6.869
SUMAN NEPAL

Q.1 We have,

$$\epsilon^2 = \sum_{u,v} \left| \hat{X}_F[u,v] - X_F[u,v] \right|^2$$

$$= \sum_{u,v} \left| X_F[u,v] - \hat{X}_F[u,v] \right|^2$$

For simplicity, we are dropping $[u,v]$

$$\Rightarrow \quad \epsilon^2 = \sum_{u,v} \left| X_F - \hat{X}_F \right|^2$$

We know, $\hat{X} = G \cdot Y$

$$\Rightarrow \quad \epsilon^2 = \sum_{u,v} \left| X_F - G \cdot Y \right|^2$$

$$= \sum_{u,v} [X_F - G \cdot Y] \cdot [X_F - GY]^*$$

$$= \sum_{u,v} [X_F - G \cdot (HX_F + N)] \cdot [X_F - G \cdot (HX_F + N)]^*$$

$$= \sum_{u,v} [X_F (1 - G \cdot H) - GN] [X_F (1 - GH) - GN]^*$$

$$= \sum_{u,v} [X_F (1 - G \cdot H) - GN] [X_F^* (1 - G^* H^*) - G^* N^*]$$

$$= \sum_{u,v} [X_F \cdot X_F^* (1 - GH)(1 - G^* N^*) - GN \cdot G^* N^*] \quad \begin{bmatrix} X_F N^* = 0 \\ X_F^* N = 0 \end{bmatrix}$$

$$= \sum_{u,v} \left[P_{xx} - P_{xx} GH - P_{xx} G^* H^* + P_{xx} H_{xx} |G|^2 + |G|^2 \sigma^2 \right]$$

Now,

$$G^* = G_R - G_I \cdot i \qquad |G|^2 = G_R^2 + G_I^2$$

$$G = G_R + G_I \cdot i \qquad H = H_R + H_I i$$

Then,

$$\epsilon^2 = \sum_{u,v} \left[P_{xx} - P_{xx} H (G_R + G_I i) - P_{xx} (G_R - G_I i)(H_R - H_I i) + P_{xx} H_{xx} (G_R^2 + G_I^2) + (G_R^2 + G_I)^2 \sigma^2 \right]$$

$$= \sum_{u,v} \left[P_{xx} - P_{xx} (H_R G_R + H_R G_I i + H_I G_R i - H_i G_i) - P_{xx} (G_R H_R - G_R H_I i - G_I H_R i - G_I H_I) + P_{xx} H_{xx} (G_R^2 + G_I^2) + (G_R^2 + G_I^2) \sigma^2 \right]$$

Then, $$\frac{\partial \epsilon^2}{\partial G_R^2} = - P_{xx} H_R - P_{xx} H_I i - P_{xx} H_R + P_{xx} H_I i + 2 P_{xx} H_{xx} G_R + 2 G_R \cdot \sigma^2$$

