P8104 Homework 3

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Problem 1

(1)

(2)

Problem 2

$$f_X(x) = 2xI\{0 < x < 1\}$$

(1)

$$Y = g(x) = 4X - 3$$

$$\frac{d}{dx}g(x) = 4 > 0 \qquad \text{thus } g(x)\text{is an increasing function}$$

$$g^{-1}(y) = \frac{y+3}{4}$$

$$f_Y(y) = f_X(g^{-1}(y))|\frac{d}{dy}g^{-1}(y)|$$

$$= f_X(\frac{y+3}{4})|\frac{1}{4}| \qquad \frac{d}{dy}g^{-1}(y) = \frac{1}{4}$$

$$= 2\frac{y+3}{4}(\frac{1}{4})$$

$$= \frac{y+3}{8} \text{ for } -3 < y < 1$$

(2)

$$Y = g(x) = 3 - X^2$$

$$\frac{d}{dy}g(x) = -2x < 0 \quad \text{monotone } \forall\ 0 < x < 1$$

$$g^{-1}(y) = \sqrt{3-y}$$

$$f_Y(y) = f_X(g^{-1}(y))|\frac{d}{dy}g^{-1}(y)|$$

$$= f_x(\sqrt{3-y})\left|-\frac{1}{2\sqrt{3-y}}\right| \quad \frac{d}{dy}g^{-1}(y) = -\frac{1}{2\sqrt{3-y}}$$

$$= f_x(\sqrt{3-y})\left(\frac{1}{2\sqrt{3-y}}\right)$$

$$= 2(\sqrt{3-y})\left(\frac{1}{2\sqrt{3-y}}\right)$$

$$= 1 \text{ for } 2 < y < 3$$

Problem 3

(1)

(2)

$$X \sim U(-1,1) \quad f_X(x) = \frac{1}{2} \text{ for } -1 \leq x \leq 1$$
$$Y = g(x) = e^x \qquad\qquad g^{-1}(y) = log(x)$$

Because $\frac{d}{dy}g(x) = e^x > 0 \ \forall x \in [-1,1]$, $g(x)$ is an increasing function and monontone