

# Physics

## Classical Mechanics

### Kinematic Relationships

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v \frac{dv}{dx}$$

→ These equations will also work for terms  $\theta, \omega, \alpha$  if substituted.

### Uniform Rectilinear Motion

$$x = x_0 + vt$$

$$\theta = \theta_0 + \omega t$$

→ Applies when  $a = 0$

### Uniformly Accelerated Motion

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

→ These equations will also work for terms  $\theta, \omega, \alpha$  if substituted. Also “ $x$ ” is subjective and could be any defined axis ( $y, z, \text{etc.}$ ).

### Circular Motion

$$\theta = \frac{s}{r}$$

$$v = \frac{2\pi r}{T} = r\omega$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$a_{tan} = r\alpha$$

$$\omega = 2\pi f$$

→ Where  $s$  is arc length,  $r$  is the radius of curvature,  $f$  is the frequency, and  $T$  is the period.

### General Force Equations

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_{ab} = -\vec{F}_{ba}$$

$$F_g = mg$$

$$F_{Spring} = -ks$$

$$\vec{F}_{static\ friction} = \mu_s \vec{F}_N$$

$$\vec{F}_{kinetic\ friction} = \mu_k \vec{F}_N$$

### Work and Energy

$$W = \int_{x_0}^{x_f} F(x) dx = \int_{s_0}^{s_f} \vec{F} \cdot d\vec{s}$$

$$W = Fd \cos \theta = \vec{F} \cdot \vec{d}$$

→ Where  $F$  is force,  $d$  is distance traveled, and  $\theta$  is the angle between the two  $\vec{F}$  and  $\vec{d}$  vectors

$$KE_1 + W_{1 \rightarrow 2} = KE_2$$

$$KE_1 + PE_1 = PE_2 + KE_2$$

$$KE = \frac{1}{2}mv^2$$

$$PE_{grav} = mgh$$

$$PE_{Elastic} = \frac{1}{2}kx^2$$

### Universal Gravitation

$$F_G = G \frac{m_1 m_2}{r^2}$$

### Efficiency

$$\eta = \frac{\text{input}}{\text{output}}$$

### Center of Mass

$$x_{CM} = \frac{1}{M} \int x dm$$

$$y_{CM} = \frac{1}{M} \int y dm$$

$$x_{CM} = \frac{\sum_{i=1}^n m_i x_i}{m_T}$$

$$y_{CM} = \frac{\sum_{i=1}^n m_i y_i}{m_T}$$

### Impulse and Momentum

$$p = mv$$

$$m\vec{v}_1 + \int \vec{F} dt = m\vec{v}_2$$

$$I = \int_{t_i}^{t_f} F(t) dt = \Delta p$$

$$I = \vec{F} \Delta t$$

$$\vec{F} \Delta t = \Delta p$$

### Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

$$L = I\omega = mrv \sin \phi$$

$$L = mr^2 \omega \sin \phi$$

$$I_1 \omega_1 = I_2 \omega_2$$

### Collisions

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

### Torque (Moments)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = F \cdot r \sin \theta$$

### Density

$$\rho = \frac{m}{V}$$

$$\rho_{Theor} = \frac{nA}{V_c N_A}$$

### Simple Harmonic Motion

$$T = 2\pi \sqrt{\frac{m}{k}} = \frac{1}{f} = 2\pi \sqrt{\frac{L}{g}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad v = \frac{\lambda}{T}$$

→ Where  $T$  is the period,  $m$  is mass,  $g$  is gravitational acceleration,  $f$  is the frequency,  $L$  is the length of the string, and  $k$  is the spring constant.

### Fluids (Basic Equations)

$$A_1 v_1 = A_2 v_2$$

$$F_2 A_1 = F_1 A_2$$

$$P = \frac{F}{A}$$

$$P_g = \rho g h$$

$$P_T = P_{atm} + \rho g h$$

$$F_{buoyant} = \rho g V_{fluid}$$

$$\frac{V}{t} = A v = Q$$

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

$$\Delta P = \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2$$

→ Where  $\rho$  is density,  $V$  is volume,  $v$  is velocity,  $g$  is gravitational acceleration,  $t$  is time,  $P$  is pressure,  $A$  is cross sectional area,  $Q$  is flow rate,  $h$  is height, and  $F$  is force.