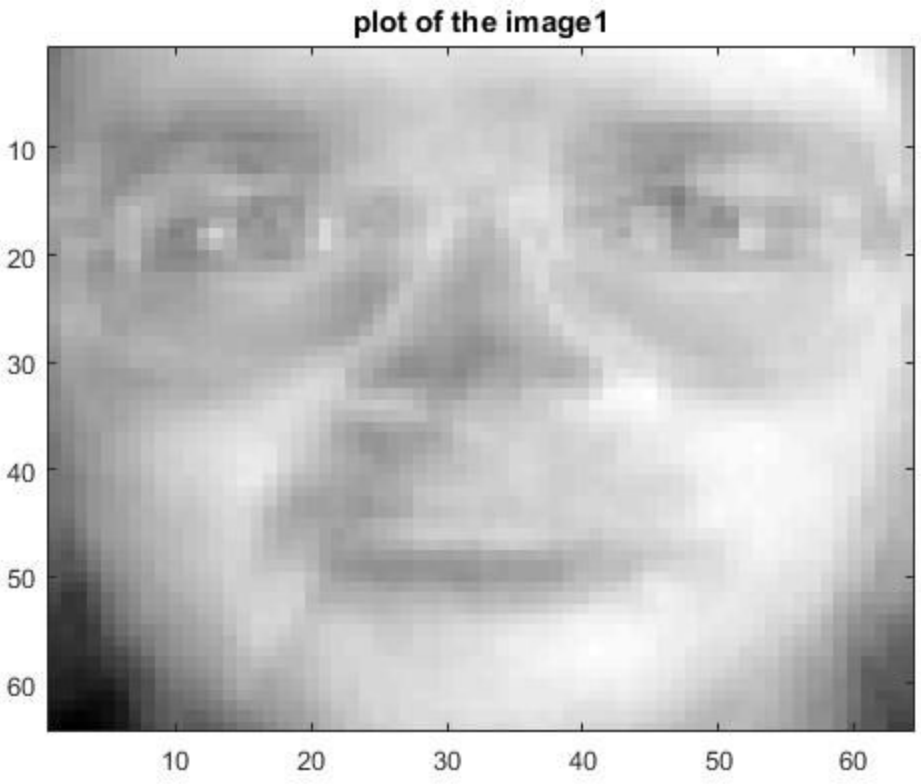
To minimize,

$$0 = - 2 P_{xx} H_R + G_R (2 P_{xx} H_{xx} + \sigma^2)$$

$$\Rightarrow G_R = \frac{P_{xx} H_R}{P_{xx} H_{xx} + \sigma^2}$$

Similarly, differentiating with respect to G_I, we get.

