypothesis**.

To test the average of 1000 hours, a pair of hypotheses are stated – one that represents the claim and the other, its complement. When one of these hypotheses is false, the other must be true.

Stating a Hypothesis

"H subzero" or "H naught"

A **null hypothesis** $\mathbf{H_0}$ is a statistical hypothesis that contains a statement of equality such as $\leq$, $=$, or $\geq$.

"H sub-a"

A **alternative hypothesis** $\mathbf{H_a}$ is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and contains a statement of inequality such as $>$, $\neq$, or $<$.

To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement.

Stating a Hypothesis

Example:
Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

A manufacturer claims that its rechargeable batteries have an average life of at least 1,000 charges.

$\mu \geq 1000$

Condition of equality

H_0: $\mu \geq 1000$ (Claim)

H_a: $\mu < 1000$

Complement of the null hypothesis

Stating a Hypothesis

Example:
Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

PES University claims that 94% of their graduates find employment within six months of graduation.

$p = 0.94$

Condition of equality

H_0: $p = 0.94$ (Claim)

H_a: $p \neq 0.94$

Complement of the null hypothesis

Types of Errors

No matter which hypothesis represents the claim, always begin the hypothesis test **assuming that the null hypothesis is true**.

At the end of the test, one of two decisions will be made:

1. reject the null hypothesis, or
2. fail to reject the null hypothesis.

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

Types of Errors

	Actual Truth of H_0	
Decision	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	Type II Error
Reject H_0	Type I Error	Correct Decision

Types of Errors

Example:
PES University claims that 94% of their graduates find employment within six months of graduation. What will a type I or type II error be?

H_0: $p = 0.94$ (Claim)

H_a: $p \neq 0.94$

A type I error is rejecting the null when it is true.
The population proportion is actually 0.94, but is rejected. (We believe it is not 0.94.)

A type II error is failing to reject the null when it is false.
The population proportion is not 0.94, but is not rejected. (We believe it is 0.94.)

Level of Significance

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a type I error. It is denoted by α, the lowercase Greek letter alpha.

Hypothesis tests are based on α.

The probability of making a type II error is denoted by β, the lowercase Greek letter beta.

By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small.

Commonly used levels of significance:

$$\alpha = 0.10 \qquad \alpha = 0.05 \qquad \alpha = 0.01$$

Statistical Tests

After stating the null and alternative hypotheses and specifying the level of significance, a random sample is taken from the population and sample statistics are calculated.

The statistic that is compared with the parameter in the null hypothesis is called the **test statistic**.

Population parameter	Test statistic	Standardized test statistic
μ	$\overline{x}$	$z\ (n \geq 30)$ $t\ (n < 30)$
p	$\hat{p}$	z
σ^2	s^2	X^2

P-values

If the null hypothesis is true, a ***P*-value** (or **probability value**) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

The *P*-value of a hypothesis test depends on the nature of the test.

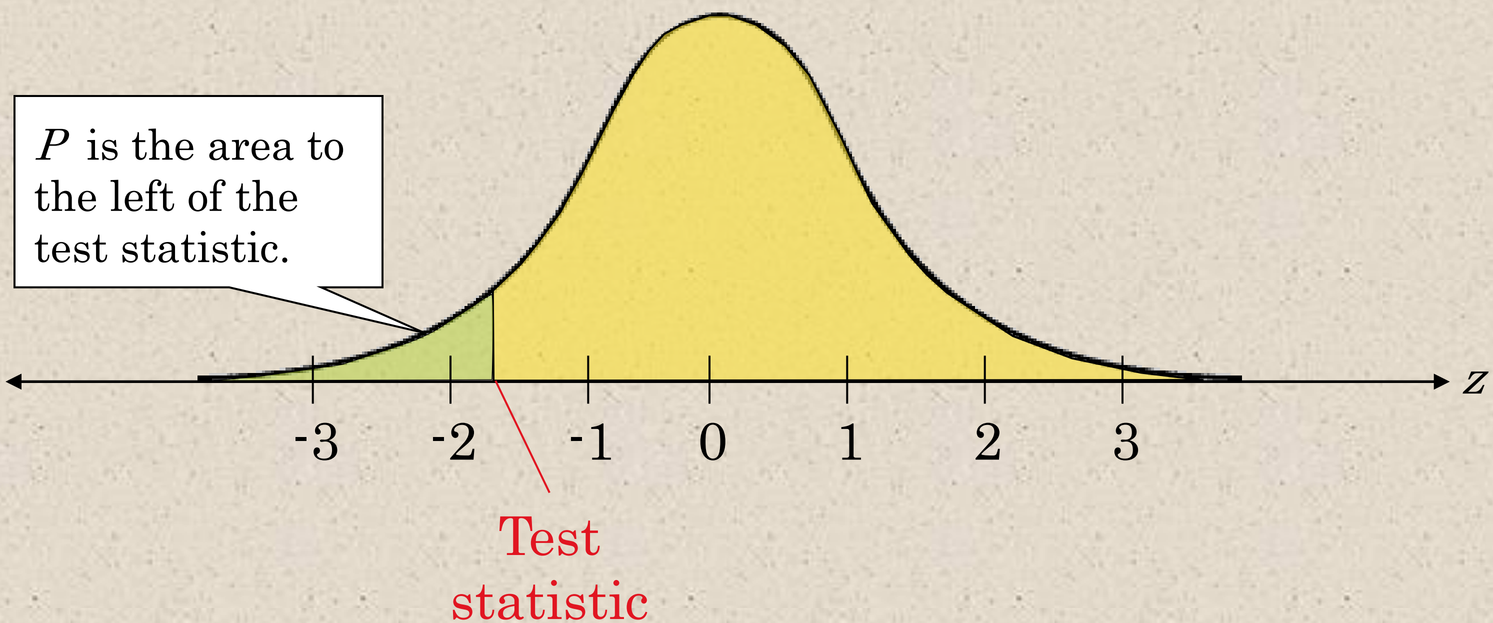
There are three types of hypothesis tests – a left-, right-, or two-tailed test. The type of test depends on the region of the sampling distribution that favors a rejection of H_0. This region is indicated by the alternative hypothesis.

