AE 5222 – Homework Assignment #1

Due date: Friday, 24-March-2017 at 5:00 PM.
Instructions: Refer to the syllabus for grading policies and other guidelines. Make one submission per group via Canvas. Do not attach any code.

Reading assignment (0 points): Read through the lecture notes for Module 1 posted on Canvas.

Problem 1 (Unconstrained optimization). (20 points)
Lift and *drag* are aerodynamic forces acting on an aircraft that are often approximated as follows for a simplified analysis of the gross motion of an aircraft:

$$L = \tfrac{1}{2}\rho V^2 S C_L, \qquad D = \tfrac{1}{2}\rho V^2 S (C_{D,0} + K C_L^2).$$

Here, L and D are the lift and drag forces, respectively, ρ is the atmospheric density, S is the wing surface area, V is the speed of the aircraft (assuming no wind), and C_L is a dimensionless quantity called the *coefficient of lift.* $C_{D,0}$ and K are dimensionless constants.

To maintain steady level flight (i.e., constant speed, constant altitude), the lift must be equal to the weight W of the aircraft, i.e. $L = W$. It is easy to show that the propulsive thrust required to maintain steady level flight is minimized when the lift-to-drag ratio, i.e., the quantity (L/D) is maximum. Due to the relationship between lift and drag, the ratio (L/D) varies with speed.

Approximate data for the Boeing B-52 Stratofortress strategic bomber aircraft are as follows:

$$W = 430,000 \text{ lb}, \qquad S = 4,000 \text{ ft}^2, \qquad C_{D,0} = 0.02, \qquad K = 0.08.$$

We are interested in steady level flight at an altitude of 30,000 ft, where $\rho = 8.9 \times 10^{-4}$ slugs/ft^3.

a) Determine an analytical expression for the speed V^* at which (L/D) is maximized, and an analytical expression for the maximum value of (L/D).

b) Determine the numerical value of V^*, and the numerical value of the maximum (L/D) using the given B-52 aircraft data.

c) Using the given B-52 aircraft data, plot a curve of (L/D) for a range of values of speed. Indicate V^*, as previously computed, on this plot.

Problem 2 (Constrained optimization). (20 points)
An open rectangular box is made by folding a thin rectangular sheet of metal. Find the width, length, and height of a box with volume V that minimizes the area of the required metal sheet.

Problem 3 (Constrained optimization, numerical). (25 points)
Minimize the objective function $f(\mathbf{x}) := x_1^3 - 6x_1^2 + 11x_1 + x_3$ in the domain $\mathbf{x} \in \mathbb{R}^3_{\geqslant 0}$ (i.e., the nonnegative octant of $\mathbb{R}^3$) subject to the constraints

$$x_1^2 + x_2^2 - x_3^2 \leqslant 0, \qquad \|\mathbf{x}\| \geqslant 2, \qquad x_3 \geqslant 5.$$

Problem 4 (Constrained optimization, numerical). (35 points)
Consider the problem of designing the cross-sectional areas of beams in a three-beam truss shown in Fig. 1. The objective is to minimize the total weight of the truss, which is given in normalized units by the formula

$$\text{Weight} = 2\sqrt{2}x_1 + x_2.$$

The minimum and maximum values for the cross section of any beam are 0.1 and 5.0 units, respectively. The constraints on the design are that the stresses induced in each beam must be lower than prespecified stress limits. Beams 1 and 2 (left and center) experience stresses in tension, given by the formulae

$$\sigma_1 = P\frac{x_2 + \sqrt{2}x_1}{\sqrt{2}x_1^2 + 2x_1x_2}, \qquad \sigma_2 = P\frac{1}{x_1 + \sqrt{2}x_2}.$$

The maximum permissible stress in tension is 20 units. Beam 3(right) experiences stress in compression, the magnitude of which is given by the formula

$$\sigma_3 = P\frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2}.$$

The magnitude of maximum permissible stress in compression is 15 units. Determine the optimal cross-sectional areas of the beams in this truss.

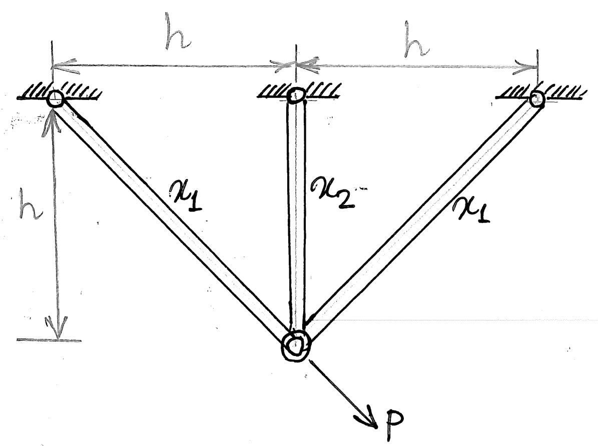

Figure 1