$$G_I = \frac{H_F^I P_{xx}}{P_{xx} P_{HH} + \sigma^2}$$


plot of the image1
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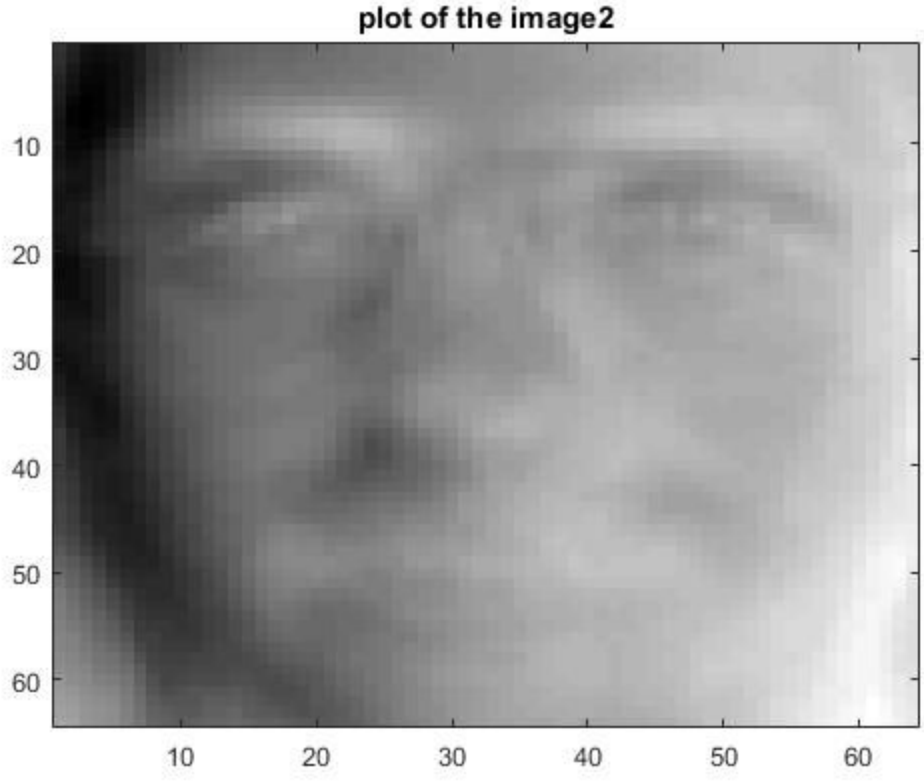
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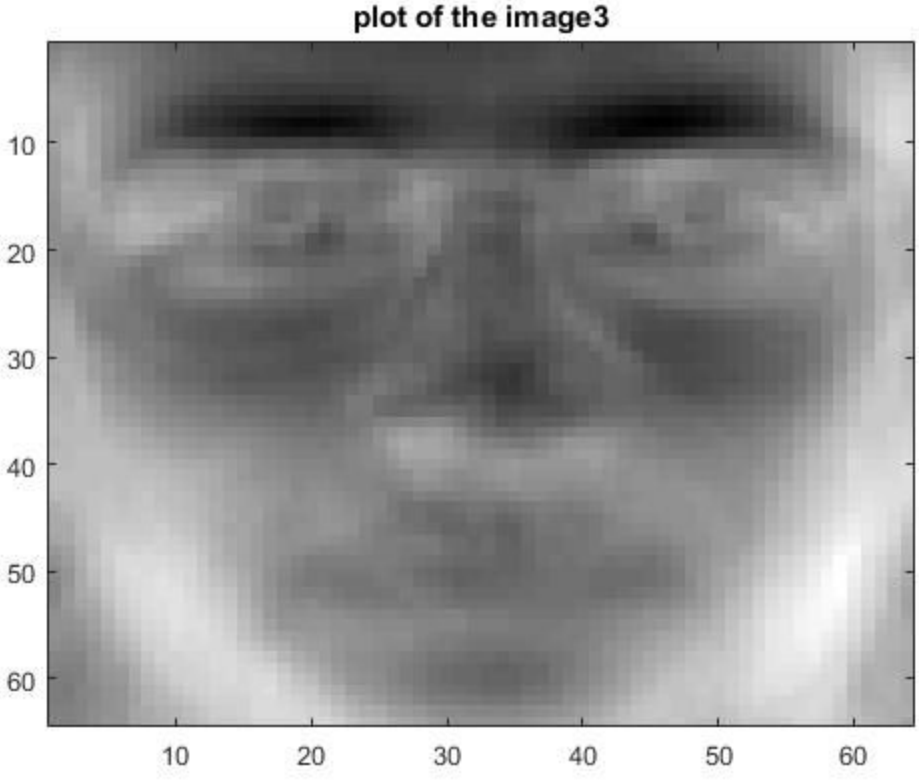
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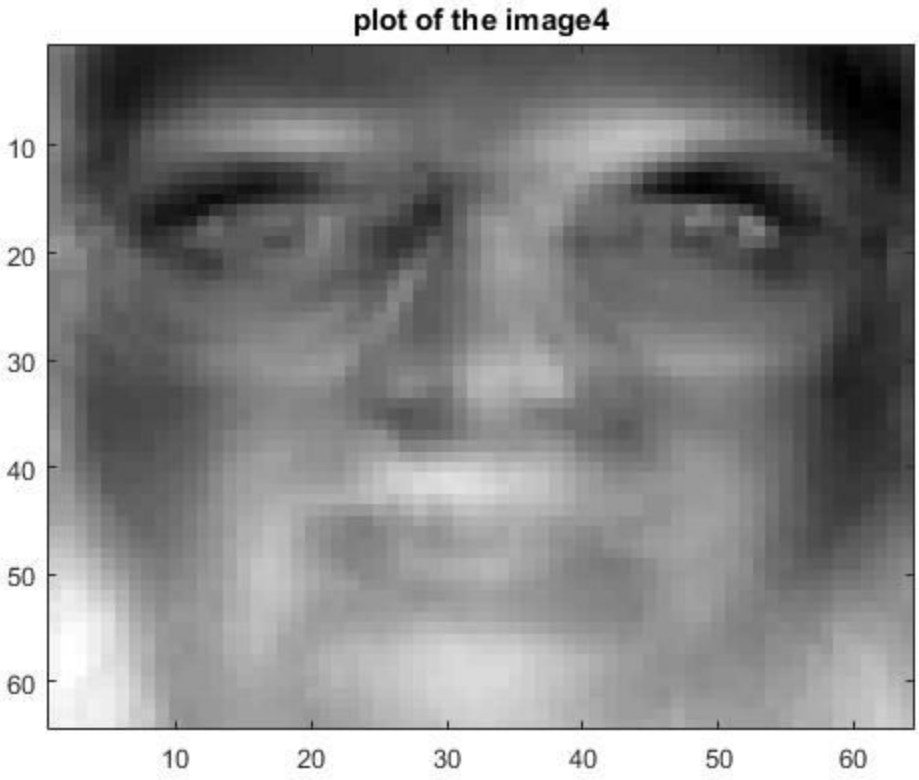
plot of the image4
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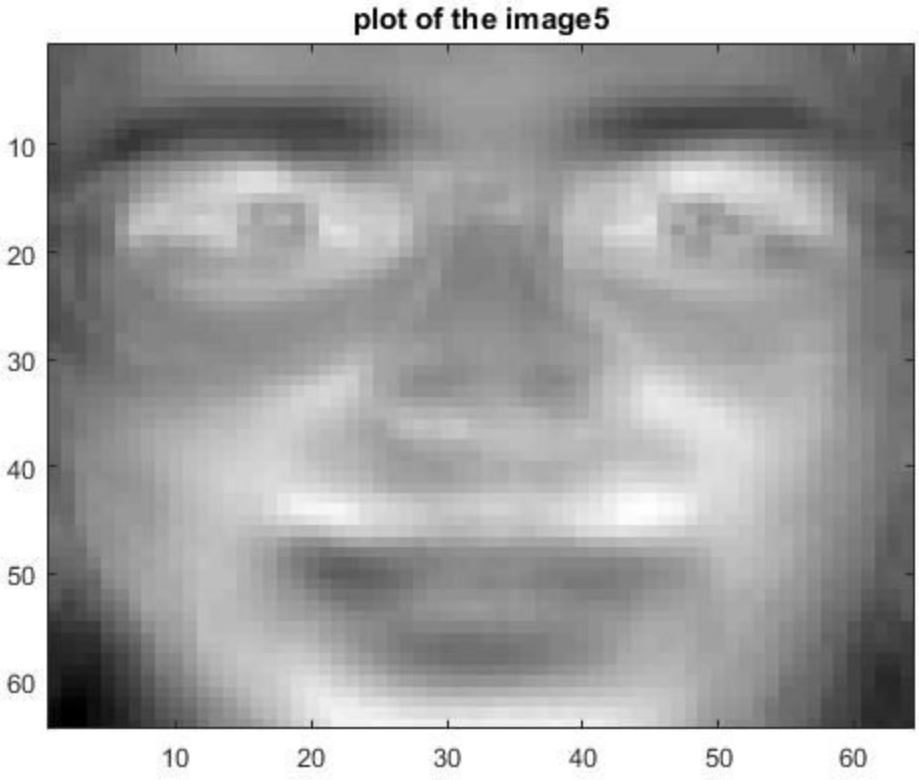


