Mathematics

Algebra

Exponent Laws

$x^a \cdot x^b = x^{a+b}$

$(x^a)^b = x^{ab}$

$(xy)^a = (x^a y^a)$

$x^{-1} = \dfrac{1}{x}$

$\dfrac{a^m}{a^n} = a^{m-n}$

Quadratic Formula

➔ Given $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Linear Slope Equations

$y = mx + b$

$y - y_1 = m(x - x_1)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Factoring

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 - b^2 = (a+b)(a-b)$

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Logarithms

$\ln A^x = x \ln A$

$\ln[A \cdot B] = \ln A + \ln B$

$\ln\left[\dfrac{A}{B}\right] = \ln A - \ln B$

$\ln\left(\dfrac{1}{x}\right) = -\ln(x)$

$\ln(1) = 0$ $\quad$ $\ln(e) = 1$

$\ln(e^x) = x$ $\quad$ $e^{\ln(x)} = x$

Vectors and Matrices

$\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{v} \cos(\theta)$

$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$

$\hat{u} \cdot \hat{v} = \cos(\theta)$

$\vec{u} = \sqrt{u_x^2 + u_y^2 + u_z^2}$

$\tan^{-1}\left(\dfrac{r_y}{r_x}\right) = \theta$

➔ Where r_x and r_y are vectors in the x-y plane

$\hat{u} = \dfrac{\vec{u}}{\vec{u}}$

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{v \quad u}$

$adj(A) = \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$

$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$A^{-1} = \dfrac{1}{ad - bc} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$

$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$

$\hat{i} \times \hat{j} = \hat{k}$ $\quad$ $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ $\quad$ $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad$ $\hat{i} \times \hat{k} = -\hat{j}$

Radicals

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Geometry

Circles

$A = \pi r^2$

$C = 2\pi r$

$r^2 = (x-a)^2 + (y-b)^2$

$s = r\theta$

$A_{Hoop} = \dfrac{\pi}{4}(d_o^2 - d_i^2)$

$A_{Hoop} = \pi(r_o^2 - r_i^2)$

➔ (a, b) is the center of the circle.

➔ θ must be in radians.

Cylinders

$A = 2\pi rl + 2\pi r^2$

$V = \pi r^2 l$

Spheres

$A = 4\pi r^2$

$V = \dfrac{4}{3}\pi r^3$

$r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$

➔ (a, b, c) is the center of the sphere and (x, y, z) are coordinates on the surface of the sphere.

Right Triangles

$A = \dfrac{1}{2}bh$

$a^2 + b^2 = c^2$

Equilateral Triangles

$A = \dfrac{\sqrt{3}}{4}a^2$

$\theta = 60°$

Trigonometry

Right Angle Ratios

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

➔ Using reciprocal identities, the ratios for sec, csc, and cot can be found.

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Tan/Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trig Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$\sin(2x) = 2\sin(x)\cdot\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$

$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$

*$\cos^n\theta = [\cos\theta]^n$

*Valid for all trigonometric functions (sin, cos, tan, cot, sec, csc).

Even/Odd Formulas

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

Double Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

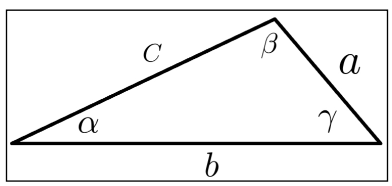
$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

Degrees to Radians

➔ Where D is an angle is degrees and R is an angle in radians.

$$R = D\cdot\frac{\pi}{180} \qquad D = R\cdot\frac{180}{\pi}$$

Law of Sines



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$a^2 = b^2 + c^2 - 2bccos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Small Angle Approx.

$\sin\theta \approx \theta$

$\cos\theta \approx 1 - \frac{\theta^2}{2} \approx 1$

$\tan\theta \approx \theta$

Calculus

Derivative Properties

$\frac{d}{dx}(c) = 0$

$(c\cdot f(x))' = c\cdot f'(x)$

$(f(x) \pm g(x))' = f'(x) \pm g'(x)$

Derivative Power Rule

$\frac{d}{dx}(x^n) = n\cdot x^{n-1}$

Derivative Product Rule

$\frac{d}{dx}\big(f(x)\cdot g(x)\big) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

Derivative Quotient Rule

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}$

Derivative Chain Rule

$\frac{d}{dx}\Big(f\big(g(x)\big)\Big) = f'\big(g(x)\big)\cdot g'(x)$

Standard Derivatives

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}\big(\sin(x)\big) = \cos(x)$

$\frac{d}{dx}\big(\cos(x)\big) = -\sin(x)$

$\frac{d}{dx}\big(\tan(x)\big) = \sec^2(x)$

$\frac{d}{dx}\big(\sec(x)\big) = \sec(x)\cdot\tan(x)$

$\frac{d}{dx}\big(\csc(x)\big) = -\csc(x)\cdot\cot(x)$

$\frac{d}{dx}\big(\cot(x)\big) = -\csc^2(x)$

$\frac{d}{dx}\big(\sin^{-1}(x)\big) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\big(\cos^{-1}(x)\big) = -\frac{1}{\sqrt{1-x^2}}$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\left(n^x\right) = n^x \cdot \ln(n)$$
$$\frac{d}{dx}\left(e^{nx}\right) = n \cdot e^{nx}$$
$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}, x > 0$$
$$\frac{d}{dx}\left(\ln \ x \ \right) = \frac{1}{x}, x \neq 0$$

Chain Rule Variations

$$\frac{d}{dx}\left(\left[f(x)\right]^n\right) = n \cdot \left[f(x)\right]^{n-1} \cdot f'(x)$$
$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x) \cdot e^{f(x)}$$
$$\frac{d}{dx}\left(\ln\left[f(x)\right]\right) = \frac{f'(x)}{f(x)}$$
$$\frac{d}{dx}\left(\sin[f(x)]\right) = f'(x) \cdot \cos\left[f(x)\right]$$
$$\frac{d}{dx}\left(\cos\left[f(x)\right]\right) = -f'(x) \cdot \sin\left[f(x)\right]$$
$$\frac{d}{dx}\left(\tan\left[f(x)\right]\right) = f'(x) \cdot \sec^2\left[f(x)\right]$$
$$\frac{d}{dx}\left(\sec\left[f(x)\right]\right) = f'(x) \cdot \sec\left[f(x)\right] \cdot \tan\left[f(x)\right]$$
$$\frac{d}{dx}\left(\tan^{-1}\left[f(x)\right]\right) = \frac{f'(x)}{1+\left[f(x)\right]^2}$$

Integral Properties

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$
$$\int_a^a dx = 0$$
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$
$$\int_a^b C \cdot f(x)\,dx = C \cdot \int_a^b f(x)\,dx$$
$$\int_a^b C \cdot \,dx = C \cdot (b-a)$$

Integral Power Rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

Standard Integrals

$$\int k\,dx = k \cdot x + C$$
$$\int e^n x\,dx = \frac{1}{n}e^x + C$$
$$\int \cos(x)\,dx = \sin(x) + C$$
$$\int \sin(x)\,dx = -\cos(x) + C$$
$$\int \sec^2(x)\,dx = \tan(x) + C$$
$$\int \csc^2(x)\,dx = -\cot(x) + C$$
$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln \ ax+b \ + C$$
$$\int \sec(x) \cdot \tan(x)\,dx = sec(x) + C$$
$$\int \csc(x) \cdot \cot(x)\,dx = -\csc(x) + C$$
$$\int \sec(x)\,dx = \ln \ \sec(x) + \tan(x) \ + C$$
$$\int \csc(x)\,dx = -\ln \ \csc(x) + \cot(x) \ + C$$
$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln \ x \ + C$$
$$\int \ln(x)\,dx = x \cdot \ln(x) - x + C$$
$$\int \tan(x)\,dx = \ln \ \sec(x) \ + C$$
$$\int \tan(x)\,dx = -\ln \ \cos(x) \ + C$$
$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

Integration Techniques

Topics include U-Sub, Integration by parts, trigonometric integrals, trig. sub, and PFD.