Left-tailed Test

1. If the alternative hypothesis contains the less-than inequality symbol (<), the hypothesis test is a **left-tailed test**.

H_0: $\mu \geq k$

H_a: $\mu < k$

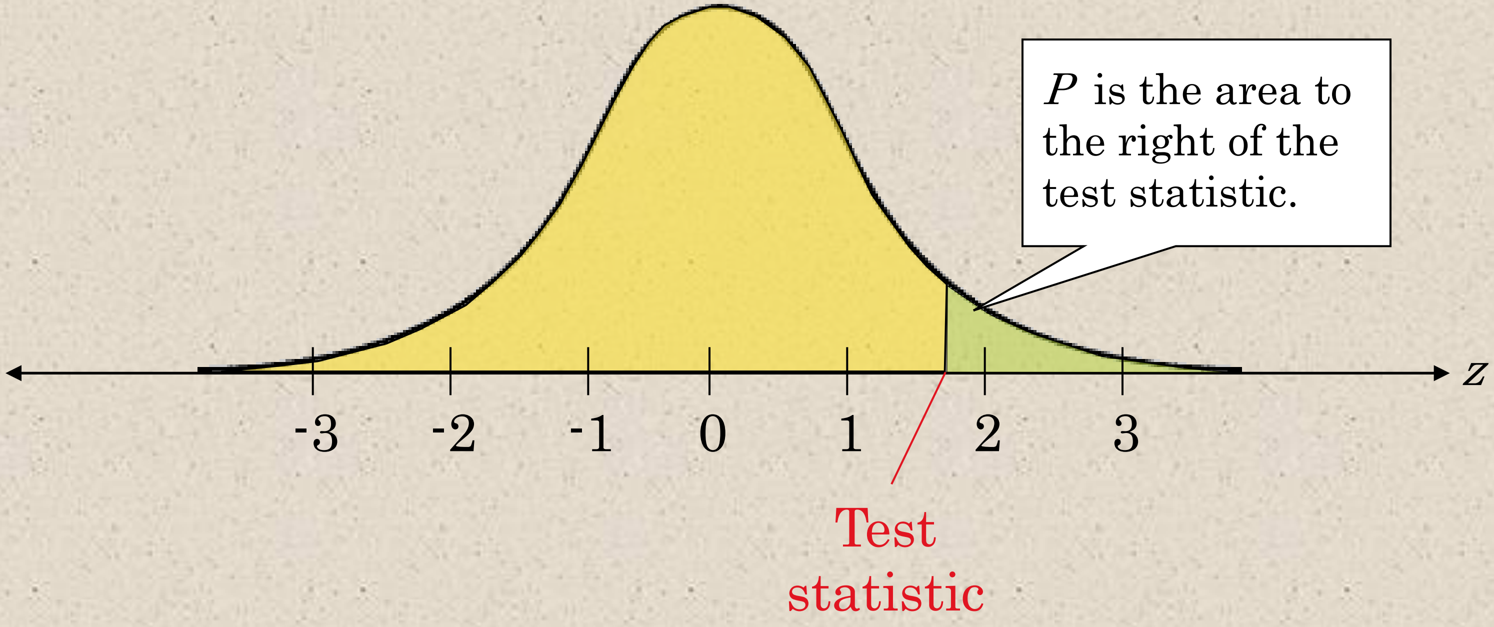


Right-tailed Test

2. If the alternative hypothesis contains the greater-than symbol (>), the hypothesis test is a **right-tailed test**.

H_0: $\mu \leq k$

H_a: $\mu > k$

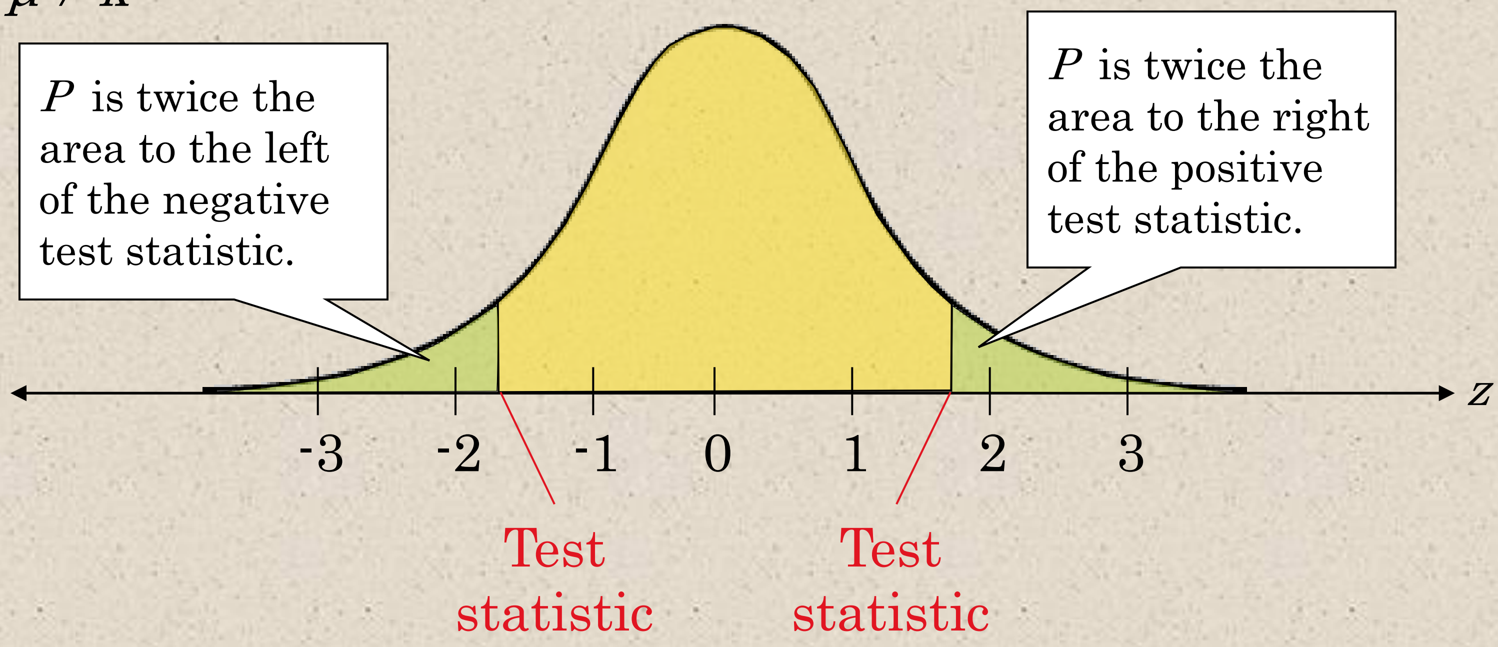


Two-tailed Test

3. If the alternative hypothesis contains the not-equal-to symbol (≠), the hypothesis test is a **two-tailed test**. In a two-tailed test, each tail has an area of $\frac{1}{2}P$.