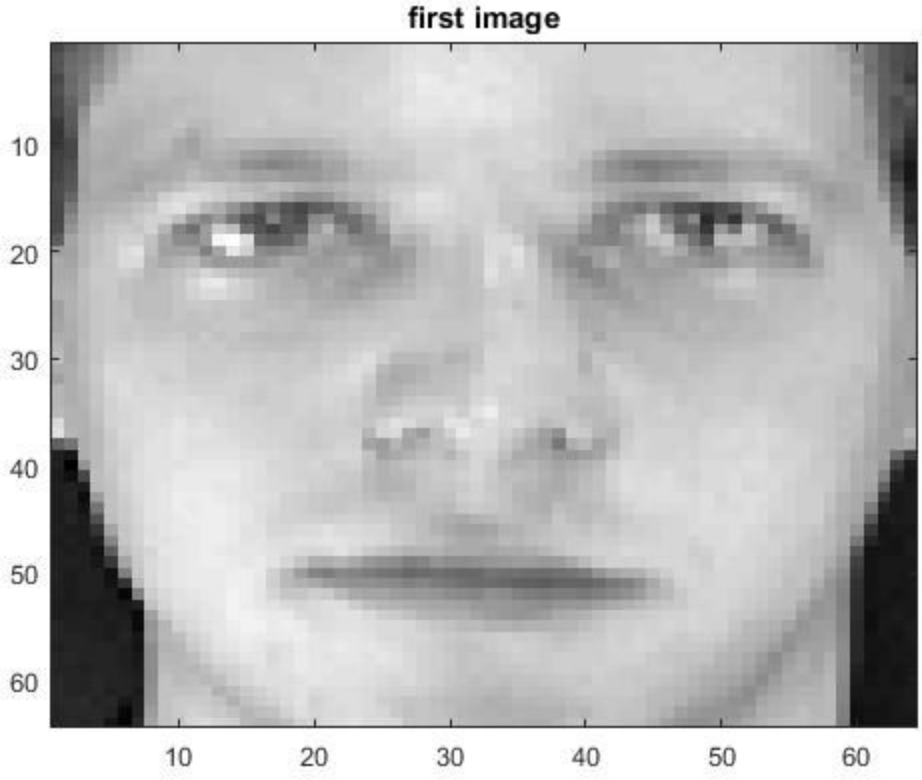
The coefficients for the first twenty basis used to reconstruct the first image are

