

Probability: Homework 2

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Problem 1:

(1) By definition, a probability mass function must satisfy $\sum^t f(t) = 1$, if $x \in \{0, 1, 2, \dots\}$

$$\text{then } \sum_{i=0}^{\infty} \frac{k}{2^x} = 1$$

by the common form of the geometric series: $a = \frac{k}{2^0}; r = \frac{1}{2}$

$$1 = \frac{a}{1-r}$$

$$1 = \frac{k}{1-1/2}$$

$$1 = \frac{k}{1/2}$$

$$k = \frac{1}{2}$$

(2)

Problem 2:

(1) For $f(t)$ to be a pdf, $\int_{-\infty}^{\infty} f(t)dt = 1$

$$\int_0^{\infty} ce^{-2t} dt = 1$$

$$c \int_0^{\infty} e^{-2t} dt = 1$$

$$c \int_0^{\infty} e^u - \frac{1}{2} du = 1$$

$$c \int_0^{\infty} \frac{-e^u}{2} du = 1$$

$$\frac{-c}{2} \int_0^{\infty} e^u du = 1$$

$$\left[-\frac{1}{2} ce^{-2t} \right]_0^{\infty} = 1$$

$$\lim_{t \rightarrow \infty} \left(-\frac{1}{2} ce^{-2t} \right) - \left(-\frac{1}{2} c \right) = 1$$

$$0 + \frac{1}{2} c = 1$$

$$c = 2$$

(2) a cdf is the integral of the corresponding pdf, thus:

$$F(x) = -\frac{1}{2}e^{-2t} = -e^{2t}, \text{ where } 0 \leq t < \infty$$