Optimal and Naive Diversification in Currency Markets*

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Abstract

DeMiguel, Garlappi, and Uppal (*Review of Financial Studies*, 22 (2009), 1915–1953) showed that in the stock market, it is difficult for an optimized portfolio constructed using mean-variance analysis to outperform a simple equally-weighted portfolio because of estimation error. In this paper, we demonstrate that portfolio optimization can be made to work in currency markets. The key difference between the two settings is that in currency markets interest rates provide a predictor of future returns that is free of estimation error, which permits the application of mean-variance analysis. We show that over the last 26 years, a mean-variance efficient portfolio constructed in this fashion has a Sharpe ratio of 0.91, versus only 0.15 for the equally-weighted portfolio. We also consider the practical implementation of this strategy.

Keywords: Carry trade, currency, mean-variance analysis, portfolio optimization.

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1 Introduction

Financial markets are challenging environments for decision-making. Markowitz's invention of mean-variance analysis offered the hope of bringing rigor to this decision-making by supposing a simple trade-off between risk and return. One of the most high-profile applications of optimization in business, it provides a simple answer to the question of how to construct a diversified portfolio of risky assets. It has since become a standard part of the business school curriculum, and Markowitz himself was awarded the Nobel Prize in economics in 1989. Despite its prominence, the empirical verdict after more than 60 years of experience in using mean-variance analysis to choose stocks has been negative. A large literature (e.g. Black and Litterman (1992), Kan and Zhou (2007)) has identified and grappled with the practical problems of using mean-variance analysis. The central challenge is handling estimation error in the parameters of the model. The most definitive statement of this difficulty is by DeMiguel, Garlappi, and Uppal (2009), which shows that using estimated means and variances performs worse than a naive strategy of simply holding equal positions in every asset. So was the optimization approach to portfolio choice a blind alley?

We show that there is hope for mean-variance analysis yet. The key discovery of DeMiguel, Garlappi, and Uppal (2009) is that if means and variances are estimated, then the negative effect of the estimation error can swamp the positive effect of exploiting the additional information contained in means and variances. If we could identify an exogenous source of information on expected returns, we could use that to improve on the equally-weighted portfolio. We show that in a different market – the currency market – we can indeed identify such an exogenous source of information, and that this allows us to improve on the equally-weighted portfolio. In this market, the equally-weighted portfolio has a Sharpe ratio of 0.15, while the mean-variance optimal portfolio using this exogenous information can improve on this substantially to a Sharpe ratio of 0.91.

What is this exogenous source of information? Short-term interest rates vary across currencies. For example in September 2008, the (annualized) monthly (interbank) interest rate for Japanese yen was 0.5%, while for the Australian dollar was 7%. Investors can (and do) try to exploit these differences as an investment strategy. The classic example of such a strategy is the carry trade – borrowing in a low interest-rate currency and using the proceeds to lend in a high interest-rate currency. The strategy is not riskless, even if the underlying debt was risk-free, because each short-term bond pays off in a different currency, and the investors face the risk of exchange rates moving against them. This risk is far from theoretical – in the wake of the Lehman Brothers bankruptcy the yen appreciated 28% in October 2008 versus the Australian dollar, completely reversing any profits made from the 6.5% difference in interest rates over a year in less than a single month.

Returns on short-term bonds thus have two components: the interest rate component, which is known in advance (unless the bond issuer defaults), and the exchange rate component, which is not. To implement mean-variance analysis in this setting, we make a simple assumption: the direction of exchange rate changes are not predictable. This gives a simple model for the expected return, the interest rate, that contains no estimation error.

Can exchange rate changes be forecast? Classically, the *uncovered interest parity* (UIP) hypothesis suggested that not only could they be forecast, but that on average exchange rates move to cancel out any advantage investors might try to derive from a difference in interest rates. If this hypothesis were true, then any attempt to make a profit on interest rate spreads would be futile, and the expected return on every short-term bond would be the domestic risk-free rate. This would be devastating to any attempt to use mean-variance analysis in this market. This theory has not fared well when confronted with the data (Tryon (1979), Fama (1984), Korajczyk (1985)), and many variants on the carry trade seem to produce positive returns. Meese and Rogoff (1983) show that many economically-motivated predictors do not help in predicting exchange rates, and have trouble outperforming a simple random walk model. The finance literature has extensively studied potential explanations for the risk premium in currencies, which are potential sources for variables that can help predict exchange rate changes. See Lustig, Roussanov, and Verdelhan (2011), Verdelhan (2012), Jurek (2014), Jurek and Xu (2014) for some of these approaches. The overall question of exchange-rate predictability remains controversial (see Rossi (2013) for a survey[1]).

Even if exchange rates have a forecastable component, we would again be subject to the challenge of estimating the precise relationship, with its attendant estimation error, so that a mean-variance portfolio that exploits forecastability of exchange rates can perform worse than one that does not. As a simple example, we compare our results with the results from incorporating an estimation of the expected change in the exchange rate. We find that over our sample this lowers

[1] In the final paragraph of her survey, Rossi (2013, Conclusions) concludes: "Overall, although some predictors (Taylor-rule fundamentals and net foreign assets) do exhibit some predictive ability at short horizons, and others (monetary fundamentals, especially in panel models) reveal some predictive ability at long horizons, none of the predictors, models, or tests systematically find empirical support for superior exchange rate forecasting ability of a predictor for all models, countries and time periods: typically, when predictability appears, it does so occasionally for some countries and for short periods of time. Thus, Meese and Rogoff's finding does not seem to be entirely and convincingly overturned."

the Sharpe ratio by 0.08. (This tendency of estimation error to lower the performance was termed the "error-maximizing" property of mean-variance analysis by Michaud (1989). A theoretical analysis can be found in Kan and Zhou (2007).) Whether or not predictors of exchange rates can be estimated with enough accuracy so as to improve the performance of the mean-variance analysis over the pure interest-rate only model for expected returns is an interesting question for further research.

While we do not need to estimate expected returns statistically, we do need to estimate variances. Experience has shown (see Poon and Granger (2003) for a review, or Chopra and Ziemba (2011)) that variances can be forecast more accurately than expected returns, as long as the number of assets is not too large. For our headline results, we compute covariances over a simple 260-day rolling window. (More sophisticated models for the covariance matrix, such as DCC-GARCH, do not appreciably change the results.) We consider even simpler techniques for covariance estimation, with fewer parameters, but find that in general the results become slightly worse. If the covariance matrix can be estimated accurately, then there are three channels by which it can potentially improve the performance of the mean-variance portfolio. For each asset in isolation, the procedure takes into account the risk-return trade-off between assets. For example, over the sample period, the three lowest interest-rate currencies have been the yen, the Singapore dollar, and the Swiss franc. The yen generally has the lowest interest rate of the three, but both the franc and Singapore dollar have had lower exchange-rate volatility. This makes the latter currencies a potentially superior choice on a risk-adjusted basis. Secondly, the mean-variance strategy exploits the correlation between the assets. In our sample, the two highest interest-rate currencies have been the Australian and New Zealand dollars. They are both highly correlated, which makes them close substitutes. An optimal strategy would take this into account. Finally, the optimal portfolio can use the aggregate risk-return to choose the total exposure. There is considerable time variation in interest rate spreads. A target mean criterion, for example, will automatically decrease exposure when the spreads are wide, and increase it when the spreads narrow.

To separate out the determinants of the mean-variance portfolio's performance, we consider several variants of the strategy. To measure the gains from weighing the individual risk-reward trade-offs of each currency considered in isolation, we compute the optimum using a covariance matrix with the off-diagonal elements artificially set to zero. This approach, with a Sharpe ratio of 0.59, offers some improvement over the Long 5 Short 5 strategy but falls short of the mean-variance optimal portfolio. Mean-variance also exploits the cross-correlation across assets. We test for the importance of this property by using the correlation matrix alone. This approach has a high Sharpe ratio of 0.76, which suggests that the correlation between assets is particularly important. Monthly rebalancing permits us to profit from volatility timing, so we consider the consequences of using a constant covariance matrix. This alternative has a Sharpe ratio of 0.75.

The mean-variance strategy has another advantage over naive strategies. The Sharpe ratio expected by the model is critically dependent on the available interest-rate spreads, which are time-varying. For example, most interest rate spreads have vanished in recent years as central banks grapple with the fallout from the global financial crisis. The spreads give us an ex-ante expectation of when investing in currency markets is attractive. We find that these ex-ante expectations are borne out. When the mean-variance portfolio has a high expected Sharpe ratio according to the model, the ex-post Sharpe ratio is also high. For example, months in which the predicted Sharpe ratio exceeds 1, the realized Sharpe ratio is 1.55.