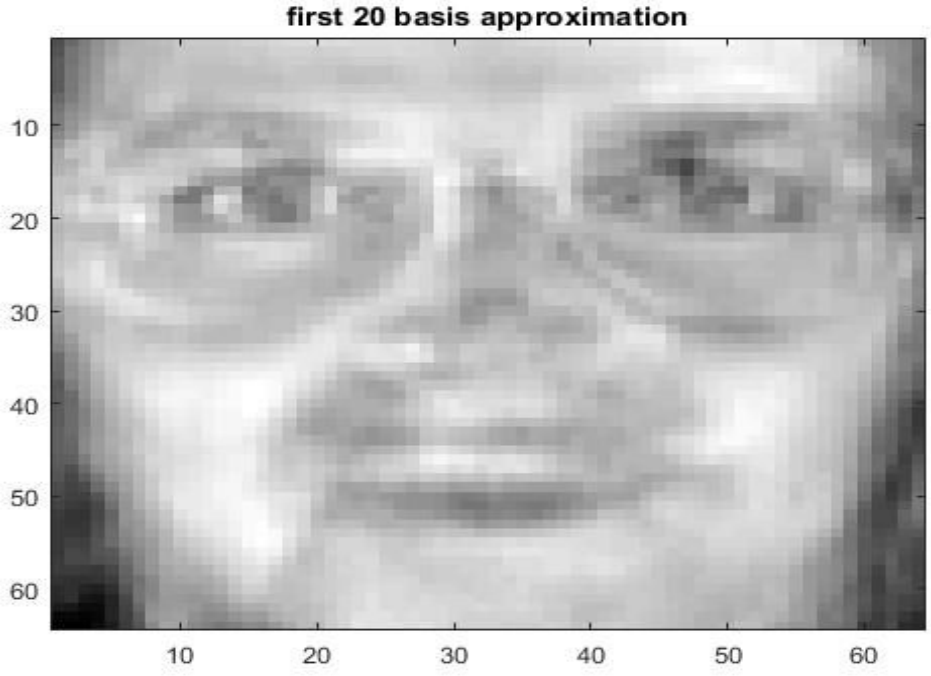
9.6838
1.8331
0.3110
0.8036
-0.6982
0.3768
-0.0488
0.4723
-0.6675
0.0867
0.6049
0.0992
0.0176
-0.3775
0.4008
-0.2817
0.3815
-0.6743
0.2079

-0.2005

Since we used PCA, the top twenty bases represents the bases that capture the dimensions or bases that capture the most variances in the data set. i.e the first basis captures the most variances seen in the dataset and in decending order of captures variances by second, third and so on.

The reconstruction of the first image is




first 20 basis approximation
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