H_0: $\mu = k$

H_a: $\mu \neq k$



Identifying Types of Tests

Example:

For each claim, state H_0 and H_a. Then determine whether the hypothesis test is a left-tailed, right-tailed, or two-tailed test.

a.) A cigarette manufacturer claims that less than one-eighth of the US adult population smokes cigarettes.

H_0: $p \geq 0.125$

H_a: $p < 0.125$ (Claim) → Left-tailed test

b.) A local telephone company claims that the average length of a phone call is 8 minutes.

H_0: $\mu = 8$ (Claim)

H_a: $\mu \neq 8$ → Two-tailed test

Making a Decision

Decision Rule Based on *P*-value

To use a *P*-value to make a conclusion in a hypothesis test, compare the *P*-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

Decision	Claim: Claim is H_0	Claim: Claim is H_a
Reject H_0	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Do not reject H_0	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Interpreting a Decision

Example:

You perform a hypothesis test for the following claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

H_0: (Claim) A cigarette manufacturer claims that less than one-eighth of the US adult population smokes cigarettes.

If H_0 is rejected, you should conclude "there is sufficient evidence to indicate that the manufacturer's claim is false."

If you fail to reject H_0, you should conclude "there is *not* sufficient evidence to indicate that the manufacturer's claim is false."

Steps for Hypothesis Testing

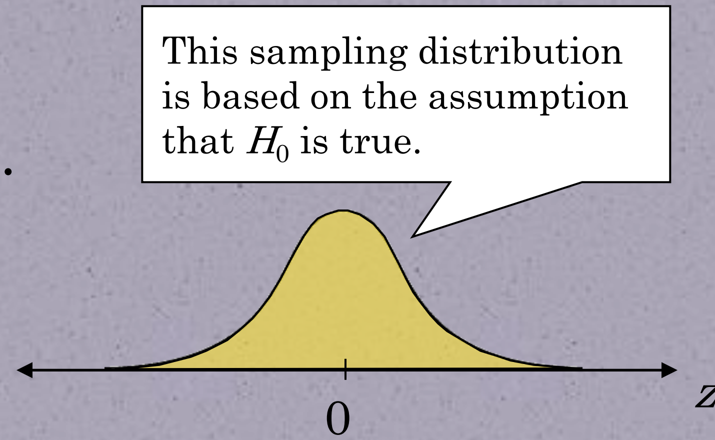
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

   H_0: ? H_a: ?

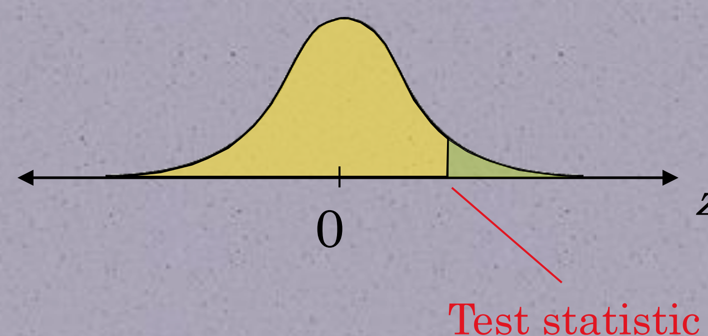
2. Specify the level of significance.

   $\alpha = ?$

3. Determine the standardized sampling distribution and draw its graph.



4. Calculate the test statistic and its standardized value. Add it to your sketch.



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Steps for Hypothesis Testing

5. Find the P-value.

6. Use the following decision rule.

Is the P-value less than or equal to the level of significance?

No → Fail to reject H_0.

Yes → Reject H_0.

7. Write a statement to interpret the decision in the context of the original claim.

These steps apply to left-tailed, right-tailed, and two-tailed tests.

Hypothesis Testing for the Mean (Large Samples)

Using P-values to Make a Decision

Decision Rule Based on P-value

To use a P-value to make a conclusion in a hypothesis test, compare the P-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

Recall that when the sample size is at least 30, the sampling distribution for the sample mean is normal.

Using *P*-values to Make a Decision

Example:

The P-value for a hypothesis test is $P = 0.0256$. What is your decision if the level of significance is

a.) 0.05,

b.) 0.01?

a.) Because 0.0256 is < 0.05, you should reject the null hypothesis.

b.) Because 0.0256 is > 0.01, you should fail to reject the null hypothesis.

Finding the P-value

After determining the hypothesis test's standardized test statistic and the test statistic's corresponding area, do one of the following to find the P-value.

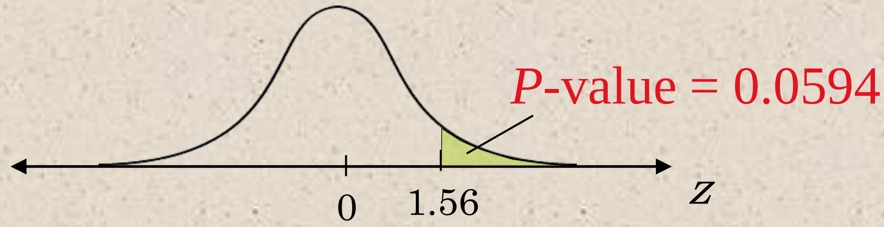
a. For a left-tailed test, $P =$ (Area in left tail).

b. For a right-tailed test, $P =$ (Area in right tail).

c. For a two-tailed test, $P = 2$(Area in tail of test statistic).