U-Substitution

Take an "X" term to be u, and then take du of that u term. Solve the integral in terms of u, and then re-substitute into the equation.

If needed, find new limits of integration using the substitution. Example:

$$\int_1^2 5x^2\cos(x^3)\,dx$$

so

$$u = x^3 \therefore du = 3x^2\,dx$$

or

$$\frac{1}{3}\,du = x^2\,dx$$

resulting in

$$= 5\int_*^{**} \frac{1}{3}\cos(u)\,du$$

Notice the substitution chosen allows for all x terms to be turned into u terms.

The integral can now easily be solved through standard methods. Once solved, replace u with the substitution above and replace the limits of integration as well. Solve as normal.

It is possible to complete u-sub without suppressing the limits of integration, you will just need to plug the given limits into the u term to find the new limits of integration.

For example, the lower would become $(1^3) = 1$ and the upper would become $(2^3) = 8$. Note that either method works and produces the same solution.

Integration by Parts

The standard formula for integration by parts is as follows:

$$\int u\, dv = uv - \int v\, du$$

Find u and dv in the original equation, then solve for du and v. Plug into the formula above and solve.
The u term can be found according to ILATE: inverse trigonometric, logarithmic, algebraic, trigonometric and exponential.

Example:

$$\int xe^{-x}\, dx$$

so

$$u = x \qquad dv = e^{-x}$$
$$du = dx \qquad v = -e^{-x}$$

using the equation above:

$$= -xe^{-x} + \int e^{-x}\, dx$$

resulting in

$$= -xe^{-x} - e^{-x} + C$$

Trigonometric Integrals

When solving an integral with trigonometric functions (usually involving powers and multiple trig functions multiplied together), a u-sub may not be able to be applied.

Instead, the integral will need to be separated into multiples of the trig function, apply a trig identity, and then complete the u-sub.

Example:

$$\int \sin^6 x \cos^3 x\, dx$$

separating $\cos^3 x$ into $\cos^2 x \cdot \cos x$ and applying an identity:

$$= \int \sin^6 x(1 - \sin^2 x)\cos x\, dx$$

take $u = \sin x \therefore du = \cos x\, dx$ and perform the remaining u-sub:

$$= \int u^6(1 - u^2)\, du$$

ending with:

$$= \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$$

Note that while $\sin^2 x + \cos^2 x = 1$ is a common substitution, it is also common for other identities such as $\tan^2 x + 1 = \sec^2 x$ to be used as well.

Trigonometric Substitution

In certain cases, an integral may contain one of the following roots. In such situation, the following substitutions and formulas will be used to solve the integral.

Case I:

$$\sqrt{a^2 - b^2x^2} \Rightarrow x = \frac{a}{b}\sin\theta$$

uses $\cos^2(\theta) = 1 - \sin^2(\theta)$

Case II:

$$\sqrt{b^2x^2 - a^2} \Rightarrow x = \frac{a}{b}\sec(\theta)$$

uses $\tan^2(\theta) = \sec^2(\theta) - 1$

Case III:

$$\sqrt{a^2 + b^2x^2} \Rightarrow x = \frac{a}{b}\tan(\theta)$$

uses $\sec^2(\theta) = \tan^2(\theta) + 1$

Example:

$$\int \frac{1}{(1 - x^2)^{3/2}}\, dx$$

Because this is a case I problem, use the substitution

$$x = \sin\theta \therefore dx = \cos\theta$$

Apply the substitution(s) back into the original equation:

$$\int \frac{1}{(1 - \sin^2(\theta))^{3/2}} \cdot \cos\theta\, d\theta$$

From here, the integral can be simplified and solved readily:

$$= \int \frac{1}{(\cos^2\theta)^{3/2}} \cdot \cos\theta\, d\theta$$

$$= \int \frac{1}{\cos^3\theta} \cdot \cos\theta\, d\theta$$

$$= \int \frac{1}{\cos^2\theta}\, d\theta$$

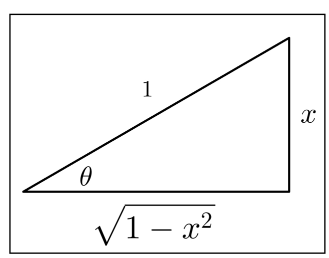
$$= \int \sec^2\theta\, d\theta$$

$$= \tan\theta + C$$

Although temping to assume so, the problem is not solved. Because a substitution was applied near the beginning, the final answer must be in terms of x, not θ.

$$\sin\theta = \frac{x}{1} = \frac{\text{opposite}}{\text{hypotenuse}}$$

By creating a right triangle with this definition, the adjacent side a can be solved:



Recall that $a^2 + b^2 = c^2$ and as such $(x)^2 + (a)^2 = (1)^2$, resulting in

$$a = \sqrt{1-x^2}$$

The final result can finally be expressed in terms of x as

$$= \frac{x}{\sqrt{1-x^2}} + C$$

Partial Fractions

Occasionally an integral will involve a fraction which may be difficult to be solved by standard substitution methods.
Using PFD, the integral can be broken up into simpler fractions which can be easier solved.

Example:

$$\int \frac{3x+2}{x^2+x}\,dx$$

This integral is difficult by itself, due to the fact that an easy u-sub is not available.

To help with this, it can be broken down into simpler integrals. Begin by observing the fraction only and factoring the denominator:

$$\frac{3x+2}{x(x+1)}$$

This fraction can now re-written, with the factors of the denominator for each fraction:

$$\frac{3x+2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

Because the numerators are not known, variables A and B are put in place. Note the original factored fraction goes on the left.

From here the denominator of the left (in this case $x(x+1)$) is multiplied through the equation:

$$^{*}\ 3x+2 = A(x+1) + B(x)$$

Make note that parts of the denominators of terms A and B canceled, resulting in a much simpler expression than what was started with.

Multiplying terms:

$$3x+2 = Ax + A + Bx$$

Group terms based on their order (or "power"):

$$3x+2 = (A+B)x + A$$

From here, the coefficient matching game is played. Match the coefficients from the left (with respect to exponents/powers) to the coefficients of the right.

$$3 = A + B$$
$$2 = A$$

Notice it is just the raw coefficients and A/B terms in the new set of equations. From here, it is seem that $A = 2$ and $B = 1$.

This conclusion could also be reached by revisiting equation *. Because the equation is true for any value of x, the equation can be solved by picking "0" as x and solving from there.

$$3(0)+2 = A(0+1) + B(0)$$
$$2 = A$$

The same could be done for finding B (notice that A cancels this time):

$$3(-1)+2 = A(-1+1) + B(-1)$$
$$-1 = -B \therefore B = 1$$

Once the numerators are realized, they can be plugged back into the first decomposition:

$$\frac{3x+2}{x(x+1)} = \frac{2}{x} + \frac{1}{x+1}$$

Because of this, the starting integral can now be replaced as well:

$$\int \frac{3x+2}{x(x+1)}\,dx = \int \frac{2}{x} + \frac{1}{x+1}\,dx$$

This is now a much easier integral, and can be readily solved using standard methods:

$$\int \frac{2}{x}\,dx + \int \frac{1}{x+1}\,dx$$

The Laplace Transform

$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st}\cdot f(t)\,dt$$

Laplace Transforms

$$\mathscr{L}\{1\} = \frac{1}{s}$$
$$\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}}, n = 1,2,3,...$$
$$\mathscr{L}\{e^{at}\} = \frac{1}{s-a}$$
$$\mathscr{L}\{\sin kt\} = \frac{k}{s^2+k^2}$$
$$\mathscr{L}\{\cos kt\} = \frac{s}{s^2+k^2}$$
$$\mathscr{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$$
$$\mathscr{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$$

Inverse Laplace Transforms

$$\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} = 1$$
$$\mathscr{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n, n = 1,2,3,...$$
$$\mathscr{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$
$$\mathscr{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} = \sin kt$$
$$\mathscr{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos kt$$
$$\mathscr{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\} = \sinh kt$$
$$\mathscr{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\} = \cosh kt$$

Physics

Classical Mechanics

Kinematic Relationships

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

➔ These equations will also work for terms θ, ω, α if substituted.