The mean-variance strategy has several other attractive properties. Christiansen, Ranaldo, and Soderlind (2011) analyze the time-varying systematic risk of the carry trade for a long 3 short 3 portfolio. They find that systematic risk of the carry trade increases when FX market volatility and TED spread are high, and to a smaller extent also when equity market volatility is high or FX market liquidity is low. (Clarida, Davis, and Pedersen (2009) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012) also consider the impact of FX market volatility.) In contrast, the mean-variance strategy has a low correlation with equity returns and innovations in the VIX. Also, the high Sharpe ratios are largely unaffected by the choice of domestic currency, so it is not an artifact of the US dollar's special role in the world economy. Brunnermeier, Nagel, and Pedersen (2009) observe that large spreads in interest rates are associated with the potential for large crashes in exchange rates. (Jurek (2014) and Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2009) consider this question further using option data.) We show that the mean-variance portfolio mitigates this problem.

The literature on the determinants of exchange rates and interest rate spreads is vast. We mention a few additional related papers. Baz, Breedon, Naik, and Peress (2001) considers the performance under mean-variance analysis for a small set of currencies in the 1990s. Della Corte, Sarno, and Tsiakas (2009) also consider mean-variance portfolios of four currencies (USD, JPY, EUR, GBP). We find that such a small set of currencies does not allow for much diversification, particularly after the 2008 financial market crisis when the spreads between the currencies have converged to zero. Barroso and Santa-Clara (2012) also considers optimized portfolios, but they pursue a much different optimization strategy. Investors optimize expected CRRA utility of returns. They use a sample of 27 currencies and many more predictors of returns than just interest-rate spreads such as three-month momentum, a predictor for long-term (5-year) reversal, real exchange rates, and current account figures. Daniel, Hodrick, and Lu (2014) report results on five different carry-trade strategies for the G10 currencies from 1976 until 2013 (except for AUD and NZD). Similar to our results, each of these strategies produces negatively skewed returns as well as Sharpe ratios exceeding those of the S&P 500.

2 Portfolio Construction

There are several financial instruments available to take advantage of the difference in interest rates across countries, such as foreign-currency denominated bonds, forward contracts, and swaps. But all other instruments can be synthesized in terms of bonds, so we explain how to construct the mean-variance portfolio in terms of bonds. The choice of instruments only matters when computing transaction costs. We consider the issue of implementation in Section 4.

We state everything explicitly for the sake of establishing notation. First we consider the expected return for a single foreign bond. Let S_t be the foreign currency exchange rate, quoted as the amount of domestic currency necessary to buy one unit of the foreign currency. (For example, if the exchange rate at time t is that 1 Euro equals 1.25 dollars and the dollar is the domestic currency, then $S_t = 1.25$, not 0.8.)

Let r_t be the interest rate when borrowing or lending in the foreign currency. Similarly, let r_t^d be the corresponding risk-free interest rate in the domestic currency. Suppose that the interest rate abroad is higher than the domestic rate, so the investor wishes to lend 1 unit of domestic currency abroad. This equals $1/S_t$ units of the foreign currency. At maturity, the investor receives $(1 + r_t)/S_t$ in the foreign currency, which when converted back to the domestic currency, is

$$\frac{S_{t+1}}{S_t}(1 + r_t).$$

At time t, the rates r_t^d and r_t as well as the exchange rate S_t are known, only the future exchange rate S_{t+1} is unknown. The expected (gross) payoff of the bond abroad in domestic currency is, therefore,

$$\frac{E_t(S_{t+1})}{S_t}(1 + r_t).$$

Under the assumption that the direction of exchange rate changes are not predictable, the expected exchange rate at time $t+1$ is just the exchange rate at time t, so $E(S_{t+1}) = S_t$ and the expected payoff would be

$$\frac{E(S_{t+1})}{S_t}(1 + r_t) = 1 + r_t, \tag{1}$$

resulting in an excess return of $r_t - r_t^d$. Under this special case, the main focus of our paper, returns do not have to be estimated.

The covariance of (gross) returns for investments in the foreign currencies i and j is

$$\text{Cov}\left[\frac{S_{t+1}^i}{S_t^i}(1+r_t^i), \frac{S_{t+1}^j}{S_t^j}(1+r_t^j)\right] = \text{Cov}\left[\frac{S_{t+1}^i}{S_t^i}, \frac{S_{t+1}^j}{S_t^j}\right](1+r_t^i)(1+r_t^j) \tag{2}$$

since the interest rates r_t^i are known ex-ante. Denoting the covariance matrix at time t by Σ_t, the portfolio variance is

$$(\sigma_t^p)^2 = w_t' \, \Sigma_t \, w_t,$$

where w_t denotes the vector of portfolio weights at time t.

Unlike the model for expected returns, there is no natural model for exchange-rate covariances, and thus they must be estimated. For our main results in Section 3.2 we re-estimate the covariance matrix each month, using a rolling window of one year's worth of daily exchange-rate movements. In Section 3.3 we consider some alternative methods.

The mean-variance portfolio optimization problem can be viewed as the choice of weights that maximize the Sharpe ratio,

$$\max_{w_t} \frac{w_t'\left(r_t - r_t^d \iota\right)}{\sqrt{w_t' \, \Sigma_t \, w_t}},$$

where $\iota = (1, 1, \ldots, 1)'$ denotes the vector of all ones.

This problem has a unique solution up to a scale factor in the weights of the risky asset. Generally this scale factor is pinned down by an auxiliary restriction, such as targeting a specific expected return or variance, or requiring the weight in the risk-free asset to be 0. Realistically, a trading desk implementing the strategy would want to limit their exposure to each currency, rather than targeting an expected return or variance. In line with our goal to accurately reflect industry practice, we scale the weights so that the gross exposure (the sum of the weights in long and short positions) is exactly 1.

From the point of view of a counterparty such as a bank, the excess return on the portfolio can be achieved entirely in terms of forwards, which require collateral up-front (which we assume is invested in the domestic risk-free asset). From this point of view, the gross exposure corresponds to a margin of 100%. This is a much more conservative margin requirement than is found in industry. For example, as of January 1, 2015, the foreign exchange broker FXCM requires as little as 2% margin[2] We discuss implementation in terms of forwards in Section 4.

[2]The current margin requirements can be found at `https://www.fxcm.com/forex/margin-requirements/`. Last accessed January 26, 2016.

3 Empirical Results

In this section we report the empirical results of our mean-variance analysis. We first describe the data and provide some summary statistics on the currencies. Then we compare the mean-variance optimal portfolio with simple naive rules, and show that in the currency market the optimal portfolio outperforms naive strategies (by a large margin the case of the equally-weighted portfolio). Subsequently we report results for alternative estimation strategies, and then finally we consider the performance of the mean-variance portfolio more closely.

3.1 Data

The availability of data has gradually increased since the end of the Bretton Woods era, as floating exchange rates have become increasingly standard. For our main results, we consider the period for which Bloomberg has forward rate data, beginning in January 1989. As we require one year data to estimate the covariance matrix, the first return can be calculated for January 1990. We prefer forward rate data, because it represents the most liquid market rate at which banks and financial institutions conduct foreign interest rate transactions. We also consider a smaller universe of currencies beginning in 1976 when exchange rates first began to float, and a larger universe starting in August 2003. One challenge in all samples is the introduction of the Euro in January 1999, which removed several major currencies from circulation. In order to keep the number of currencies fixed, we use the Deutsche Mark as a proxy for the Euro.

For the main sample, the currency universe is the US dollar (USD), Swiss franc (CHF), Euro (EUR), Japanese yen (JPY), British pound (GBP), Australian dollar (AUD), Canadian dollar (CAD), Norwegian krone (NOK), Swedish krona (SEK), Singapore dollar (SGD), and New Zealand dollar (NZD).[3] For the smaller universe, we extend the data to 1976 using data from Datastream, but we are forced to omit the Japanese yen, as well as the Australian, New Zealand, and Singapore dollars. For the larger universe, we add the Hungarian forint (HUF), the Polish zloty (PLN), Czech koruna (CZK), South African rand (ZAR), the Turkish lira (TRY), Israeli shekel (ILS), Mexican peso (MXN), Icelandic krona (ISK), Thai baht (THB) and Russian ruble (RUB), which are less liquid but also freely tradeable since 2003.

[3] We initially included the Danish krone, but this actually made our results too good, since the krone is pegged to the Euro, but interest rates do not match perfectly. This makes the krone a terrific hedge for the euro, but at the same time causes numerical problems for mean-variance analysis since the covariance matrix becomes almost singular. We elected to drop the krone to solve the problem. Likewise for much of the sample the Hong Kong dollar is pegged to the US dollar.

