Problem Set #6

MACS 30150, Dr. Evans

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In [168]:

```
import numpy as np
import pandas as pd
from pandas import Series, DataFrame
import statsmodels.api as sm
import sklearn
from sklearn import preprocessing
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import train_test_split
from sklearn import metrics
from sklearn.metrics import classification_report
from sklearn.metrics import confusion_matrix
from pylab import rcParams
from sklearn import datasets, neighbors, linear_model
import matplotlib.pyplot as plt
import seaborn as sb
%matplotlib inline
rcParams['figure.figsize'] = 10, 8
sb.set_style('whitegrid')
```

Question 1

1(a)

In [169]:

```
url = ('https://raw.githubusercontent.com/nt546/persp-model-econ_W19/' +
       'master/ProblemSets/PS6/data/Auto.csv')
df = pd.read_csv(url, na_values='?')
df.dropna(inplace=True)
print(df.shape)
df.head()
```

```
(392, 9)
```

Out[169]:

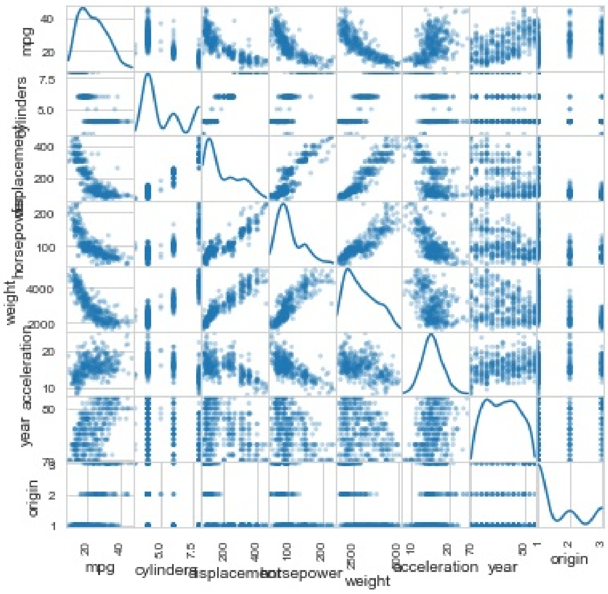
	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin	name
0	18.0	8	307.0	130.0	3504	12.0	70	1	chevrolet chevelle malibu
1	15.0	8	350.0	165.0	3693	11.5	70	1	buick skylark 320
2	18.0	8	318.0	150.0	3436	11.0	70	1	plymouth satellite
3	16.0	8	304.0	150.0	3433	12.0	70	1	amc rebel sst
4	17.0	8	302.0	140.0	3449	10.5	70	1	ford torino

1(b)

In [170]:

```
from pandas.plotting import scatter_matrix
df_quant=df[['mpg','cylinders','displacement','horsepower',
```

```
                'weight','acceleration','year','origin']]
scatter_matrix(df_quant, alpha=0.3, figsize=(6, 6),diagonal='kde')
plt.show()
```



1(c)

In [171]:

```
df_quant.corr()
```

Out[171]:

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
mpg	1.000000	-0.777618	-0.805127	-0.778427	-0.832244	0.423329	0.580541	0.565209
cylinders	-0.777618	1.000000	0.950823	0.842983	0.897527	-0.504683	-0.345647	-0.568932
displacement	-0.805127	0.950823	1.000000	0.897257	0.932994	-0.543800	-0.369855	-0.614535
horsepower	-0.778427	0.842983	0.897257	1.000000	0.864538	-0.689196	-0.416361	-0.455171
weight	-0.832244	0.897527	0.932994	0.864538	1.000000	-0.416839	-0.309120	-0.585005
acceleration	0.423329	-0.504683	-0.543800	-0.689196	-0.416839	1.000000	0.290316	0.212746
year	0.580541	-0.345647	-0.369855	-0.416361	-0.309120	0.290316	1.000000	0.181528
origin	0.565209	-0.568932	-0.614535	-0.455171	-0.585005	0.212746	0.181528	1.000000

1(d)