Example:

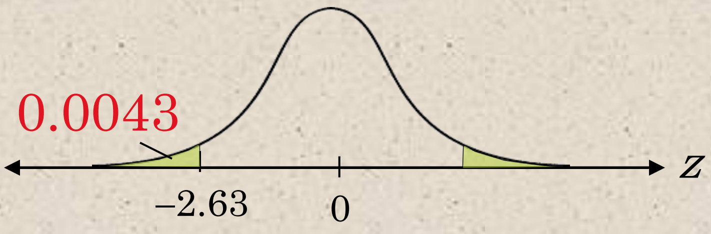
The test statistic for a right-tailed test is $z = 1.56$. Find the P-value.



The area to the right of z = 1.56 is 1 – .9406 = 0.0594.

Finding the P-value

Example:
The test statistic for a two-tailed test is $z = -2.63$.
Find the P-value.



The area to the left of $z = -2.63$ is 0.0043.
The P-value is $2(0.0043) = 0.0086$

Using *P*-values for a *z*-Test

The **z-test for the mean** is a statistical test for a population mean. The z-test can be used when the population is normal and σ is known, or for any population when the sample size n is at least 30.

The **test statistic** is the sample mean $\bar{x}$ and the **standardized test statistic** is z.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \qquad \frac{\sigma}{\sqrt{n}} = \text{standard error} = \sigma_{\bar{x}}$$

When $n \geq 30$, the sample standard deviation s can be substituted for σ.

Using P-values for a z-Test

Using P-values for a z-Test for a Mean μ

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the standardized test statistic.	$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
4. Find the area that corresponds to z.	Use Z Score Table

Continued.

Using P-values for a z-Test

Using P-values for a z-Test for a Mean μ

In Words | *In Symbols*

5. Find the P-value.
   a. For a left-tailed test, $P =$ (Area in left tail).
   b. For a right-tailed test, $P =$ (Area in right tail).
   c. For a two-tailed test, $P = 2$(Area in tail of test statistic).

6. Make a decision to reject or fail to reject the null hypothesis.

7. Interpret the decision in the context of the original claim.

Reject H_0 if P-value is less than or equal to α. Otherwise, fail to reject H_0.

Hypothesis Testing with P-values

Example:

A manufacturer claims that its rechargeable batteries are good for an average of more than 1,000 charges. A random sample of 100 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$?

H_0: $\mu \leq 1000$ H_a: $\mu > 1000$ (Claim)

The level of significance is $\alpha = 0.01$.

The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{1002 - 1000}{14 / \sqrt{100}}$$

$$\approx 1.43$$

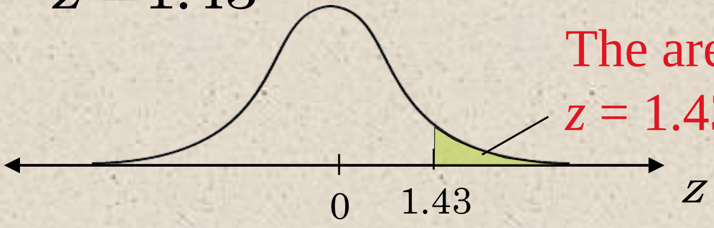
Continued.

Hypothesis Testing with *P*-values

Example continued:
A manufacturer claims that its rechargeable batteries are good for an average of more than 1,000 charges. A random sample of 100 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$?

H_0: $\mu \leq 1000$ H$_a$: $\mu > 1000$ (Claim)

$z = 1.43$



The area to the right of $z = 1.43$ is $P = 0.0764$.

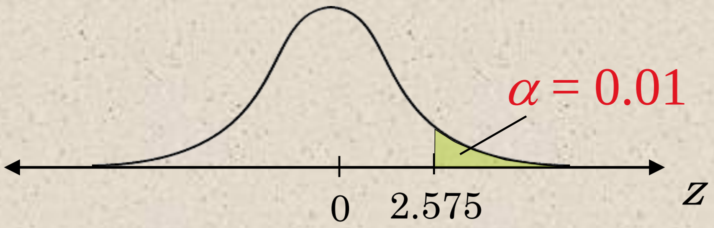
P-value is greater than $\alpha = 0.01$, fail to reject H_0.

At the 1% level of significance, there is not enough evidence to support the claim that the rechargeable battery has an average life of at least 1000 charges.

Rejection Regions and Critical Values

A **rejection region** (or **critical region**) of the sampling distribution is the range of values for which the null hypothesis is not probable. If a test statistic falls in this region, the null hypothesis is rejected. A critical value z_0 separates the rejection region from the nonrejection region.

Example:
Find the critical value and rejection region for a right tailed test with $\alpha = 0.01$.



The rejection region is to the right of $z_0 = 2.575$.

