Designing an Olympic Archery Algorithm

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Introduction

Using projectile motion, an angle of launch can be determined and applied in an Olympic archery setting. Paired with an Arduino microcontroller, the derivations can be used to create a device that will assist an archer in training their muscle memory. The purpose of this document is to explain derivations as well as electronic components, should future modifications need to be adjusted/implemented.

A word on Consistency

To help reduce inconsistencies, the same 20 lb bow and 0.635 oz arrow will be used for testing of the device. Additionally, markers and laser levels will be used to ensure an accurate measurement of target distance.

Definitions and derivations

For the purpose of the following calculations, these definitions are made:

v = Velocity (ft/s)

a = Acceleration due to gravity (ft/s^2)

(x, y) = Final coordinates of target relative to tip of arrow (ft)

s = Deformation of bowstring (ft)

θ = Launch angle relative to horizontal axis (°)

m = Mass of arrow (lb·s^2/ft)

k = Spring constant (lb/ft)

Determining velocity

With a modern recurve bow, manufacturing processes have solidified a linear relationship between draw length (how far the string is drawn back) and draw weight (force applied to the string). Because of this, the string can be modeled as a spring in which Hooke's law can be applied.

$$F = ks \therefore k = \frac{F}{s} = \frac{\text{Draw Weight}}{\text{Draw Length}}$$

Take $T_1 = 0$ and T_2 to be the point at which the arrow leaves the bow. Using energy methods,

$$T_1 + V_1 = T_2 + V_2$$

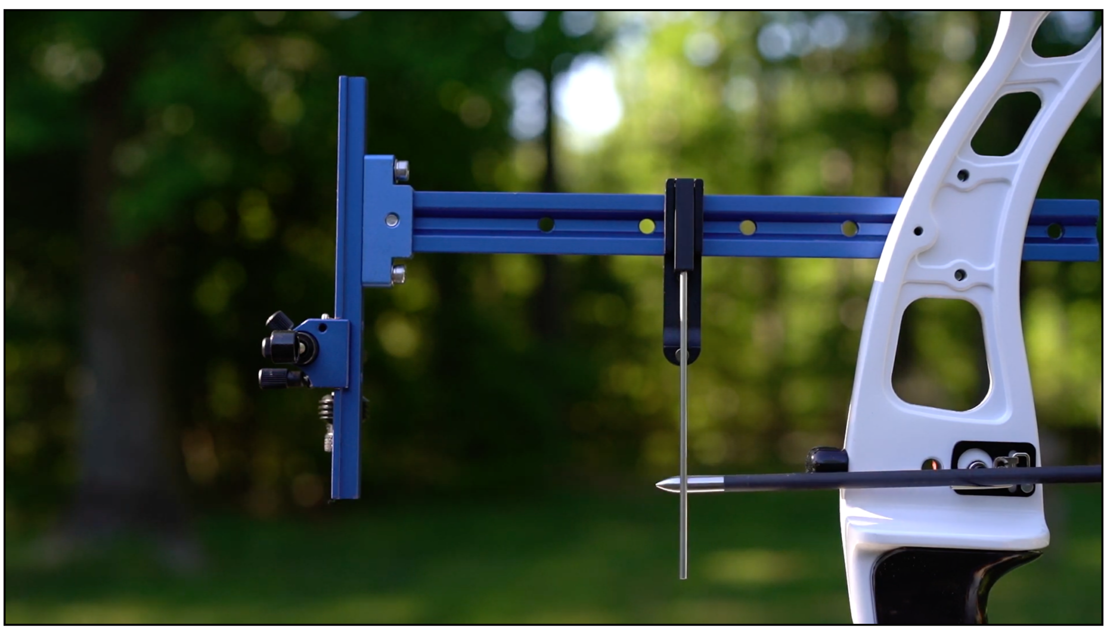
$$0 + \frac{1}{2}ks^2 = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{\frac{ks^2}{m}}$$

$$v = \sqrt{\frac{\frac{F}{s}s^2}{m}}$$

$$v = \sqrt{\frac{Fs}{m}}$$

A concern may arise that it would be hard to ensure the archer pulls back the bow the same distance s each shot, but this problem is already dealt with and solved in Olympic archery. To counteract this, Olympic archers will use a device called a "clicker". As the archer pulls back the arrow, eventually it will be pulled back far enough in which a lever near the tip will "click" down onto the bow. With this attached to the bow, the same draw length can be achieved very consistently. Archers will also try to pull back to a set point (their nose, under their chin, etc) called the anchor point. Additionally, the arrow may be marked to achieve a similar result.