Uniform Rectilinear Motion

$$x = x_0 + vt$$

$$\theta = \theta_0 + \omega t$$

➔ Applies when $a = 0$

Uniformly Accelerated Motion

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

➔ These equations will also work for terms θ, ω, α if substituted. Also *"x"* is subjective and could be any defined axis (*y, z, etc*).

Circular Motion

$$\theta = \frac{s}{r}$$

$$v = \frac{2\pi r}{T} = r\omega$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$a_{tan} = r\alpha$$

$$\omega = 2\pi f$$

➔ Where s is arc length, r is the radius of curvature, f is the frequency, and T is the period.

General Force Equations

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_{ab} = -\vec{F}_{ba}$$

$$F_g = mg$$

$$F_{Spring} = -ks$$

$$\vec{F}_{static\ friction} = \mu_s \vec{F}_N$$

$$\vec{F}_{kinetic\ friction} = \mu_k \vec{F}_N$$

Work and Energy

$$W = \int_{x_0}^{x_f} F(x)\,dx = \int_{s_0}^{s_f} F \cdot ds$$

$$W = Fd\cos\theta = \vec{F} \cdot \vec{d}$$

➔ Where F is force, d is distance traveled, and θ is the angle between the two $\boldsymbol{F}$ and $\boldsymbol{d}$ vectors

$$KE_1 + W_{1\to 2} = KE_2$$

$$KE_1 + PE_1 = PE_2 + KE_2$$

$$KE = \frac{1}{2}mv^2$$

$$PE_{grav} = mgh$$

$$PE_{Elastic} = \frac{1}{2}kx^2$$

Universal Gravitation

$$F_G = G\frac{m_1 m_2}{r^2}$$

Efficiency

$$\eta = \frac{\text{input}}{\text{output}}$$

Center of Mass

$$x_{CM} = \frac{1}{M}\int x\,dm$$

$$y_{CM} = \frac{1}{M}\int \frac{y}{2}dm$$

$$x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{m_T}$$

$$y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{m_T}$$

Impulse and Momentum

$$p = mv$$

$$m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2$$

$$I = \int_{t_i}^{t_f} F(t)dt = \Delta p$$

$$I = \vec{F}\Delta t$$

$$\vec{F}\Delta t = \Delta p$$

Angular Momentum

$$\vec{L} = \vec{r} \text{ x } \vec{p} = m(\vec{r} \text{ x } \vec{v})$$

$$L = I\omega = mrv\sin\phi$$

$$L = mr^2\omega\sin\phi$$

$$I_1\omega_1 = I_2\omega_2$$

Collisions

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Torque (Moments)

$$\vec{\tau} = \vec{r} \text{ x } \vec{F}$$

$$\tau = F \cdot r\sin\theta$$

Density

$$\rho = \frac{m}{V}$$

$$\rho_{Theor} = \frac{nA}{V_c N_A}$$

Simple Harmonic Motion

$$T = 2\pi\sqrt{\frac{m}{k}} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad v = \frac{\lambda}{T}$$

➔ Where T is the period, m is mass, g is gravitational acceleration, f is the frequency, L is the length of the string, and k is the spring constant.

Fluids (Basic Equations)

$$A_1 v_1 = A_2 v_2$$
$$F_2 A_1 = F_1 A_2$$
$$P = \frac{F}{A}$$
$$P_g = \rho g h$$
$$P_T = P_{atm} + \rho g h$$
$$F_{buoyant} = \rho g V_{fluid}$$
$$\frac{V}{t} = Av = Q$$
$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$
$$\Delta P = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

➔ Where ρ is density, V is volume, v is velocity, g is gravitational acceleration, t is time, P is pressure, A is cross sectional area, Q is flow rate, h is height, and F is force.

Temperature

$$T_F = \frac{9}{5}T_c + 32$$
$$T_c = \frac{5}{9}(T_F - 32)$$
$$T_K = T_C + 273$$

Thermal Expansion

$$L_f = L_i(1 + \alpha \Delta T)$$
$$V_f = V_i(1 + \beta \Delta T)$$

Electricity and Magnetism

Electrostatic Equations

Coulomb's Law

$$\vec{F}_{1,2} = \frac{k\ q_1 q_2}{r_{1,2}^2}\hat{r}$$

Electric Field (Discrete Charges)

$$\vec{E} = \frac{\vec{F}}{q_0}$$
$$\vec{E}_i = \frac{kq}{r_{(i,0)}^2}\hat{r}_{(r,0)}$$
$$E(x) = \Sigma E_i(x) = E_1(x) + E_2(x) + \ldots$$

Electric Field (Continuous Charges)

$$\lambda = \frac{q}{L} \qquad \sigma = \frac{q}{A} \qquad \rho = \frac{q}{V}$$
$$\vec{E} = \int_{V,A,L} \frac{k\,dq}{r^2}\hat{r}$$

Line: $dq = \lambda dx$

Surface: $dq = \sigma dA$

Volume: $dq = \rho dV$

Finite Line Charge - Parallel

$$E_x = \frac{kQ}{a(a+L)}$$

➔ Where L is length, and a is the distance from the end

Finite Line Charge - Perpendicular

$$E_y = \frac{kQ}{y\sqrt{y^2 + \left(\frac{L}{2}\right)^2}}$$

Infinite Line

$$E = \frac{2k\lambda}{y}$$

Infinite Sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

Ring of Charge (radius R)

$$E_z = \frac{kzQ}{(z^2 + R^2)^{3/2}}$$

Disk of Charge

$$E_z = 2\pi k\sigma\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Electric Flux

$$E_n = E \cdot \hat{n} = E\cos\theta$$
$$\phi = E_n A$$
$$\phi_{net} = \int_S E_n dA$$

Gauss's Law

$$\phi_{net} = E_n A = \frac{Q_{in}}{\epsilon_0}$$

Electric Field Near a Conductor

$$\vec{E} = \frac{\sigma}{\epsilon_0}(\hat{r})$$
$$\sigma_{total} = \sigma_{charge} + \sigma_{induced}$$
$$E_{total} = \frac{\sigma_{total}}{\epsilon_0} = E_{external} + E_{charge}$$

Electric Potential

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{x}$$
$$E = -\frac{dV}{dx}$$

Coulomb Potential

$$V = \frac{kq}{r}$$
$$V = \Sigma_i \frac{kq_i}{r_i}$$

$$V = \int \frac{k\,dq}{r}$$

Line charge:

$dq = \lambda dx$

Plane charge:

$dq = \sigma dx dy$

Disk charge:

$dq = \sigma\rho d\theta d\rho = 2\pi\sigma\rho d\rho$

Potential Energy

$$U = q_0 V = \frac{kq_0 q}{r}$$

DC Circuit Equations

Capacitance

$$C = \frac{Q}{V}$$

$$C_{ParallelPlates} = \frac{\epsilon_0 A}{d}$$

$$C_{Cylindrical} = \frac{2\pi\epsilon_0 L}{\ln(r_2/r_1)}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

➔ Where C is capacitance, Q is charge, U is energy in a capacitor, and V is electric potential.

Dielectrics

$$\kappa = \frac{\epsilon}{\epsilon_0}$$

$$C = \kappa C_0$$

$$E = \frac{E_0}{\kappa}$$

➔ Where E is the field with the dielectric, E_0 is the field without the dielectric, and κ is the dielectric constant.