Rejection Regions and Critical Values

Finding Critical Values in a Normal Distribution

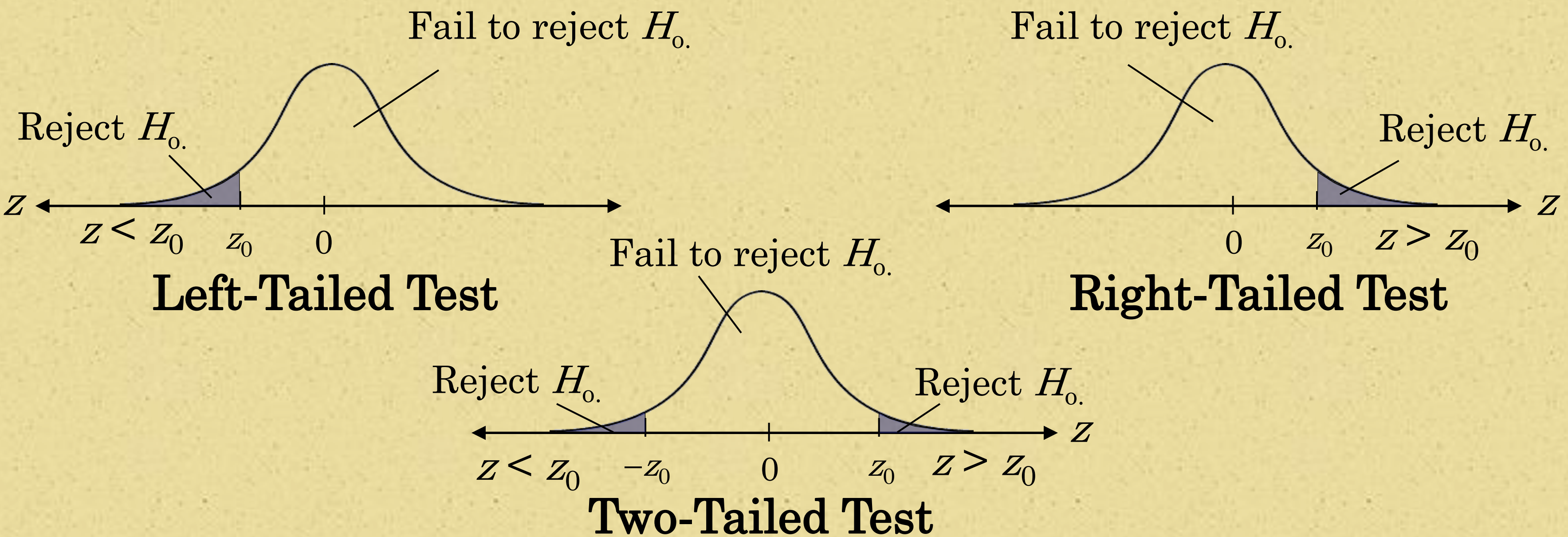
1. Specify the level of significance α.
2. Decide whether the test is left-, right-, or two-tailed.
3. Find the critical value(s) z_0. If the hypothesis test is
   a. left-tailed, find the z-score that corresponds to an area of α,
   b. right-tailed, find the z-score that corresponds to an area of $1 - \alpha$,
   c. two-tailed, find the z-score that corresponds to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution. Draw a vertical line at each critical value and shade the rejection region(s).

Rejection Regions for a z-Test

Decision Rule Based on Rejection Region

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic, z. If the standardized test statistic

1. is in the rejection region, then reject H_0.
2. is *not* in the rejection region, then fail to reject H_0.



Rejection Regions for a *z*-Test

Using Rejection Regions for a *z*-Test for a Mean μ

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Sketch the sampling distribution.	
4. Determine the critical value(s).	Use Z score Table
5. Determine the rejection regions(s).	

Continued.

Rejection Regions for a *z*-Test

Using Rejection Regions for a z-Test for a Mean μ

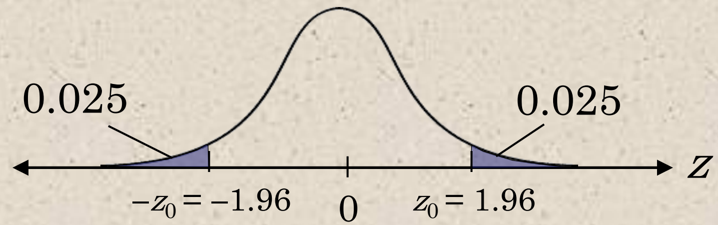
In Words	*In Symbols*
6. Find the standardized test statistic.	$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ or if $n \geq 30$ use $\sigma \approx s$.
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Testing with Rejection Regions

Example:
A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 58 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

H_0: $\mu = 8$ (Claim) $\qquad$ H_a: $\mu \neq 8$

The level of significance is $\alpha = 0.05$.



Continued.

Testing with Rejection Regions

Example continued:

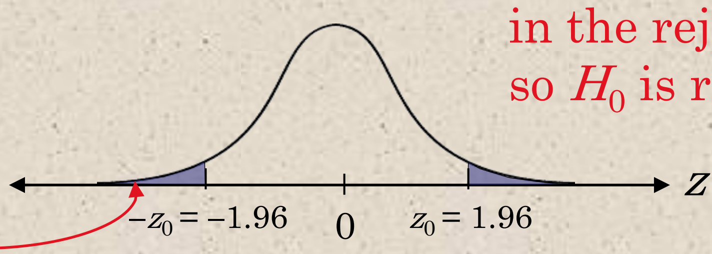
A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 58 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

H_0: $\mu = 8$ (Claim) $\quad$ H_a: $\mu \neq 8$

The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{7.8 - 8}{0.5 / \sqrt{58}}$$

$$\approx -3.05.$$



The test statistic falls in the rejection region, so H_0 is rejected.

At the 5% level of significance, there is enough evidence to reject the claim that the average length of a phone call is 8 minutes.

Hypothesis Testing for the Mean (Small Samples)

Critical Values in a t-Distribution

Finding Critical Values in a t-Distribution

1. Identify the level of significance α.
2. Identify the degrees of freedom d.f. = $n - 1$.
3. Find the critical value(s) using t Table in the row with $n - 1$ degrees of freedom. If the hypothesis test is
   a. left-tailed, use "One Tail, α" column with a negative sign,
   b. right-tailed, use "One Tail, α" column with a positive sign,
   c. two-tailed, use "Two Tails, α" column with a negative and a positive sign.

Finding Critical Values for t

Example:
Find the critical value t_0 for a right-tailed test given $\alpha = 0.01$ and $n = 24$.

The degrees of freedom are d.f. = $n - 1 = 24 - 1 = 23$.

To find the critical value, use t Table with d.f. = 23 and 0.01 in the “One Tail, α“ column. Because the test is a right-tail test, the critical value is positive.

$t_0 = 2.500$

Finding Critical Values for t

Example:
Find the critical values t_0 and $-t_0$ for a two-tailed test given $\alpha = 0.10$ and $n = 12$.

The degrees of freedom are d.f. = $n - 1 = 12 - 1 = 11$.

To find the critical value, use Table 5 with d.f. = 11 and 0.10 in the "Two Tail, α" column. Because the test is a two-tail test, one critical value is negative and one is positive.

$-t_0 = -1.796$ and $t_0 = 1.796$

t-Test for a Mean μ ($n < 30$, σ Unknown)

The **t-test for the mean** is a statistical test for a population mean. The t-test can be used when the population is normal or nearly normal, σ is unknown, and $n < 30$.

The **test statistic** is the sample mean $\bar{x}$ and the **standardized test statistic** is t.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. = $n - 1$.

t-Test for a Mean μ ($n < 30$, σ Unknown)

Using the t-Test for a Mean μ (Small Sample)

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Identify the degrees of freedom and sketch the sampling distribution.	d.f. = $n - 1$.
4. Determine any critical values.	Use t Table
5. Determine any rejection region(s).	

