AE5222 Exam 2

Nathaniel Goldfarb

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Problem 1

The Traveling sales person (TSP) problem is a NP hard problem that can be viewed as a dynamic programing problem. Dijkstra algorithm can be used to solve this problem. The cost to move from city to city can be seen in below. A online C++ function was used found here: http://www.sanfoundry.com/cpp-program-implement-traveling-salesman-nearest-neighbour-algorithm/

city	Boston, MA	Providence, RI	New York, NY	Albany, NY	Buffalo, NY
Boston, MA	0	51	217	169	454
Providence, RI	51	0	182	163	449
New York, NY	217	182	0	151	373
Albany, NY	169	163	151	0	189
Buffalo, NY	454	449	373	289	0

To simplify the the problem we are going to we are going to rename the node as the following.

city	ID
Boston, MA	0
Providence, RI	1
New York, NY	2
Albany, NY	3
Buffalo, NY	4

Modifying the C++ cited above find the path with the least cost to be $[0, 4, 2, 3, 1, 0]$ and has a cost of 1064.

Problem 2

Part A

The problem involves LQR and the Algebraic Riccati equation. The first step in solving this equation is setting up the Hamiltonian.

$$H = 0.5(x(k)^T Qx(k) + u(k)^T Ru(k)) + p^T \dot{x} \tag{1}$$

The necessary conditions for this are the following:

$$\begin{aligned} -\frac{\partial H}{\partial \lambda} &= x(k+1) \\ \frac{\partial H}{\partial u(k)} &= 0 = p^T B + Ru(k) \\ p(k+1) &= -\frac{\partial H}{\partial x}^T = -Qx(k) - A^T p \end{aligned} \tag{2}$$

Part B

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ u &= -R^{-1}B^T p(k) \\ x(k+1) &= Ax(k) - BR^{-1}B^T p(x) \\ p(k+1) &= -Qx(k) - A^T p(k)S(k)x(k) \\ x(k+1) &= Ax(x+1) - BR^{-1}BS(k)x(k) \\ S^{-1}(k+1)p(k+1) &= Ax(x+1) - BR^{-1}BS(k)x(k) \\ S^{-1}(k+1)[Qx(k) - A^T p(k)] &= Ax(x+1) - BR^{-1}BS(k)x(k) \\ S^{-1}(k+1)[Qx(k) - A^T S(k)x(k)] &= Ax(x+1) - BR^{-1}BS(k)x(k) \\ S(k) &= A^T (S^{-1}(k+1) + BR^{-1}B^T)^{-1}A + Q \end{aligned} \tag{3}$$

Part C

Plugging into matlab and recursively solving yields.

p	x	u
[9;23]	[2;-23]	-23
[-44;-86]	[-23;86]	86
[149;-275]	[86;-275]	-275
[-464;-842]	[-275;842]	842
[1409;2543]	[842;-2543]	-2543