Current

$$I_{Avg} = \frac{\Delta Q}{\Delta t}$$

$$J = \frac{I}{A}$$

$$J = qnv_D$$

➔ Where J is current density, I is current, A is cross-sectional area, q is charge per particle, n is particle density, v_d is drift velocity, and Q is overall charge.

Power

$$P = IV = I^2R = \frac{V^2}{R}$$

➔ Where V is voltage, R is resistance, and I is current.

Ohm's Law

$$\vec{J} = \sigma\vec{E}$$

$$\sigma = \frac{1}{\rho}$$

$$V = IR$$

$$R = \rho\frac{L}{A}$$

➔ Where $\vec{J}$ is the electric current density (at a point), $\vec{E}$ is the applied electric field, σ is the conductivity of a material, and ρ is the material resistivity.

Batteries

$$V_{terminal} = V_{EMF} - IR_{internal}$$

Resistor Networks

$$R_{Series} = \Sigma_i R_i = R_1 + R_2 + \ldots$$

$$\frac{1}{R_{Parallel}} = \Sigma_i \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

Capacitor Networks

$$\frac{1}{C_{Series}} = \Sigma_i \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

$$C_{Parallel} = \Sigma_i C_i = C_1 + C_2 + \ldots$$

Kirchhoff's Circuit Rules

Loop Rule: $\Sigma V = 0$

Junction Rule: $\Sigma I_{in} = \Sigma I_{out}$

RC Circuits

$\tau = RC$

Discharging:

$$Q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

Charging:

$$Q(t) = CV_{EMF}(1 - e^{-t/RC})$$
$$= Q_{max}(1 - e^{-t/\tau})$$

$$I(t) = \frac{V_0}{R}e^{-t/RC} = I_0 e^{-t/RC}$$

Magnetism Equations

Moving Point Charge

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \text{ x } \hat{r}}{r^2}$$

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl \text{ x } \hat{r}}{4\pi r^2}$$

B for Various Configurations

$$B_{solenoid} = \mu_0 \frac{N}{l} I$$

$$B_{loop} = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

$$B_{loop,center} = \frac{\mu_0 I}{2R}$$

$$B_{\infty wire} = \frac{\mu_0}{2\pi}\frac{I}{R}$$

Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

Point Charge in B

$$\overrightarrow{F} = q\vec{v} \text{ x } B$$

$$r = \frac{mv}{qB}$$

$$f_{cyclotron} = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$v = \frac{E}{B} \text{ (velocity selector)}$$

Force on Current Carrying Wires

$$\overrightarrow{F} = \vec{I}dl \text{ x } \overrightarrow{B}$$

$$\frac{\overrightarrow{F}}{L} = \frac{\mu_0 I_1 I_2}{2\pi R}$$

Torque on a Loop

$$\overrightarrow{\mu} = NIA\hat{n}$$

$$\tau = \vec{u} \text{ x } \overrightarrow{B}$$

$$U = -\mu B\cos\theta = -\mu \cdot B$$

Magnetic Flux

$$\phi_B = \int_{Surface} \overrightarrow{B} \cdot \hat{n}\, dA$$

$$\phi_B = NBA\cos\theta$$

Faraday's Law

$$EMF = -\frac{d\phi_B}{dt}$$

Inductance

$$L = \frac{\phi_B}{I}$$

$$L_{solenoid} = \frac{\mu_0 N^2 A}{l}$$

$$EMF_i = -L\frac{dl}{dt}$$

$$U = \frac{1}{2}LI^2$$

LR Circuits (DC)

$$\tau = \frac{L}{R}$$

$$I(t) = I_{max}(1 - e^{-t/\tau}) \text{ (closes)}$$

$$I(t) = I_{max}e^{-t/\tau} \text{ (``opens'')}$$

LC Circuits (DC)

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\pi f$$

$$I(t) = -\omega Q_{max}\sin(\omega t)$$

AC Circuits and EM Waves Equations

Displacement Current

$$I_d = \epsilon_0 \frac{d\phi_e}{dt}$$

Maxwell's Equations

$$\oint_S \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q}{\epsilon}$$

$$\oint_S \overrightarrow{B} \cdot d\overrightarrow{A} = 0$$

$$\oint_C \overrightarrow{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint_C \overrightarrow{B} \cdot d\vec{l} = \mu\left(I + \epsilon\frac{d\phi_E}{dt}\right)$$

Lorentz Force

$$\overrightarrow{F} = q\overrightarrow{E} + q(\vec{v} \text{ x } \overrightarrow{B})$$

AC Circuits

$$\Delta V = NBA\omega\sin(\omega t)$$

$$\omega = 2\pi f$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$EMF = \Delta V_{EMF} = IZ$$

$$\phi_{\text{phase angle}} = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$P_{ave} = I_{RMS}^2 R$$

$$P_{ave} = I_{RMS}V_{RMS}\cos\phi$$

EM Waves

$$E_x(z,t) = E_0\sin(kz - \omega t)$$

$$B_y(z,t) = \frac{k}{\omega}E_0\sin(kz - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$c = \lambda f = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

Poynting Vector

$$\vec{S} \equiv \frac{1}{\mu_0}\overrightarrow{E} \text{ x } \overrightarrow{B}$$

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

Intensity

$$I = \frac{\text{Power}}{\text{Area}} = \frac{EB}{2\mu_0} = \frac{E^2}{2c\mu_0}$$

Light Equations

Light

$$c = \lambda f = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

$$E = hf = \frac{hc}{\lambda}$$

$$\text{Intensity} = \frac{E_{max}B_{max}}{2\mu_0}$$

$$n_{medium} = \frac{c}{v_{medium}}$$

Polarization

$$I_{transmitted} = I_{incident}\cos^2\theta$$
$$\tan\theta_{brewster} = \frac{n_2}{n_1}$$

Snell's Law

$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\sin\theta_{crit} = \frac{n_2}{n_1}$$

Mirrors

$$f = \frac{1}{2}R$$
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$
$$m = \frac{h_{img}}{h_{obj}} = -\frac{s'}{s}$$

Lenses

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$
$$m = -\frac{n_1 s'}{n_2 s}$$
$$s' = -\frac{n_2}{n_1}s$$
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Thin Lens Equation

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

Interference

Constructive:

$\Delta L = 0, \lambda, 2\lambda \ldots = m\lambda$

Destructive:

$\Delta L = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda \ldots = (m + \frac{1}{2})\lambda$

Phase

$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda}$$

$= 0, 2\pi, 4\pi \ldots$ const.

$= \pi, 3\pi, 5\pi \ldots$ dest.

Double Slit

Max: $d\sin\theta = m\lambda$

Min: $d\sin\theta = (m + \frac{1}{2})\lambda$

Single Slit

Min: $a\sin\theta = m\lambda$

Grating

Max: $d\sin\theta = m\lambda$

Resolution

$\alpha_{Rayleigh} = \frac{\lambda}{D}$ (rads)

Thin Film

$$\Delta\phi = 2\pi\frac{2t}{\lambda'}$$

$\Delta\phi = \pi$ at $n_2 > n_1$

Modern Physics/ Quantum Mechanics

Heat

$$W = -\int_{V_i}^{V_f} P\,dV$$
$$Q = mc\Delta T$$

$Q = \pm mL_f$ or $Q = \pm mL_v$

$$Q_{gained} = -Q_{lost}$$
$$\Delta E_{int} = nC_V\Delta T = Q_{in} + W$$

Gas laws

$$n = \frac{N}{N_A} = \frac{m \text{ (in g)}}{m_{\text{mol}}}$$
$$PV = nRT = Nk_BT$$
$$P_iV_i^\gamma = P_fV_f^\gamma$$
$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$
$$\gamma = \frac{C_p}{C_V}$$
$$K_{avg} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$$
$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