Continued.

t-Test for a Mean μ ($n < 30$, σ Unknown)

Using the t-Test for a Mean μ (Small Sample)

In Words	*In Symbols*
6. Find the standardized test statistic.	$t = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
7. Make a decision to reject or fail to reject the null hypothesis.	If t is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Testing μ Using Critical Values

Example:

A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 18 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

H_0: $\mu = 8$ (Claim) $\quad$ H_a: $\mu \neq 8$

The level of significance is $\alpha = 0.05$.

The test is a two-tailed test.

Degrees of freedom are d.f. = 18 – 1 = 17.

The critical values are $-t_0 = -2.110$ and $t_0 = 2.110$

Continued.

Testing μ Using Critical Values

Example continued:

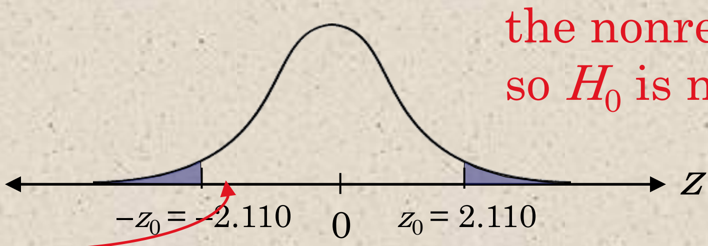
A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 18 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

H_0: $\mu = 8$ (Claim) $\qquad$ H_a: $\mu \neq 8$

The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{7.8 - 8}{0.5/\sqrt{18}}$$

$$\approx -1.70.$$



The test statistic falls in the nonrejection region, so H_0 is not rejected.

At the 5% level of significance, there is not enough evidence to reject the claim that the average length of a phone call is 8 minutes.

Testing μ Using P-values

Example:

A manufacturer claims that its rechargeable batteries have an average life greater than 1,000 charges. A random sample of 10 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$?

H_0: $\mu \le 1000$ H$_a$: $\mu > 1000$ (Claim)

The level of significance is $\alpha = 0.01$.

The degrees of freedom are d.f. = $n - 1 = 10 - 1 = 9$.

The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1002 - 1000}{14/\sqrt{10}}$$

$$\approx 0.45$$

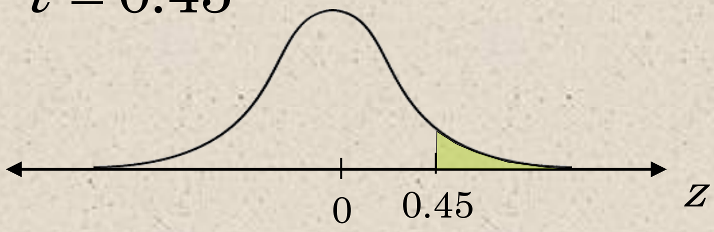
Continued.

Testing μ Using P-values

Example continued:
A manufacturer claims that its rechargeable batteries have an average life greater than 1,000 charges. A random sample of 10 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$?

H_0: $\mu \leq 1000$ H_a: $\mu > 1000$ (Claim)

$t = 0.45$



Using the d.f. = 9 row from Table 5, you can determine that P is greater than $\alpha = 0.25$ and is therefore also greater than the 0.01 significance level. H_0 would fail to be rejected.

At the 1% level of significance, there is not enough evidence to support the claim that the rechargeable battery has an average life of at least 1000 charges.

Hypothesis Testing for Proportions

z-Test for a Population Proportion

The **z-test for a population** is a statistical test for a population proportion. The z-test can be used when a binomial distribution is given such that $np \geq 5$ and $nq \geq 5$.

The **test statistic** is the sample proportion $\hat{p}$ and the **standardized test statistic** is z.

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Hypothesis Test for Proportions

Using a *z*-Test for a Proportion *p*

Verify that $np \geq 5$ and $nq \geq 5$.

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Sketch the sampling distribution.	
4. Determine any critical values.	Use z table

Continued.

Hypothesis Test for Proportions

Using a z-Test for a Proportion p

Verify that $np \geq 5$ and $nq \geq 5$.

In Words	*In Symbols*
5. Determine any rejection regions.	
6. Find the standardized test statistic.	$z = \dfrac{\hat{p} - p}{\sqrt{pq/n}}$
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Hypothesis Test for Proportions

Example:
Statesville college claims that more than 94% of their graduates find employment within six months of graduation. In a sample of 500 randomly selected graduates, 475 of them were employed. Is there enough evidence to support the college's claim at a 1% level of significance?

Verify that the products np and nq are at least 5.

$np = (500)(0.94) = 470$ and $nq = (500)(0.06) = 30$

H_0: $p \leq 0.94$ $\quad$ H_a: $p > 0.94$ (Claim)

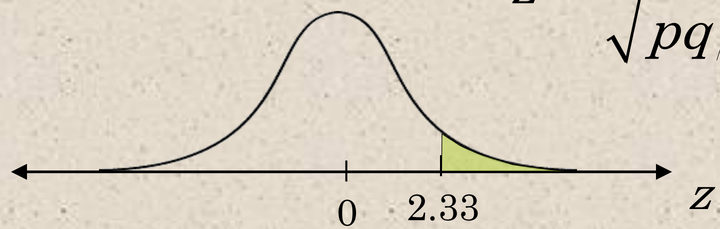
Continued.

Hypothesis Test for Proportions

Example continued:
Statesville college claims that more than 94% of their graduates find employment within six months of graduation. In a sample of 500 randomly selected graduates, 475 of them were employed. Is there enough evidence to support the college's claim at a 1% level of significance?

H_0: $p \leq 0.94$ H_a: $p > 0.94$ (Claim)

Because the test is a right-tailed test and $\alpha = 0.01$, the critical value is 2.33.



