Noname manuscript No.
(will be inserted by the editor)

Social Cost of Carbon under stochastic tipping points

When does risk play a role?

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The is the accepted version of the article published in *Environmental and Resource Economics*.
DOI: https://doi.org/10.1007/s10640-021-00549-x

Abstract Is climate change concerning because of its expected damages, or because of the risk that damages could be very high? Climate damages are uncertain, in particular they depend on whether the accumulation of greenhouse gas emissions will trigger a tipping point. In this article, we investigate how much risk contributes to the Social Cost of Carbon in the presence of a tipping point inducing a higher-damage regime. To do so, we decompose the effect of a tipping point as an increase in expected damages plus a zero-mean risk on damages. First, using a simple analytical model, we show that the SCC is primarily driven by expected damages, while the effect of pure risk is only of second order. Second, in a numerical experiment using a stochastic Integrated Assessment Model, we show that expected damages account for most of the SCC when the tipping point induces a productivity shock lower than 10%, the high end of the range commonly used in the literature. It takes both a large productivity shock and high risk aversion for pure risk to significantly contribute to the SCC. Our analysis suggests that the *risk aversion puzzle*, which is the usual finding that risk aversion has a surprisingly little effect on the SCC, occurs since the SCC is well estimated using expected damages only. However, we show that the risk aversion puzzle does not hold for large productivity shocks, as pure risk greatly contributes to the SCC in these cases.

Keywords Climate change · Tipping points · Expected utility · Integrated Assessment Models · Risk · Social Cost of Carbon

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1 Introduction

Climate change will induce damages in the future, although their magnitude remains uncertain (Diaz and Moore, 2017). A key uncertainty is whether emissions will trigger non-marginal or abrupt changes, often referred to as "tipping points" (Lenton et al., 2008; Alley et al., 2003; Steffen et al., 2018). Examples of large-scale regime shifts include the shutdown of thermohaline circulation, the melting of the Arctic sea-ice or the die back of the Amazonian rainforest. Such shifts could also stem from the limited ability of social and economic systems to cope with climate conditions beyond some threshold. Assessing the present social value of damages from an additional ton of CO_2 released in the atmosphere, i.e. the Social Cost of Carbon (SCC), requires to account for the possibility of triggering such high-impact events as the planet warms.

Early assessments of the SCC relied on deterministic models balancing abatement costs with benefits from avoided damages, assuming they are known to the social planner (Nordhaus, 1994). They used a damage function capturing best-guess value of the level of damage for each additional degree of warming. Concerns about catastrophic damages and tipping points have first been addressed in a deterministic fashion. Many studies considered how alternative damage functions, meant to reflect that damages may be more convex or present abrupt jumps, affect the results (Pizer, 2003; Dumas and Ha-Duong, 2005; Ackerman and Stanton, 2012; Wouter Botzen and van den Bergh, 2012; Dietz and Stern, 2015; Weitzman, 2012).

More recently, studies have included in an endogenous fashion different types of climate damages risks, i.e. that damages could differ from their expected value, in Integrated Assessment Models (IAMs). First, some have considered uncertainty over the damage function by considering a social planner facing a distribution of damage functions rather than a single estimate (see for instance Crost and Traeger (2013)). A second type of risks is recurring shocks or volatility in damages which hit the economy, notably as a way to reflect the occurrence of disasters (Pindyck and Wang, 2013; Bretschger and Vinogradova, 2019). There is an emerging literature on how these risks affect the SCC, also in combination with other uncertain dimensions, such as climate sensitivity or growth (Jensen and Traeger, 2016; Lemoine and Rudik, 2017; Van Den Bremer and van der Ploeg, 2018).

Tipping points belong to another category of risk. They entail that emissions can irreversibly shift the world from a low- to a high-damage regime. Pioneering works on the management of thresholds show how optimal abatement is affected by the risk that pollution may trigger a catastrophic event (Clarke and Reed, 1994; Tsur and Zemel, 1996). Studies which model tipping points as a stochastic risk in IAMs suggest that accounting for those raises the SCC (Lemoine and Traeger, 2014; Cai and Lontzek, 2019; Diaz and Keller, 2016) or near-term abatement (Keller et al., 2004; Belaia et al., 2014). These studies consider tipping points which induce either a permanent loss of productivity (Belaia et al., 2014; van der Ploeg and de Zeeuw, 2018), the destruction of most productive capacities (Bretschger and Vinogradova, 2018), or an increased damage regime (Lemoine and Traeger, 2014), and provide insightful analysis, notably on how time and risk preferences affect optimal policy.

When considering risk, one naturally wishes to compare how results differ from a risk-free situation only relying on the expected outcome. In financial analysis, this comparison allows to derive the risk premium of an asset. Similarly, for the SCC, the gap between a risky and a risk-free situation indicates how much risk compounds the diminution of social welfare brought by emitting carbon. This is typically done by comparing results with risk to results when all the parameters are set to their expected value (Crost and Traeger, 2013; Van Den Bremer and van der Ploeg, 2018). However, for tipping points, the standard practice is to compare the SCC with and without a tipping point. Because the additional damage a tipping point could cause does not have a zero mean, comparing cases with and without tipping points conflates the effect of increased expected damages resulting from the introduction of a tipping point, and that from the dispersion of possible damages, i.e., a zero mean risk.

In this paper, we propose a simple method to distinguish the contributions of expected damages and risk to the SCC in the case of a tipping point triggering a shift to a higher-damage regime. We decompose a tipping point on damages as an increase in expected damages plus a (zero-mean) pure risk on damages. We compare the SCC with a tipping point to the SCC under expected damages, i.e. when there is no risk and damages are set at their expected level. First, using a simplified model, we demonstrate that in the case of a stochastic tipping point, the SCC is at first order equal to the SCC under expected damage. Risk introduces a correction that is only of second order, and that is proportional to risk aversion. We then investigate numerically in an IAM the gap between the SCC with a stochastic tipping point and the SCC under expected damages. We introduce the tipping point as a stochastic risk whose hazard rate depends on temperature, leading to a permanent drop in productivity. We analyse how preferences of the social planner (i.e., risk aversion, resistance to intertemporal substitution) and damages from the tipping point affect the comparison. We find that the SCC under expected damages closely approximates the SCC for low values of the productivity shock. Thus, risk plays a minor role when introducing a tipping point: the increase in SCC is due to the increase of expected damages. This result holds as long as the magnitude of the productivity shock is less than 10%. However, risk becomes important with higher productivity shocks and under high risk aversion. The results are also robust when considering a tipping point leading to more convex damage.

Our article contributes to a wider literature on how climate damage risks affect optimal climate policies. Evidence is mixed about whether risk is a fundamental part of optimal abatement, or just a second order correction. On the one hand, some authors argue that hedging against catastrophic outcomes should be the primary driver of abatement (Weitzman, 2009; Pindyck, 2013; Dietz, 2011). However, several studies show that risk has a surprisingly limited effect on the optimal policy – this has been called the *risk aversion puzzle*. Risk aversion, when disentangled from the elasticity of intertemporal substitution, is found to play a modest role in IAMs (Ackerman et al., 2013), even in the case of a tipping point (Belaia et al., 2014; van der Ploeg, 2016). Our contribution isolates the role of risk in the case of a stochastic tipping point, and shows under which assumptions the effect of risk

is of second order. In our numerical simulations, risk only matters when there is possible exposure to catastrophic damages, while the SCC is primarily driven by expected damages for moderate productivity shocks triggered by the tipping point. This explains why studies find a modest role of risk aversion on the optimal policy, as productivity shocks usually considered in the literature remain below the 10% threshold. We add to this literature and provide orders of magnitude of the range of shocks and risk aversion it takes for risk aversion to significantly affect the SCC.

Our article also contributes to understanding the effect of different representations for climate damages on optimal policy. Since climate damages are the least-grounded aspect of IAMs (Diaz and Moore, 2017; Revesz et al., 2014; Howard, 2014) and have a strong impact on the SCC (as large as discounting (van den Bijgaart et al., 2016)), it is essential to build rigorous methodologies that compare how different representations of damages affect the SCC (Pottier et al., 2015; Guivarch and Pottier, 2018). Here, we compare indeed two different settings to represent a tipping point: one where tipping points are represented as a truly stochastic event, one where tipping points are represented as a mere increase in the damage function, so that it reflects expected damages. In other words, we examine how the endogenous introduction of a tipping point in an IAM compares to simply using a more convex function, i.e., the method that was first used to represent catastrophic risks in such models. Our results show in which cases the two methods lead to different outcomes.

We illustrate in section 2 the effects of increased expected damages and pure risk due to a tipping point in a simplified one-period model, and show that the former is the primary driver of the SCC. In section 3, we lay out the stochastic IAM we use and present how we compute the SCC under expected damages. Numerical comparison between the SCC under expected damages and the SCC with a stochastic tipping point are discussed in section 4. Section 5 concludes.

2 A simple model of a stochastic tipping point

We present a simple one-period model of a climate economy to highlight the intuition behind the comparison between the SCC with a stochastic tipping point, and the SCC under expected damages. To that end we will consider successively optimal abatement and the resulting SCC in three situations:

- When there is no tipping point (SCC^*),
- When there is a tipping point (SCC^t),
- When damages are set at their expected level (SCC^{ed}).

We show that there is a first-order difference between the SCC with tipping points (SCC^t) and without (SCC^*), difference that is captured when the SCC is calculated with damages set at their expected level (SCC^{ed}). We further demonstrate that risk introduces a second-order correction, proportional to risk aversion and to the variance of damages.

Let us consider an economy producing gross output Y. Gross output is reduced because of damage $d(E)$ due to emissions E, and costs $c(a)$ of abatement measures a. Net output is entirely consumed,

so that aggregate consumption is given by $C(a,E) = Y - c(a) - d(E)$. With σ the (unabated) carbon intensity of production, emissions and abatement are linked by $E = \sigma Y(1-a)$. Let $u(C)$ be the social welfare function that depends only on aggregate consumption, the program of the social planner is:

$$\max_{a,E} \; u\left(C\left(a,E\right)\right), \quad s.t. \; E = \sigma Y(1-a) \tag{1}$$

so that optimal abatement satisfies $c'(a^*) = \sigma Y d'(E^*)$ (we star all variables related to the optimal point in this case).

We can define the SCC at the optimal point as the marginal social damages occurring due to an extra emission, measured in consumption units.

$$SCC^* = -\frac{\partial_E u(C(a,E))|_{a^*,E^*}}{\partial_C u(C)|_{C^*}} = -\frac{-u'(C^*)d'(E^*)}{u'(C^*)} = d'(E^*) \tag{2}$$

Note that (provided the constraints $0 \leq a \leq 1$ are not binding) the equation for optimal abatement can be written as:

$$\partial_C u|_{C^*} .(-c'(a^*)) + \partial_E u(C(a,E))|_{C^*} .(-\sigma Y) = 0 \tag{3}$$

so that $SCC^* = \frac{c'(a^*)}{\sigma Y}$: Thus the SCC is also a measure of marginal abatement costs.

2.1 SCC with a stochastic tipping point

We now introduce stochastic tipping points to our setting. There are several states of nature ω, depending on whether or not tipping points have been crossed. The damage function is $d_\omega(E)$ in state ω. The probability of having triggered tipping points, i.e. on being in state ω depends on aggregate emissions $p_\omega(E)$. A simple case is when there are one tipping point and two states of nature: 1 the pre-tipping point world where the damage function is the d_1, and 2 the post tipping point world that leads to a higher damage regime $d_2 \geq d_1$. The tipping point induces a jump in damages $d_2 - d_1$. The probability of being in the post-tipping world is an increasing function of the quantities of emission released $p_2(E)$. Our framework accommodates for more general formulations, such as multiple points or a single tipping point leading to different post-tipping damage functions.