### Temperature

$$T_F = \frac{9}{5} T_C + 32$$

$$T_C = \frac{5}{9} (T_F - 32)$$

$$T_K = T_C + 273$$

### Thermal Expansion

$$L_f = L_i (1 + \alpha \Delta T)$$

$$V_f = V_i (1 + \beta \Delta T)$$

## Electricity and Magnetism

### Electrostatic Equations

#### Coulomb's Law

$$\vec{F}_{1,2} = \frac{k}{r_{1,2}^2} q_1 q_2 \hat{r}$$

#### Electric Field (Discrete Charges)

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E}_i = \frac{k q}{r_{(i,0)}^2} \hat{r}_{(r,0)}$$

$$E(x) = \Sigma E_i(x) = E_1(x) + E_2(x) + \dots$$

#### Electric Field (Continuous Charges)

$$\lambda = \frac{q}{L} \quad \sigma = \frac{q}{A} \quad \rho = \frac{q}{V}$$

$$\vec{E} = \int_{V,A,L} \frac{k dq}{r^2} \hat{r}$$

$$\text{Line: } dq = \lambda dx$$

$$\text{Surface: } dq = \sigma dA$$

$$\text{Volume: } dq = \rho dV$$

#### Finite Line Charge - Parallel

$$E_x = \frac{kQ}{a(a+L)}$$

→ Where  $L$  is length, and  $a$  is the distance from the end

#### Finite Line Charge - Perpendicular

$$E_y = \frac{kQ}{y \sqrt{y^2 + \left(\frac{L}{2}\right)^2}}$$

#### Infinite Line

$$E = \frac{2k\lambda}{y}$$

#### Infinite Sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

#### Ring of Charge (radius R)

$$E_z = \frac{kzQ}{(z^2 + R^2)^{3/2}}$$

#### Disk of Charge

$$E_z = 2\pi k \sigma \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

#### Electric Flux

$$E_n = E \cdot \hat{n} = E \cos \theta$$

$$\phi = E_n A$$

$$\phi_{net} = \int_S E_n dA$$

#### Gauss's Law

$$\phi_{net} = E_n A = \frac{Q_{in}}{\epsilon_0}$$

#### Electric Field Near a Conductor

$$\vec{E} = \frac{\sigma}{\epsilon_0} (\hat{r})$$

$$\sigma_{total} = \sigma_{charge} + \sigma_{induced}$$

$$E_{total} = \frac{\sigma_{total}}{\epsilon_0} = E_{external} + E_{charge}$$

#### Electric Potential

$$\Delta V = V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{x}$$

$$E = - \frac{dV}{dx}$$

#### Coulomb Potential

$$V = \frac{kq}{r}$$

$$V = \Sigma_i \frac{kq_i}{r_i}$$

$$V = \int \frac{k dq}{r}$$

Line charge:

$$dq = \lambda dx$$

Plane charge:

$$dq = \sigma dx dy$$

Disk charge:

$$dq = \sigma \rho d\theta d\rho = 2\pi\sigma\rho d\rho$$

### Potential Energy

$$U = q_0 V = \frac{k q_0 q}{r}$$

## DC Circuit Equations

### Capacitance

$$C = \frac{Q}{V}$$

$$C_{\text{Parallel Plates}} = \frac{\epsilon_0 A}{d}$$

$$C_{\text{Cylindrical}} = \frac{2\pi\epsilon_0 L}{\ln(r_2/r_1)}$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

→ Where C is capacitance, Q is charge, U is energy in a capacitor, and V is electric potential.

### Dielectrics

$$\kappa = \frac{\epsilon}{\epsilon_0}$$

$$C = \kappa C_0$$

$$E = \frac{E_0}{\kappa}$$

→ Where E is the field with the dielectric, E<sub>0</sub> is the field without the dielectric, and κ is the dielectric constant.

### Current

$$I_{\text{Avg}} = \frac{\Delta Q}{\Delta t}$$

$$J = \frac{I}{A}$$

$$J = qn v_D$$

→ Where J is current density, I is current, A is cross-sectional area, q is charge per particle, n is particle density, v<sub>d</sub> is drift velocity, and Q is overall charge.

### Power

$$P = IV = I^2 R = \frac{V^2}{R}$$

→ Where V is voltage, R is resistance, and I is current.

### Ohm's Law

$$\vec{J} = \sigma \vec{E}$$

$$\sigma = \frac{1}{\rho}$$

$$V = IR$$

$$R = \rho \frac{L}{A}$$

→ Where  $\vec{J}$  is the electric current density (at a point),  $\vec{E}$  is the applied electric field, σ is the conductivity of a material, and ρ is the material resistivity.