$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.95 - 0.94}{\sqrt{(0.94)(0.06)/500}}$$

$$\approx 0.94$$ Test statistic

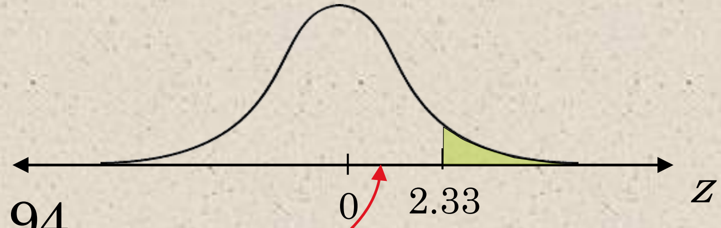
Continued.

Hypothesis Test for Proportions

Example continued:
Statesville college claims that more than 94% of their graduates find employment within six months of graduation. In a sample of 500 randomly selected graduates, 475 of them were employed. Is there enough evidence to support the college's claim at a 1% level of significance?

H_0: $p \leq 0.94$ H_a: $p > 0.94$ (Claim)

$z = 0.94$



The test statistic falls in the nonrejection region, so H_0 is not rejected.

At the 1% level of significance, there is not enough evidence to support the college's claim.

Hypothesis Test for Proportions

Example:
A cigarette manufacturer claims that one-eighth of the US adult population smokes cigarettes. In a random sample of 100 adults, 5 are cigarette smokers. Test the manufacturer's claim at $\alpha = 0.05$.

Verify that the products np and nq are at least 5.

$$np = (100)(0.125) = 12.5 \quad \text{and} \quad nq = (100)(0.875) = 87.5$$

H_0: $p = 0.125$ (Claim) $\qquad$ H_a: $p \neq 0.125$

Because the test is a two-tailed test and $\alpha = 0.05$, the critical values are $\pm$ 1.96.

Continued.

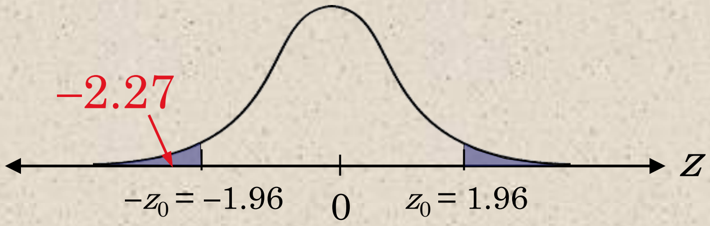
Hypothesis Test for Proportions

Example continued:

A cigarette manufacturer claims that one-eighth of the US adult population smokes cigarettes. In a random sample of 100 adults, 5 are cigarettes smokers. Test the manufacturer's claim at $\alpha = 0.05$.

H_0: $p = 0.125$ (Claim)

H_a: $p \neq 0.125$



The test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.05 - 0.125}{\sqrt{(0.125)(0.875)/100}}$$

$$\approx -2.27$$ Reject H_0.

At the 5% level of significance, there is enough evidence to reject the claim that one-eighth of the population smokes.

Hypothesis Testing for Variance and Standard Deviation

Critical Values for the χ^2-Test

Finding Critical Values for the χ^2-Distribution

1. Specify the level of significance α.
2. Determine the degrees of freedom d.f. = $n - 1$.
3. The critical values for the χ^2-distribution are found in Table 6 of Appendix B. To find the critical value(s) for a
   a. right-tailed test, use the value that corresponds to d.f. and α.
   b. left-tailed test, use the value that corresponds to d.f. and $1 - \alpha$.
   c. two-tailed test, use the values that corresponds to d.f. and $\frac{1}{2}\alpha$ and d.f. and $1 - \frac{1}{2}\alpha$.

Finding Critical Values for the χ^2

Example:

Find the critical value for a left-tailed test when $n = 19$ and $\alpha = 0.05$.

There are 18 d.f. The area to the right of the critical value is $1 - \alpha = 1 - 0.05 = 0.95$.

From ChiTable, the critical value is $\chi^2_0 = 9.390$.

Example:

Find the critical value for a two-tailed test when $n = 26$ and $\alpha = 0.01$.

There are 25 d.f. The areas to the right of the critical values are $\frac{1}{2}\alpha = 0.005$ and $1 - \frac{1}{2}\alpha = 0.995$.

From ChiTable, the critical values are $\chi^2_L = 10.520$ and $\chi^2_R = 46.928$.

The Chi-Square Test

The **χ^2-test for a variance or standard deviation** is a statistical test for a population variance or standard deviation. The χ^2-test can be used when the population is normal.

The **test statistic** is s^2 and the **standardized test statistic**

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

follows a chi-square distribution with degrees of freedom d.f. = $n - 1$.

The Chi-Square Test

Using the χ^2-Test for a Variance or Standard Deviation

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the degrees of freedom and sketch the sampling distribution.	d.f. = $n - 1$
4. Determine any critical values.	Use chiTable

Continued.

The Chi-Square Test

Using the χ2-Test for a Variance or Standard Deviation

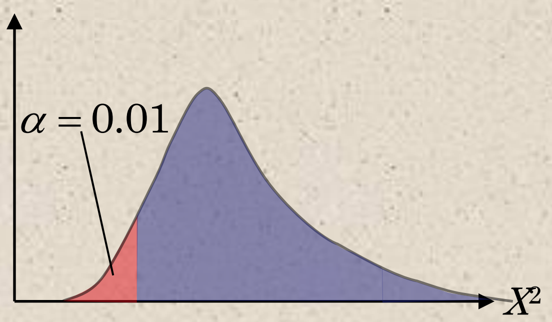
In Words	*In Symbols*
5. Determine any rejection regions.	
6. Find the standardized test statistic.	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$
7. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Hypothesis Test for Standard Deviation

Example:
A college professor claims that the standard deviation for students taking a statistics test is less than 30. 10 tests are randomly selected and the standard deviation is found to be 28.8. Test this professor's claim at the $\alpha = 0.01$ level.

H_0: $\sigma \geq 30$ $\qquad$ H_a: $\sigma < 30$ (Claim)

This is a left-tailed test with d.f.= 9 and $\alpha = 0.01$.



Continued.