The social planner maximizes the expected social welfare, where we write $C_\omega(a,E) = Y - c(a) - d_\omega(E)$ the aggregate consumption in state ω:

$$\max_{a,E} \; \mathbb{E}_\omega\left[u(C_\omega(a,E))\right], \quad s.t. \; E = \sigma Y(1-a) \tag{4}$$

We note a^t, E^t the optimal abatement and emissions of this program. The SCC is the marginal social damages of an extra emission measured in consumption units, so, in this framework:

$$SCC^t = -\frac{\partial_E \mathbb{E}_\omega\left[u(C_\omega(a,E))\right]|_{a^t,E^t}}{\mathbb{E}_\omega\left[u'(C_\omega(a,E))\right]|_{a^t,E^t}} \tag{5}$$

Note that for tipping points, the probability of being in a given state of nature is endogenous to the action of the planner, in contrast with exogenous risk (for instance on the value of climate sensitivity). The optimal abatement a^t of this program solves (provided the constraints on it are not binding):

$$\mathbb{E}_\omega \left[u'(C_\omega(a,E))\right]|_{a^t,E^t}.(-c'(a^t)) + (\partial_E \mathbb{E}_\omega \left[u(C_\omega(a,E))\right]|_{a^t,E^t}).(-\sigma Y) = 0 \tag{6}$$

so that the SCC is still given by $\frac{c'(a^t)}{\sigma Y}$. Specifying to the case of a single tipping point, we have $\mathbb{E}_\omega[u(C_\omega(a,E))] = (1-p_2(E)).u(C_1(a,E)) + p_2(E).u(C_2(a,E))$, and the SCC of equation (5) can be written as:

$$\begin{aligned} SCC^t = \frac{c'(a^t)}{\sigma Y} &= d_1'(E^t) \\ &+ (d_2'(E^t) - d_1'(E^t))\frac{p_2(E^t)u'(C_2(a^t))}{\mathbb{E}_\omega[u'(C_\omega(a,E))]} \\ &+ p_2'(E^t)\frac{u(C_1(a^t)) - u(C_2(a^t))}{\mathbb{E}_\omega[u'(C_\omega(a,E))]} \end{aligned} \tag{7}$$

As in van der Ploeg and de Zeeuw (2019), the previous equation offers a decomposition of the SCC into three terms:

- the marginal damage term, before reaching the tipping point,
- the additional marginal damage in case the tipping point is reached, weighted by the probability of tipping,
- the damage due to a marginal increase in the probability of triggering the tipping point.

Introducing tipping points in this way conflates a level effect and a risk effect. Damages are indeed increased in the same time as risk is introduced, because the effect of the tipping point is not a zero-mean risk on consumption. Indeed, compared to the situation without a tipping point, the mean additional risk of a tipping point is $(1-p(E))(d_1-d) + p(E)(d_2-d)$, which is equal to $p(E)(d_2-d_1)$ when the pre-tipping damage function is the same as the damage function without a tipping point ($d_1 = d$). As a consequence, the increase of the SCC found when a tipping point is introduced cannot be attributed to risk alone, it may be simply the effect of higher expected damages. The next subsection will investigate this point.

2.2 SCC under expected damages

To understand the effect of risk induced by a tipping point, we need a way to decompose the SCC so that we can disentangle what comes from expected damages from what is due to a zero-mean risk.

Let us go back to equation (5). We consider the expected damage function $\tilde{d}(E) = \mathbb{E}_\omega[d_\omega(E)]$. One can write: $C_\omega(a,E) = \tilde{C}(a,E) + \epsilon_\omega(E)$ with $\tilde{C}(a,E) = \mathbb{E}_\omega[C_\omega(a,E)] = Y - c(a) - \tilde{d}(E)$. This

decomposes the effect of tipping point on consumption as an effect on expected damages plus a zero-mean risk $\epsilon_\omega(E) = d_\omega(E) - \tilde{d}(E)$. We note $V_\epsilon(E) = \mathbb{E}_\omega[\epsilon_\omega^2(E)]$ the variance of damages of the zero-mean risk ϵ.

In the case of a single tipping point, expected damages are $\tilde{d} = p_1 d_1 + p_2 d_2 = d_1 + p_2.(d_2 - d_1)$. The risk ϵ on consumption is $\epsilon_1 = -p_2(d_2 - d_1)$ in state 1, with probability $1-p_2$, and $\epsilon_2 = (1-p_2)(d_2-d_1)$ in state 2 with probability p_2 (where all symbols are functions of emissions E). So, compared to a world without tipping points, in which the damage function is the pre-tipping damages function, introducing a tipping point increases expected damages by $p_2.(d_2 - d_1)$ and adds risk. Thus, the risk of the tipping point is that damages are less than expected if the world does not tip (state 1, pre-tipping) and that damages are above expectations if the world does tip (state 2, post-tipping). It is a zero-mean risk whose variance is $V_\epsilon(E) = p_2(1-p_2).(d_2 - d_1)^2$. The risk is of the same order of magnitude as the jump in damages $d_2 - d_1$.

Let us assume that the risk ϵ is small and make a Taylor-expansion at the second-order in formula (5):

$$SCC^t = -\frac{\partial_E \mathbb{E}_\omega \left[u(\tilde{C}(a,E)) + u'(\tilde{C}(a,E))\epsilon_\omega(E) + u''(\tilde{C}(a,E))\frac{\epsilon_\omega^2(E)}{2} \right]\Big|_{a^t,E^t}}{\mathbb{E}_\omega \left[u'(C_\omega(a,E)) \right]|_{a^t,E^t}} \tag{8}$$

$$= \frac{u'(\tilde{C}(a^t,E^t))\tilde{d}'(E^t)}{\mathbb{E}_\omega \left[u'(C_\omega(a,E)) \right]|_{a^t,E^t}} - \frac{\partial_E \left(u''(\tilde{C}(a,E))\frac{V_\epsilon(E)}{2} \right)\Big|_{a^t,E^t}}{\mathbb{E}_\omega \left[u'(C_\omega(a,E)) \right]|_{a^t,E^t}} \tag{9}$$

We show in appendix A that we can make a Taylor expansion also of the denominator and, after reordering, we obtain the following decomposition of the SCC (ignoring higher-than-second-order terms):

$$SCC^t = \tilde{d}'(E^t) + \gamma(\tilde{C}(a^t,E^t))\frac{\tilde{C}(a^t,E^t)}{2}\ \partial_E V_\epsilon(E)|_{E^t} \tag{10}$$

The SCC in the presence of a tipping point is thus shown to be the sum of marginal expected damages plus a second-order correction that is proportional to the Arrow-Pratt measure of relative risk aversion $\gamma(C) = -u''(C)/(u'(C).C)$ of the utility u, and to the marginal increase in the variance of risk $\partial_E V_\epsilon(E)$. When the effect of tipping point is split into an increase in expected damage plus a zero-mean risk, this decomposition shows that changes in expected damages drive the effect on the SCC whereas the zero-mean risk introduces only a second-order correction.

To make this statement more precise, we introduce the SCC under expected damages, that is the SCC in a deterministic model similar to equation 1 but with the damage function replaced by the expected damages of the tipping point $\tilde{d}$. We note a^{ed}, E^{ed} the optimal abatement and emissions of this program under expected damages. Thus the SCC under expected damages SCC^{ed} is given by:

$$SCC^{ed} = \frac{c'(a^{ed})}{\sigma Y} = \tilde{d}'(E^{ed}) \tag{11}$$

We have the following proposition:

Proposition 1 *The difference between the SCC without a tipping point (SCC^*) and the SCC with a tipping point (SCC^t) is proportional to the difference between marginal expected damages and marginal damage in the pre-tipping point state of the world. It is first-order in the magnitude of risk induced by the tipping point.*

On the contrary, the difference between the SCC under expected damages (SCC^{ed}) and the SCC with a tipping point (SCC^t) is only second-order. It is proportional to risk aversion and the marginal increase in the variance of risk induced by the tipping point.

Proof See appendix B

To summarize, we have demonstrated that introducing a tipping point raises the SCC, compared to a SCC without a tipping point, where the damage function is given by the pre-tipping point damage function. This first order difference is explained by an increase in expected damages. Whereas comparing the SCC with and without a tipping point introduces a first-order correction in the SCC, the correction is only of second order when we compare the SCC with a tipping point and the SCC under expected damages.

This means first that it is misleading to compare the SCC with and without stochastic tipping point, and second that the increase due to the tipping point is, in this simple model, a matter of expected level rather than risk. The SCC under expected damages captures most of the value of the SCC, as long as the damage shocks are small. Risk introduces a correction that increases with risk aversion. Interestingly, this result holds under rather general conditions. We did not assume a specific form for the change in damages induced by the tipping point. Hence, it applies generally, and in particular, in the case of one tipping point, it applies when the tipping point induces a jump in damage ($d_2 - d_1$ is constant) or more convex damage, i.e. higher marginal damage ($d_2 - d_1$ is an increasing function of temperature).

The key insight of this simple one-period model is that we can think of the SCC as being composed of two parts: SCC under expected damages and a risk premium proportional to risk aversion. For small tipping point damages, the SCC under expected damages will be close to the actual SCC. In the next section, we define the SCC under expected damages for an intertemporal model and use an IAM to explore numerically the gap between both SCC.

3 Contributions of expected damages vs. risk in a stochastic IAM

We now use an IAM to numerically compare the SCC under a stochastic tipping point and the SCC under expected damages, in order to quantify the contribution of pure risk to the SCC. This allows us to examine whether the intuition from the simple model holds in a multi-period framework, and also for larger damages. We present in section 3.1 the climate-economy model and in section 3.2 the social welfare functions we use. We then explain how we construct in this model the SCC under

expected damages (section 3.3), and the values we explore for the parameters of the model (section 3.4).

3.1 The climate-economy model

An IAM is meant to capture the main crossed interactions between the economy and the climate system. On the one hand, the economy, depending on growth, mitigation policies and technological choices, produces greenhouse gas emissions which interfere with the climate system. On the other, these changes in the climate system cause damages to the economy. An IAM allows to derive an optimal emission path from the point of view of a social planner balancing the costs of mitigation and damages of climate change, and to calculate the social value of intertemporal marginal damages caused by emissions – the SCC.

We use a classical DICE-like model, building on the Ramsey-Cass-Koopmans framework (Guivarch and Pottier, 2018). The economy produces a single good in quantity Q_t using two factors, capital K_t and labour L_t through a Cobb-Douglas function. The productivity is affected by climate change via a damage factor[1] Ω_t that depends on temperature T_t, so that final production Y_t writes:

$$Y_t = \Omega(T_t) A_t K_t^{\alpha} L_t^{1-\alpha} \tag{12}$$

Production induces emissions, which can be mitigated at a certain cost. The social planner trades off consumption, mitigation costs (which represent a share Λ_t of production), and investment in capital (share s_t of production)

$$C_t = Y_t(1 - \Lambda_t - s_t) \tag{13}$$

$$\Lambda_t = \theta_1(t)\mu_t^{\theta_2} \tag{14}$$

$$K_{t+1} - K_t = -\delta K_t + Y_t s_t \tag{15}$$

where δ is capital depreciation, and μ_t the abatement rate. $\theta_1(t)$ measures total mitigation costs and decreases exogenously due to technical progress.

The difference with DICE equations concerns the climate system. It has been shown that DICE's climate model implies a lag between CO_2 emissions and warming that is too long, i.e., the temperature rises too slowly in response to emissions (National Academies of Sciences, 2016; Mattauch et al., 2019) and is inconsistent at long timescales (Glotter et al., 2014). We adopt a simple linear formula linking temperature change to cumulative CO_2 emissions, as in Guivarch and Pottier (2018);

[1] For notational convenience, we use damage factor Ω instead of damage function D. The correspondence is simply $\Omega = 1 - D$.