Table 1 reports summary statistics for the 11 currencies in the main data set, sorted by average interest rate. Average interest rates range from 1.030% for the JPY to 6.222% for the NZD. In addition, the table reports the mean, standard deviation, skewness, and kurtosis of the exchange rate changes of each currency versus the USD. For each currency, a large component of the return

Table 1: Currency Summary Statistics

	IR	FX Change				Total
	Mean	Mean	SD	Skewness	Kurtosis	Return
JPY	1.030	0.692	10.810	0.658	6.411	1.722
CHF	2.086	1.678	11.200	0.082	4.121	3.764
SGD	2.591	1.116	5.644	-0.403	5.901	3.706
USD	3.332	0	0	0	0	3.332
EUR	3.472	-0.244	10.405	-0.221	3.873	3.228
CAD	3.863	-0.680	7.912	-0.357	7.280	3.183
SEK	4.534	-1.183	11.580	-0.364	4.503	3.351
GBP	4.868	-0.346	9.219	-0.617	5.696	4.522
NOK	4.983	-1.121	10.863	-0.266	3.778	3.862
AUD	5.710	-0.304	11.365	-0.319	4.875	5.406
NZD	6.222	0.534	11.732	-0.203	5.258	6.756

This table presents summary statistics for the 11 currencies in our sample from January 1990 to December 2015. IR is the annualized interest rate implied by one-month forward rates using covered interest parity. FX Change is annualized change in the exchange rate versus the US dollar. (A positive value means the currency appreciated on average.) Total Return is the return in US dollars. The mean and standard deviation are annualized. Skewness and kurtosis are monthly figures. Currencies are sorted by average interest rate. Interest rates and mean and standard deviation of FX change are in percent.

from a one-month bond is given by the interest rate return. For example, the yen has the lowest average interest rate, and the lowest average total return, while the New Zealand dollar had the highest total return, and the highest interest rate return. The interest rate does provide a proxy, albeit imperfect, for the realized return.

The relationship between interest rates and skewness observed by Brunnermeier, Nagel, and Pedersen (2009) is readily apparent – lower interest-rate currencies have more positively skewed changes in the exchange rates. For example, historically the yen is more likely to sharply appreciate than sharply depreciate, while the opposite is true for the New Zealand dollar. A strategy that is just long NZD and short JPY will have negatively skewed returns.

Table 2: Forecasting Error in Expected Returns

	Interest-only	FX Trend	% Increase
CHF	0.0323	0.0336	4.2089
EUR	0.0300	0.0313	4.4397
JPY	0.0312	0.0319	2.2861
GBP	0.0266	0.0281	5.5388
AUD	0.0330	0.0344	4.5329
CAD	0.0228	0.0235	2.7030
NOK	0.0314	0.0328	4.6263
SEK	0.0334	0.0350	4.6103
SGD	0.0164	0.0169	3.0884
NZD	0.0340	0.0354	4.1740

This table compares the forecasting error for returns one month ahead, as measured by root mean squared error (RMSE). The column "Interest-only" reports the RMSE for the forecasted return using only the current interest rates, while the column "FX Trend" reports the RMSE when using a 260-day rolling window to estimate the average change in exchange rates. The rightmost column "% Increase" shows the percentage increase in RMSE by including the trend.

Table 2 provides a small example of the perils of estimation error on forecasting. The summary data in Table 1 suggests that there is a small trend in exchange rates over the 26-year sample period. This observation suggests that estimating a trend will improve our ability to forecast expected returns. In sample, adding an extra variable always improves the fit, but as the third column illustrates, including the trend estimation increases the forecast error by about 4% on average.

3.2 Optimal Versus Naive Rules

For the calculation of the optimal portfolio each month, we compute the expected excess return for each currency under the random walk hypothesis, which is just the difference between the currency's interest rate and the U.S. risk-free rate (see equation (1)). We use equation (2) to compute the entries in the covariance matrix for each currency. For each month, we re-estimate the daily covariance matrix by using the last 260 observations, which we then convert to monthly figures by scaling by 22.[4] Each month we rebalance the portfolio.

We consider a range of naive currency trading strategies to compare with the optimal portfolio. The equally-weighted portfolio of short-term bonds is the natural analogue of the equally-weighted stock portfolio of DeMiguel, Garlappi, and Uppal (2009). We also consider simple carry trade strategies, long-short strategies where investors borrow in the low interest-rate currencies, and lend in the high interest-rate currencies. "Long 1 Short 1" denotes the simplest carry trade with the highest expected return, borrowing in the lowest and lending in the highest interest rate currency. "Long 5 Short 5" denotes a equally weighted and diversified form of the carry trade with 5 long and 5 short positions, while the weight of USD is 1. Table 3 reports the results.

Table 3: Full Sample Performance Comparison

	Equally-Weighted	Long 1 Short 1	Long 5 Short 5	Mean-Variance	S&P 500
Total gross (in %)	3.91	5.85	5.39	5.05	9.99
Total excess (in %)	0.57	2.51	2.05	1.70	6.65
Volatility (in %)	3.79	6.72	3.32	1.87	14.60
Sharpe ratio	0.15	0.37	0.62	0.91	0.46
Adj. Sharpe ratio	0.15	0.34	0.59	0.79	0.43
Skewness	-0.19	-1.22	-0.41	-0.55	-0.59
Kurtosis	1.01	4.01	0.54	1.53	1.21

The table shows the performance of five different investment strategies from January 1990 to December 2015. "Equally-Weighted" is an investment with identical positions in all foreign currencies. "Long 1 Short 1" is an investment that is long 0.5 in the highest interest in our sample of currencies and short -0.5 in the lowest interest rate. "Long 5 Short 5" is an investment that is long in the 5 highest interest rate currencies with a weight of 1/10, and short in the 5 lowest interest rate currencies with a weight of -1/10. "Mean-Variance" is the mean-variance portfolio described in this paper with the maximum Sharpe ratio. The first five rows report annualized figures.

The Sharpe ratio of the equally-weighted portfolio is the lowest. The Long 1 Short 1 strategy has a Sharpe ratio of 0.37 over the 1990–2015 period, comparable to the S&P 500, but with noticeably larger skewness. The Long 5 Short 5 strategy improves the Sharpe ratio to 0.62, which

[4] We also tried more complicated procedures to estimate the covariance, such as multivariate GARCH models, but these did not produce substantially better results, so we report the results from the simplest procedure. We also investigated the possibility of using option-implied volatilities, but there was insufficient data for our sample of currencies.

shows that even a mechanical diversification strategy can improve on the very simple strategies. The mean-variance strategy outperforms all three strategies by a considerably margin, with a Sharpe ratio of 0.91.

The large skewness is consistent with the pattern observed in Table 1 and Brunnermeier, Nagel, and Pedersen (2009) that lower interest-rate currencies have more positively skewed changes in the exchange rate. A long-short strategy based on interest rates would then be expected to be negatively skewed. Interestingly, the pattern is much less pronounced for the more-diversified strategies.

We test for the presence of statistically significant differences in Sharpe ratios between strategies. We use the robust Sharpe ratio test of Ledoit and Wolf (2008), which is a bootstrapped p-value that accounts for non-normally distributed returns and time-series autocorrelation. The test sharply rejects the hypothesis that the equally-weighted portfolio and the optimal portfolio have equal Sharpe ratios. Furthermore, the mean-variance strategy has significantly higher Sharpe ratios than the Long 1 Short 1 strategy at the 5% significance level (p-value 0.023), while the p-value for the comparison with the Long 5 Short 5 strategy is 0.187. The mean-variance strategy has a higher Sharpe ratio than the S&P 500 (p-value 0.164), while the Sharpe ratios of the Long 1 Short 1 and Long 5 Short 5 are not clearly significantly different (p-values of 0.765 and 0.541, respectively) from the Sharpe ratio of the S&P 500.

We also report the adjusted Sharpe ratio (Pezier and White (2006)), which modifies the Sharpe ratio to account for skewness and kurtosis deviating from normal distribution,

$$\begin{aligned}\text{Adj. Sharpe Ratio} &= \text{Sharpe Ratio} \times \\ &\quad (1 + \frac{\text{Skewness}}{6} \times \text{Sharpe Ratio} - \frac{\text{Kurtosis}}{24} \times \text{Sharpe Ratio}^2).\end{aligned}$$

The formula derives from the Taylor series approximation to exponential utility. The impact of this modification is relatively modest for our results.

3.3 Alternative Optimization Procedures

Since the total return on a one-period foreign bond consists of the interest-rate return (which is known) and the return from appreciation or depreciation of the foreign currency, we can estimate next period returns by estimating any trend in currency changes. In theory, this may improve the forecast of expected returns. We use a 260-day window to estimate expected exchange rate changes, and compute the associated mean-variance portfolios. Table 4 reports the results. Note that the Sharpe ratio is lower. This result is in line with the measurement of forecasting error for

Table 4: Results for Forecasting FX Trend

	Mean-Variance
Total gross (in %)	5.16
Total excess (in %)	1.82
Volatility (in %)	2.19
Sharpe ratio	0.83

The table describes the performance of the mean-variance strategy when estimating expected return as a combination of the interest rate and the estimated change in exchange rates (using the average change over the previous 260 days) from January 1990 to December 2015. "Mean-Variance" is the mean-variance portfolio described in this paper with the maximum Sharpe ratio. All figures are annualized.

each currency in Table 2. Forecasting exchange rate changes introduces estimation error, which worsens the performance of the mean-variance portfolio. This result is in line with the findings of DeMiguel, Garlappi, and Uppal (2009) for the stock market, but the robust Sharpe ratio test is unable to reject the possibility that the two portfolios have the same Sharpe ratio (see Table 5).

