

$$C = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ s_1 & s_2 & s_3 \end{bmatrix} ; \quad \begin{aligned} \vec{L} &= (l_1, l_2, l_3) \\ \vec{M} &= (m_1, m_2, m_3) \\ \vec{S} &= (s_1, s_2, s_3) \end{aligned}$$

We define P as,

$$P = \begin{bmatrix} \frac{l_1}{|\vec{L}|^2} & \frac{m_1}{|\vec{M}|^2} & \frac{s_1}{|\vec{S}|^2} \\ \frac{l_2}{|\vec{L}|^2} & \frac{m_2}{|\vec{M}|^2} & \frac{s_2}{|\vec{S}|^2} \\ \frac{l_3}{|\vec{L}|^2} & \frac{m_3}{|\vec{M}|^2} & \frac{s_3}{|\vec{S}|^2} \end{bmatrix}$$

$$\vec{L} \cdot \vec{M} = \vec{M} \cdot \vec{S} = \vec{S} \cdot \vec{L} = 0.$$

Since C is positive, P is also positive. We are just dividing by square of magnitude.

Now,

$$C \cdot P = \begin{bmatrix} \frac{1}{|\vec{L}|^2} \cdot (l_1^2 + l_2^2 + l_3^2) & 0 & 0 \\ 0 & \frac{1}{|\vec{M}|^2} \cdot (m_1^2 + m_2^2 + m_3^2) & 0 \\ 0 & 0 & \frac{1}{|\vec{S}|^2} \cdot (s_1^2 + s_2^2 + s_3^2) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow C \cdot P = I$$

\Rightarrow This proves P is valid set of primaries.