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AE5222:Exam 4

```
syms r a x0 x1 x2 T u0 u1 J1 J2 J0
```

Define the states

```
x1 = (1 - a)*x0 + a*u0;
x2 = (1 - a)*x1 + a*u1;
```

Get the cost at each state

```
J2 = r*(x2 - T)^2
```

$$J2 = r\,\left(T - a\,u_1 + \left(a\,u_0 - x_0\,\left(a - 1\right)\right)\,\left(a - 1\right)\right)^2$$

```
J1 = (u1^2 + J2)
```

$$J1 = r\,\left(T - a\,u_1 + \left(a\,u_0 - x_0\,\left(a - 1\right)\right)\,\left(a - 1\right)\right)^2 + u_1^{\,2}$$

```
J0 =  (u0^2 + J1)
```

$$J0 = r\,\left(T - a\,u_1 + \left(a\,u_0 - x_0\,\left(a - 1\right)\right)\,\left(a - 1\right)\right)^2 + u_0^{\,2} + u_1^{\,2}$$

FInd the minum by seting the partical dertives to 0

```
du1 = diff(J0,u1);
du0 = diff(J0,u0);
U = solve( du0==0,du1==0, [ u0, u1]);
```

Solution

```
u0_star = collect(U.u0,[x0 ,r])
```

u0_star =

$$\frac{\left(a^4 - 3\,a^3 + 3\,a^2 - a\right)\,x_0\,r + \left(T\,a - T\,a^2\right)\,r}{\left(a^4 - 2\,a^3 + 2\,a^2\right)\,r + 1}$$

```
u1_star = collect(U.u1,[x0 ,r])
```

u1_star =

$$\frac{\left(-a^3 + 2\,a^2 - a\right) x_0\, r + \left(T\, a\right) r}{\left(a^4 - 2\,a^3 + 2\,a^2\right) r + 1}$$