Entropy

$$S = k_B\ln\Omega$$
$$\Delta S = \int_i^f \frac{dQ_r}{T}$$

Efficiency and Heat Engines

$$\epsilon = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H} \le 1 - \frac{T_C}{T_H}$$
$$COP = \frac{Q_C}{W_{in}} \le \frac{T_C}{T_H - T_C}$$

Wien's Law

$$\lambda_{peak} = \frac{2.90 \cdot 10^6 \text{ nm} \cdot \text{K}}{T}$$

Mean Free Path

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

Waves

$$v = \lambda f$$
$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$
$$k = \frac{2\pi}{\lambda}$$
$$y(x,t) = A\sin(kx \pm \omega t)$$

$$v_s = \sqrt{\frac{F_T}{\mu}}$$

$$v_s = 331 \text{ m/s} \sqrt{1 + \frac{T_C}{273 \text{ C}}}$$

$$v = \sqrt{\frac{B}{\rho}}$$

Sound Intensity

$$I = \frac{P}{area}$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

Sound and Standing Waves

$$f' = f\left(\frac{v_{sound} \pm v_{obs}}{v_{sound} \pm v_{source}}\right)$$

$$f_n = n\frac{v}{2L}; n = 1,2,3,...$$

$$f_n = n\frac{v}{4L}; n = 1,3,5,...$$

$$f_{heard} = \frac{1}{2}(f_1 + f_2)$$

$$f_{beat} = \quad f_1 - f_2$$

Photoelectric Effect

$$E = hf = \frac{hc}{\lambda}$$

$$K_{max} = eV_{stop} = hf - \phi$$

$$E_H - E_L = hf$$

Schrödinger Equation

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{(h/2\pi)^2}[E - U(x)]\psi(x)$$

Penetration Distance

$$\eta = \frac{(h/2\pi)}{\sqrt{2m(U_0 - E)}}$$

Uncertainty Equation

$$\Delta x \Delta p_x \geq \frac{h/2\pi}{2}$$

Normalization

$$(\text{Prob. xL to xR}) = \int_{x_L}^{x_R} P(x)dx$$

$$\int_{x_L}^{x_R} P(x)dx = \int_{x_L}^{x_R} \psi(x)^{\ 2} dx = 1$$

Particle in a Box

$$E_n = n^2\frac{h^2}{8mL^2}$$

Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_p$$

$$L = \frac{L_p}{\gamma}$$

$$u = \frac{u' \pm v}{1 \pm u' \cdot \frac{v}{c^2}}$$

Relativistic Momentum

$$p = \gamma m_0 v$$

Relativistic Kinetic Energy

$$K = (\gamma - 1)E_0$$

Relativistic Mass

$$E^2 = (pc)^2 = E_0$$

Rest Energy

$$E_0 = m_0 c^2$$

Total Energy

$$E = \gamma m_0 c^2$$

Disintegration Energy

$$B = (Zm_H + Nm_n - m_{atom})^2$$

Half Life

$$t_{1/2} = \tau \ln 2$$

$$N(t) = N_0 e^{-t/\tau}$$

Decay Activity

$$R = R_0 e^{-t/\tau}$$

$$R_0 = \frac{N_0}{\tau}$$

Dynamics

Kinematic Relationships

$v = \frac{dx}{dt}$

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$

➔ These equations will also work for terms θ, ω, α if substituted.

Uniform Rectilinear Motion

$x = x_0 + vt$

$\theta = \theta_0 + \omega t$

➔ Applies when $a = 0$

Uniformly Accelerated Motion

$v = v_0 + at$

$x = x_0 + v_0t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(x - x_0)$

➔ These equations will also work for terms θ, ω, α if substituted. Also "*x*" is subjective and could be any defined axis (*y, z, etc*).

3D Rectangular Motion

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$

$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

2D Motion - Tangential & Normal Components

$\vec{v} = v\hat{e}_t$

$\vec{a} = a_t\hat{e}_t + a_n\hat{e}_n$

➔ Where $a_t = \frac{dv}{dt}$ and $a_n = \frac{v^2}{\rho}$

2D Motion - Radial & Transverse Components

$\vec{r} = r\hat{e}_r$

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

Newtown's Second Law

$\sum \vec{F} = m\vec{a}$

$\sum \vec{M} = I\vec{\alpha}$

Work & Energy

$T_1 + U_{1\to 2} = T_2$

➔ Where $U_{1\to 2} = \int_{A_1}^{A_2} \vec{F} \cdot d\vec{r}$

Mechanical Energy

$T_1 + V_1 = T_2 + V_2$

$V_g = Wy$ $\qquad$ $V_e = \frac{1}{2}kx^2$

$T_1 + V_1 + U_{NC1\to 2} = T_2 + V_2$

Power

$P = \frac{dU}{dt}$

$P = \vec{F} \cdot \vec{v}$

Efficiency

$\eta = \frac{U_{out}}{U_{in}}$

Linear Impulse & Momentum

$m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2$

Coefficient of Restitution

$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$

Rigid Body Kinematics

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$

For a 2D slab:

$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2\vec{r}_{B/A}$

Rotating Frames

$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F}$

$\vec{a}_P = \vec{a}_{p'} + \vec{a}_{P/F} + \vec{a}_{cor}$

➔ Where $\vec{a}_{cor} = 2\vec{\Omega} \times \vec{v}_{P/F}$

Rotational Kinetic Energy

$T_{rot} = \frac{1}{2}\bar{I}\omega^2$

Moments of Inertia

$\bar{I}_{disk} = \frac{1}{2}mr^2$ $\qquad$ $\bar{I}_{rod} = \frac{1}{12}ml^2$

$\bar{I}_{hoop,symmetry\ axis} = mr^2$

$\bar{I}_{sphere} = \frac{2}{5}mr^2$

Angular Momentum

$\vec{H}_G = \bar{I}\vec{\omega}$

Mechanics

Average normal/axial stress

$$\sigma_{avg} = \frac{F}{A}$$

Average Shear Stress

$$\tau_{avg} = \frac{V}{A_s}$$

Average Normal/Axial Strain

$$\epsilon_{avg} = \frac{\Delta L}{L_0}$$

Thermal Strain

$$\epsilon_T = \alpha \Delta T$$

Shear Strain

$$\gamma = \frac{\pi}{2} - \theta$$

$$\gamma = \tan^{-1}\left(\frac{\delta}{L}\right)$$

Poisson's Ratio

$$\nu = -\frac{\epsilon_L}{\epsilon_{Ax}} = -\frac{\epsilon_{trans}}{\epsilon_{long}}$$

Shear Modulus

$$G = \frac{E}{2(1+\nu)}$$

Factor of Safety

$$F_s = \frac{\sigma_{failure}}{\sigma_{allow}}$$

Hooke's Law (1D)

$$\sigma_x = E\epsilon_x$$

Hooke's Law (plane stress)