Dietz and Venmans (2019). Indeed, the ratio of global temperature increase to cumulative emissions has been shown to be almost independent of time and of emission pathways in simulations of the response to a range of emission scenarios with climate models, as well as in observations (Matthews et al., 2009; Zickfeld et al., 2009, 2013; Gillett et al., 2013; Collins et al., 2013). There are physical explanations to this near-linear dependence between warming and cumulative carbon emissions, due to the compensating effects of oceanic uptake of heat and carbon (Solomon et al., 2009; Goodwin et al., 2015; MacDougall and Friedlingstein, 2015). There are also limitations to such a simple climate representation, for instance Leduc et al. (2015) have shown that the linear relationship between temperature change and cumulative emissions is no longer valid for high emission pathways such as the RCP 8.5.

Our equation for temperature change is thus:

$$T_t = \beta(CE_0 + \sum_{s=0}^{t} E_s) \tag{16}$$

where T_t is the global temperature increase at time t, CE_0 is cumulated emissions up to the first period of the model and E_s the emissions at time s. The current stock of cumulated emissions is $S_t = CE_0 + \sum_{s=0}^{t} E_s$.

$$E_t = \sigma_t Y_t (1 - \mu_t) \tag{17}$$

where σ_t is the carbon content of production that decreases exogenously over time, and μ_t the abatement rate.

In our central estimates, we model the tipping point as a stochastic process with an endogenous hazard rate h_t, leading to a permanent productivity shock, in line with Cai and Lontzek (2019); van der Ploeg and de Zeeuw (2018); Belaia et al. (2014). Such a change in the damage function can potentially apply to a large range of tipping points inducing larger damages than expected. It can be direct impact on the economy, either caused by the melting of ice caps, leading to severe sea-level rise; a slowing down of thermohaline circulation; or a social tipping point beyond which adaptation is no longer possible. Before the tipping point, the damage factor is:

$$\Omega_1(T) = \frac{1}{1 + \pi T^2} \tag{18}$$

In our central case, once the tipping point has been reached, the damage factor writes:

$$\Omega_2(T) = \frac{1 - J}{1 + \pi T^2} \tag{19}$$

J is the magnitude of the productivity shock, ranging from 0 to 1. In this representation, the tipping point increases the level of damages, but it does not affect the convexity of the damage

function. Indeed, damages go approximatively from $\approx \pi T^2$ to $\approx \pi T^2 + J$. Uncertainty about the convexity of damages may lead to greater changes than uncertainty about the level of damages (Crost and Traeger, 2013), so we also consider in the Annex the case of a tipping point increasing marginal damages.

At each period, the tipping point occurs with a hazard rate h_t which depends on the temperature level. We assume that the location of the tipping point is unknown. The initial prior is that the tipping point is uniformly distributed between $T_{\min}$ and $T_{\max}$. At each time $t-1$ with temperature T_{t-1}, the social planner learns whether the tipping point has been reached or not, as in Lemoine and Traeger (2014). If it has not been reached, this means that it can only occur when temperature is above T_{t-1}, so that the social planner updates prior for the next period. Hence the probability to reach the tipping point at t conditional to not reaching it at $t-1$ is given by:

$$h_t(T_t, T_{t-1}) = \begin{cases} 0 & \text{if } T_t \leq T_{t-1} \text{ or } T_t \leq T_{\min} \\ \frac{T_t - \max(T_{\min}, T_{t-1})}{T_{\max} - \max(T_{\min}, T_{t-1})} & \text{if } T_t > T_{t-1} \text{ and } T_{\min} \leq T_t \leq T_{\max} \\ 1 & \text{if } T_t > T_{t-1} \text{ and } T_t \geq T_{\max} \end{cases} \tag{20}$$

Note that the marginal hazard rate tends to increase (i.e., $\partial_2 \partial_1 h_t \geq 0$), as experienced temperatures increase. This setting differs from the representation chosen in Cai and Lontzek (2019); van der Ploeg and de Zeeuw (2018), in which the hazard rate depends solely on current temperature with no learning. In that case the tipping point is therefore unavoidable in the long term. In our setting however, the tipping point is avoided with certainty if temperature stabilizes below $T_{\min}$; it can be avoided - but with no certainty - if temperature stabilizes between $T_{\min}$ and $T_{\max}$, and it is triggered with certainty if $T_{\max}$ is exceeded. This representation does not consider possible processes where a tipping point would be triggered with some lag.

3.2 Social welfare functions

We study two types of social welfare functions: expected utilitarianism with Constant Relative Risk Aversion (CRRA), and an Epstein-Zin social welfare function. In the CRRA representation, time and risk preferences are embedded in a single parameter, elasticity of marginal utility, which conflates the resistance to intertemporal substitution and the risk aversion. However, the resistance to intertemporal substitution and risk aversion can have opposite effects in the presence of risk (Ha-Duong and Treich, 2004): while the former favours the consumption of present generations, the latter encourages more abatement in the present in order to lower the risk of triggering the tipping point. For this reason, we also apply Epstein-Zin preferences, which disentangle intertemporal substitution and risk aversion.

Welfare after time t, U_t, is defined recursively:

– For CRRA preferences

$$U_t = \left(1 - \frac{1}{1+\rho}\right) u_t + \frac{1}{1+\rho}\mathbb{E}[U_{t+1}] \tag{21}$$

where ρ is the pure time preference rate, and utility at each time step is given by:

$$u_t = L_t \frac{(C_t/L_t)^{1-\eta}}{1-\eta} \tag{22}$$

η is the elasticity of marginal utility.

So that we can define Bellman functions as follows:

$$V_t(x_t) = \max_{y_t} \left[u(x_t, y_t) + \frac{1}{1+\rho}\mathbb{E}[V_{t+1}(G(x_t, y_t))] \right] \tag{23}$$

where $x_t = (S_t, K_t)$ are state variables, $y_t = (\mu_t, s_t)$ are control variables, and $x_{t+1} = G(x_t, y_t)$ is the transfer function.

– For Epstein Zin preferences:[2]

$$U_t = \left(\left(1 - \frac{1}{1+\rho}\right) u_t + \frac{1}{1+\rho}\mathbb{E}[U_{t+1}^{1-\gamma}]^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1}{1-\theta}} \tag{24}$$

$$u_t = L_t \frac{(C_t/L_t)^{1-\theta}}{1-\theta} \tag{25}$$

For the sake of clarity, we use different notations in the Epstein-Zin case. We denote θ the resistance to intertemporal substitution (the inverse of the elasticity of intertemporal substitution), and γ the risk aversion parameter.

We can define Bellman functions in order to solve this dynamic program: $V_t = \frac{U_t^{1-\theta}}{1-\frac{1}{1+\rho}}$.

$$V_t(x_t) = \max_{y_t} [u(x_t, y_t) + \frac{1}{1+\rho} f(V_{t+1}(G(x_t, y_t)))] \tag{26}$$

f accounts for the decision maker's attitude towards the risk of reaching a tipping point.[3] $f(V_{t+1}) = [\mathbb{E}(V_{t+1}^{\frac{1-\gamma}{1-\theta}})]^{\frac{1-\theta}{1-\gamma}}$. It is the same formula as for CRRA preferences, in which $f = \mathbb{E}$.

Using dynamic programming, we first approximate Bellman functions in the post-tipping world, and then in the pre-tipping world using expectations over the temperature at which the tipping point occurs.

[2] The formula holds for $\theta<1$. Otherwise when $\theta > 1$ utility function is negative, so that $U_t = -(-(1 - \frac{1}{1+\rho})u + \frac{1}{1+\rho}[\mathbb{E}_t(-U_{t+1})^{1-\gamma}]^{\frac{1-\theta}{1-\gamma}})^{\frac{1}{1-\theta}}$

[3] when $0 < \theta < 1$, the recursive formula involves $u_t - \frac{1}{1+\rho} f(-V_{t+1})$

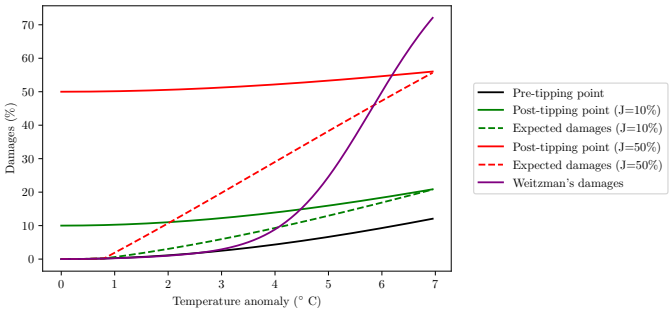


Fig. 1
Comparison between the two approaches. If the stochastic tipping point is triggered, the damage function jumps from pre-tipping point level (black curve) to a post-tipping point level (the green curve for a productivity shock of 10%, the red curve for a shock of 50%). The expected damage function of a tipping point is the prior expected damage level at each temperature (dashed curves), given that the tipping point is uniformly distributed between $T_{min} = 0.8°C$ and $T_{max} = 7°C$. The sextic damage function proposed in Weitzman (2012) is pictured for comparison.

3.3 Comparing the SCC for a stochastic tipping point and the SCC under expected damages

If S is the stock of emissions (* denotes that control variables y_0 are optimally chosen given x_0.), the SCC (at initial time) under a stochastic tipping point writes:

$$SCC = -\frac{1}{1+\rho}\frac{\partial_S \mathbb{E}[V_1]|_{x_1}}{\partial_C V_0|_{(x_0,y_0^*)}} \tag{27}$$

We use a modified damage factor $\Omega_{ed}(T)$ to calculate the SCC under expected damages. This modified damage factor represents the expected damage factor given the prior on the temperature at which the tipping point occurs. Let us note $p(T)$ the prior probability of having reached the tipping point at temperature T. The expected damage factor writes:

$$\Omega_{ed}(T) = (1 - p(T))\Omega_1(T) + p(T)\Omega_2(T) \tag{28}$$

Damages at a given temperature are set at their expected level given the prior knowledge on the temperature at which the tipping point occurs. Figure 1 shows the resulting expected damages for two productivity shocks ($J = 10\%$ and $J = 50\%$). Even though the tipping point only affects the level of damages, the expected damage function is more convex than the pre-tipping point damage function. Indeed, whereas the tipping point changes the damage function from $\approx \pi T^2$ to $\approx \pi T^2 + J$, the expected damage function is $\approx \pi T^2 + p(T)J$, to be compared to $\approx \pi T^2$ without a tipping point.

For each computation of the SCC for a stochastic tipping SCC, we can compute the corresponding SCC under expected damages, noted SCC^{ed}, which is the SCC of a deterministic run with damages set at their expected level. The SCC in a stochastic setting represents the full effect of the tipping point, whereas we take SCC^{ed} as representing the level effect of the tipping point, absent risk. We

will analyse the ratio SCC^{ed}/SCC, which is the part of the SCC that is explained by expected damages. Its complement is the part of the SCC that is due to a pure risk effect, purged of any level effect.

3.4 Calibration of the parameters

We summarize in table 1 the values and range explored for the parameters of interest. We use typical ranges of possible values for parameters related to attitudes towards risk and time. The pure rate of time preference is set at 1.5% (Nordhaus, 2008), — a lower value of 0.5% is explored in the Appendix. The elasticity of marginal utility ranges from 0.8 to 3 in the CRRA case. In the Epstein-Zin case, θ is set at 0.8, and we perform a sensitivity analysis in the Appendix. Considering the range used in the literature (Ackerman et al., 2013; Crost and Traeger, 2013; Jensen and Traeger, 2014; Cai and Lontzek, 2019), we explore risk aversion (γ) from 0.5 to 20.