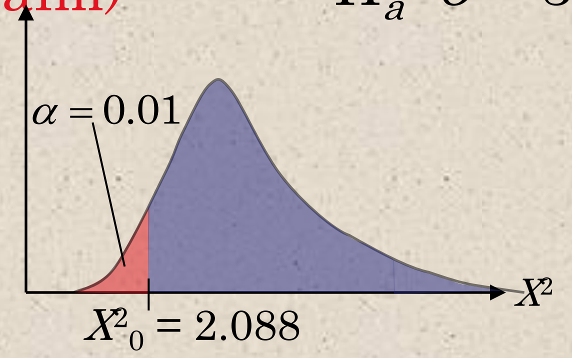
Hypothesis Test for Standard Deviation

Example continued:

A college professor claims that the standard deviation for students taking a statistics test is less than 30. 10 tests are randomly selected and the standard deviation is found to be 28.8. Test this professor's claim at the $\alpha = 0.01$ level.

H_0: σ <=30 (Claim) H_a: $\sigma > 30$

$X^2{}_0 = 2.088$



$$X^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)(28.8)^2}{30^2}$$

$$\approx 8.29$$

Fail to reject H_0.

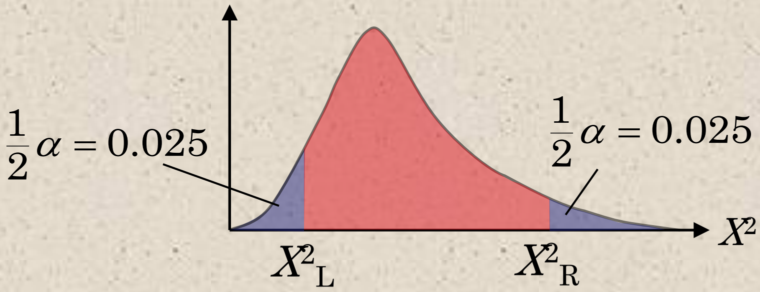
At the 1% level of significance, there is not enough evidence to support the professor's claim.

Hypothesis Test for Variance

Example:
A local balloon company claims that the variance for the time its helium balloons will stay afloat is 5 hours. A disgruntled customer wants to test this claim. She randomly selects 23 customers and finds that the variance of the sample is 4.5 seconds. At $\alpha = 0.05$, does she have enough evidence to reject the company's claim?

H_0: $\sigma^2 = 5$ (Claim) $\quad\quad$ H_a: $\sigma^2 \neq 5$

This is a two-tailed test with d.f.= 22 and $\alpha = 0.05$.



Continued.

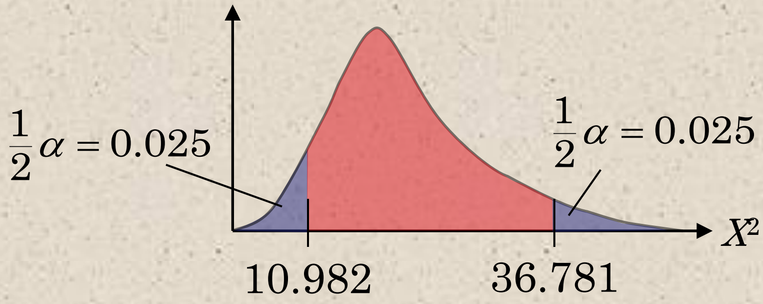
Hypothesis Test for Variance

Example continued:

A local balloon company claims that the variance for the time its helium balloons will stay afloat is 5 hours. A disgruntled customer wants to test this claim. She randomly selects 23 customers and finds that the variance of the sample is 4.5 seconds. At $\alpha = 0.05$, does she have enough evidence to reject the company's claim?

H_0: $\sigma^2 = 5$ (Claim) H$_a$: $\sigma^2 \neq 5$

The critical values are $\chi^2_L = 10.982$ and $\chi^2_R = 36.781$.



Continued.

Hypothesis Test for Variance

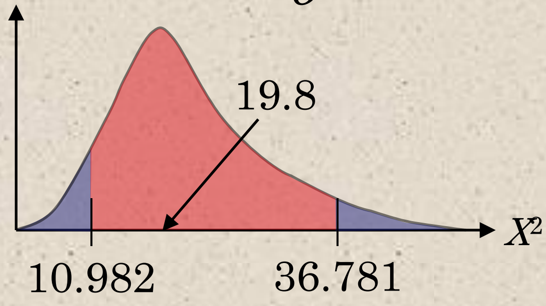
Example continued:

A local balloon company claims that the variance for the time one of its helium balloons will stay afloat is 5 hours. A disgruntled customer wants to test this claim. She randomly selects 23 customers and finds that the variance of the sample is 4.5 seconds. At $\alpha = 0.05$, does she have enough evidence to reject the company's claim?

H_0: $\sigma^2 = 5$ (Claim) H$_a$: $\sigma^2 \neq 5$

$$X^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(23-1)(4.5)}{5} = 19.8$$

Fail to reject H_0.



At $\alpha = 0.05$, there is not enough evidence to reject the claim that the variance of the float time is 5 hours.

Hypothesis Testing in R

Test	Type	Critical value(CV)	Test Statistic	P
z	Lower	qnorm(1-alpha) [add minus before the value]	$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$	pnorm(z)
	Upper	qnorm(1-alpha)		pnorm(z, lower.tail=FALSE)
	Two	qnorm(1-alpha/2) [consider one plus value and one minus value]		2*pnorm(z)
t	Lower	qt(1-alpha,df) [add minus before the value]	$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$	pt(t,df)
	Upper	qt(1-alpha,df)		pt(t,df, lower.tail=FALSE)
	Two	qt(1-alpha/2,df) [consider one plus value and one minus value]		2*pt(t)

Test	Type	Critical value(CV)	Test Statistic	P
Prop. z	Lower	qnorm(1-alpha) [add minus before the value]	$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$	pnorm(z)
	Upper	qnorm(1-alpha)		pnorm(z, lower.tail=FALSE)
	Two	qnorm(1-alpha/2) [consider one plus value and one minus value]		2*pnorm(z)
chi	Lower	qchisq(1-alpha,df)	$X^2 = \frac{(n-1)s^2}{\sigma^2}$	pchisq(chi_test,df)
	Upper	qchisq(alpha, df)		
	Two	**Left tail** qchisq(alpha/2,df) **Right tail** qchisq(1-alpha/2,df)		pchisq(chi_test,df) pchisq(chi_test,df)
F test (two samples)	Upper	df(1-alpha,df1,df2)	F=var1/var2	pf(f,df1,df2)