# Probability: Homework 2

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## Problem 1:

(1) By definition, a probability mass function must satisify $\sum^{t} f(t) = 1$, if $x \in \{0, 1, 2, ...\}$

$$\text{then } \sum_{i=0}^{\infty} \frac{k}{2^x} = 1$$

by the common form of the geometric series: $a = \frac{k}{2^0}; r = \frac{1}{2}$

$$\begin{aligned}
1 &= \frac{a}{1-r} \\
1 &= \frac{k}{1-1/2} \\
1 &= \frac{k}{1/2} \\
k &= \frac{1}{2}
\end{aligned}$$

(2)

## Problem 2:

(1) For $f(t)$ to be a pdf, $\int_{-\infty}^{\infty} f(t)dt = 1$

$$\begin{aligned}
1 &= \int_0^{\infty} ce^{-2t} dt \\
&= c\int_0^{\infty} e^{-2t} dt \\
&= c\int_0^{\infty} e^{u} - \frac{1}{2} du \\
&= c\int_0^{\infty} \frac{-e^{u}}{2} du \\
&= \frac{-c}{2}\int_0^{\infty} e^{u} du \\
&= \Big[ -\frac{1}{2}ce^{-2t} \Big]_0^{\infty} \\
&= \lim_{t\to\infty} \big( -\frac{1}{2}ce^{-2t} \big) - \big( -\frac{1}{2}c \big) \\
&= 0 + \frac{1}{2}c \\
c &= 2
\end{aligned}$$

(2) a cdf is the integral of the corresponding pdf, thus:

$$F(x) = P(X \leq x) = -\frac{1}{2}2e^{-2t} = -e^{2t}, \text{ where } 0 < t < \infty$$

## Problem 3:

If $f(t)$ and $g(t)$ are pdfs then:

$$\int_{-\infty}^{\infty} f(t)dt = 1 \text{ and } \int_{-\infty}^{\infty} g(t)dt = 1$$

If $a \geq 0$ and $b \geq 0$ are constants satisfying $a + b = 1$ then, $a = 1 - b$. If $af(t) + bg(t)$ is a pdf then:

$$\begin{aligned}
1 &= af(t) + bg(t) \\
&= \int_{-\infty}^{\infty} af(t) + \int_{-\infty}^{\infty} bg(t) \\
&= a\int_{-\infty}^{\infty} f(t) + b\int_{-\infty}^{\infty} g(t) \\
&= (1-b)\int_{-\infty}^{\infty} f(t) + b\int_{-\infty}^{\infty} g(t) \\
&= (1-b)(1) + b(1) \\
&= 1 - b + b \\
&= 1
\end{aligned}$$

## Problem 4:

(1) For $F(t) = (1 - \frac{1}{1+t})\ I\{t \geq 0\}$ to be a cdf, it must satisfy

a. $\lim_{t\to\infty} F(t) = 1$ and $\lim_{t\to-\infty} F(t) = 0$:

$$\begin{aligned}
1 &= \lim_{t\to\infty}(1 - \frac{1}{1+t}) & 0 &= \lim_{t\to-\infty}(1 - \frac{1}{1+t}) \\
&= 1 - \lim_{t\to\infty}\frac{1}{1+t} & &= 1 - \lim_{t\to-\infty}\frac{1}{1+t} \\
&= 1 - \lim_{t\to\infty}\frac{1}{1+t} & &= 1 - \lim_{t\to-\infty}\frac{1}{1+t} \\
&= 1 + 0 \\
&= 1
\end{aligned}$$

**Problem 5:**

(1)

$$f(x) = P(X = x) = \begin{cases} c(x+1) & \text{for } 0, 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

by the definition of a probability mass function:

$$\begin{aligned} 1 &= \sum_x f(x) \\ &= \sum_x c(x+1) \\ &= \sum_{x=0}^{5} = c(x+1) \\ &= c + 2c + 3c + 4c + 5c + 6c \\ &= 21c \\ c &= \frac{1}{21} \end{aligned}$$

(2)

$$F(x) = P(X \leq x) = \sum_{X \leq x} \frac{x+1}{21}$$

(3)

$$\begin{aligned} P(X > 2) = P(X \geq 3) &= 1 - P(X \leq 2) \\ &= 1 - \sum_{x=0}^{2} \frac{x+1}{21} \\ &= 1 - \frac{1}{21} + \frac{2}{21} + \frac{3}{21} \\ &= 0.7143 \end{aligned}$$