We also consider the impact of different strategies for estimating the covariance matrix. In theory, mean-variance analysis exploits both the individual asset variances as well as the covariances between the assets. To gain insight into the importance of each component, we consider a variant of the strategy where the off-diagonal elements of the covariance matrix are set to zero. Exchange rates with high volatilities will receive lower weights than they would with a naive diversification strategy. The correlation between currencies is often positive, so the individual variances will tend to understate the true risk of the portfolio. On the positive side, the strategy is not exposed to correlation estimation errors. We refer to this strategy as 'No Correlation'. Using only individual variances does not lead to an improvement in Sharpe ratio compared to the Long 5 Short 5 portfolio – the Sharpe ratio slightly decreases from 0.62 to 0.59, as both return and volatility decrease. Much better results can be achieved by using the correlation matrix, which results in a Sharpe ratio of 0.76, but still worse than the dynamic mean-variance strategy (Sharpe ratio 0.91). This improvement suggests that exploiting the cross-correlation between assets is more important than the individual variances.

We finally compute the mean-variance strategy with a constant covariance matrix, which uses the average covariance over the entire sample period. We are aware that this is a non-investable strategy, as the sample covariance matrix was unknown during the sample period. Nevertheless this exercise has an interesting feature as it shows the effect of the volatility timing: the Sharpe ratio is 0.75, which suggests that volatility timing and the ability to diversify are of roughly equal importance. Figure 1 displays the excess returns and volatilities for different currency strategies, including the different covariance estimation strategies.

[FIGURE 1 ABOUT HERE]

We report the results of the robust Sharpe ratio test for each of these portfolios in Table 5. The test is unable to reject the hypothesis that the Sharpe ratios of each of these portfolios are identical with our optimal portfolio, but it is able to reject that they are identical with the equally-weighted portfolio.

Table 5: Results of Robust Sharpe Ratio Test

	Sharpe Ratio	p-value of Difference Vs.	
		Main Portfolio	Equally-Weighted
Estimated FX trend	0.83	0.714	0.037
No Correlation	0.59	0.096	0.050
Correlation matrix	0.76	0.185	0.026
Sample Covariance	0.75	0.253	0.031

The table reports p-values for the Sharpe ratio test of four alternative estimation strategies for mean-variance analysis. The strategy "Estimated FX Trend" uses both the interest rate and the estimated trend in exchange rate changes as the expected return. The strategy "No Correlation" uses a covariance-matrix where the off-diagonal elements are zero. The strategy "Correlation matrix" uses the correlation matrix instead of the covariance matrix. Finally, the strategy "Sample Covariance" employs the constant ex-post covariance matrix computed over the whole sample. All portfolios use the 11-currency portfolio from January 1990 to December 2015.

There is considerable time variation in the ex-ante desirability of the carry trade, in that spreads narrow and widen over time. For example, the average spread between interest rates (as measured from the absolute value of the difference from the mean) was 1.86% from 1990–2008 and 1.03% from 2009–2015. This large change should lead to time variation in the Sharpe ratios. We investigate the consequences of variation in the ex-ante Sharpe ratio for the profitability of the strategy. Table 6 compares the results for the mean-variance strategy for different minimum levels of the expected Sharpe ratio. High ex-ante expected Sharpe ratio periods successfully predict high realized Sharpe ratios ex-post. When restricted to the subsample of expected Sharpe ratio at least 1, the actual realized Sharpe ratio is a lofty 1.55. In contrast, the Sharpe ratio for the months with expected Sharpe ratio below 0.8 in the sample is 0.29. Thus, the expected Sharpe ratio provides some guidance to the actual realized Sharpe ratio.

Table 6: Results with Expected Sharpe Ratio Constraints

Expected Sharpe Ratio	< 0.8	>= 0.8	>= 1
Total excess (in %)	0.64	2.31	2.53
Volatility (in %)	2.22	1.64	1.64
Sharpe ratio	0.29	1.41	1.55
Skewness	-0.29	-0.65	-0.86
Kurtosis	0.38	2.69	3.89
Number of Months	110	202	155

The performance of mean-variance carry trade in subperiods of January 1990 to December 2015, divided by expected Sharpe ratio ex ante. (The expected Sharpe ratio is computed via the model.) For example, ">= 0.8" means the expected Sharpe ratio must be at least 0.8 for the subsequent month to be considered as part of the subperiod. All figures in the first four rows are annualized.

Finally, we consider some smaller samples of currencies that previously appeared in the literature. Table 7 shows the results for the mean-variance strategy for the 5-currency universe of Baz, Breedon, Naik, and Peress (2001) and the 4-currency universe of Della Corte, Sarno, and Tsiakas (2009) from January 1990 to December 2015. Obviously, the Sharpe ratio of the strategy

Table 7: Results for Small Currency Portfolios

	Mean-Variance 5 Currencies	Mean-Variance 4 Currencies
Total gross (in %)	4.70	5.67
Total excess (in %)	1.36	2.33
Volatility (in %)	3.24	4.44
Sharpe ratio	0.42	0.52

The performance of mean-variance carry trade for the 5-currency portfolio (USD, CHF, JPY, EUR, GBP) from January 1990 to December 2015 as in Baz, Breedon, Naik, and Peress (2001) and for the 4-currency portfolio (USD, JPY, EUR, GBP) as in Della Corte, Sarno, and Tsiakas (2009). "Mean-Variance" is the mean-variance portfolio described in this paper. All figures are annualized.

of Baz, Breedon, Naik, and Peress (2001) and Della Corte, Sarno, and Tsiakas (2009) is much lower compared to the results of our strategy in this paper. (The Sharpe ratio for the 5-currency portfolio is considerably lower than for the period measured by Baz, Breedon, Naik, and Peress (2001).) When compared to the time period of the 21-currency portfolio, 2004–2015, the difference is even starker: the excess return (and therefore Sharpe ratio) on both portfolios is negative. This result is driven by the convergence of the interest rates on USD, CHF, JPY, EUR and GBP to (essentially) zero in the aftermath of 2008.

3.4 Performance Analysis

Our preliminary analysis suggests that our model-driven mean-variance analysis can outperform more naive strategies. In this section we consider the performance of mean-variance portfolio more carefully.

We consider two alternative samples. Datastream has currency data available since 1977, but to use it we must drop the Japanese yen as well as the Australian, New Zealand, and Singapore dollars. Note that all four are important carry trade currencies: the yen and Singapore dollar are usually low interest-rate currencies, while the Australian and New Zealand dollars are high interest-rate currencies. Bloomberg has additional forward data available for developing nations, but for shorter time horizons. As of August 2004, 21 currencies are available. We also report the results for this wider universe of currencies. Table 8 reports the performance statistics of the optimal mean-variance strategy for both alternate samples, as well as the performance of the original 11-currency portfolio for the shorter time period. The smaller currency universe suffers

Table 8: Results for Alternative Samples

	7 Currencies 1977–2015	11 Currencies 2004–2015	21 Currencies 2004–2015
Total gross (in %)	6.77	2.77	3.40
Total excess (in %)	1.27	0.87	1.50
Volatility (in %)	2.61	2.06	2.18
Sharpe ratio	0.49	0.42	0.69

The table shows the performance of two alternative samples for mean-variance optimal currency portfolios. The column "7 Currencies" reports results for the 7-currency portfolio (USD, CAD, GBP, EUR, CHF, NOK, SEK) from January 1977 to December 2015. The column "21 Currencies" shows figures for the 21-currency portfolio (which includes HUF, PLN, CZK, ZAR, TRY, ILS, MXN, ISK, THB and RUB) from August 2004 to December 2015. The column "11 Currencies" shows the results for the original 11 currencies, again from August 2004 to December 2015. All figures are annualized.

from the absence of two of the lowest and two of the highest interest rate currencies, so unsurprisingly it has a lower Sharpe ratio, but it is still substantially larger than the equally-weighted portfolio. Over the shorter time period, the 11 currency portfolio performs worse than it does for the full sample (we discuss this point further below), but the 21 currency portfolio outperforms it. Together, this suggests that the mean-variance portfolio can take practical advantage of diversification opportunities.

Figure 2 shows the time series of monthly excess returns and Figure 3 the cumulated excess return of the mean-variance carry trade strategy for the 11 currency portfolio. In a significant majority of months, the excess returns are positive. There is a large increase in the variance around the financial crisis period.