For the parameters describing the tipping point, most of the published literature use impacts under a 10% decrease of productivity. Nevertheless, we acknowledge that the impacts of such a phenomenon are difficult to quantify and could be very large, so we explore a much larger window for the productivity shock J, from 0 to 50%. The temperature at which the tipping point would occur is uncertain, and we assume it is distributed between current temperature ($T_{\min} = 0.8$) and $T_{\max} = 7°$C (Lemoine and Traeger (2014) consider the upper bound for the temperature threshold between 3 and 9°C). This means for instance that there is a 19% probability of triggering a tipping point between 0.8 and 2°C. [4]

Table 1
Main parameters for the stochastic tipping point numerical exercise. All other parameters are calibrated according to Guivarch and Pottier (2018)

Parameter	Value (Sensitivity test)
Pure rate of time preference (ρ)	1.5% (0.5)
Elasticity of marginal utility (η)	from 0.8 to 3
Resistance to intertemporal substitution (θ)	0.8 (0.5 and 1.5)
Risk aversion (γ)	from 0.5 to 20
Productivity shock (J)	from 0 to 50%
Minimum temperature threshold ($T_{\min}$)	0.8
Maximum temperature threshold ($T_{\max}$)	7 (10)

[4] A sensitivity test using $T_{\max} = 10$ is performed in the Appendix. As said above, we also explore in the Appendix the case of a tipping point affecting the convexity of the damage function, rather than its level. Results are similar to those presented in the main text.

4 Results

We present the comparison between the SCC under a stochastic tipping point and the SCC under expected damages, first with CRRA preferences, then with Epstein-Zin preferences where risk aversion and resistance to intertemporal substitution differ. We finally discuss the significance of our results and lessons that can be drawn for the *risk aversion puzzle*.

4.1 With CRRA preferences

We compute the SCC for a stochastic tipping point for different values of the elasticity of marginal utility (η) and productivity shocks (J) in the range specified in section 3.4, as well as the SCC under expected damages SCC^{ed}.

One striking result is that the ratio SCC^{ed}/SCC is very close to one for low productivity shocks and low risk aversion (see figure 2, panel b), meaning that most of the SCC stems from expected damages enhanced by the tipping point rather than from the risk on damages the tipping point introduces. As J increases, aversion to the risk of high damages make the SCC rise faster than the SCC under expected damages, so that the ratio decreases.

The share of SCC due to expected damages also decreases as the elasticity of marginal utility η increases. The role of the elasticity of marginal utility η on the SCC is *a priori* ambiguous. Indeed, CRRA preferences conflate intertemporal trade-offs and risk aversion, and η has opposing effects on the SCC. On the one hand, a higher η favours present consumption relative to future consumption of wealthier generations (intertemporal substitution), which decreases the SCC. On the other hand, it encourages mitigation to reduce the risk induced by reaching a tipping point (risk aversion), which increases the SCC. In the case of SCC^{ed}, only the intertemporal substitution effect is at play, not the countervailing risk aversion effect. Thus the SCC under expected damages (SCC^{ed}) decreases faster with η than the SCC does, and the ratio SCC^{ed}/SCC decreases with η. Note that in the stochastic case, the intertemporal substitution effect outweighs the risk aversion effect: for a given productivity shock J, the SCC decreases when η increases (figure 2, panel a).

Though SCC varies by more than an order of magnitude with the ranges of shocks and preferences explored here, most of this variation is explained by expected damages. For example, at $\eta = 2$, introducing a tipping point with a shock of $J = 10\%$ trebles the SCC from 34 to 103 \$/tCO_2. However, this increase is not related to risk but to the simple fact that expected damages have increased – indeed, SCC^{ed} is 97 \$/tCO_2. A mere 6 percent of the SCC is due to a pure risk effect.

Numerically, it takes both a high productivity shock and a high elasticity of marginal utility for SCC^{ed} to significantly underestimate SCC. Expected damages explain more than 90% of the SCC, as long as the productivity shock is inferior to 10%, whatever the value of risk aversion in the range explored here. Only with productivity shocks higher than 40%, jointly with an elasticity of marginal utility higher than 2, does risk contribute to around half of the SCC.

The same pattern is found with a lower pure time preference rate (ρ). Though a lower ρ significantly raises the level of the SCC, it does so to the same extent in the stochastic case and under expected damages, so that the part of the SCC explained by expected damages is similar in the case of a lower ρ (see graph 7 in the Appendix).

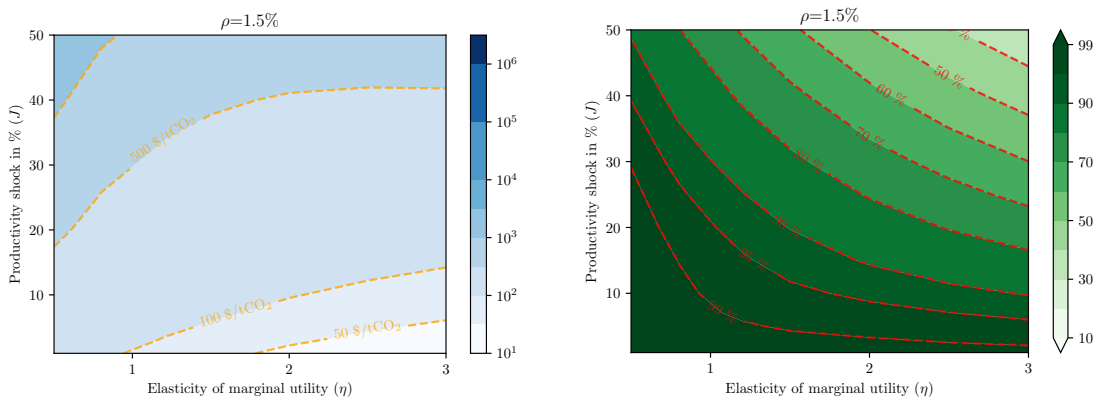
4.2 With Epstein-Zin preferences

We perform the same exercise using Epstein-Zin preferences, i.e., disentangling preferences for risk and intertemporal substitution. We set the resistance to intertemporal substitution (θ) at 0.8, and present the results when the risk aversion γ and the productivity shock J vary in figure 3. The corresponding graphs for different values of resistance to intertemporal substitution and pure rate of time preference can be found in the Appendix as a sensitivity check, as well as alternative values for $T_{\max}$.

The part of the SCC explained by expected damages has a similar pattern as in the case of CRRA preferences. As expected, the ratio SCC^{ed}/SCC decreases with risk aversion γ and productivity shock J. Values are somewhat similar to the case with CRRA preferences (with a correspondence $\eta \sim \gamma$) but as we explore a much larger range in risk aversion, the part explained by expected damages can become much lower. Interestingly, the SCC does not significantly increase with risk aversion for low values of the productivity shock (see figure 3, panel a.), hence the low horizontal gradient below $J = 10\%$. However, for higher productivity shocks, the SCC shows a high sensitivity to risk aversion, and a higher γ leads to SCC orders of magnitude greater. This suggests that risk aversion only matters when the economy is exposed to catastrophic risks.

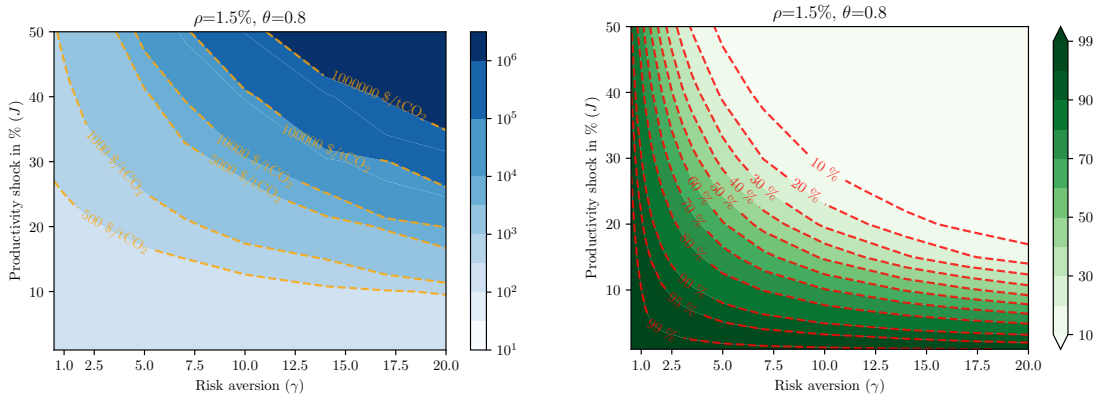
For instance, for a productivity shock equal to 10%, 90% of the SCC is explained by expected damages up to a risk aversion of 4 (as in the CRRA case), but the part explained is only 60% when $\gamma = 15$. Productivity shocks higher than 25%, combined with risk aversion higher than 5, lead to the ratio SCC^{ed}/SCC below 50%. For a productivity shock equal to 40% and a risk aversion parameter equal to 5, the SCC under expected damages only makes 20% of the SCC.

The same holds for the sensitivity checks we explore in the Appendix, i.e., for alternative values of the pure time preference rate ($\rho = 0.5$) and of the elasticity of intertemporal substitution ($\theta = 0.5,\ 1.5$). A decrease in the elasticity of substitution (a higher θ) tends to decrease the SCC, but it does not affect how much the SCC is explained by expected damages. Indeed, θ plays a similar role in both deterministic and stochastic settings, as it governs the trade-off between future and present consumption. This is a strong indication that our construction of the expected damages has correctly isolated the level effect of the tipping point. It is graphically confirmed with panel b. of figure 4 in Appendix, where the iso-lines for the ratio SCC^{ed}/SCC are almost flat in the θ direction. For the same reason, changes in the pure time preference rate (ρ) or using a higher temperature threshold $T_{\max}$ do not affect much the shape or position of the contours of the ratio. Even when considering a tipping point affecting the convexity of the damage function, it takes both very high



(a) (b)

Fig. 2
Comparison of the Social Cost of Carbon to a risk-free SCC under expected damages for CRRA preferences: Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC^{ed}/SCC). The closer the ratio to 100 %, the less risk plays a role. 90% of the SCC comes from expected damages for shocks lower than 10%, while it takes shocks greater than 40% for risk to explain at most half of the SCC.



(a) (b)

Fig. 3
Comparison of the Social Cost of Carbon to a risk-free SCC under expected damages for CRRA preferences: Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC^{ed}/SCC). The ratio decreases with risk aversion and with the magnitude of the shock, but remains above 90% for productivity shocks lower than 10%, unless risk aversion is greater than 10.

post-tipping point marginal damages and high risk aversion for the SCC to deviate from the value given by expected damages.

4.3 Significance of our findings and discussion of the *risk aversion puzzle*

A productivity shock of 10% is in the higher range of those typically considered in the literature. For instance, Lontzek et al. (2015), with a similar framework, do not consider shocks above $J = 10\%$. Belaia et al. (2014) only considers productivity shocks below 4.5% when thermohaline circulation collapses. Other modelling choices, in Lemoine and Traeger (2014), assume a tipping point induces a change from a quadratic to a sextic damage function, i.e., Weitzman's damage function that relies on an expert panel explicitly considering physical tipping points. At 4°C, this corresponds to a change of damage factor from $\Omega_1 = 0.96$ to $\Omega_2 = 0.91$ (so equivalent to a productivity shock of 5%). Our results thus suggest that the increase of SCC found in studies considering tipping points is mostly due to an increase in expected damages, rather than to the risk itself.