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) + \alpha\Delta T$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) + \alpha\Delta T$$

$$\epsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

Elongation in Axial Members

$$e = \int_0^L \frac{F(x)\cdot dx}{A(x)\cdot E(x)}$$

$$e = \frac{FL}{EA}$$

$$e = \frac{FL}{EA} + \alpha\Delta TL$$

Angle of Twist

$$\phi = \int_0^L \frac{T(x)\,dx}{I_p(x)\,G(x)}$$

$$\phi = \frac{TL}{I_P G}$$

Maximum Shear Stress

$$\tau_{max} = \frac{Tr}{I_p}$$

Polar Moment of Inertia

$$I_p = \int_A \rho^2\,dA \text{ (General)}$$

$$I_p = \frac{\pi d^4}{32} \text{ (Circular Shaft)}$$

$$I_P = \frac{\pi}{32}(d_o^4 - d_i^4) \text{ (Tubular Shaft)}$$

Equilibrium Beam Relationships

$$\Delta V = P$$

$$\Delta V = V_2 - V_1 = \int_{x_1}^{x_2} p(x)\,dx$$

$$\frac{dV}{dx} = p(x)$$

$$\Delta M = -M$$

$$\Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V(x)\,dx$$

$$\frac{dM}{dx} = V(x)$$

Flexure Stress in Bending Beams

$$\sigma_x = \frac{-My}{I_z}$$

$$S = \frac{I_z}{c}$$

$$S_{design} \geq \frac{M_{max}}{\sigma_{allow}}$$

Centroids

$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$$

Moment of Inertia

$$I_z = \int_A y^2\,dA$$

Rectangle:

$$I_z = \frac{bh^3}{12}$$

Solid Circular Section:

$$I_z = \frac{\pi d^4}{64}$$

Hollow Circular Section:

$$I_z = \frac{\pi}{64}(d_0^4 - d_i^4)$$

Parallel Axis Theorem

$$I_{axis,any} = I_c + d^2 A$$

$$I = \sum_i (I_{c,i} + d_i^2 A_i)$$

Shear Stress in Bending Beams

$$\tau = \frac{VQ}{It}$$

$$Q = \sum_i \bar{y}_i{}' A_i'$$

Solid Circular Section:

$$Q_{max} = \frac{1}{12} d^3$$

Hollow Circular Section:

$$Q_{max} = \frac{1}{12}[d_o^3 - d_i^3]$$

Shear Flow

$$q = \frac{VQ}{I}$$

$$n_f \tau_f A_f \geq q \Delta s$$

Stress Transformations

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\sigma_x + \sigma_y = \sigma_n + \sigma_t$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

Principal (max/min) Stresses

$$\sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \sigma_{avg} + R$$

$$\sigma_2 = \sigma_{avg} - R$$

$$\tau_{max} = \pm R$$

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

If $(\sigma_x - \sigma_y) > 0,\ \theta_P = \theta_{p,1}$

If $(\sigma_x - \sigma_y) < 0,\ \theta_P = \theta_{p,2}$

$$\tan(2\theta_s) = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$$\theta_s = \theta_p \pm 45$$

If $\theta_p > 0,\ \theta_s = \theta_p - 45$

If $\theta_p < 0,\ \theta_s = \theta_p + 45$

Pressure Vessels

$$\sigma_{sphere} = \frac{pr}{2t}$$

$$\sigma_{axial} = \frac{pr}{2t}$$

$$\sigma_{hoop} = \frac{pr}{t}$$

$$\tau_{abs,max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Differential Equations of the Deflection Curve

Deflection $= v(x)$

Slope $= \dfrac{dv}{dx} = \theta(x)$

Moment, $M(x) = EI\dfrac{d^2v}{dx^2}$

Shear, $V(x) = \dfrac{dM}{dx} = EI\dfrac{d^3v}{dx^3}$

Load, $w(x) = \dfrac{dV}{dx} = EI\dfrac{d^4}{dx^4}$

Materials Engineering

Lattice Paramaters

SC : $a = 2R$

BCC : $a\sqrt{3} = 4R$

FCC : $a\sqrt{2} = 4R$

DC : $a = \dfrac{8R}{\sqrt{3}}$

Points, Directions and Planes Notation

Points : $x,\ y,\ z$

Direction (singular) : $[h\ k\ l]$

Direction (family) : $< h\ k\ l >$

Planes (singular) : $(h\ k\ l)$

Planes (family) : $\{h\ k\ l\}$

Density (metals)

$$\rho = \frac{nA}{V_c N_A}$$

%Ionic Character

$$\%\,\mathrm{IC} = [1 - \exp[-(0.25)(X_A - X_B)^2] \times 100$$

Linear Density

$$LD = \frac{L_{\text{occupied}}}{L_{\text{total}}}$$

Planar Density

$$PD = \frac{A_{\text{occupied}}}{A_{\text{total}}}$$

Atomic Packing Factor

$$APF = \frac{V_{\text{occupied}}}{V_{\text{total}}}$$

Bragg's Law

$$n\lambda = 2d_{hkl}\sin\theta$$

Inter-Planar Spacing (for cubic symmetries)

$$d_{dkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Vacancies/Unit Volume

$$N_v = N\exp\left(-\frac{Q_v}{kT}\right)$$

Atomic Sites/Unit Volume

$$N = \frac{N_A \rho}{A}$$

Composition (weight %)

$$C_w = \frac{m_1}{m_1 + m_2} \times 100$$

Composition (atomic %)

$$C_a = \frac{n_{m1}}{n_{m1} + n_{m2}} \times 100$$

Mean Intercept Length

$$\bar{l} = \frac{L_T}{PM}$$

Diffusion

Diffusion Flux

$$J = \frac{M}{At}$$

Fick's First Law

$$J = -D\frac{dC}{dx}$$

Fick's Second Law

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

$$\frac{C_x - C_0}{C_s - C_0} = 1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Temperature Dependence of Diffusion Coefficient

$$D = D_0\exp\left(-\frac{Q_d}{RT}\right)$$

Mechanical Properties

Engineering Stress

$$\sigma = \frac{F}{A_0}$$

Engineering Strain

$$\epsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}$$

Hooke's Law

$$\sigma = E\epsilon$$

Poisson's Ratio

$$v = -\frac{\epsilon_x}{\epsilon_z} = \frac{\epsilon_y}{\epsilon_z}$$

Ductility % Elongation

$$\%\,EL = \left(\frac{l_f - l_0}{l_0}\right) \times 100$$

Ductility % Area Reduction

$$\%\,RA = \left(\frac{A_0 - A_f}{A_0}\right) \times 100$$

True Stress

$$\sigma_T = \frac{F}{A_i}$$

True Strain

$$\epsilon_T = \ln \frac{l_i}{l_0}$$

Plastic Region to the Point of Necking

$$\sigma_T = K\epsilon_T^n$$

Resolved Shear Stress

$$\tau_R = \sigma \cos\phi \cos\lambda$$

Critical Resolved Shear Stress

$$\tau_{crss} = \sigma_y(\cos\phi \cos\lambda)_{max}$$

Hall-Petch Equation

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

Percent Cold Work

$$\%\,CW = \left(\frac{A_0 - A_d}{A_0}\right) \times 100$$

Average Grain Size

$$d^n - d_0^n = Kt$$

➔ During grain growth

Failure/Fracture Mechanics

Elliptical Cracks

$$\sigma_m = 2\sigma_0\left(\frac{a}{p_t}\right)^{1/2}$$

➔ Where ‘a’ is the length of an edge external crack, and ‘2a’ is the length of an internal crack.

Fracture Toughness

$$K_c = Y\sigma_c\sqrt{\pi a}$$

Critical Stress

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}}$$

Maximum Flaw Size

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma Y}\right)^2$$

Mean Stress (fatigue tests)

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Stress Amplitude (fatigue tests)

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Range of Stress (fatigue tests)

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

Max. Stress for Fatigue Rotating-Bending Tests

$$\sigma = \frac{16FL}{\pi d_0^3}$$

Thermal Stress

$$\sigma = \alpha_l E \Delta T$$

Steady-State Creep Rate

$$\dot{\epsilon}_s = K_1\sigma^n \text{ (constant T)}$$

$$\dot{\epsilon}_s = K_2\sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

Larson-Miller Parameter

$$m = T(C + \log t_r)$$

Phase Diagrams

Mass Fraction - Liquid Phase

$$W_L = \frac{C_\alpha - C_0}{C_\alpha - C_L}$$

➔ For binary isomorphous

Mass Fraction - Solid Phase

$$W_\alpha = \frac{C_0 - C_L}{C_\alpha - C_L}$$

➔ For binary isomorphous

Phase Transformations

Critical Radius for Stable Solid Particle

$$r^* = \left(-\frac{2\gamma T_m}{\Delta H_f}\right)\left(\frac{1}{T_m - T}\right)$$