[FIGURE 2 ABOUT HERE] [FIGURE 3 ABOUT HERE]

We next take a closer look at the returns during the period from January 2007 to December 2009 to address the turmoil on world financial markets. Table 9 reports the performance statistics of the same four portfolios during the crisis. The stock market crashed over this period, as did

Table 9: Performance Comparison During the Financial Crisis

	Equally-Weighted	Long 1 Short 1	Long 5 Short 5	Mean-Variance	S&P 500
Total gross (in %)	4.59	-0.59	3.51	3.36	-3.78
Total excess (in %)	1.75	-3.43	0.68	0.53	-6.62
Volatility (in %)	5.24	10.34	3.95	2.16	20.02
Sharpe ratio	0.33	-0.33	0.17	0.24	-0.33
Adj. Sharpe ratio	0.32	-0.35	0.17	0.25	-0.34
Skewness	-0.39	-1.28	-0.39	0.50	-0.71
Kurtosis	1.11	2.20	1.22	-0.53	0.70

The performance of five different investment strategies from January 2007 to December 2009. "Equally-Weighted" is an investment with identical positions in all foreign currencies. "Long 1 Short 1" is an investment strategy that is long 0.5 in the highest interest in our sample of currencies and short -0.5 in the lowest interest rate. "Long 5 Short 5" is an investment that is long in the 5 highest interest rate currencies with a weight of 1/10, and short in the 5 lowest interest rate currencies with a weight of -1/10. "'Mean-Variance" is the mean-variance portfolio described in this paper with the maximum Sharpe ratio. The first five rows are annualized figures.

the simple Long 1 Short 1 strategy. In contrast, the Long 5 Short 5 and mean-variance strategies still had positive returns, though not as pronounced. The equally weighted strategy also had positive return and a surprisingly high Sharpe ratio due to the extensive depreciation of the US Dollar against several currencies during the crisis. The return on the 21-currency portfolio was even higher, with a Sharpe ratio of 0.39.

Figure 4 shows the time series of expected means and volatilities for the mean-variance portfolio. In the wake of the Lehman crisis, a small increase in volatility and a large decrease in expected returns is clearly visible. The decline in expected returns is driven by developed world central banks, almost all of which have forced interest rates towards zero, and interest rate spreads with them. Interestingly, the effect is not nearly as significant for the 21-currency portfolio, which has a Sharpe ratio of 0.57 since 2009.

[FIGURE 4 ABOUT HERE]

Next we provide additional results on the whole sample period 1990–2015. Table 10 reports the mean and standard deviation of the weights for each currency over the whole sample period.

Table 10: Mean and Standard Deviation of the Monthly Currency Weights

	Mean	Std. Dev
JPY	-0.036	0.049
CHF	-0.114	0.090
SGD	-0.113	0.159
USD	1.040	0.159
EUR	0.015	0.122
CAD	-0.015	0.093
SEK	0.000	0.088
NOK	0.076	0.121
GBP	0.042	0.072
AUD	0.051	0.094
NZD	0.054	0.064

Description of the monthly average currency weights and their standard deviations of the Mean-Variance carry trade strategy. Negative weights indicate short positions. High standard deviation indicates that the month-to-month change of the weights is high. The risk free allocation is weight 1 in USD and weight 0 in the other currencies, as USD is the domestic currency.

In addition, the graphs in Figures 5–7 show the monthly portfolio weights for all eleven currencies over the entire sample period. We observe that the U.S. dollar is usually the largest position in the portfolio, and frequently above 1. The USD weight falls below 1 during times of below-average USD interest rates such as after the internet bubble burst as well as during the recent recession (the "Great Recession"). The standard deviations of the portfolio weights in

Table 10 as well as the graphs in Figures 5–7 show that there is substantial portfolio rebalancing from month to month. The two currencies with the largest average short positions have been the CHF and SGD. Most of the time the portfolio has had long positions in the USD, EUR, AUD, and NZD. Interestingly, and perhaps contrary to conventional wisdom, most of the time the mean-variance portfolio puts only a modestly negative weight on the JPY.

[FIGURES 5–7 ABOUT HERE]

The graphs in Figures 5–7 clearly portray the turmoil in currency markets during and after the financial crisis. The optimal portfolio reacts strongly to changes in interest rates as well as correlations between currencies. For example, note that the desirability of the AUD as a large long currency position is a rather recent phenomenon. Unsurprisingly, currency markets saw a large increase in the volatility of exchange rates during the financial crisis.

Table 11 compares the results of the mean-variance strategy with the S&P 500, the US 10 Year Treasury, the GSCI Commodity Index and a balanced portfolio of 50% S&P 500 and 50% the US 10 Year Treasury. Of the five different strategies, the mean-variance currency portfolio

Table 11: Comparison with Other Asset Classes

	Mean-Variance	S&P 500	US 10 Year Treasury	GSCI Commodity	Balanced Portfolio
Total gross (in %)	5.05	9.99	6.25	2.85	8.12
Total excess (in %)	1.70	6.65	2.91	-0.49	4.78
Volatility (in %)	1.87	14.60	7.16	21.41	7.82
Sharpe ratio	0.91	0.46	0.41	-0.02	0.61

The performance of mean-variance carry trade strategy in comparison to other asset classes over the entire sample period, January 1990 to December 2015. All indices are total return indices including dividends or coupons. The balanced portfolio is an equally-weighted mix of the S&P 500 and the US 10 Year Treasury Index. All figures are annualized.

has by far the highest Sharpe ratio (0.91), double of that of the S&P 500 (0.46), and a clearly higher Sharpe ratio than for the balanced bond and equity strategy (0.61).

Next, Table 12 shows the correlation of carry trade investment strategies with other assets and risk measures. The mean-variance strategy has a low correlation with other asset classes. This is a surprising contrast to both the Long 1 Short 1 and Long 5 Short 5 strategies, which both have larger correlations with the other asset classes. There is little correlation with the innovations in the TED spread, which was an effective proxy for systematic risk during the banking crisis.

Table 12: Correlation with Other Assets

	Equally-Weighted	Long 1 Short 1	Long 5 Short 5	Mean-Variance
S&P 500	0.31 (6.07)	0.20 (3.59)	0.34 (6.82)	0.13 (2.24)
US 10 YR Treasury	0.09 (1.52)	-0.02 (-0.32)	-0.06 (-1.03)	-0.01 (-0.17)
GSCI Commodity	0.43 (9.34)	0.19 (3.51)	0.25 (4.75)	0.02 (0.32)
Balanced Portfolio	0.33 (6.52)	0.17 (3.18)	0.29 (5.63)	0.11 (2.01)
Δ VIX	-0.29 (-5.51)	-0.20 (-3.72)	-0.33 (-6.59)	-0.15 (-2.71)
Δ FX Volatility	-0.23 (-4.24)	-0.39 (-8.09)	-0.42 (-8.88)	-0.23 (-4.30)
Δ TED Spread	-0.08 (-1.50)	-0.02 (-0.33)	-0.15 (-2.80)	-0.04 (-0.62)

Report on the correlation of four different currency investment strategies with other risk factors over the entire sample period and t-values in brackets, January 1990 to December 2015. "Equally-Weighted" is an investment with identical positions in all foreign currencies. "Long 1 Short 1" is an investment that is long 0.5 in the highest interest in our sample of currencies and short -0.5 in the lowest interest rate. "Long 5 Short 5" is an investment that is long in the 5 highest interest rate currencies with a weight of 1/10, and short in the 5 lowest interest rate currencies with a weight of -1/10. "Mean-Variance" is the mean-variance portfolio described in this paper with the maximum Sharpe ratio. The VIX Index is the S&P 500 option implied volatility index. The FX Volatility is measured with the G7 option implied currency volatility index by JP Morgan. The TED spread is the difference between the 3 month LIBOR and the 3 month treasury bill and is an indicator for credit risk and interbank market liquidity.

(Interestingly, while the "Long 1 Short 1" strategy has a low correlation with innovations in the TED spread, the "Long 5 Short 5" strategy, which in theory is more diversified, has a higher correlation.) All three strategies are negatively correlated with innovations in systematic FX volatility risk, or volatility risk in general (as measured by Δ VIX), which is consistent with Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Clarida, Davis, and Pedersen (2009), and Menkhoff, Sarno, Schmeling, and Schrimpf (2012), but the correlation with the mean-variance portfolio is lower.