Identifying that most of the SCC in the presence of a small tipping point is due to expected damages and not risk sheds light on the so-called *risk aversion puzzle*. It has been found in previous work that risk aversion has a surprisingly little effect on the SCC (Ackerman et al., 2013), even in the case of a tipping point (Belaia et al., 2014). When risk aversion is higher, the SCC does not change much. This seems counter-intuitive, as risk aversion, especially after it has been disentangled from the elasticity of substitution in Epstein-Zin preferences, is expected to significantly increase the SCC.

The simple model of section 2 reveals why this may be the case. Indeed, the SCC is equal to the SCC under expected damages plus a risk premium proportional to risk aversion. Moreover, the smaller the tipping point, the smaller the risk premium. For instance, for a productivity shock $J = 5\%$ and a risk aversion $\gamma = 1$, the SCC under expected damages represents 99.5% of the value of the SCC. Based on the simplified model, increasing γ from 1 to 20 increases the SCC from a base $100 = 99.5 + 0.5$ to $109.5 = 99.5 + 20 \times 0.5$, i.e., an increase of less than 10%. However, for a productivity shock $J = 50\%$, the SCC under expected damages represents around 70% of the SCC at $\gamma = 1$. Setting $\gamma = 20$ increases the SCC from a base $100 = 70 + 30$ to $670 = 70 + 20 \times 30$. In our numerical results, the effects of risk turns out to be more than proportional to risk aversion, but the pattern is as described by the simple model of section 2.

While the numerical results explain away the *risk aversion puzzle* for small shocks, they also show that this puzzle does not always hold. We quantify the magnitude of the shock it takes for risk aversion to play a role. Shocks leading to a minimum of a 10% drop in productivity are necessary for risk aversion to impact the SCC.

Only few studies explore the possibility of large shocks, which are required for risk aversion to significantly affect the SCC: for instance, Dietz (2011) explores damages as large as 90 % of

consumption; Méjean et al. (2020) explore possible extinction, though with a very low probability. In such cases, risk aversion is expected to play a significant role.

5 Conclusion

When considering climate damages, one might wonder whether it is their expected level or their possible dispersion (i.e., risk) that warrants undertaking mitigation actions. This question has been studied for many types of risks, for instance regarding climate sensitivity or other critical aspects of the climate-economy system, but has not been applied to tipping points in damage functions.

Our research fills this gap. We model a tipping point as an endogenous risk, with a hazard rate increasing with temperature, leading to a permanent productivity drop. First, we demonstrate in a simple setting that the SCC with a stochastic tipping point is at the first order the SCC with damages set at their expected level. We thus isolate the effect of the increased expected damages brought by the introduction of a tipping point, so that the difference between the two methods can be attributed to the sole effect of risk. Numerically, when risk aversion is equal to resistance to intertemporal substitution (CRRA preferences), the SCC under expected damages closely approximates the SCC for a low productivity shock and a low risk aversion. Even when disentangling resistance to intertemporal substitution and risk aversion, using Epstein-Zin preferences, the share of SCC attributable to risk remains limited. For both Social Welfare Functions, risk contributes less than 10% to the SCC, as long as shocks remain below 10% of production and risk aversion is below 10. We test the robustness of our results with a number of sensitivity runs regarding the parametrization of the tipping point and preferences. In particular, the results are qualitatively unchanged when considering a tipping point that leads to more convex damages rather than higher damages.

Providing realistic values for the damage shocks triggered by tipping points is beyond the scope of the article, but we nevertheless provide orders of magnitude for the type of damages it may take for risk to play a role. This sheds light on the *risk aversion puzzle*. We show that risk aversion only plays a role when the economy is exposed to very high post-tipping damages. When low post-tipping point impacts are considered, the SCC is sensitive to expected damages, so that risk aversion plays a moderate role, in particular compared to time preferences. Thus, the low level of possible damages considered in the literature explains the *risk aversion puzzle*. However, this stresses once again that the shape of the damage function is central in IAMs. In particular, provided that the risk of triggering a tipping point increases with temperature, expected damages are more convex when considering tipping points. Thus, simply using more convex damage functions in a deterministic fashion may be a good proxy to determine the SCC, as was done for instance by Pizer (2003); Ackerman and Stanton (2012); Wouter Botzen and van den Bergh (2012); Dietz and Stern (2015); Weitzman (2012).

These results can be extended to other situations. For instance, multiple tipping points (Lemoine and Traeger, 2016b) are equivalent to a single tipping point with compounding damages. Our approach could be applied to this case, provided that these combined effects are accounted for (i.e.,

including the increased probability of triggering another tipping point if one is reached). Damages affecting growth can also result into a higher SCC (Moore and Diaz, 2015), but they would make the comparison with expected damages more difficult, because of the different time profiles of damages, with losses of potential growth adding up each year (Guivarch and Pottier, 2018). Thus, we leave for future research the question of whether the risk effect plays a greater role for a tipping point affecting growth.

Because the role of preferences is critical to assess the optimal strategy under uncertainty, a direct extension of our work would be to study how the comparison between methods is affected by alternative preferences for the social planner. Epstein-Zin preferences have recently received a lot of attention in this literature, but they may violate first-order stochastic dominance (Bommier and Le Grand, 2014). Bommier et al. (2015) propose another class of preferences, which could be used in the case of catastrophic risks. The social planner may also be ambiguity averse to differing worldviews about tipping points (Berger et al., 2016; Lemoine and Traeger, 2016a). Though we have focused here on how uncertainty about the tipping points affects optimal policy, this uncertainty could be combined with other uncertainties at stake, such as climate sensitivity or the volality of growth (Cai and Lontzek, 2019; Van Den Bremer and van der Ploeg, 2018). Finally, even if the damage function is the weakest point of IAMs, they also embed a number of assumptions about the climate-economy system that may be regarded as simplistic and questionable (Pindyck, 2013).

Deterministic approaches using best-guess expected damages (together with sensitivity analyses) are currently used to set a value for the SCC for policy evaluations (IAWG, 2010), and benefit from a lower computational burden than a fully fledged stochastic model. Knowing when deterministic approaches can be used as a good proxy for computing the SCC under risk can guide policy-making. Our results show that the SCC is primarily driven by the expected level of damages, when the shock induced by a potential tipping point remains lower than 10% or so. In that case, the effects of tipping points are well captured by updating the damage function typically used to a more convex function, reflecting the increasing probability to trigger a tipping point as the Earth warms.

References

Ackerman F, Stanton E (2012) Climate risks and carbon prices: Revising the social cost of carbon. Economics: The Open-Access, Open-Assessment E-Journal 6:10

Ackerman F, Stanton EA, Bueno R (2013) Epstein–Zin utility in DICE: Is risk aversion irrelevant to climate policy? Environmental and Resource Economics 56(1):73–84

Alley RB, Marotzke J, Nordhaus WD, Overpeck JT, Peteet DM, Pielke RA, Pierrehumbert RT, Rhines PB, Stocker TF, Talley LD (2003) Abrupt climate change. science 299(5615):2005–2010

Belaia M, Funke M, Glanemann N (2014) Global Warming and a Potential Tipping Point in the Atlantic Thermohaline Circulation: The Role of Risk Aversion. Environmental and Resource Economics pp 1–33

Berger L, Emmerling J, Tavoni M (2016) Managing Catastrophic Climate Risks Under Model Uncertainty Aversion. Management Science 63(3):749–765, DOI 10.1287/mnsc.2015.2365, URL https://pubsonline.informs.org/doi/abs/10.1287/mnsc.2015.2365, publisher: INFORMS

van den Bijgaart I, Gerlagh R, Liski M (2016) A simple formula for the social cost of carbon. Journal of Environmental Economics and Management 77:75–94, DOI 10.1016/j.jeem.2016.01.005, URL http://www.sciencedirect.com/science/article/pii/S0095069616000061

Bommier A, Le Grand F (2014) A Robust Approach to Risk Aversion: Disentangling Risk Aversion and Elasticity of Substitution without Giving Up Preference Monotonicity. SSRN Scholarly Paper ID 2376339, Social Science Research Network, Rochester, NY, DOI 10.2139/ssrn.2376339, URL https://papers.ssrn.com/abstract=2376339

Bommier A, Lanz B, Zuber S (2015) Models-as-usual for unusual risks? On the value of catastrophic climate change. Journal of Environmental Economics and Management 74:1–22, DOI 10.1016/j.jeem.2015.07.003, URL http://www.sciencedirect.com/science/article/pii/S0095069615000625

Bretschger L, Vinogradova A (2018) Escaping Damocles' Sword: Endogenous Climate Shocks in a Growing Economy. SSRN Scholarly Paper ID 3187976, Social Science Research Network, Rochester, NY, DOI 10.2139/ssrn.3187976, URL https://papers.ssrn.com/abstract=3187976

Bretschger L, Vinogradova A (2019) Best policy response to environmental shocks: Applying a stochastic framework. Journal of Environmental Economics and Management 97:23–41, DOI 10.1016/j.jeem.2017.07.003, URL http://www.sciencedirect.com/science/article/pii/S0095069616302893

Cai Y, Lontzek TS (2019) The Social Cost of Carbon with Economic and Climate Risks. Journal of Political Economy 127(6):2684–2734, DOI 10.1086/701890, URL https://www.journals.uchicago.edu/doi/abs/10.1086/701890

Clarke HR, Reed WJ (1994) Consumption/pollution tradeoffs in an environment vulnerable to pollution-related catastrophic collapse. Journal of Economic Dynamics and Control 18(5):991–1010

Collins M, Knutti R, Arblaster J, Dufresne JL, Fichefet T, Friedlingstein P, Gao X, Gutowski WJ, Johns T, Krinner G (2013) Long-term climate change: projections, commitments and irreversibility. In: Climate Change 2013-The Physical Science Basis: Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, Cambridge University Press, pp 1029–1136

Crost B, Traeger CP (2013) Optimal climate policy: uncertainty versus Monte Carlo. Economics Letters 120(3):552–558

Diaz D, Keller K (2016) A potential disintegration of the West Antarctic Ice Sheet: Implications for economic analyses of climate policy. American Economic Review 106(5):607–11

Diaz D, Moore F (2017) Quantifying the economic risks of climate change. Nature Climate Change 7(11):774

Dietz S (2011) High impact, low probability? An empirical analysis of risk in the economics of climate change. Climatic Change 108(3):519–541

Dietz S, Stern N (2015) Endogenous growth, convexity of damage and climate risk: how Nordhaus' framework supports deep cuts in carbon emissions. The Economic Journal 125(583):574–620, iSBN: 0013-0133 Publisher: Oxford University Press Oxford, UK

Dietz S, Venmans F (2019) Cumulative carbon emissions and economic policy: In search of general principles. Journal of Environmental Economics and Management 96:108–129, DOI 10.1016/j.jeem.2019.04.003, URL `http://www.sciencedirect.com/science/article/pii/S0095069618302122`

Dumas P, Ha-Duong M (2005) An abrupt stochastic damage function to analyze climate policy benefits. In: The Coupling of Climate and Economic Dynamics, Springer, pp 97–111

Gillett NP, Arora VK, Matthews D, Allen MR (2013) Constraining the ratio of global warming to cumulative CO2 emissions using CMIP5 simulations*. Journal of Climate 26(18):6844–6858

Glotter MJ, Pierrehumbert RT, Elliott JW, Matteson NJ, Moyer EJ (2014) A simple carbon cycle representation for economic and policy analyses. Climatic Change 126(3):319–335, DOI 10.1007/s10584-014-1224-y, URL `https://doi.org/10.1007/s10584-014-1224-y`

Goodwin P, Williams RG, Ridgwell A (2015) Sensitivity of climate to cumulative carbon emissions due to compensation of ocean heat and carbon uptake. Nature Geoscience 8(1):29–34

Guivarch C, Pottier A (2018) Climate Damage on Production or on Growth: What Impact on the Social Cost of Carbon? Environmental Modeling & Assessment 23(2):117–130