### Batteries

$$V_{\text{terminal}} = V_{\text{EMF}} - IR_{\text{internal}}$$

### Resistor Networks

$$R_{\text{Series}} = \sum_i R_i = R_1 + R_2 + \dots$$

$$\frac{1}{R_{\text{Parallel}}} = \sum_i \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

### Capacitor Networks

$$\frac{1}{C_{\text{Series}}} = \sum_i \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

$$C_{\text{Parallel}} = \sum_i C_i = C_1 + C_2 + \dots$$

### Kirchhoff's Circuit Rules

Loop Rule:  $\sum V = 0$

Junction Rule:  $\sum I_{\text{in}} = \sum I_{\text{out}}$

### RC Circuits

$$\tau = RC$$

Discharging:

$$Q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

Charging:

$$Q(t) = C V_{\text{EMF}} (1 - e^{-t/RC}) = Q_{\text{max}} (1 - e^{-t/\tau})$$

$$I(t) = \frac{V_0}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

## Magnetism Equations

### Moving Point Charge

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$$

### Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Idl \times \hat{r}}{4\pi r^2}$$

### B for Various Configurations

$$B_{\text{solenoid}} = \mu_0 \frac{N}{l} I$$

$$B_{\text{loop}} = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

$$B_{\text{loop,center}} = \frac{\mu_0 I}{2R}$$

$$B_{\text{wire}} = \frac{\mu_0}{2\pi} \frac{I}{R}$$

### Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

### Point Charge in B

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$r = \frac{mv}{qB}$$

$$f_{cyclotron} = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$v = \frac{E}{B} \text{ (velocity selector)}$$

### Force on Current Carrying Wires

$$\vec{F} = I\vec{dl} \times \vec{B}$$

$$\frac{\vec{F}}{L} = \frac{\mu_0 I_1 I_2}{2\pi R}$$

### Torque on a Loop

$$\vec{\mu} = NIA\hat{n}$$

$$\tau = \vec{u} \times \vec{B}$$

$$U = -\mu B \cos \theta = -\mu \cdot \vec{B}$$

### Magnetic Flux

$$\phi_B = \int_{\text{Surface}} \vec{B} \cdot \hat{n} dA$$

$$\phi_B = NBA \cos \theta$$

### Faraday's Law

$$EMF = -\frac{d\phi_B}{dt}$$

### Inductance

$$L = \frac{\phi_B}{I}$$

$$L_{solenoid} = \frac{\mu_0 N^2 A}{l}$$

$$EMF_i = -L \frac{dl}{dt}$$

$$U = \frac{1}{2} LI^2$$

### LR Circuits (DC)

$$\tau = \frac{L}{R}$$

$$I(t) = I_{max}(1 - e^{-t/\tau}) \text{ (closes)}$$

$$I(t) = I_{max}e^{-t/\tau} \text{ ("opens")}$$

### LC Circuits (DC)

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\pi f$$

$$I(t) = -\omega Q_{max} \sin(\omega t)$$

### AC Circuits and EM

#### Waves Equations

#### Displacement Current

$$I_d = \epsilon_0 \frac{d\phi_e}{dt}$$

### Maxwell's Equations

$$\oint_s \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon}$$

$$\oint_s \vec{B} \cdot d\vec{A} = 0$$

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint_c \vec{B} \cdot d\vec{l} = \mu \left( I + \epsilon \frac{d\phi_E}{dt} \right)$$

### Lorentz Force

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

### AC Circuits

$$\Delta V = NBA\omega \sin(\omega t)$$

$$\omega = 2\pi f$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$EMF = \Delta V_{EMF} = IZ$$

$$\phi_{\text{phase angle}} = \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$$

$$P_{ave} = I_{RMS}^2 R$$

$$P_{ave} = I_{RMS} V_{RMS} \cos \phi$$

### EM Waves

$$E_x(z, t) = E_0 \sin(kz - \omega t)$$

$$B_y(z, t) = \frac{k}{\omega} E_0 \sin(kz - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$c = \lambda f = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

### Poynting Vector

$$\vec{S} \equiv \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

### Intensity

$$I = \frac{\text{Power}}{\text{Area}} = \frac{EB}{2\mu_0} = \frac{E^2}{2c\mu_0}$$

### Light Equations

#### Light

$$c = \lambda f = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$E = hf = \frac{hc}{\lambda}$$

$$\text{Intensity} = \frac{E_{max} B_{max}}{2\mu_0}$$

$$n_{medium} = \frac{c}{v_{medium}}$$

### Polarization

$$I_{transmitted} = I_{incident} \cos^2 \theta$$

$$\tan \theta_{brewster} = \frac{n_2}{n_1}$$

### Snell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_{crit} = \frac{n_2}{n_1}$$

### Mirrors

$$f = \frac{1}{2}R$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{h_{img}}{h_{obj}} = -\frac{s'}{s}$$

### Lenses

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

$$m = -\frac{n_1 s'}{n_2 s}$$

$$s' = -\frac{n_2}{n_1} s$$

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

### Thin Lens Equation

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

### Interference

Constructive:

$$\Delta L = 0, \lambda, 2\lambda \dots = m\lambda$$

Destructive:

$$\Delta L = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda \dots = (m + \frac{1}{2})\lambda$$