The results in the previous section use the USD as the domestic currency, and therefore short-term USD as the risk-free asset. Could the results be driven by idiosyncratic features of the dollar? For example, most currency transactions are conducted in dollars. To answer this question, we repeat the analysis with each currency in our sample as the home currency. Table 13 reports the results. The results are very similar across the different domestic currencies. The gross returns differ mainly due to different average levels in domestic risk free interest rates, but the excess

Table 13: Results for Different Domestic Currencies

	USD	CHF	EUR	JPY	GBP	AUD	CAD	NOK	SEK	SGD	NZD
Total gross (in %)	5.05	3.83	5.18	2.79	6.57	7.38	5.56	6.68	6.23	4.30	7.89
Total excess (in %)	1.70	1.74	1.70	1.75	1.68	1.6	1.68	1.68	1.68	1.71	1.65
Volatility (in %)	1.87	1.88	1.88	1.87	1.88	1.88	1.88	1.88	1.90	1.88	1.88
Sharpe ratio	0.91	0.92	0.90	0.94	0.90	0.88	0.89	0.89	0.88	0.91	0.88

The performance of mean-variance carry trade strategy for difference domestic currencies from January 1990 to December 2015. "Mean-Variance" is the mean-variance portfolio described in this paper with the maximum Sharpe ratio. All figures are annualized.

returns and volatilities are almost identical.

4 Practical Implementation

4.1 Choice of Instruments

We described the carry trade in terms of domestic and foreign bonds, but this is not the most cost-effective way to implement the trade in practice. Currency markets feature very deep markets in instruments such as forwards and FX swaps that allow the trade to be implemented with very low transaction costs. In 2010, for example, forwards had a daily turnover of \$475 billion and foreign exchange swaps of \$1765 billion (BIS 2010). (In contrast, the US stock market turnover is around \$200 billion.)

A *forward* is a contract to buy or sell a specific amount of currency at a specific date in the future at a fixed price today. No cash changes hands today. The price of a forward, F_t^i, is quoted as the amount of domestic currency necessary to buy one unit of the foreign currency (at a specific date in the future). A forward can be synthesized in terms of one spot transaction and two bonds, which allows the price of the forward to be calculated as

$$F_t^i = S_t^i \frac{1 + r_t^d}{1 + r_t^i}, \tag{3}$$

a result known as *covered interest parity.*

An *FX swap* is a contract to buy or sell a specific amount of currency today at the spot price, and a reverse transaction to buy or sell the same amount of currency at a specific date in the future at the forward price. Despite its complexity, the FX swap is one of the central transactions of currency markets. A forward is generally synthesized in terms of a spot transaction followed by an FX swap that reverses the spot transaction. Since it is such a high-volume transaction, it has very low transaction costs, and so it is an important tool in a low-cost carry trade implementation.

Suppose an investor invests 1 unit of domestic currency into a forward. The investor enters into a contract to buy $1/F_t^i$ units of foreign currency for 1 unit of domestic currency. At the future date, the investor converts the foreign currency at the prevailing spot, S_{t+1}^i, for a net gain of

$$\frac{S_{t+1}^i}{F_t^i} - 1,$$

denominated in future currency. The investor receives a net gain of

$$\frac{S_{t+1}^i}{S_t^i}\frac{1+r_t^i}{1+r_t^d} - 1$$

in future currency values. In present value terms, this yields an excess return of

$$\frac{S_{t+1}^i}{S_t^i}(1+r_t^i) - (1+r_t^d).$$

The forward is equivalent to a zero net investment portfolio of being long in the foreign currency, and short in the domestic currency, and thus has the same return.

Forward prices are usually quoted in terms of *forward points*, which is just the difference between the spot and the forward price,

$$F_t^i - S_t^i = S_t^i\left(\frac{1+r_t^d}{1+r_t^i} - 1\right). \tag{4}$$

Forward points are also known as the swap rate, since they determine the difference between the two sides of the FX swap. This quoting convention highlights the role the difference in interest rates plays. If the domestic interest rate is higher, then the forward points are positive. If the foreign interest rate is higher, then the forward points are negative.

4.2 Transaction Costs

In the final step of our analysis, we analyze the impact of transaction cost. In particular, we want to examine whether the mean-variance carry trade strategy remains very profitable in the face of transaction cost.

A low-cost implementation of the carry trade involves two types of transactions, with distinct transaction costs – spot transactions and FX swaps. Swaps have much lower cost than spot transactions, so the choice of transaction is important. We report transaction cost data from the Zurich Kantonalbank in Zurich, Switzerland in Table 14. The figures represent one half of the average bid-ask spread for spot transactions and one-month FX swaps quoted to the bank's institutional clients in 2009. (The transaction cost for a forward is the sum of the transaction costs for the spot market trade and the FX swap.) Since 2009 transaction costs have fallen due to lower volatility and an increasingly electronic trading share. Thus these transaction costs overestimate current transaction costs.

Table 14: Half Bid-Ask Spreads (in %) for One-Month Currency Forwards

	CHF/USD	**EUR/USD**	**JPY/USD**	**GBP/USD**	**AUD/USD**
Spot	0.016	0.013	0.019	0.012	0.025
FX Swap	0.004	0.002	0.002	0.003	0.010
	CAD/USD	**NOK/USD**	**SEK/USD**	**SGD/USD**	**NZD/USD**
Spot	0.014	0.025	0.022	0.050	0.060
FX Swap	0.005	0.012	0.006	0.002	0.017

An investor entering a currency forward pays a single transaction cost for the forward which is the sum of the transaction costs for the necessary transactions on the spot market and the FX swap market. The table reports half of the bid-ask spreads (in percent) for the two transactions. (Source: Zurich Kantonalbank trading desk.)

For a one-time carry trade, the distinction between the two sources of transaction costs is unimportant, but it becomes important in a dynamic portfolio strategy. If an investor wants to roll over the currency forward for another month, then the initial spot transaction can be avoided. (Rolling over a position includes a spot transaction today and a reverse spot transaction in a month that exactly offsets today's amount.) Thus the investor only pays the much lower transaction costs for the FX swap. Spot transaction costs occur only if the weight in a currency changes.

Here is a simple example of implementation. An investor wishes to construct a USD-CHF forward for 1 month using a spot transaction and an FX swap. The transaction cost for the initial spot transaction is 0.016% for spot and 0.004% to enter into the swap. The total transaction cost is then 0.02%. To close the position the next month, the investor again performs a spot transaction. Suppose the investor wants to do the same transaction the next month. The investor could close out the current position and repeat the previous month's transaction for a total cost of 0.036%. Alternatively, the investor could achieve the same effect by entering into another swap, which only costs 0.004%, slightly more than one-tenth the cost.

Table 15 shows the effect of transaction costs on different variants of the carry trade. The annual transaction cost are -0.16% for the mean-variance strategy, which is 9.61% of the excess

Table 15: Results including Transaction Costs

	Long 1 Short 1	Long 5 Short 5	Mean-Variance
Total gross (in %)	5.85	5.39	5.05
Total excess (in %)	2.51	2.05	1.70
Spot transaction costs (in %)	-0.11	-0.04	-0.10
Swap transaction costs (in %)	-0.09	-0.07	-0.06
Total transaction costs (in %)	-0.20	-0.11	-0.16
Transaction costs in % of excess	7.89	5.47	9.61
Total excess after transaction costs (in %)	2.31	1.94	1.54
Volatility excess after transaction costs(in %)	6.71	3.32	1.87
Sharpe ratio after transaction costs	0.34	0.58	0.82

The performance of three different carry trade strategies including transaction costs from January 1990 to December 2015. "Long 1 Short 1" is an investment that is long 0.5 in the highest interest in our sample of currencies and short -0.5 in the lowest interest rate. "Long 5 Short 5" is an investment that is long in the 5 highest interest rate currencies with a weight of 1/10, and short in the 5 lowest interest rate currencies with a weight of -1/10. "Mean-Variance" is the mean-variance portfolio described in this paper with the maximum Sharpe ratio. All figures are annualized.

return. This is similar to the Long 1 Short 1 strategy, where transaction costs are 7.89% of excess return. For the Long 5 Short 5 strategy, the share of transaction costs is somewhat lower with 5.47% of excess return. So, the effect of transaction costs do not negate the superior performance of the mean-variance strategy. For a naive implementation on the spot market only, the transaction costs would be significantly higher: for example, for the mean-variance strategy, the annual transaction cost is -0.34% (20% of excess return), which lowers the Sharpe ratio to 0.73.

5 Conclusion

While theoretically attractive, mean-variance analysis has proven challenging to implement in practice. Estimation error, particularly in means, has made the practical use of the technique troublesome. The most extreme evidence of this difficulty was provided by DeMiguel, Garlappi, and Uppal (2009), which showed that it was difficult for a mean-variance optimal portfolio to outperform a simple equally-weighted portfolio in equity markets.

We extend this analysis to the currency market. Here we find that mean-variance analysis can work well. The key element of the currency market is that a large component of the return, the interest rate, is known a priori, and is not subject to estimation error. We find that mean-variance analysis using the interest rate as expected return outperforms an equally-weighted portfolio by a considerable margin: a Sharpe ratio of 0.91 versus 0.15.