Ha-Duong M, Treich N (2004) Risk aversion, intergenerational equity and climate change. Environmental and Resource Economics 28(2):195–207

Howard P (2014) Omitted Damages: What's Missing from the Social Cost of Carbon. Electronic copy available at: http://costofcarbon org/files/Omitted_Damages_Whats_Missing_From_the_Social_Cost_of_Carbon pdf

IAWG U (2010) Technical support document: Social cost of carbon for regulatory impact analysis under executive order 12866. Interagency Working Group on Social Cost of Carbon, United States

Government, Washington, DC

Jensen S, Traeger CP (2014) Optimal climate change mitigation under long-term growth uncertainty: Stochastic integrated assessment and analytic findings. European Economic Review 69:104–125, DOI 10.1016/j.euroecorev.2014.01.008, URL http://www.sciencedirect.com/science/article/pii/S0014292114000105

Jensen S, Traeger CP (2016) Pricing climate risk. UC Berkeley

Keller K, Bolker BM, Bradford DF (2004) Uncertain climate thresholds and optimal economic growth. Journal of Environmental Economics and Management 48(1):723–741

Leduc M, Matthews HD, de Elía R (2015) Quantifying the limits of a linear temperature response to cumulative CO2 emissions. Journal of Climate 28(24):9955–9968

Lemoine D, Rudik I (2017) Managing Climate Change Under Uncertainty: Recursive Integrated Assessment at an Inflection Point. Annual Review of Resource Economics 9(1):117–142, DOI 10.1146/annurev-resource-100516-053516, URL https://doi.org/10.1146/annurev-resource-100516-053516, _eprint: https://doi.org/10.1146/annurev-resource-100516-053516

Lemoine D, Traeger C (2014) Watch your step: Optimal policy in a tipping climate. American Economic Journal: Economic Policy 6(1):137–166

Lemoine D, Traeger CP (2016a) Ambiguous tipping points. Journal of Economic Behavior & Organization 132:5–18, iSBN: 0167-2681 Publisher: Elsevier

Lemoine D, Traeger CP (2016b) Economics of tipping the climate dominoes. Nature Climate Change 6(5):514–519, DOI 10.1038/nclimate2902, URL https://www.nature.com/articles/nclimate2902

Lenton TM, Held H, Kriegler E, Hall JW, Lucht W, Rahmstorf S, Schellnhuber HJ (2008) Tipping elements in the Earth's climate system. Proceedings of the National Academy of Sciences 105(6):1786–1793

Lontzek TS, Cai Y, Judd KL, Lenton TM (2015) Stochastic integrated assessment of climate tipping points indicates the need for strict climate policy. Nature Climate Change 5(5):441–444

MacDougall AH, Friedlingstein P (2015) The origin and limits of the near proportionality between climate warming and cumulative CO2 emissions. Journal of Climate 28(10):4217–4230

Mattauch L, Matthews HD, Millar R, Rezai A, Solomon S, Venmans F (2019) Steering the climate system: Comment. American Economic Review

Matthews HD, Gillett NP, Stott PA, Zickfeld K (2009) The proportionality of global warming to cumulative carbon emissions. Nature 459(7248):829

Méjean A, Pottier A, Fleurbaey M, Zuber S (2020) Catastrophic climate change, population ethics and intergenerational equity. Climatic Change 163(2):873–890, DOI 10.1007/s10584-020-02899-9, URL https://doi.org/10.1007/s10584-020-02899-9

Moore FC, Diaz DB (2015) Temperature impacts on economic growth warrant stringent mitigation policy. Nature Climate Change 5(2):127–131

Nordhaus W (2008) A question of balance. Yale University Press New Haven

Nordhaus WD (1994) Managing the global commons: the economics of climate change, vol 31. MIT press Cambridge, MA

Pindyck RS (2013) Climate change policy: What do the models tell us? Tech. rep., National Bureau of Economic Research

Pindyck RS, Wang N (2013) The Economic and Policy Consequences of Catastrophes. American Economic Journal: Economic Policy 5(4):306–339, DOI 10.1257/pol.5.4.306, URL https://www.aeaweb.org/articles?id=10.1257/pol.5.4.306

Pizer WA (2003) Climate change catastrophes. Resources for the Future

van der Ploeg F (2016) Climate change economics: Reacting to multiple tipping points. Nature Climate Change

van der Ploeg F, de Zeeuw A (2018) Climate Tipping and Economic Growth: Precautionary Capital and the Price of Carbon. Journal of the European Economic Association 16(5):1577–1617, DOI 10.1093/jeea/jvx036, URL https://academic.oup.com/jeea/article/16/5/1577/4565430, publisher: Oxford Academic

van der Ploeg F, de Zeeuw A (2019) Pricing Carbon and Adjusting Capital to Fend Off Climate Catastrophes. Environmental and Resource Economics 72(1):29–50, DOI 10.1007/s10640-018-0231-2, URL https://doi.org/10.1007/s10640-018-0231-2

Pottier A, Espagne E, Fabert BP, Dumas P (2015) The comparative impact of integrated assessment models' structures on optimal mitigation policies. Environmental Modeling & Assessment 20(5):453–473

Revesz RL, Howard PH, Arrow K, Goulder LH, Kopp RE, Livermore MA, Oppenheimer M, Sterner T (2014) Global warming: Improve economic models of climate change. Nature 508(7495):173–175

National Academies of Sciences EaM (2016) Assessment of approaches to updating the social cost of carbon: Phase 1 report on a near-term update. National Academies Press

Solomon S, Plattner GK, Knutti R, Friedlingstein P (2009) Irreversible climate change due to carbon dioxide emissions. Proceedings of the national academy of sciences 106(6):1704–1709

Steffen W, Rockström J, Richardson K, Lenton TM, Folke C, Liverman D, Summerhayes CP, Barnosky AD, Cornell SE, Crucifix M (2018) Trajectories of the Earth System in the Anthropocene. Proceedings of the National Academy of Sciences 115(33):8252–8259, iSBN: 0027-8424 Publisher: National Acad Sciences

Tsur Y, Zemel A (1996) Accounting for global warming risks: Resource management under event uncertainty. Journal of Economic Dynamics and Control 20(6):1289–1305, DOI 10.1016/0165-1889(95)00900-0, URL http://www.sciencedirect.com/science/article/pii/0165188995009000

Van Den Bremer T, van der Ploeg R (2018) Pricing Carbon Under Economic and Climactic Risks: Leading-Order Results from Asymptotic Analysis. SSRN Scholarly Paper ID 3112266, Social Science Research Network, Rochester, NY, URL https://papers.ssrn.com/abstract=3112266

Weitzman ML (2009) On modeling and interpreting the economics of catastrophic climate change. The Review of Economics and Statistics 91(1):1–19

Weitzman ML (2012) GHG targets as insurance against catastrophic climate damages. Journal of Public Economic Theory 14(2):221–244

Wouter Botzen WJ, van den Bergh JCJM (2012) How sensitive is Nordhaus to Weitzman? Climate policy in DICE with an alternative damage function. Economics Letters 117(1):372–374, DOI 10.1016/j.econlet.2012.05.032, URL http://www.sciencedirect.com/science/article/pii/S016517651200300X

Zickfeld K, Eby M, Matthews HD, Weaver AJ (2009) Setting cumulative emissions targets to reduce the risk of dangerous climate change. Proceedings of the National Academy of Sciences 106(38):16129–16134

Zickfeld K, Eby M, Weaver AJ, Alexander K, Crespin E, Edwards NR, Eliseev AV, Feulner G, Fichefet T, Forest CE (2013) Long-term climate change commitment and reversibility: an EMIC intercomparison. Journal of Climate 26(16):5782–5809

Acknowledgements The authors would like to thank two anonymous referees who contributed to improving the paper.

A Proof of equation 10

Recall that we note $V_\epsilon(E) = \mathbb{E}_\omega[\epsilon_\omega^2(E)]$ the variance of damages of the zero-mean risk ϵ, which is of second order in $|\epsilon|$ (a norm of the risk ϵ). Let us start with Taylor expansion at second order of the term evaluated in the denominators of equation (8):

$$\mathbb{E}_\omega\left[u'(C_\omega(a,E))\right] = \mathbb{E}_\omega\left[u'(\tilde{C}(a,E)) + u''(\tilde{C}(a,E))\epsilon_\omega(a,E) + u'''(\tilde{C}(a,E))\frac{\epsilon_\omega^2(E)}{2} + o(|\epsilon|^2)\right] \tag{29}$$

$$= u'(\tilde{C}(a,E)) + u'''(\tilde{C}(a,E))\mathbb{E}_\omega\left[\frac{\epsilon_\omega^2(E)}{2}\right] + o(|\epsilon|^2) \tag{30}$$

$$= u'(\tilde{C}(a,E)) + u'''(\tilde{C}(a,E))\frac{V_\epsilon(E)}{2} + o(|\epsilon|^2) \tag{31}$$

$$\tag{32}$$

So the denominator is finally:

$$\mathbb{E}_\omega\left[u'(C_\omega(a,E))\right]\big|_{a^t,E^t} = u'(\tilde{C}(a^t,E^t)) + u'''(\tilde{C}(a^t,E^t))\frac{V_\epsilon(E^t)}{2} + o(|\epsilon|^2) \tag{33}$$

Let us go back to equation (8). It is a sum of two terms, the first one is of zero-order, whereas a second term is of second order. In this second term, we can simply replace the denominator by its zero-order approximation as any correction would induce terms with orders higher than 2. For the first term of the sum, we have to keep the full Taylor expansion of the denominator. We then reorder terms of the Taylor expansion.

$$SCC^t = \frac{u'(\tilde{C}(a^t,E^t))\tilde{d}'(E^t)}{\mathbb{E}_\omega\left[u'(C_\omega(a,E))\right]|_{a^t,E^t}} - \frac{\partial_E\left(u''(\tilde{C}(a,E))\frac{V_\epsilon(E)}{2}\right)\Big|_{a^t,E^t}}{\mathbb{E}_\omega\left[u'(C_\omega(a,E))\right]|_{a^t,E^t}} + o(|\epsilon|^2) \tag{34}$$

$$= \frac{u'(\tilde{C}(a^t,E^t))\tilde{d}'(E^t)}{u'(\tilde{C}(a^t,E^t)) + u'''(\tilde{C}(a^t,E^t))\frac{V_\epsilon(E^t)}{2}} - \frac{\partial_E\left(u''(\tilde{C}(a,E))\frac{V_\epsilon(E)}{2}\right)\Big|_{a^t,E^t}}{u'(\tilde{C}(a^t,E^t))} + o(|\epsilon|^2) \tag{35}$$

$$= \frac{\tilde{d}'(E^t)}{1 + \frac{u'''}{u'}\left(\tilde{C}(a^t,E^t)\right)\frac{V_\epsilon(E^t)}{2}} - \frac{u''(\tilde{C}(a^t,E^t))\,\partial_E\left(\frac{V_\epsilon(E)}{2}\right)\Big|_{E^t} - u'''(\tilde{C}(a^t,E^t))\tilde{d}'(E^t)\frac{V_\epsilon(E^t)}{2}}{u'(\tilde{C}(a^t,E^t))} + o(|\epsilon|^2) \tag{36}$$

$$= \tilde{d}'(E^t)\left(1 - \frac{u'''}{u'}\left(\tilde{C}(a^t,E^t)\right)\frac{V_\epsilon(E^t)}{2}\right) - \frac{u''(\tilde{C}(a^t,E^t))\,\partial_E\left(\frac{V_\epsilon(E)}{2}\right)\Big|_{E^t}}{u'(\tilde{C}(a^t,E^t))} + \frac{u'''(\tilde{C}(a^t,E^t))\tilde{d}'(E^t)\frac{V_\epsilon(E^t)}{2}}{u'(\tilde{C}(a^t,E^t))} + o(|\epsilon|^2) \tag{37}$$

$$= \tilde{d}'(E^t) - \frac{u''(\tilde{C}(a^t,E^t))\,\partial_E\left(\frac{V_\epsilon(E)}{2}\right)\Big|_{E^t}}{u'(\tilde{C}(a^t,E^t))} + o(|\epsilon|^2) \tag{38}$$

$$= \tilde{d}'(E^t) + \gamma(\tilde{C}(a^t,E^t))\frac{\tilde{C}(a^t,E^t)}{2}\,\partial_E V_\epsilon(E)|_{E^t} + o(|\epsilon|^2) \tag{39}$$

At the last line, we have introduced the Arrow-Pratt measure of relative risk aversion $\gamma(C) = -u''(C)/(u'(C).C)$ of the utility u to get equation (10).