### Phase

$$\Delta\phi = 2\pi \frac{\Delta L}{\lambda}$$

$$= 0, 2\pi, 4\pi \dots \text{const.}$$

$$= \pi, 3\pi, 5\pi \dots \text{dest.}$$

### Double Slit

$$\text{Max: } d \sin \theta = m\lambda$$

$$\text{Min: } d \sin \theta = (m + \frac{1}{2})\lambda$$

### Single Slit

$$\text{Min: } a \sin \theta = m\lambda$$

### Grating

$$\text{Max: } d \sin \theta = m\lambda$$

### Resolution

$$\alpha_{Rayleigh} = \frac{\lambda}{D} \text{ (rads)}$$

### Thin Film

$$\Delta\phi = 2\pi \frac{2t}{\lambda'}$$

$$\Delta\phi = \pi \text{ at } n_2 > n_1$$

## Modern Physics/ Quantum Mechanics

### Heat

$$W = - \int_{V_i}^{V_f} P dV$$

$$Q = mc\Delta T$$

$$Q = \pm mL_f \text{ or } Q = \pm mL_v$$

$$Q_{gained} = -Q_{lost}$$

$$\Delta E_{int} = nC_V\Delta T = Q_{in} + W$$

### Gas laws

$$n = \frac{N}{N_A} = \frac{m \text{ (in g)}}{m_{mol}}$$

$$PV = nRT = Nk_B T$$

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\gamma = \frac{C_p}{C_v}$$

$$K_{avg} = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_B T$$

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

### Entropy

$$S = k_B \ln \Omega$$

$$\Delta S = \int_i^f \frac{dQ_r}{T}$$

### Efficiency and Heat Engines

$$\epsilon = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H} \leq 1 - \frac{T_C}{T_H}$$

$$COP = \frac{Q_C}{W_{in}} \leq \frac{T_C}{T_H - T_C}$$

### Wien's Law

$$\lambda_{peak} = \frac{2.90 \cdot 10^6 \text{ nm} \cdot \text{K}}{T}$$

### Mean Free Path

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

### Waves

$$v = \lambda f$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$k = \frac{2\pi}{\lambda}$$

$$y(x, t) = A \sin(kx \pm \omega t)$$

$$v_s = \sqrt{\frac{F_T}{\mu}}$$

$$v_s = 331 \text{ m/s} \sqrt{1 + \frac{T_C}{273 \text{ C}}}$$

$$v = \sqrt{\frac{B}{\rho}}$$

#### Sound Intensity

$$I = \frac{P}{\text{area}}$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

#### Sound and Standing Waves

$$f' = f \left( \frac{v_{\text{sound}} \pm v_{\text{obs}}}{v_{\text{sound}} \pm v_{\text{source}}} \right)$$

$$f_n = n \frac{v}{2L}; n = 1, 2, 3, \dots$$

$$f_n = n \frac{v}{4L}; n = 1, 3, 5, \dots$$

$$f_{\text{heard}} = \frac{1}{2}(f_1 + f_2)$$

$$f_{\text{beat}} = f_1 - f_2$$

#### Photoelectric Effect

$$E = hf = \frac{hc}{\lambda}$$

$$K_{\text{max}} = eV_{\text{stop}} = hf - \phi$$

$$E_H - E_L = hf$$

#### Schrödinger Equation

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{(\hbar/2\pi)^2}[E - U(x)]\psi(x)$$

#### Penetration Distance

$$\eta = \frac{(\hbar/2\pi)}{\sqrt{2m(U_0 - E)}}$$

#### Uncertainty Equation

$$\Delta x \Delta p_x \geq \frac{\hbar/2\pi}{2}$$

#### Normalization

$$(\text{Prob. xL to xR}) = \int_{x_L}^{x_R} P(x) dx$$

$$\int_{x_L}^{x_R} P(x) dx = \int_{x_L}^{x_R} \psi(x)^2 dx = 1$$

#### Particle in a Box

$$E_n = n^2 \frac{h^2}{8mL^2}$$

#### Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_p$$

$$L = \frac{L_p}{\gamma}$$

$$u = \frac{u' \pm v}{1 \pm u' \cdot \frac{v}{c^2}}$$

#### Relativistic Momentum

$$p = \gamma m_0 v$$

#### Relativistic Kinetic Energy

$$K = (\gamma - 1)E_0$$

#### Relativistic Mass

$$E^2 = (pc)^2 = E_0^2$$

#### Rest Energy

$$E_0 = m_0 c^2$$

#### Total Energy

$$E = \gamma m_0 c^2$$

#### Disintegration Energy

$$B = (Zm_H + Nm_n - m_{\text{atom}})^2$$

#### Half Life

$$t_{1/2} = \tau \ln 2$$

$$N(t) = N_0 e^{-t/\tau}$$

#### Decay Activity

$$R = R_0 e^{-t/\tau}$$

$$R_0 = \frac{N_0}{\tau}$$