In [172]:

```
mpg_ = df_quant['mpg']
df_quant['constants'] = 1
model_ = df_quant[['constants', 'cylinders', 'displacement', 'horsepower', 'weight',
                               'acceleration', 'year', 'origin']]
l_reg = sm.OLS(mpg_, model_).fit()
print(l_reg.params)
print(l_reg.summary())
```

```
constants       -17.218435
cylinders        -0.493376
displacement      0.019896
horsepower       -0.016951
weight           -0.006474
acceleration      0.080576
year              0.750773
origin            1.426140
dtype: float64
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.821
Model:                            OLS   Adj. R-squared:                  0.818
Method:                 Least Squares   F-statistic:                     252.4
Date:                Wed, 20 Feb 2019   Prob (F-statistic):          2.04e-139
Time:                        10:56:48   Log-Likelihood:                -1023.5
No. Observations:                 392   AIC:                             2063.
Df Residuals:                     384   BIC:                             2095.
Df Model:                           7
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
constants      -17.2184      4.644     -3.707      0.000     -26.350      -8.087
cylinders       -0.4934      0.323     -1.526      0.128      -1.129       0.142
displacement     0.0199      0.008      2.647      0.008       0.005       0.035
horsepower      -0.0170      0.014     -1.230      0.220      -0.044       0.010
weight          -0.0065      0.001     -9.929      0.000      -0.008      -0.005
acceleration     0.0806      0.099      0.815      0.415      -0.114       0.275
year             0.7508      0.051     14.729      0.000       0.651       0.851
origin           1.4261      0.278      5.127      0.000       0.879       1.973
==============================================================================
Omnibus:                       31.906   Durbin-Watson:                   1.309
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               53.100
Skew:                           0.529   Prob(JB):                     2.95e-12
Kurtosis:                       4.460   Cond. No.                     8.59e+04
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 8.59e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

i. The coefficients of displacement, weight, year and origin are statistically significant at 1% level.

ii. The coefficients of cylinders, horsepower and acceleration aren't statistically significant at 10% level

iii. The coefficient of year(i) is 0.7508 which means that everything else being equal, an increase in year by one results in 0.7508 units increase in mpg.

1(e)

i.

In [191]:

```python
df_quant['constants'] = 1
df_quant['weight_2'] = df_quant['weight'] ** 2
df_quant['displacement_2'] = df_quant['displacement'] ** 2
df_quant['horsepower_2'] = df_quant['horsepower'] ** 2
df_quant['acceleration_2'] = df_quant['acceleration'] ** 2

mpg_ = df_quant['mpg']
model_1 = df_quant[['constants', 'cylinders', 'displacement', 'displacement_2', 'horsepower', 'horsepower_2',
                    'weight', 'weight_2', 'acceleration', 'acceleration_2', 'year', 'origin']]

l_reg_1 = sm.OLS(mpg_, model_1).fit()
print(l_reg_1.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.870
Model:                            OLS   Adj. R-squared:                  0.866
Method:                 Least Squares   F-statistic:                     230.2
Date:                Wed, 20 Feb 2019   Prob (F-statistic):          1.75e-160
Time:                        10:57:27   Log-Likelihood:                -962.02
No. Observations:                 392   AIC:                             1948.
Df Residuals:                     380   BIC:                             1996.
Df Model:                          11
```

```
Df Model:                          11
Covariance Type:            nonrobust
===================================================================================
                      coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------
constants          20.1084      6.696      3.003      0.003       6.943      33.274
cylinders           0.2519      0.326      0.773      0.440      -0.389       0.893
displacement       -0.0169      0.020     -0.828      0.408      -0.057       0.023
displacement_2   2.257e-05   3.61e-05      0.626      0.532   -4.83e-05    9.35e-05
horsepower         -0.1635      0.041     -3.971      0.000      -0.244      -0.083
horsepower_2        0.0004      0.000      2.943      0.003       0.000       0.001
weight             -0.0136      0.003     -5.069      0.000      -0.019      -0.008
weight_2         1.514e-06   3.69e-07      4.105      0.000    7.89e-07    2.24e-06
acceleration       -2.0884      0.557     -3.752      0.000      -3.183      -0.994
acceleration_2      0.0576      0.016      3.496      0.001       0.025       0.090
year                0.7810      0.045     17.512      0.000       0.693       0.869
origin              0.6104      0.263      2.320      0.021       0.093       1.128
==============================================================================
Omnibus:                       33.614   Durbin-Watson:                   1.576
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               77.985
Skew:                           0.438   Prob(JB):                     1.16e-17
Kurtosis:                       5.002   Cond. No.                     5.13e+08
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.13e+08. This might indicate that there are
strong multicollinearity or other numerical problems.
```

ii. The adj. R^2 value of new model is 0.866. The adj. R^2 value of previous model was 0.818 which means that the adjusted R^2 value of the new model has improved from the previous model.

iii. The displacement variable is no longer statistically significant. The displacement_2 (square of the displacement variable) is also not statistically significant.

iv. The cylinders variable is also not statistically significant under the new model.

1(f)

In [174]:

```
l_reg_1.predict([1, 6, 200, 200**2, 100, 100**2, 3100, 3100**2, 15.1, 15.1**2, 99, 1])
```

Out[174]:

```
array([38.7321111])
```

The predicted miles per gallon mpg of the given car would be 38.732 mpg.

Question 2

2(a)

In [175]:

```
def distance_KNN(x, y):
    x1, x2, x3 = x
    y1, y2, y3 = y
    return np.sqrt((x1 - y1)**2 + (x2 - y2)**2 + (x3 - y3)**2)
```

In [176]:

```
x = (0,0,0)

print("Euclidean distance between observation 1 and the test point is", distance_KNN(x, (0, 3,
0)))
```

```
print("Euclidean distance between observation 2 and the test point is", distance_KNN(x, (2, 0,
0)))
print("Euclidean distance between observation 3 and the test point is", distance_KNN(x, (0, 1,
3)))
print("Euclidean distance between observation 4 and the test point is", distance_KNN(x, (0, 1,
2)))
print("Euclidean distance between observation 5 and the test point is", distance_KNN(x, (-1, 0,
1)))
print("Euclidean distance between observation 6 and the test point is", distance_KNN(x, (1, 1,
1)))
```

```
Euclidean distance between observation 1 and the test point is 3.0
Euclidean distance between observation 2 and the test point is 2.0
Euclidean distance between observation 3 and the test point is 3.1622776601683795
Euclidean distance between observation 4 and the test point is 2.23606797749979
Euclidean distance between observation 5 and the test point is 1.4142135623730951
Euclidean distance between observation 6 and the test point is 1.7320508075688772
```

2(b)

KNN prediction for K = 1 is green as the observation 5 (green color) is the closest neighbor to the test point x.

2(c)

KNN prediction for K = 3 is red as out of the three closest neighbors (5, 6 and 2), observation 5 is green while observations 6 and 2 are red. Hence, the probability of red (2/3) is greater than the probability of green (1/3).

2(d)

If the Bayes (optimal) Decision Boundary is highly nonlinear, then the best value for K is expected to be small. This is because as K gets small, the boundary line gets rough while it is smoother for larger value of K (which is good for linear boundary).

2(e)

In [177]:

```
data = {'X1' : [0, 2, 0 , 0, -1, 1], 'X2' : [3, 0, 1, 1, 0, 1], \
           'X3' : [0, 0, 3, 2, 1, 1], 'Y' : ['Red', 'Red', 'Red', 'Green', 'Green', 'Red']}

knn = pd.DataFrame.from_dict(data)

y = knn['Y']
x = np.column_stack((knn['X1'], knn['X2'], knn['X3']))

knn_classifier_model = sklearn.neighbors.KNeighborsClassifier(n_neighbors=2)
knn_pred = knn_classifier.fit(x, y)
prediction = knn_pred.predict([(1, 1, 1)])
prediction
```

Out[177]:

```
array(['Green'], dtype=object)
```

In [178]:

```
print('The estimated KNN classifier of the test point (1, 1, 1) with K = 2 is ', prediction)
```

```
The estimated KNN classifier of the test point (1, 1, 1) with K = 2 is  ['Green']
```

Problem 3

In [179]:

```
df_quant.head()
```

Out[179]:

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin	constants	weight_2	displacement_2	horsepower_2
0	18.0	8	307.0	130.0	3504	12.0	70	1	1	12278016	94249.0	16900.0
1	15.0	8	350.0	165.0	3693	11.5	70	1	1	13638249	122500.0	27225.0
2	18.0	8	318.0	150.0	3436	11.0	70	1	1	11806096	101124.0	22500.0
3	16.0	8	304.0	150.0	3433	12.0	70	1	1	11785489	92416.0	22500.0
4	17.0	8	302.0	140.0	3449	10.5	70	1	1	11895601	91204.0	19600.0

In [180]:

```
print(df_quant['mpg'].median())
```

```
22.75
```

In [181]:

```
df_quant['mpg_high'] = (df_quant['mpg'] >= df_quant['mpg'].median()).astype(int)
```

In [182]:

```
df_quant.dropna()
df_quant.head()
```

Out[182]:

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin	constants	weight_2	displacement_2	horsepower_2
0	18.0	8	307.0	130.0	3504	12.0	70	1	1	12278016	94249.0	16900.0
1	15.0	8	350.0	165.0	3693	11.5	70	1	1	13638249	122500.0	27225.0
2	18.0	8	318.0	150.0	3436	11.0	70	1	1	11806096	101124.0	22500.0
3	16.0	8	304.0	150.0	3433	12.0	70	1	1	11785489	92416.0	22500.0
4	17.0	8	302.0	140.0	3449	10.5	70	1	1	11895601	91204.0	19600.0

3(a)

In [183]:

```
import warnings
warnings.filterwarnings("ignore")

cols = ['cylinders','displacement','horsepower','weight','acceleration','year','origin']
x = df_quant[cols]
x['constant'] = 1
y = df_quant['mpg_high']

LogitModel = sm.Logit(y, x, missing='drop')
LogitReg_sm = LogitModel.fit()
LogitReg_sm.summary()
```

```
Optimization terminated successfully.
         Current function value: 0.200944
         Iterations 9
```

Out[183]:

Logit Regression Results

Dep. Variable:	mpg_high	**No. Observations:**	392

Model:	Logit	**Df Residuals:**	384
Method:	MLE	**Df Model:**	7
Date:	Wed, 20 Feb 2019	**Pseudo R-squ.:**	0.7101
Time:	10:56:51	**Log-Likelihood:**	-78.770
converged:	True	**LL-Null:**	-271.71
		LLR p-value:	2.531e-79

	coef	std err	z	P>|z|	[0.025	0.975]
cylinders	-0.1626	0.423	-0.384	0.701	-0.992	0.667
displacement	0.0021	0.012	0.174	0.862	-0.021	0.026
horsepower	-0.0410	0.024	-1.718	0.086	-0.088	0.006
weight	-0.0043	0.001	-3.784	0.000	-0.007	-0.002
acceleration	0.0161	0.141	0.114	0.910	-0.261	0.293
year	0.4295	0.075	5.709	0.000	0.282	0.577
origin	0.4773	0.362	1.319	0.187	-0.232	1.187
constant	-17.1549	5.764	-2.976	0.003	-28.452	-5.858

Possibly complete quasi-separation: A fraction 0.14 of observations can be perfectly predicted. This might indicate that there is complete quasi-separation. In this case some parameters will not be identified.

The regressors weight and year are statistically significant at the 5% level.

3(b)

In [184]:

```
X_train, X_test, y_train, y_test = train_test_split(x, y, test_size=0.5, random_state=10)
```

3(c)

In [185]:

```
import warnings
warnings.filterwarnings("ignore")

LogReg = LogisticRegression()
LogReg.fit(X_train, y_train)
```

Out[185]:

```
LogisticRegression(C=1.0, class_weight=None, dual=False, fit_intercept=True,
          intercept_scaling=1, max_iter=100, multi_class='warn',
          n_jobs=None, penalty='l2', random_state=None, solver='warn',
          tol=0.0001, verbose=0, warm_start=False)
```

In [186]:

```
LogReg.coef_
```

Out[186]:

```
array([[-0.33709221,  0.000466  , -0.05200648, -0.00490103, -0.18191715,
         0.30925357, -0.09542257, -0.032587  ]])
```

In [187]:

```
LogReg.intercept_
```

Out[187]:

```
array([-0.032587])
```

The coefficients are:
cylinders : -0.33709
displacement : 0.00046
horsepower : -0.05200
weight : -0.00490
acceleration : -0.18191
year : 0.30925
origin : -0.09542
constant : -0.03258

3(d)

In [188]:

```
y_pred = LogReg.predict(X_test)
```

In [189]:

```
from sklearn.metrics import confusion_matrix
c_m = confusion_matrix(y_test, y_pred)
c_m
```

Out[189]:

```
array([[86, 13],
       [12, 85]])
```

In [190]:

```
print(classification_report(y_test, y_pred))
```

```
              precision    recall  f1-score   support

           0       0.88      0.87      0.87        99
           1       0.87      0.88      0.87        97

   micro avg       0.87      0.87      0.87       196
   macro avg       0.87      0.87      0.87       196
weighted avg       0.87      0.87      0.87       196
```

The precision values for both low mpg and high mpg are almost similar but precision value for low mpg (0.88) is slightly higher than that of high mpg value (0.87). Hence, the model predicts low mpg slightly better than high mpg.

In [ ]: