AE 5222 – Homework Assignment #2

Due date: Saturday, 01-April-2017.
Instructions: Refer to the syllabus for grading policies and other guidelines. Make one submission per group via Canvas.

Reading assignment (0 points): Read through Chapter 2 of the lecture notes posted on Canvas.

Problem 1 (**Modeling with graphs).** (20 points)
An autonomous forklift in a warehouse is to perform the task of shelving 5 pallets of goods, marked $p_1, p_2, \ldots, p_5$ in the bins where they belong. These bins are marked $b_1, b_2, \ldots, b_5$. Unfortunately, a previous forklift mistakenly placed pallets in wrong bins as follows:

p_1	p_2	p_3	p_4	p_5
b_2	b_5	b_1	b_3	b_4

The new forklift is required to correct the mistake by picking up pallets and placing them in the correct bins. At any given instant of time, the forklift can lift only one pallet, each bin can hold only one pallet, and each pallet must be either in a bin or on the forklift. There is an spare bin b^{T} where pallets can be placed temporarily.

The objective is to enable the forklift to complete this task while traveling a minimum distance. The distances of travel between bins are known.

Identify a graph (i.e., define vertices and edges) that can be used to model this problem.

Extra credit (50 points): For the given inter-bin distances, solve this problem.

	b_1	b_2	b_3	b_4	b_5	b^{T}
b_1	–	66	94	64	132	124
b_2	66	–	36	54	70	145
b_3	94	36	–	52	72	140
b_4	64	54	52	–	118	92
b_5	132	70	72	118	–	209
b^{T}	124	145	140	92	209	–

The next two problems are related to the problem of navigating a threat field with minimum threat exposure. A vehicle moves in the square region $\mathcal{W} = [-1, 1] \times [-1, 1]$. We denote by $\mathsf{x} = (x, y)$ the position coordinates of any point in this region. It is common to model threat fields by a series expansion of the form:

$$c(\mathsf{x}) = c_{\text{offset}} + \sum_{n=1}^{N_{\mathrm{P}}} \theta_n \phi_n(\mathsf{x}) \tag{1}$$

where c_{offset} and N_{P} are given constants and $\phi_n(\mathsf{x})$ are prespecified spatial basis functions of the form

$$\phi_n(\mathsf{x}) := \frac{1}{\sqrt{2\pi\nu_n}} \exp(-\frac{1}{2\nu_n} \cdot (\mathsf{x} - \bar{\mathsf{x}}_n)^{\mathrm{T}}(\mathsf{x} - \bar{\mathsf{x}}_n)),$$

where $\nu_n \in \mathbb{R}_{>0}$ and $\bar{\mathsf{x}}_n \in \mathcal{W}$ are given constants for each $n = 1, \ldots, N_{\mathrm{P}}$. In a real-world application, where the "threat" could be, say, a weather system, the parameters θ_n are estimated using measurements of the field. For additional details, see for example the paper by Cooper & Cowlagi available on Canvas. For this assignment, consider θ_n as known constants.

Problem 2 (Dijkstra's algorithm). (60 points)
Formulate a grid consisting of N_{G}^2 uniformly placed in N_{G} rows and N_{G} columns. Label these grid points starting from the bottom left, similar to Example 2.4, p. 21 in the lecture notes. The coordinates the i^{th} grid point are denoted by x^i, for each $i = 1, ..., N_{\mathrm{G}}^2$. The vehicle is assumed to traverse grid points according to a "$4-$connectivity rule." i.e., it can travel from a grid point to immediately adjacent grid points in the same row or the same column.

The attached MATLAB® files provides values of the threat field at any point in the region $\mathcal{W}$, and include values of the various constants introduced above. This is the same threat field used in Example 2.4 in the lecture notes.

Write a MATLAB® program that executes Dijkstra's algorithm, and apply this program to find a path in the given threat field from the bottom left corner ($\mathsf{x} = (-1, -1)$) to the top right corner ($\mathsf{x} = (1, 1)$.) Find such a path in three different cases: (1) $N_{\mathrm{G}} = 3$: This will replicate the results of Example 2.4, and help you validate the correctness of your code. (2) $N_{\mathrm{G}} = 15$. (3) $N_{\mathrm{G}} = 101$.

In each case, report (1) the optimal path found, (2) the cost of this path, and (3) the number of iterations for which the algorithm executed. One batch of execution of Lines 3–12 in Fig. 2.3 of the lecture notes is counted as one iteration. **Notes:**

- Your implementation of Dijkstra's algorithm does not necessarily have to be in MATLAB®. You are free use a different programming language, as long as you are able to translate the given threat field calculations accordingly.
- If you can find an open-source implementation of Dijkstra's algorithm online in any programming language, you are free to use that implementation. If you do so, you must cite the source from which you obtained this implementation. **Sharing code across homework groups is not allowed.**
- If you write your own code, attach it with your homework submission. Otherwise do not attach it, and cite the source. □

Problem 3 (Optimal control in discrete stages). (20 points)
Consider the problem of selecting a sequence of heading directions of the vehicle to find a "piecewise straight" path (as discussed in class). The start and goal locations are the same. We can formulate this problem as an N-stage optimal control problem; the in-class example used $N = 1$. For the following discretized vehicle kinematic model and cost function, write down the set of necessary conditions to find a sequence of headings that minimize the cost.

$$\text{Kinematics: } x(k+1) = x(k) + V\cos\psi(k), \quad y(k+1) = y(k) + V\sin\psi(k), \quad k = 0, \ldots, N-1.$$

$$\text{Cost: } \mathcal{J} = \tfrac{1}{2}(x(N)-1)^2 + \tfrac{1}{2}(y(N)-1)^2 + \sum_{k=1}^{N} c(x(k), y(k)).$$

Extra credit (50 points): For $N = 10$ and threat field as in the previous problem (details in MATLAB® file), solve the necessary conditions to find a sequence of headings. □