A recurve bow clicker. Image source: Archery 360, "Beginner's Guide to Recurve Clickers", 2020. Accessed via https://www.youtube.com/watch?v=U_0zRWZiQ8E Used with permission.

Determining launch angle

The projectile motion of an arrow can be modeled using the following equations;

$$t\,v\cos(\theta) = x$$

$$y = y_0 + t\,v\sin(\theta) + \frac{1}{2}a t^2$$

Rearranging and substituting,

$$t = \frac{x}{v\cos(\theta)}$$

$$y = 0 + \frac{x\,v\sin(\theta)}{v\cos(\theta)} + \frac{1}{2}\frac{a x^2}{v^2\cos^2(\theta)}$$

$$y = 0 + x\tan(\theta) + \frac{1}{2}\frac{a x^2}{v^2\cos^2(\theta)}$$

From here, a substitution is made. Take $T = \tan(\theta)$, and apply the trigonometric identity $\frac{1}{\cos^2(\theta)} = 1 + \tan^2(\theta)$. In doing so, the equation then becomes

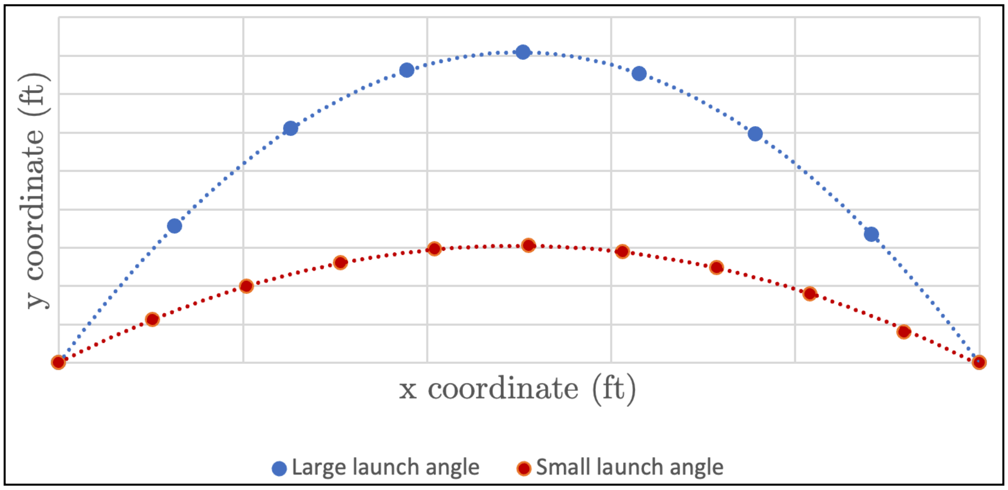
$$y = 0 + xT + \frac{a x^2}{2v^2}(1 + T^2)$$

$$\frac{-a x^2}{2v^2}T^2 - xT + \left(y - \frac{a x^2}{2v^2}\right) = 0$$

This is a quadratic equation, which can be solved with the quadratic formula. Replacing T and solving, an expression for theta is found;

$$\theta = \arctan\left(\frac{x \pm \sqrt{x^2 + \frac{2ax^2}{v^2}y - \frac{a^2x^4}{v^4}}}{\frac{-ax^2}{v^2}}\right)$$

Note that there will be two solutions given, in which case we will be interested in the smaller angle. If the solution is imaginary, then the velocity is not strong enough to hit the target.



