Erdős Institute
Summer 2025 Quant Finance Boot Camp

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Mini-Projects Presentation: Nick Switala

Project Format

4 assigned “mini-projects”, plus an extra side project I worked on at the same time.

Each project is contained in a Jupyter notebook (all code for the projects was written in Python). The notebooks are a combination of exposition, mathematical calculations, Python code, and plots.

(Mini Projects 1 & 2 were closely related enough that I put them together in one notebook.)

Mini Project 1: Portfolio construction

Global Industry Classification Standard: division of economy into 11 sectors

Portfolio 1 (“higher-risk”): top 10 stocks in “information technology” sector by market cap

Portfolio 2 (“lower-risk”): of the top 5 stocks in each sector by market cap, take the one with lowest volatility during Q1 2025 (both portfolios equally weighted)

What would have happened to $10,000 invested in one of these portfolios during Q1 2025? What about all of 2024?

Mini Project 1 (cont.)

$10,000 invested in Portfolio 1:

Would have grown to $17,606 during the year 2024 (21.6% volatility)

Would have *shrunk(!)* to $9,323 during Q1 2025 (30.7% annualized volatility)

$10,000 invested in Portfolio 2:

Would have grown to $11,649 during the year 2024 (8.8% volatility)

Would have grown to $10,957 during Q1 2025 (10.7% annualized volatility)

For 2024 as well as Q1 2025, Portfolio 2's volatility was less than that of the overall S&P 500.

Mini Project 2: Normality of log returns

Are the daily returns of Portfolios 1 and 2 lognormally distributed?

I used the Shapiro-Wilk test (with $p < 0.05$ threshold)

Portfolio 1: no reason to reject lognormal distribution hypothesis, whether we look at any individual quarter of 2024 or the whole year

Portfolio 2: no reason to reject during Q1, Q2, or Q4, but returns not lognormally distributed during Q3 and also not lognormally distributed over the whole year

Mini Project 3: Price- and time-sensitivity of options

If $C(S,T)$ is the price of a call option on a stock with current price S and time to expiration T, the partial derivatives of C with respect to S and T are called the Delta and Theta of the option respectively.

(Other parameters on which C depends are strike, risk-free interest rate, and volatility. There are other "Greek letters" as well.)

Under Black-Scholes assumptions, the Greeks can be computed explicitly. They can also be simulated using the finite difference method.

In addition to calculus and code for Monte Carlo simulations, the bulk of the mini-project was plotting and visualization, including 2-D heatmaps showing price- and time-dependence simultaneously.

Mini Project “3.5” (not assigned): path generation

For some options (e.g. Asian options) or hedging strategies, we need to simulate the stock price at several points between now and expiration, not just the end points.

It is possible to “draw” an entire path at once using the fact that we know in advance what the covariance matrix C must be.

This requires calculating a *pseudo square root* of C (a matrix A such that $AA^T = C$), which can be done in several ways (Cholesky decomposition, orthogonal diagonalization, etc.)

With orthogonal diagonalization, it is possible to draw paths that are approximately correct from a smaller-dimensional space (“principal components analysis”), speeding up the rate of convergence of Monte Carlo simulations.

In my mini-project I simulated the price of an Asian option by Monte Carlo three different ways: incremental path generation, orthogonal diagonalization, and the truncated/approximate method, comparing run times and looking at convergence rates.

Mini Project 4: stochastic volatility

The Black-Scholes model assumes that the volatility of the underlying stock is constant throughout the option's life, which is not an accurate real-world assumption.

A first step in the direction of making the model more accurate is to model volatility itself as a random time-dependent process. In the boot camp lectures, we saw one way to do this: the Heston model.

I used an alternative model: a GARCH(1,1) process, in which volatility at each time step depends on the previous step's volatility as well as the volatility *of the volatility*.

A GARCH(1,1) process has three parameters that can be estimated using the maximum likelihood method (MLE). I used MLE to fit a GARCH(1,1) process to the previous year's daily S&P 500 log returns, then simulated a periodically Delta-hedged call option over the next six months assuming volatility continued to follow this GARCH process.