Dynamics

Kinematic Relationships

$$v = \frac{dx}{dt}$$
$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

➔ These equations will also work for terms θ, ω, α if substituted.

Uniform Rectilinear Motion

$$x = x_0 + vt$$
$$\theta = \theta_0 + \omega t$$

➔ Applies when $a = 0$

Uniformly Accelerated Motion

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

➔ These equations will also work for terms θ, ω, α if substituted. Also *"x"* is subjective and could be any defined axis (*y, z, etc*).

3D Rectangular Motion

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$
$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

2D Motion - Tangential & Normal Components

$$\vec{v} = v\hat{e}_t$$
$$\vec{a} = a_t\hat{e}_t + a_n\hat{e}_n$$

➔ Where $a_t = \frac{dv}{dt}$ and $a_n = \frac{v^2}{\rho}$

2D Motion - Radial & Transverse Components

$$\vec{r} = r\hat{e}_r$$
$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

Newtown's Second Law

$$\sum \vec{F} = m\vec{a}$$
$$\sum \vec{M} = I\vec{\alpha}$$

Work & Energy

$$T_1 + U_{1\to 2} = T_2$$

➔ Where $U_{1\to 2} = \int_{A_1}^{A_2} \vec{F} \cdot d\vec{r}$

Mechanical Energy

$$T_1 + V_1 = T_2 + V_2$$
$$V_g = Wy \qquad V_e = \frac{1}{2}kx^2$$
$$T_1 + V_1 + U_{NC1\to 2} = T_2 + V_2$$

Power

$$P = \frac{dU}{dt}$$
$$P = \vec{F} \cdot \vec{v}$$

Efficiency

$$\eta = \frac{U_{out}}{U_{in}}$$

Linear Impulse & Momentum

$$m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2$$

Coefficient of Restitution

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

Rigid Body Kinematics

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$
$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

For a 2D slab:

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2\vec{r}_{B/A}$$

Rotating Frames

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F}$$
$$\vec{a}_P = \vec{a}_{p'} + \vec{a}_{P/F} + \vec{a}_{cor}$$

➔ Where $\vec{a}_{cor} = 2\vec{\Omega} \times \vec{v}_{P/F}$

Rotational Kinetic Energy

$$T_{rot} = \frac{1}{2}\bar{I}\omega^2$$

Moments of Inertia

$$\bar{I}_{disk} = \frac{1}{2}mr^2 \qquad \bar{I}_{rod} = \frac{1}{12}ml^2$$
$$\bar{I}_{hoop,symmetry\ axis} = mr^2$$
$$\bar{I}_{sphere} = \frac{2}{5}mr^2$$

Angular Momentum

$$\vec{H}_G = \bar{I}\vec{\omega}$$