In comparison, if we take a more statistical approach by estimating the expected return from the exchange rate component, this worsens the performance of the portfolio somewhat (to a Sharpe ratio of 0.83). This is in line with the estimation error story, since out-of-sample the interest rate alone is a somewhat lower error predictor of returns than the combination of interest rate and estimated exchange rate change. The mean-variance portfolio in this market has other attractive properties. It has a higher Sharpe ratio than the classic carry trade strategy of going long the highest interest-rate currency and short the lowest interest-rate currency, with a lower skewness. It also had a positive return over the financial crisis, over which time the classic carry trade strategy did poorly. It also has a low correlation with equity returns as well as innovations in the VIX. The performance is largely independent of the choice of currency used for the risk-free rate. It also has one other attractive feature: since interest rate spreads change over time, the expected Sharpe ratio changes over time. We find that periods in which the expected Sharpe ratio is high, the realized Sharpe ratio is high, and when the expected Sharpe ratio is low, the realized Sharpe ratio is low. Thus the mean-variance portfolio is a practical tool for determining when currency market investment is attractive.

We also consider the question of practical implementation. The currency market has well-developed derivatives with low transaction costs, such as FX swaps. We show that with a careful selection of instruments, the transaction costs can be minimized. The Sharpe ratio of the mean-variance portfolio after transaction costs is 0.82, which is still substantial.

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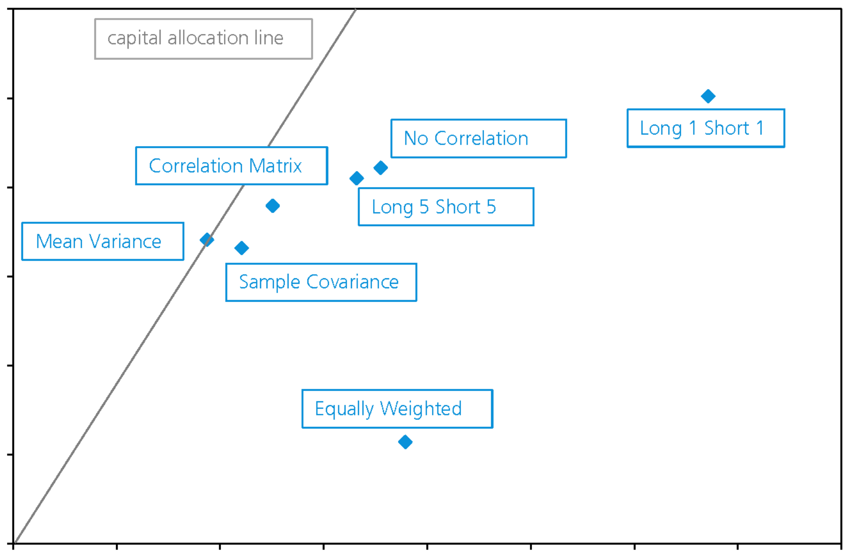
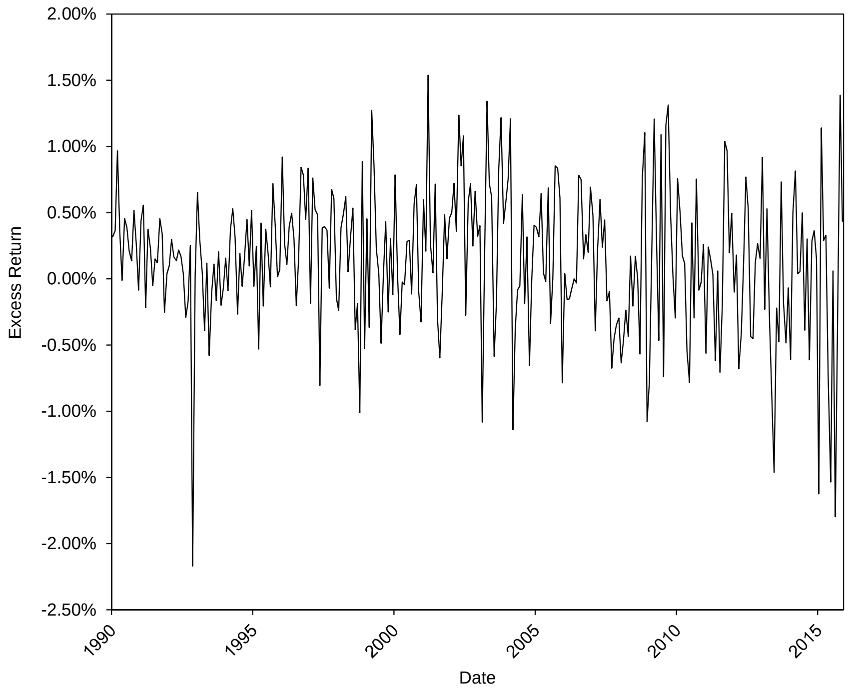
Figure 1: Excess Return and Volatility Comparison for Different Carry Trade Strategies



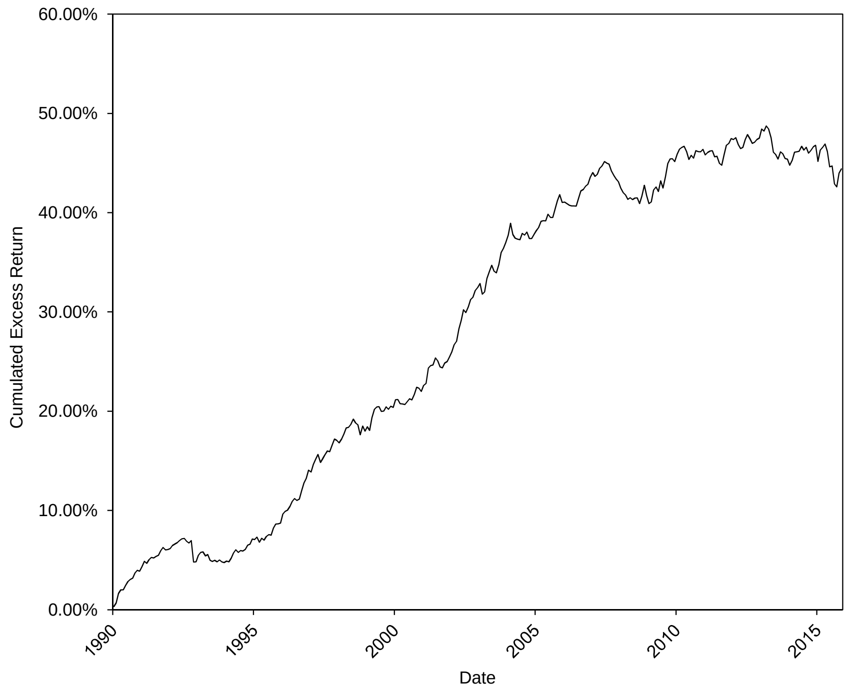
Excess return and volatility comparison for different carry trade strategies over the entire sample period, January 1990 to December 2015. “Long 1 Short 1” is an investment that is long 0.5 in the highest interest in our sample of currencies and short -0.5 in the lowest interest rate. “Long 5 Short 5” is an investment that is long in the 5 highest interest rate currencies with a weight of 1/10, and short in the 5 lowest interest rate currencies with a weight of -1/10. “Mean-Variance” is the mean-variance portfolio described in this paper. “No Correlation” is a mean-variance portfolio, but calculated with a covariance-matrix where the off-diagonal elements are zero. “Correlation-matrix” is a mean-variance portfolio, but calculated with the correlation-matrix instead of the covariance matrix. “Sample Covariance” is a mean-variance portfolio calculated with the constant ex-post covariance matrix computed over the whole sample.

Figure 2: Monthly Portfolio Excess Returns



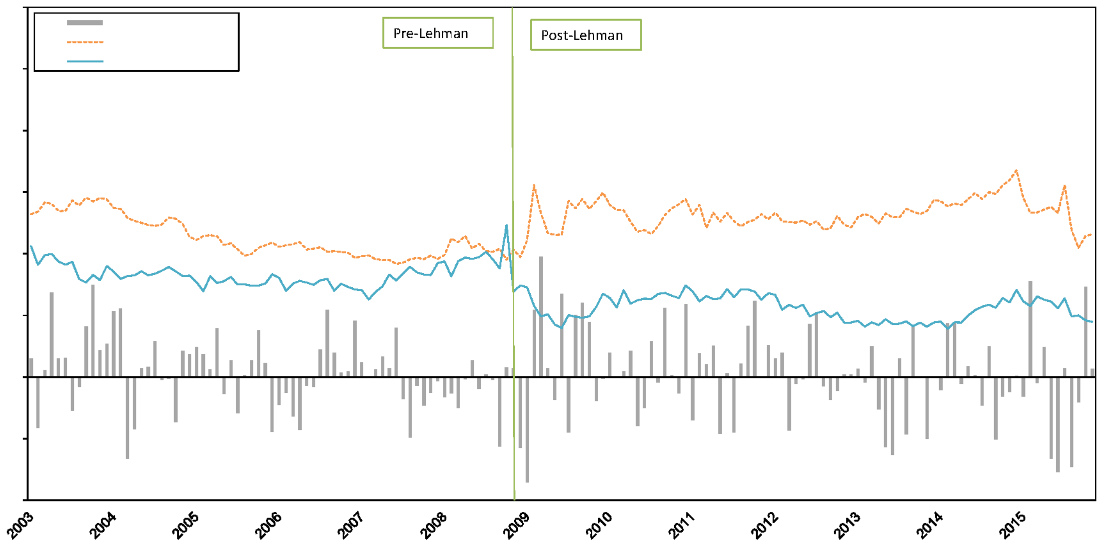
The performance of dynamic carry trade strategy from January 1990 to December 2015. The figure shows the monthly excess returns.