B Proof of proposition 1

We are first interested in the difference between SCC with a tipping point SCC^t, given by equation (5) and SCC without a tipping point SCC^*, given by (2), with damage function d egal to damage function in the pre-tipping state of the world (we call 1 this state).

The proof is a little bit more complicated than just comparing the equations (2), (10) and (11). Indeed, one has to take into account not only that there are additional terms but also that these are not evaluated at the same point. This is because the planner reacts to additional damages terms and thus the optimal emissions change accordingly (it is respectively E^*, E^t, E^{ed}).

We have, at first order in the magnitude of risk ϵ, thanks to (10):

$$SCC^t - SCC^* = \tilde{d}'(E^t) - d(E^*) + o(|\epsilon|) \tag{40}$$

The optimal abatement and emission levels solve:

$$\frac{c'(a^*)}{\sigma Y} = d'(E^*) \tag{41}$$

$$\frac{c'(a^t)}{\sigma Y} = \tilde{d}'(E^t) + o(|\epsilon|) \tag{42}$$

Let us write $a^t = a* + h$, then $E^t = E* - \sigma Y h$ and assume that h is at first-order in $|\epsilon|$. We make a Taylor-expansion of the last line in h:

$$\frac{c'(a^*)}{\sigma Y} + \frac{c''(a^*)}{\sigma Y} h = \tilde{d}'(E^*) - \tilde{d}''(E^*)\sigma Y h + o(|\epsilon|) \tag{43}$$

Hence

$$\left(\frac{c''(a^*)}{\sigma Y} + \tilde{d}''(E^*)\sigma Y\right) h = \tilde{d}'(E^*) - d'(E^*) + o(|\epsilon|) \tag{44}$$

The right hand side is simply $-\epsilon_1'(E^*)$, the marginal increase in risk ϵ in state 1. Thus h is correctly at first-order in $|\epsilon|$ and the difference between the SCCs is given by:

$$SCC^t - SCC^* = \tilde{d}'(E^*) - d'(E^*) - d''(E^*)\sigma Y h + o(|\epsilon|) \tag{45}$$

$$= \left(\tilde{d}'(E^*) - d'(E^*)\right) \frac{\frac{c''(a^*)}{\sigma Y}}{\frac{c''(a^*)}{\sigma Y} + \tilde{d}''(E^*)\sigma Y} + o(|\epsilon|) \tag{46}$$

$$= -\epsilon_1'(E^*) \frac{\frac{c''(a^*)}{\sigma Y}}{\frac{c''(a^*)}{\sigma Y} + \tilde{d}''(E^*)\sigma Y} + o(|\epsilon|) \tag{47}$$

This proves our first claim. We proceed similarly for the second. By definition and thanks to (10),

$$SCC^t - SCC^{ed} = \tilde{d}'(E^t) + \gamma(\tilde{C}(a^t, E^t)) \frac{\tilde{C}(a^t, E^t)}{2} \partial_E V_\epsilon(E)|_{E^t} - \tilde{d}'(E^{ed}) + o(|\epsilon|^2) \tag{48}$$

The optimal abatement and emission levels solve:

$$\frac{c'(a^{ed})}{\sigma Y} = \tilde{d}'(E^{ed}) \tag{49}$$

$$\frac{c'(a^t)}{\sigma Y} = \tilde{d}'(E^t) + \gamma(\tilde{C}(a^t, E^t)) \frac{\tilde{C}(a^t, E^t)}{2} \partial_E V_\epsilon(E)|_{E^t} + o(|\epsilon|^2) \tag{50}$$

Let us write $a^t = a^{ed} + g$, then $E^t = E^{ed} - \sigma Y g$ and assume that g is at second-order in $|\epsilon|$. We make a Taylor-expansion of the last line in g:

$$\frac{c'(a^{ed})}{\sigma Y} + \frac{c''(a^{ed})}{\sigma Y} h = \tilde{d}'(E^{ed}) - \tilde{d}''(E^{ed})\sigma Y g + \gamma(\tilde{C}(a^t, E^t)) \frac{\tilde{C}(a^t, E^t)}{2} \partial_E V_\epsilon(E)|_{E^t} + o(|\epsilon|^2) \tag{51}$$

Hence:

$$\left(\frac{c''(a^{ed})}{\sigma Y} + \tilde{d}''(E^{ed})\sigma Y \right) g = \gamma(\tilde{C}(a^t, E^t)) \frac{\tilde{C}(a^t, E^t)}{2} \partial_E V_\epsilon(E)|_{E^t} + o(|\epsilon|^2) \tag{52}$$

Thus g is correctly at second-order in $|\epsilon|$ (as $\partial_E V_\epsilon(E)|_{E^t}$) is) and given by:

$$g = \frac{\gamma(\tilde{C}(a^t, E^t)) \frac{\tilde{C}(a^t, E^t)}{2} \partial_E V_\epsilon(E)|_{E^t}}{\frac{c''(a^{ed})}{\sigma Y} + \tilde{d}''(E^{ed})\sigma Y} + o(|\epsilon|^2) \tag{53}$$

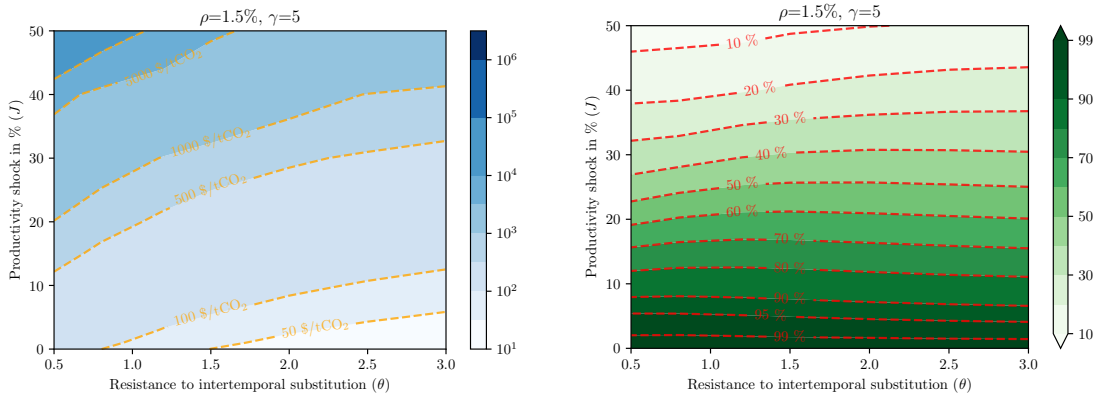
$$= \frac{\gamma(\tilde{C}(a^{ed}, E^{ed})) \frac{\tilde{C}(a^{ed}, E^{ed})}{2} \partial_E V_\epsilon(E)|_{E^{ed}}}{\frac{c''(a^{ed})}{\sigma Y} + \tilde{d}''(E^{ed})\sigma Y} + o(|\epsilon|^2) \tag{54}$$

The difference between the SCCs is given by:

$$SCC^t - SCC^{ed} = \tilde{d}'(E^{ed}) - \tilde{d}''(E^{ed})\sigma Y g + \gamma(\tilde{C}(a^{ed}, E^{ed})) \frac{\tilde{C}(a^{ed}, E^{ed})}{2} \partial_E V_\epsilon(E)|_{E^{ed}} - \tilde{d}'(E^{ed}) + o(|\epsilon|^2) \tag{55}$$

$$= \gamma(\tilde{C}(a^{ed}, E^{ed})) \frac{\tilde{C}(a^{ed}, E^{ed})}{2} \partial_E V_\epsilon(E)|_{E^{ed}} \frac{\frac{c''(a^{ed})}{\sigma Y}}{\frac{c''(a^{ed})}{\sigma Y} + \tilde{d}''(E^{ed})\sigma Y} \tag{56}$$

C Additional graph: sensitivity to resistance to intertemporal substitution



(a) (b)

Fig. 4
Epstein-Zin preference: influence of the resistance to intertemporal substitution θ. Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC_{ed}/SCC), for different values of resistance to intertemporal substitution (θ) and productivity shocks (J). Horizontal lines in panel b. indicate that preferences for intertemporal substitution does not affect the comparison between SCC and SCC under expected damage.

Two parameters are involved in welfare evaluation at each time step: risk aversion (γ) and resistance to intertemporal substitution (θ). In the main text, we have analyzed the influence of risk aversion combined with the value of the shock. On figure 4, we display how resistance to intertemporal substitution affects the comparison of determinsitic and stochastic methods. A change in θ does not affect the share of the SCC explained by expected damages, the contour lines on the graph are horizontal.

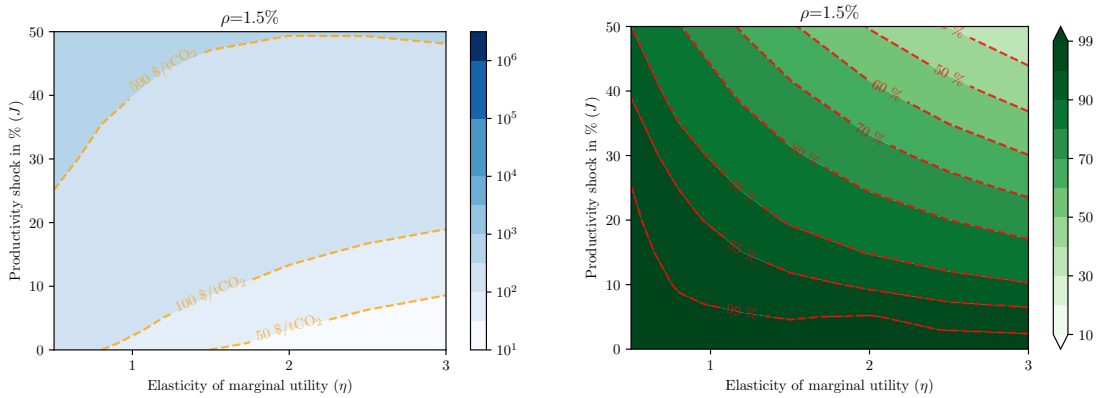
D Robustness checks

We perform a sensitivity analysis on several parameters of the model:

- The maximum temperature threshold for the tipping point $T_{\max}$. We look at $T_{\max} = 10$ instead of 7.
- Pure rate of time preference ρ. We run the model for lower ρ (0.5%)
- Elasticity of intertemporal substitution $(1/\theta)$ in the Epstein-Zin case. We consider $\theta = 0.5$ and $\theta = 1.5$.