Two possible trajectories a projectile could take to get to (x,y)

Arduino and device functionality

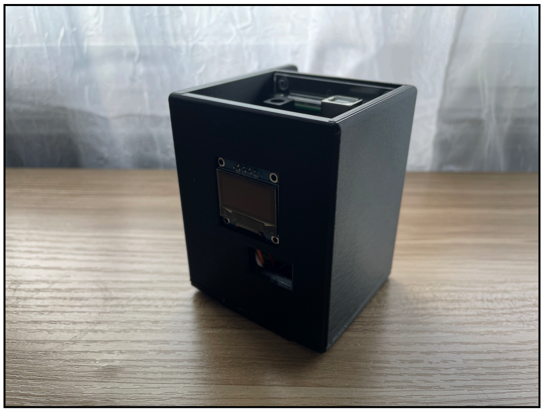
The current target distance can be imputed using the buttons located on the front of the device. The Arduino will then perform a simple comparison to the calculated angle (see *Determining launch angle*), and will display if the user needs to raise or lower the bow accordingly. Once the launch angle is reached, the user will release the arrow. Over time, the goal is that the archer will train their muscle memory and will no longer need the training device.

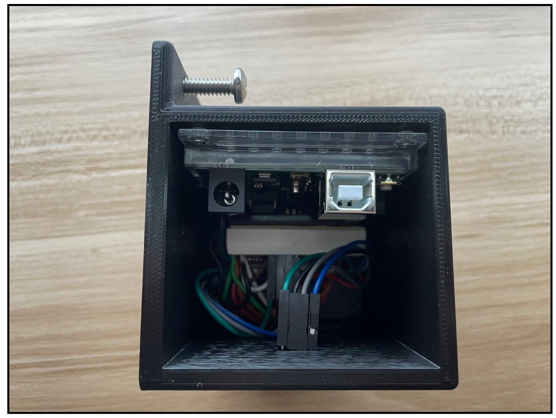
Code

The full Arduino code can be located here. Additionally, part of the gyroscope code was modified and taken from here.

Attachment of device

The attachment of the device was done by creating a 3D printed chamber using SolidWorks, in which the Arduino and components could rest. Using the pre-drilled holes in the bow riser (which would normally be used for a sight), two 10-24 x ¾ screws can be used to attach the chamber.





Since this attachment is likely to tilt the gyroscope inside the chamber (the holes on the riser are not aligned vertically), it is necessary to correct this adjustment in line 71 of the Arduino code;

```
y= RAD_TO_DEG * (atan2(-xAng, -zAng)+PI)-97.5;
```

The current angle should now read 0 when held in line with the horizon.

Full attachment of the device, with the code running.

Practicality

Trying the device with a newer archer, it was clear that the device could help train muscle memory. Using the device to help guide their aim, they were able to consistently hit within four inches of the target center at 45 ft. Most new archers would struggle to reliably hit the target at this distance, so it is a noteworthy improvement.

A new archer trying out the device at 45 ft.

Although this improvement was welcomed, after some time one archer started showing signs of target panic. This is a common occurrence (even at an Olympic level) and it is possible that this was unrelated to the device. However, the archer who was showing signs of target panic started to improve their form once the device was removed, mentioning “It was very hard to focus on my form and the screen at the same time”. This may be an indication that the device may not be suited for archers who still need to focus on their form, as it could potentially build up too much stress and cause target panic.

The device may hold a place between beginner and intermediate archers, but in it’s current state it seems it is suited for only a select skill range of archers; those who have good form and could focus on a screen. While this seems applicable, if an archer is able to produce good form and focus from the start, they would likely not need the device to begin with.

A simplified model

The author wishes to note that although this model has proven itself for shorter distances, it is a drastic simplification of archery and the physics behind it. The device cannot yet take into account crosswinds and the drag on the arrow, both which can have drastic implications at farther distances. Additionally, the current model assumes all energy from the string is transferred directly into the arrow, neglecting any absorption from the limbs of the bow.

Furthermore, the gyroscope currently only measures the vertical angle, and cannot tell if the bow is being tilted too far left or right. This could lead to an archer following the device correctly, but still missing the target.

While certain variables can be easily changed in the code (draw weight, arrow mass, archer height), there are still many improvements to be made to not only refine the device at shorter distances, but also expand its capabilities and overall range effectiveness.

Conclusion

While the device is very capable at shorter distances, there are still many improvements to be made. The device has applications for newer archers, and with further implements it could expand its reliably and accessibility to the archery world.

Acknowledgments

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