Mechanics

Average normal/axial stress

$$\sigma_{avg} = \frac{F}{A}$$

Average Shear Stress

$$\tau_{avg} = \frac{V}{A_s}$$

Average Normal/Axial Strain

$$\epsilon_{avg} = \frac{\Delta L}{L_0}$$

Thermal Strain

$$\epsilon_T = \alpha \Delta T$$

Shear Strain

$$\gamma = \frac{\pi}{2} - \theta$$

$$\gamma = \tan^{-1}\left(\frac{\delta}{L}\right)$$

Poisson's Ratio

$$\nu = -\frac{\epsilon_L}{\epsilon_{Ax}} = -\frac{\epsilon_{trans}}{\epsilon_{long}}$$

Shear Modulus

$$G = \frac{E}{2(1+\nu)}$$

Factor of Safety

$$F_s = \frac{\sigma_{failure}}{\sigma_{allow}}$$

Hooke's Law (1D)

$$\sigma_x = E\epsilon_x$$

Hooke's Law (plane stress)

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) + \alpha\Delta T$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) + \alpha\Delta T$$

$$\epsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

Elongation in Axial Members

$$e = \int_0^L \frac{F(x)\cdot dx}{A(x)\cdot E(x)}$$

$$e = \frac{FL}{EA}$$

$$e = \frac{FL}{EA} + \alpha\Delta T L$$

Angle of Twist

$$\phi = \int_0^L \frac{T(x)\,dx}{I_p(x)\,G(x)}$$

$$\phi = \frac{TL}{I_P G}$$

Maximum Shear Stress

$$\tau_{max} = \frac{Tr}{I_p}$$

Polar Moment of Inertia

$$I_p = \int_A \rho^2\,dA \text{ (General)}$$

$$I_p = \frac{\pi d^4}{32} \text{ (Circular Shaft)}$$

$$I_P = \frac{\pi}{32}(d_o^4 - d_i^4) \text{ (Tubular Shaft)}$$

Equilibrium Beam Relationships

$$\Delta V = P$$

$$\Delta V = V_2 - V_1 = \int_{x_1}^{x_2} p(x)\,dx$$

$$\frac{dV}{dx} = p(x)$$

$$\Delta M = -M$$

$$\Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V(x)\,dx$$

$$\frac{dM}{dx} = V(x)$$

Flexure Stress in Bending Beams

$$\sigma_x = \frac{-My}{I_z}$$

$$S = \frac{I_z}{c}$$

$$S_{design} \geq \frac{M_{max}}{\sigma_{allow}}$$

Centroids

$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$$

Moment of Inertia

$$I_z = \int_A y^2\,dA$$

Rectangle:

$$I_z = \frac{bh^3}{12}$$

Solid Circular Section:

$$I_z = \frac{\pi d^4}{64}$$

Hollow Circular Section:

$$I_z = \frac{\pi}{64}(d_0^4 - d_i^4)$$

Parallel Axis Theorem

$$I_{axis,any} = I_c + d^2 A$$

$$I = \sum_i (I_{c,i} + d_i^2 A_i)$$

Shear Stress in Bending Beams

$$\tau = \frac{VQ}{It}$$

$$Q = \sum_i \bar{y}_i{}' A_i'$$

Solid Circular Section:

$$Q_{max} = \frac{1}{12} d^3$$

Hollow Circular Section:

$$Q_{max} = \frac{1}{12} [d_o^3 - d_i^3]$$

Shear Flow

$$q = \frac{VQ}{I}$$

$$n_f \tau_f A_f \geq q \Delta s$$

Stress Transformations

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\sigma_x + \sigma_y = \sigma_n + \sigma_t$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

Principal (max/min) Stresses

$$\sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \sigma_{avg} + R$$

$$\sigma_2 = \sigma_{avg} - R$$

$$\tau_{max} = \pm R$$

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

If $(\sigma_x - \sigma_y) > 0,\ \theta_P = \theta_{p,1}$

If $(\sigma_x - \sigma_y) < 0,\ \theta_P = \theta_{p,2}$

$$\tan(2\theta_s) = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$$\theta_s = \theta_p \pm 45$$

If $\theta_p > 0,\ \theta_s = \theta_p - 45$

If $\theta_p < 0,\ \theta_s = \theta_p + 45$

Pressure Vessels

$$\sigma_{sphere} = \frac{pr}{2t}$$

$$\sigma_{axial} = \frac{pr}{2t}$$

$$\sigma_{hoop} = \frac{pr}{t}$$

$$\tau_{abs,max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Differential Equations of the Deflection Curve

Deflection $= v(x)$

Slope $= \dfrac{dv}{dx} = \theta(x)$

Moment, $M(x) = EI\dfrac{d^2v}{dx^2}$

Shear, $V(x) = \dfrac{dM}{dx} = EI\dfrac{d^3v}{dx^3}$

Load, $w(x) = \dfrac{dV}{dx} = EI\dfrac{d^4}{dx^4}$