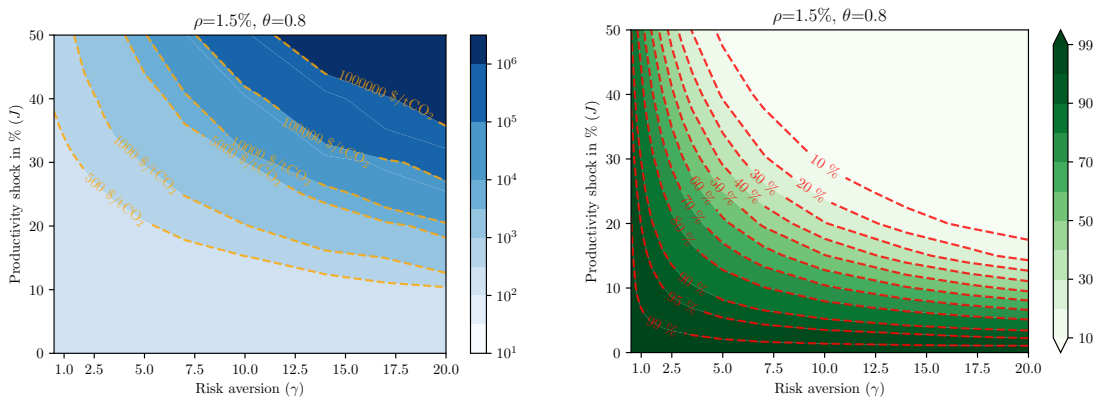
The graphs show that the shapes of the curves are not affected by a change in these parameters, and our finding that most of the SCC is still explained by expected damages as long as the shock remain under 10%.

D.1 Parameter $T_{\max}$



(a) (b)

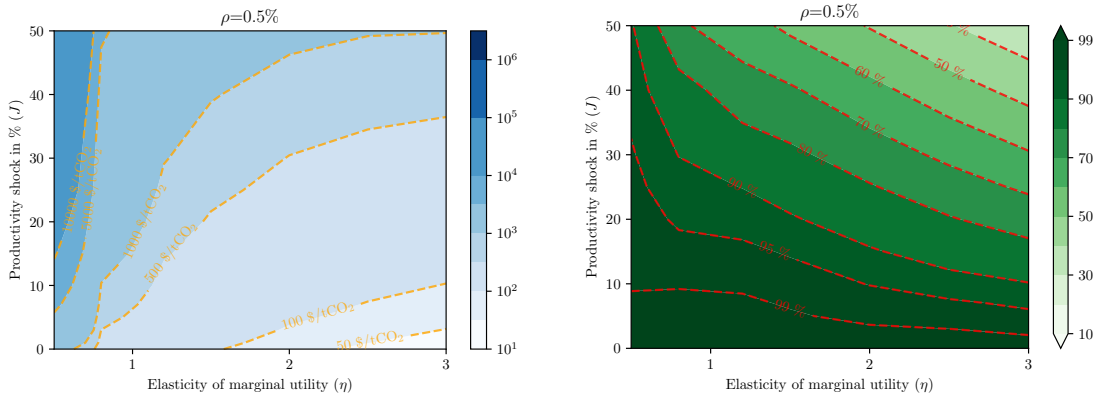
Fig. 5
Sensitivity analysis for $T_{\max} = 10°C$ (CRRA preferences). Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC_{ed}/SCC).



(a) (b)

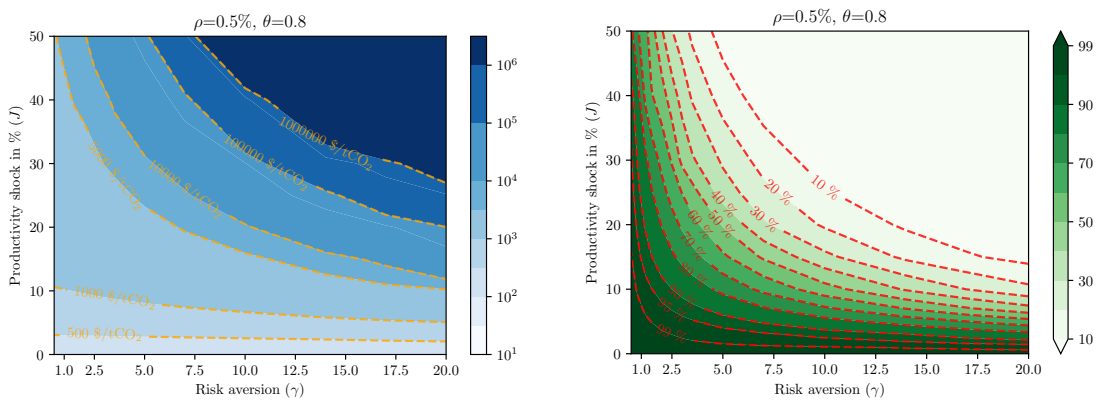
Fig. 6
Sensitivity analysis for $T_{\max} = 10°C$ (Epstein-Zin preferences). Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC_{ed}/SCC).

D.2 Parameter ρ



(a) (b)

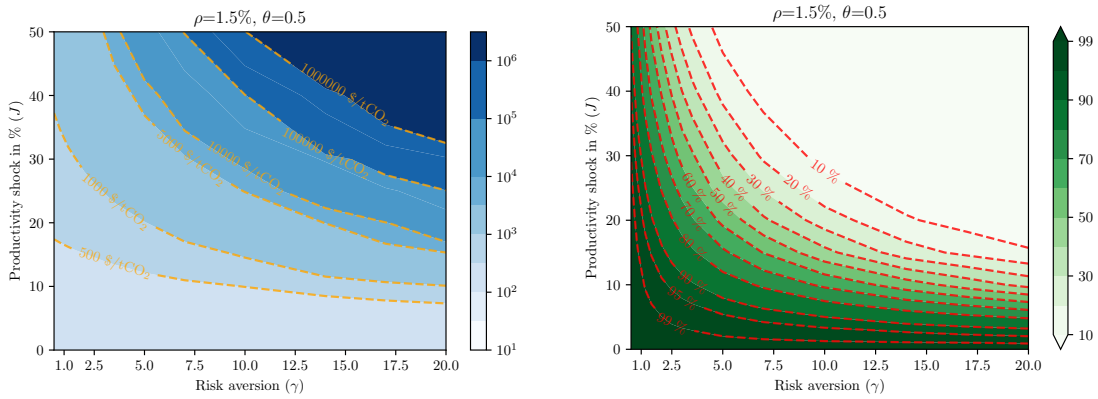
Fig. 7
Sensitivity analysis for $\rho = 0.5\%$ (CRRA preferences). Social Cost of Carbon under Epstein-Zin preferences. Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC_{ed}/SCC).



(a) (b)

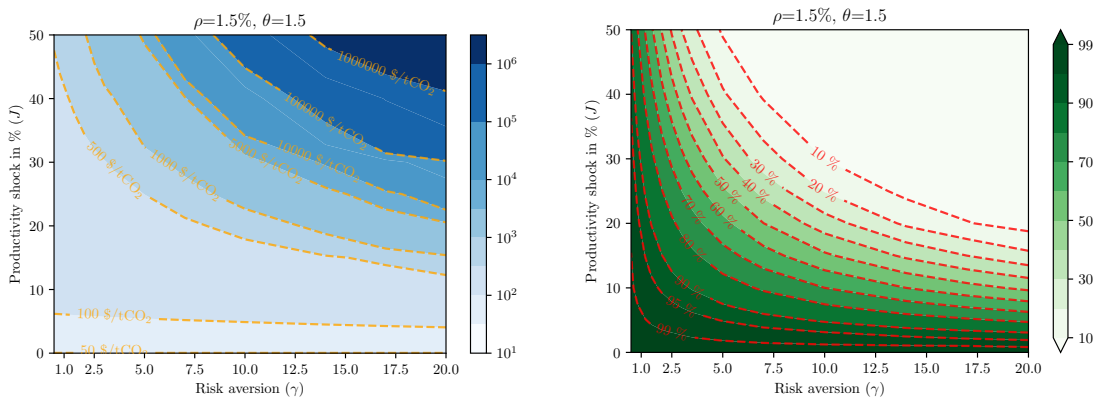
Fig. 8
Sensitivity analysis for $\rho = 0.5\%$ (Epstein-Zin preferences). Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC_{ed}/SCC).

D.3 Parameter θ



(a) (b)

Fig. 9
Sensitivity analysis for $\theta = 0.5$ (Epstein-Zin preferences). Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC_{ed}/SCC).



(a) (b)

Fig. 10
Sensitivity analysis for $\theta = 1.5$ (Epstein-Zin preferences). Heatmap of the Social Cost of Carbon (panel a., in US$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC_{ed}/SCC).

D.4 A tipping point affecting the convexity of damages

We plot the same graphs when the tipping point affects the convexity of the damage function. Thus, the tipping point increases marginal damages rather solely damage level. We assume that the coefficient π in the damage factor $\Omega = \frac{1}{1+\pi T^2}$ can jump from its initial value (π_1=0.00028) to a higher value π_2 (see figure 11 for an illustration of the effect on the damage function)

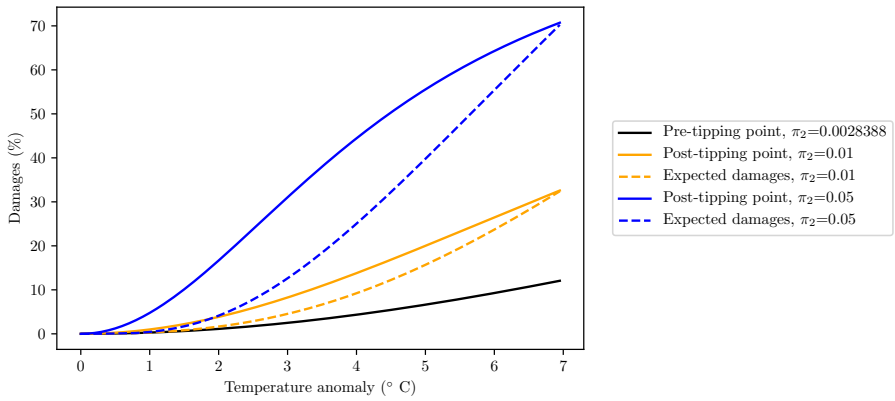
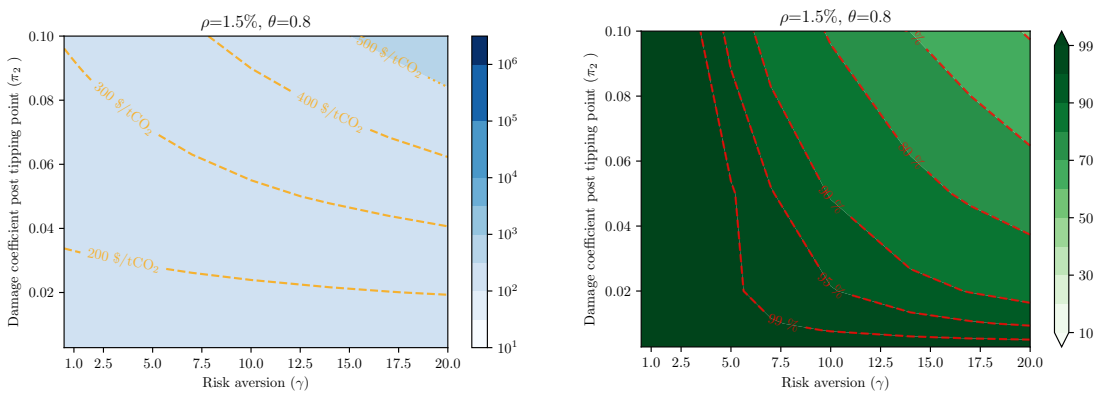


Fig. 11
Comparison between stochastic damage function and expected damages approaches, for a tipping point which affects π.



(a) (b)

Fig. 12
Sensitivity analysis for a tipping point affecting the convexity of the damage function (Epstein-Zin). Heatmap of the Social Cost of Carbon (panel a., in US\$2005) and the share of its value that can be explained by expected damages (panel b., ratio SCC_{ed}/SCC).