Homogenous nucleation:

$$r^* = -\frac{2\gamma}{\Delta G_v}$$

Heterogenous nucleation:

$$r^* = -\frac{2\gamma_{SL}}{\Delta G_v}$$

Activation Free Energy for Formation of Stable Solid Particle

$$\Delta G^* = \left(\frac{16\pi\gamma^3 T_m^2}{3\Delta H_f^2}\right)\frac{1}{(T_m - T)^2}$$

Homogenous nucleation:

$$\Delta G^* = \frac{16\pi\gamma^3}{3(\Delta G_v)^2}$$

Heterogenous nucleation:

$$\Delta G^* = \left(\frac{16\pi\gamma_{SL}^3}{3(\Delta G_v)^2}\right)S(\theta)$$

Interfacial Energies with Heterogenous Nucleation

$\gamma_{IL} = \gamma_{SI} + \gamma_{SL} \cos\theta$

Fraction of Transformation

$y = 1 - \exp(-kt^n)$

Transformation Rate

$$\text{rate} = \frac{1}{t_{0.5}}$$

Ceramic Properties

Density (Ceramics)

$$\rho = \frac{n'\left(\sum A_C + \sum A_A\right)}{V_C N_A}$$

Flexural Strength

Rectangular cross section:

$$\sigma_{fs} = \frac{3F_f L}{2bd^2}$$

Circular cross section:

$$\sigma_{fs} = \frac{F_f L}{\pi R^3}$$

Porous Ceramics

Elastic modulus:

$E = E_0(1 - 1.9P + 0.9P^2)$

Flexural strength:

$\sigma_{fs} = \sigma_0 \exp(-nP)$

Polymer Properties

Molecular Weight

Number-Average:

$\bar{M}_n = \sum x_i M_i$

Weight-Average:

$\bar{M}_w = \sum w_i M_i$

For copolymers, average repeat unit molecular weight:

$\bar{m} = \sum f_i m_i$

Degree of Polymerization

$$n = \frac{\bar{M}_n}{m}$$

% Crystallinity by Weight

$$\%\,C = \frac{\rho_c(\rho_s - \rho_\alpha)}{\rho_s(\rho_c - \rho_\alpha)} \times 100$$

Diffusion Flux

$$J = -P_M \frac{\Delta P}{\Delta x}$$

➔ For steady state diffusion through a polymer membrane

Relaxation Modulus

$$E_r(t) = \frac{\sigma(t)}{\epsilon_0}$$

Polymer Tensile Strength

$$TS = TS_\infty - \frac{A}{\bar{M}_n}$$

Composites

Rules of Mixtures

Transverse properties:

$$E_c^{trans} = \frac{E_m E_f}{E_m V_f + E_f V_m}$$

Axial/longitudinal properties:

$E_c^{long} = E_m V_m + E_p V_p$

Critical Fiber Length

$$l_c = \frac{\sigma_f^* d}{2\tau_c}$$

Longitudinal Tensile Strength

For continuous and aligned fibrous composite:

$\sigma_{cl}^* = \sigma_m'(1 - V_f) + \sigma_f^* V_f$

For discontinuous and aligned fibrous composite and $l > l_c$:

$$\sigma_{cd}^* = \sigma_f^* V_f\left(1 - \frac{l_c}{2l}\right) + \sigma_m'(1 - V_f)$$

For discontinuous and aligned fibrous composite and $l < l_c$:

$$\sigma_{cd'}^* = \frac{l\tau_c}{d} V_f + \sigma_m'(1 - V_f)$$

Corrosion and Degradation

Electrochemical Cell Potential

For two standard half-cells:

$$\Delta V^0 = V_2^0 - V_1^0$$

For two nonstandard half-cells:

$$\Delta V = (V_2^0 - V_1^0) - \frac{RT}{n\mathcal{F}} \ln \frac{[M_1^{n+}]}{[M_2^{n+}]}$$

For two nonstandard half-cells, room temperature:

$$\Delta V = (V_2^0 - V_1^0) - \frac{0.0592}{n} \log \frac{[M_1^{n+}]}{[M_2^{n+}]}$$

Corrosion Penetration Rate

$$\mathrm{CPR} = \frac{KW}{\rho A t}$$

Corrosion Rate

$$r = \frac{i}{n\mathcal{F}}$$

Overvoltage

For activation polarization:

$$\eta_\alpha = \pm \beta \log \frac{i}{i_0}$$

For concentration polarization:

$$\eta_c = \frac{2.3RT}{n\mathcal{F}} \log\left(1 - \frac{i}{i_L}\right)$$

Pilling-Bedworth Ratio

For divalent metals:

$$\text{P-B ratio} = \frac{A_o \rho_M}{A_M \rho_o}$$

For other than divalent metals:

$$\text{P-B ratio} = \frac{A_o \rho_M}{a A_M \rho_o}$$

Metal Oxidation

Parabolic rate expression:

$$W^2 = K_1 t + K_2$$

Linear rate expression:

$$W + K_3 t$$

Logarithmic rate expression:

$$W = K_4 \log(K_5 t + K_6)$$

Electrical Properties

Ohm's Law

$$V = IR$$

Electrical Resistivity

$$\rho = \frac{RA}{l}$$

Electrical Conductivity

$$\sigma = \frac{1}{\rho}$$

Current Density

$$J = \sigma \mathcal{E}$$

Electrical Field Intensity

$$\mathcal{E} = \frac{V}{l}$$

(Metals) Conductivity for *n*-Type Extrinsic Semiconductor

$$\sigma = n \;\; e \;\; \mu_e$$

(Metals) Matthiessen's Rule

$$\rho_{total} = \rho_t + \rho_i + \rho_d$$

Thermal Resistivity Contribution

$$\rho_t = \rho_0 + aT$$

Impurity Resistivity Contribution

Impurity resistivity contribution, single-phase alloy:

$$\rho_i = A c_i (1 - c_i)$$

Impurity resistivity contribution, two-phase alloy:

$$\rho_i = \rho_\alpha V_\alpha + \rho_\beta V_\beta$$

Conductivity for Intrinsic Semiconductor

$$\sigma = n \;\; e \;\; \mu_e + p \;\; e \;\; \mu_h$$
$$= n_i \;\; e \;\; (\mu_e + \mu_h)$$

Conductivity for *p*-Type Extrinsic Semiconductor

$$\sigma \cong p \;\; e \;\; \mu_h$$

Capacitance

$$C = \frac{Q}{V}$$

Parallel-plate capacitor (vacuum):

$$C = \epsilon_0 \frac{A}{l}$$

Parallel-plate capacitor (dielectric medium between plates):

$$C = \epsilon \frac{A}{l}$$

Dielectric Constant

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Dielectric Displacement

$D = \epsilon_0 \mathscr{E} + P$

In a vacuum:

$D_0 = \epsilon_0 \mathscr{E}$

In a dielectric material:

$D = \epsilon \mathscr{E}$

Polarization

$P = \epsilon_0(\epsilon_r - 1)\mathscr{E}$

Thermal Properties

Heat Capacity

$C = \frac{dQ}{dT}$

Linear Coefficient of Thermal Expansion

$\frac{l_f - l_0}{l_0} = \alpha_l(T_f - T_0)$

$\frac{\Delta l}{l_0} = \alpha_l \Delta T$

Volume Coefficient of Thermal Expansion

$\frac{\Delta V}{V_0} = \alpha_v \Delta T$

Thermal Conductivity

$q = -k\frac{dT}{dx}$

Thermal Stress

$\sigma = E\alpha_l(T_0 - T_f)$

$= E\alpha_l \Delta T$

Thermal Shock Resistance Parameter

$\mathrm{TSR} \cong \frac{\sigma_f k}{E\alpha_l}$

Magnetic Properties

Magnetic Field Strength - Coil

$H = \frac{NI}{l}$

Magnetic Flux Density

$B = \mu_0 H + \mu_0 M$

In a material:

$B = \mu H$

In a vacuum:

$B_0 = \mu_0 H$

For a ferromagnetic material:

$B \cong \mu_0 M$

Relative Permeability

$\mu_r = \frac{\mu}{\mu_0}$

Magnetization

$M = X_m H$

Magnetic Susceptibility

$X_m = \mu_r - 1$

Saturation Magnetization

For Ni:

$M_s = 0.60\mu_B N$

For a ferrimagnetic material:

$M_s = N'\mu_B$

Optical Properties

Velocity of Light

In a vacuum:

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

In a medium:

$v = \frac{1}{\sqrt{\epsilon \mu}}$

Velocity of Electromagnetic Radiation

$c = \lambda v$

Index of Refraction

$n = \frac{c}{v} = \sqrt{\epsilon_r \mu_r}$

Reflectivity

$R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$

Intensity of Transmitted Radiation

$I'_T = I'_0 e^{-\beta x}$

➔ Reflection losses not taken into account)

Intensity of Radiation Transmitted

$I_T = I_0(1 - R)^2 e^{-\beta l}$

➔ Reflection losses taken into account)

Many equations were referenced from *Callister, W. D. (2012). Materials science and engineering: An introduction 9E. John Wiley & Sons.*

Appendix

Greek Characters

Symbol	Name
α	Alpha
β	Beta
χ	Chi
$\Gamma\ \gamma$	Gamma
$\Delta\ \delta$	Delta
ϵ	Epsilon
ϵ_0	Epsilon Nought
ζ	Zeta
η	Eta
$\Theta\ \theta$	Theta
κ	Kappa
$\Lambda\ \lambda$	Lambda
μ	Mu
μ_0	Mu Nought
ν	Nu
$\Xi\ \xi$	Xi
$\Pi\ \pi$	Pi
ρ	Rho
$\Sigma\ \sigma$	Sigma
τ	Tau
$\Phi\ \varphi\ \phi$	Phi
$\Psi\ \psi$	Psi
$\Omega\ \omega$	Omega

SI Base Units

Name	Symbol	Measure	Dim. Analysis Symbol
Second	s	Time	T
Meter	m	Length	L
Kilogram	kg	Mass	M
Ampere	A	Electric Current	I
Kelvin	K	Temp	Θ
Mole	mol	Amount of substance	N
Candela	cd	Luminous Intensity	J

SI Prefixes

Prefix	Symbol	Factor	Meaning
Pico	p	10^{-12}	Trillionth
Nano	n	10^{-9}	Billionth
Micro	μ	10^{-6}	Millionth
Milli	m	10^{-3}	Thousandth
Centi	c	10^{-2}	Hundredth
Deci	d	10^{-1}	Tenth
Kilo	K	10^{3}	Thousand
Mega	M	10^{6}	Million
Giga	G	10^{9}	Billion
Tera	T	10^{12}	Trillion

Constants

Gravitational Constant

$G = 6.67430 \text{ x } 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$

Earth Topics

$m_{Earth} = 5.97 \text{ x } 10^{24}$ kg
$r_{Earth} = 6.38 \text{ x } 10^{6}$ m

Gravity on Earth

$g = 9.81\ \mathrm{m/s^2}$ or $32.17\ \mathrm{ft/s^2}$

Atmospheric Pressure

1 atm = 101325 pa = 760.00 mmHg

Avogadro Constant

$N_A = 6.022 \text{ x } 10^{23}\ \mathrm{mol^{-1}}$

Gas Constant

$R = 8.31\ \mathrm{J/(mol \cdot K)}$

Boltzmann Constant

$k_b = 1.38 \text{ x } 10^{23}\ \mathrm{J/K}$

Speed of Sound

$v_s = 343\ \mathrm{m/s}$
➔ When on earth at 20° C or 68° F

Reference Sound Intensity

$I_0 = 10^{-12}\ \mathrm{W/m^2}$
➔ Where I_0 is the lowest sound intensity able to be heard by an undamaged human ear (in room conditions)

Elementary Charge

$e = 1.602 \text{ x } 10^{-19}$ C
➔ This could be the charge of a single proton, or the magnitude of a single electron

Coulomb Constant

$k_e = 8.988 \text{ x } 10^{9}\ \mathrm{N \cdot m^2 \cdot C^{-2}} = \frac{1}{4\pi\epsilon_0}$

Vacuum Permittivity

$\epsilon_0 = 8.854 \text{ x } 10^{-12}\ \mathrm{F \cdot m^{-1}}$

Permeability of Free Space

$4\pi \cdot 10^{-7} \dfrac{Tm}{A}$

Mass of a Proton

$m_{proton} = 1.672 \text{ x } 10^{-27}$ kg = 938.27 MeV/c^2

Mass of an Electron

$m_{electron} = 9.11 \text{ x } 10^{-31}$ kg = 0.511 MeV/c^2

Speed of Light (vacuum)

$c = 2.998 \text{ x } 10^8$ m/s

Planck's Constant

$h = 6.626 \text{ x } 10^{-34}$ J · s

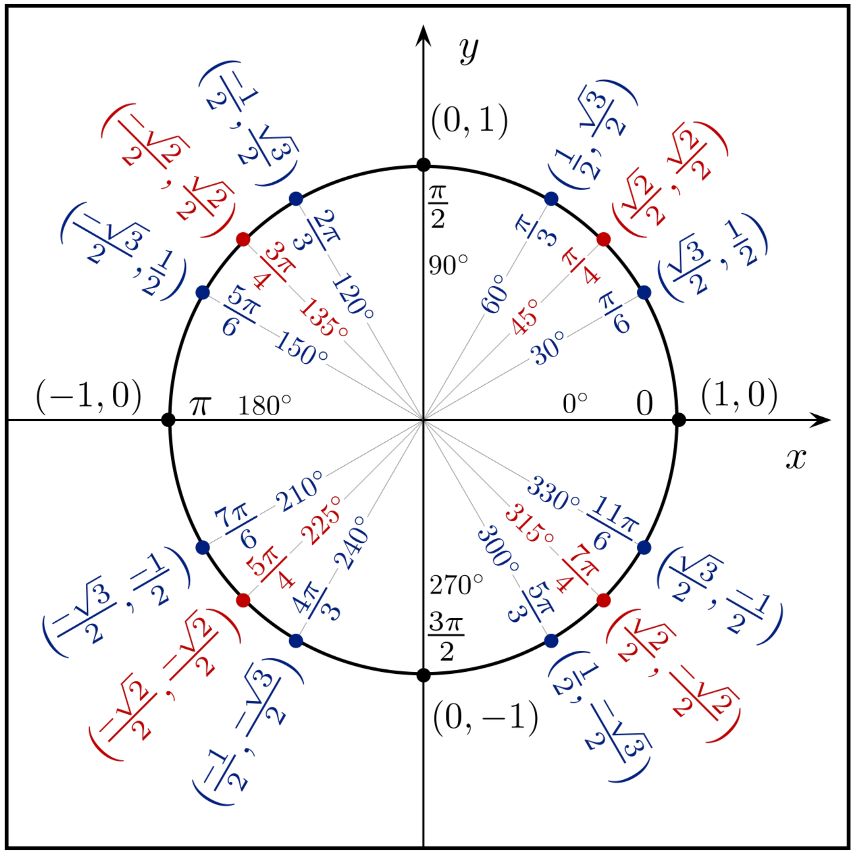
$h = 4.14 \text{ x } 10^{-15}$ eV · s

$\hbar = \dfrac{h}{2\pi}$

Bohr Radius

$a_b = 0.0529$ nm

Unit Circle



https://en.wikipedia.org/wiki/Unit_circle