Probability: Homework 2

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Problem 1:

(1) By definition, a probability mass function must satisify $\sum^{t} f(t) = 1$, if $x \in \{0, 1, 2, ...\}$

$$\text{then } \sum_{i=0}^{\infty} \frac{k}{2^x} = 1$$

by the common form of the geometric series: $a = \frac{k}{2^0}; r = \frac{1}{2}$

$$\begin{aligned} 1 &= \frac{a}{1-r} \\ 1 &= \frac{k}{1-1/2} \\ 1 &= \frac{k}{1/2} \\ k &= \frac{1}{2} \end{aligned}$$

(2)

Problem 2:

(1) For $f(t)$ to be a pdf, $\int_{-\infty}^{\infty} f(t)dt = 1$

$$\begin{aligned} \int_0^{\infty} ce^{-2t} dt &= 1 \\ c \int_0^{\infty} e^{-2t} dt &= 1 \\ c \int_0^{\infty} e^u - \frac{1}{2} du &= 1 \\ c \int_0^{\infty} \frac{-e^u}{2} du &= 1 \\ \frac{-c}{2} \int_0^{\infty} e^u du &= 1 \\ \Big[-\frac{1}{2} ce^{-2t} \Big]_0^{\infty} &= 1 \\ \lim_{t \to \infty} \big(-\frac{1}{2} ce^{-2t} \big) - \big(-\frac{1}{2} c \big) &= 1 \\ 0 + \frac{1}{2} c &= 1 \\ c &= 2 \end{aligned}$$

(2) a cdf is the integral of the corresponding pdf, thus:

$$F(x) = -\frac{1}{2}2e^{-2t} = -e^{2t}, \text{ where } 0 < t < \infty$$

Problem 3:

If $f(t)$ and $g(t)$ are pdfs then:

$$\int_{-\infty}^{\infty} f(t)dt = 1 \text{ and } \int_{-\infty}^{\infty} g(t)dt = 1$$

If $a \geq 0$ and $b \geq 0$ are constants satisfying $a + b = 1$ then, $a = 1 - b$. If $af(t) + bg(t)$ is a pdf then:

$$\begin{aligned}
1 &= af(t) + bg(t) \\
1 &= \int_{-\infty}^{\infty} af(t) + \int_{-\infty}^{\infty} bg(t) \\
1 &= a\int_{-\infty}^{\infty} f(t) + b\int_{-\infty}^{\infty} g(t) \\
1 &= (1-b)\int_{-\infty}^{\infty} f(t) + b\int_{-\infty}^{\infty} g(t) \\
1 &= (1-b)(1) + b(1) \\
1 &= 1 - b + b = 1
\end{aligned}$$