Figure 3: Cumulated Portfolio Excess Returns



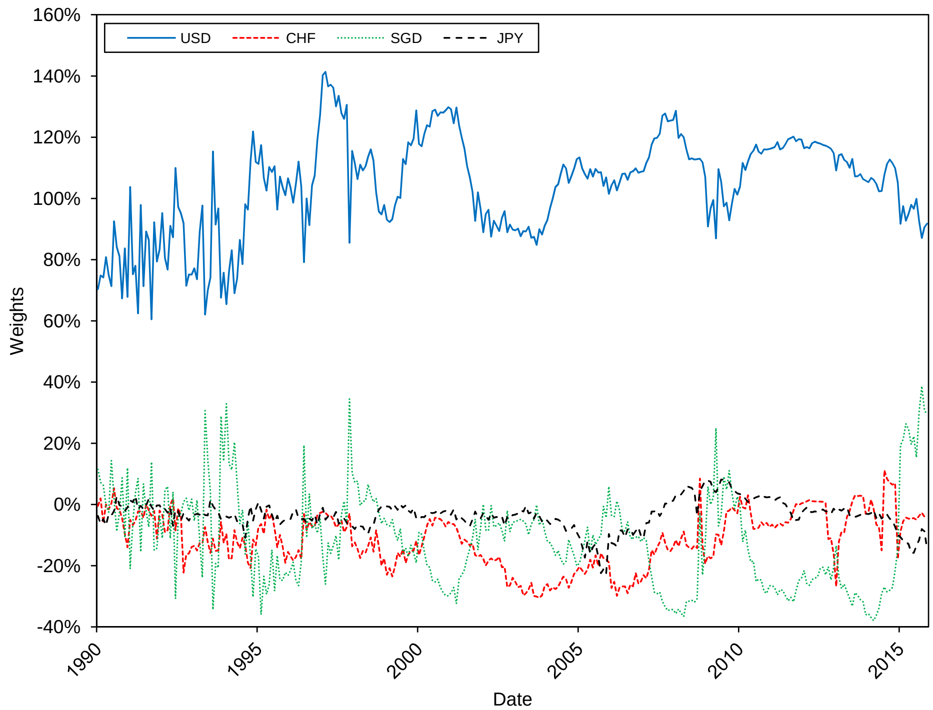
The performance of dynamic carry trade strategy from January 1990 to December 2015. The figure shows the cumulated excess returns.

Figure 4: Expected Return and Volatility



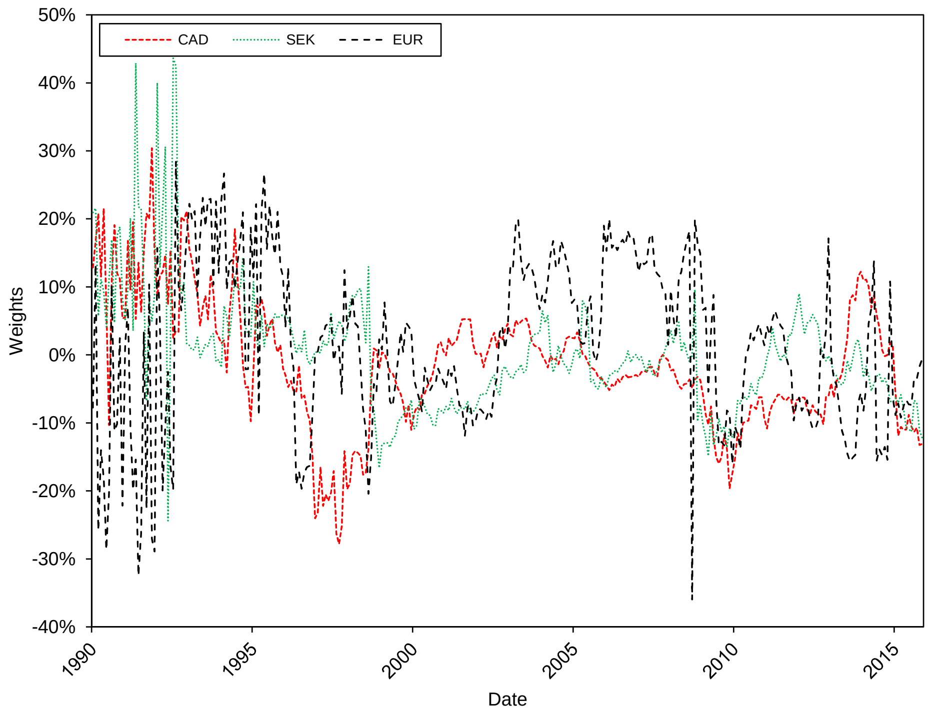
Monthly values of the realized return, as well as the expected return, and the estimated volatility for the mean-variance portfolio from July 2003 to December 2015. The vertical bars in the chart show the realized monthly returns of the mean-variance portfolio. The solid line shows the expected return for the portfolio calculated from the individual expected returns and the weights. The dotted line shows the expected volatility for the portfolio calculated from the estimated covariance matrix and the weights.

Figure 5: Weights for Low Interest Rate Currencies



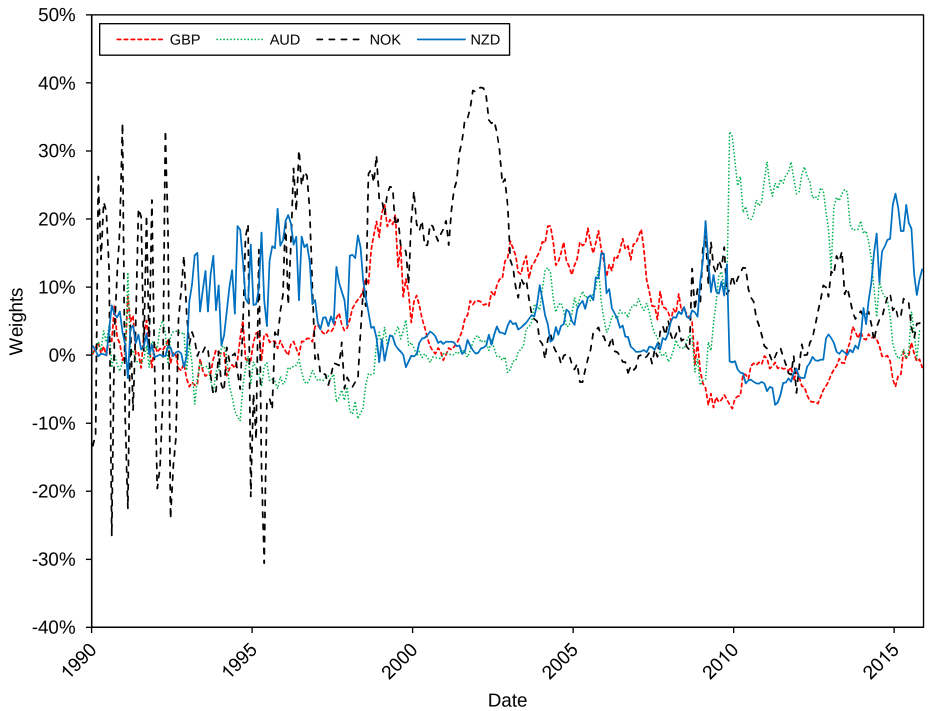
Monthly portfolio weights of the four currencies, JPY, CHF, SGD, and USD in the mean-variance portfolio over the entire sample period, January 1990 to December 2015.

Figure 6: Weights for Medium Interest Rate Currencies



Monthly portfolio weights of the three currencies, EUR, CAD, and SEK in the mean-variance portfolio over the entire sample period, January 1990 to December 2015.

Figure 7: Weights for High Interest Rate Currencies



Monthly portfolio weights of the four currencies, NOK, GBP, AUD, and NZD in the mean-variance portfolio over the entire sample period, January 1990 to December 2015.