Physics

Classical Mechanics

Kinematic Relationships

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

➔ These equations will also work for terms θ, ω, α if substituted.

Uniform Rectilinear Motion

$$x = x_0 + vt$$

$$\theta = \theta_0 + \omega t$$

➔ Applies when $a = 0$

Uniformly Accelerated Motion

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

➔ These equations will also work for terms θ, ω, α if substituted. Also *"x"* is subjective and could be any defined axis (*y, z, etc*).

Circular Motion

$$\theta = \frac{s}{r}$$

$$v = \frac{2\pi r}{T} = r\omega$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$a_{tan} = r\alpha$$

$$\omega = 2\pi f$$

➔ Where s is arc length, r is the radius of curvature, f is the frequency, and T is the period.

General Force Equations

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_{ab} = -\vec{F}_{ba}$$

$$F_g = mg$$

$$F_{Spring} = -ks$$

$$\vec{F}_{static\ friction} = \mu_s \vec{F}_N$$

$$\vec{F}_{kinetic\ friction} = \mu_k \vec{F}_N$$

Work and Energy

$$W = \int_{x_0}^{x_f} F(x)\,dx = \int_{s_0}^{s_f} F \cdot ds$$

$$W = Fd\cos\theta = \vec{F} \cdot \vec{d}$$

➔ Where F is force, d is distance traveled, and θ is the angle between the two $\boldsymbol{F}$ and $\boldsymbol{d}$ vectors

$$KE_1 + W_{1\to 2} = KE_2$$

$$KE_1 + PE_1 = PE_2 + KE_2$$

$$KE = \frac{1}{2}mv^2$$

$$PE_{grav} = mgh$$

$$PE_{Elastic} = \frac{1}{2}kx^2$$

Universal Gravitation

$$F_G = G\frac{m_1 m_2}{r^2}$$

Efficiency

$$\eta = \frac{\text{input}}{\text{output}}$$

Center of Mass

$$x_{CM} = \frac{1}{M}\int x\,dm$$

$$y_{CM} = \frac{1}{M}\int \frac{y}{2}dm$$

$$x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{m_T}$$

$$y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{m_T}$$

Impulse and Momentum

$$p = mv$$

$$m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2$$

$$I = \int_{t_i}^{t_f} F(t)dt = \Delta p$$

$$I = \vec{F}\Delta t$$

$$\vec{F}\Delta t = \Delta p$$

Angular Momentum

$$\vec{L} = \vec{r} \text{ x } \vec{p} = m(\vec{r} \text{ x } \vec{v})$$

$$L = I\omega = mrv\sin\phi$$

$$L = mr^2\omega\sin\phi$$

$$I_1\omega_1 = I_2\omega_2$$

Collisions

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Torque (Moments)

$$\vec{\tau} = \vec{r} \text{ x } \vec{F}$$

$$\tau = F \cdot r\sin\theta$$

Density

$$\rho = \frac{m}{V}$$

$$\rho_{Theor} = \frac{nA}{V_c N_A}$$

Simple Harmonic Motion

$$T = 2\pi\sqrt{\frac{m}{k}} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad v = \frac{\lambda}{T}$$

➔ Where T is the period, m is mass, g is gravitational acceleration, f is the frequency, L is the length of the string, and k is the spring constant.

Fluids (Basic Equations)

$$A_1 v_1 = A_2 v_2$$
$$F_2 A_1 = F_1 A_2$$
$$P = \frac{F}{A}$$
$$P_g = \rho g h$$
$$P_T = P_{atm} + \rho g h$$
$$F_{buoyant} = \rho g V_{fluid}$$
$$\frac{V}{t} = Av = Q$$
$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$
$$\Delta P = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

➔ Where ρ is density, V is volume, v is velocity, g is gravitational acceleration, t is time, P is pressure, A is cross sectional area, Q is flow rate, h is height, and F is force.

Temperature

$$T_F = \frac{9}{5}T_c + 32$$
$$T_c = \frac{5}{9}(T_F - 32)$$
$$T_K = T_C + 273$$

Thermal Expansion

$$L_f = L_i(1 + \alpha \Delta T)$$
$$V_f = V_i(1 + \beta \Delta T)$$

Electricity and Magnetism

Electrostatic Equations

Coulomb's Law

$$\vec{F}_{1,2} = \frac{k\ \ q_1 q_2}{r_{1,2}^2}\hat{r}$$

Electric Field (Discrete Charges)

$$\vec{E} = \frac{\vec{F}}{q_0}$$
$$\vec{E}_i = \frac{kq}{r_{(i,0)}^2}\hat{r}_{(r,0)}$$
$$E(x) = \Sigma E_i(x) = E_1(x) + E_2(x) + \ldots$$

Electric Field (Continuous Charges)

$$\lambda = \frac{q}{L} \qquad \sigma = \frac{q}{A} \qquad \rho = \frac{q}{V}$$
$$\vec{E} = \int_{V,A,L} \frac{k\,dq}{r^2}\hat{r}$$

Line: $dq = \lambda dx$

Surface: $dq = \sigma dA$

Volume: $dq = \rho dV$

Finite Line Charge - Parallel

$$E_x = \frac{kQ}{a(a+L)}$$

➔ Where L is length, and a is the distance from the end

Finite Line Charge - Perpendicular

$$E_y = \frac{kQ}{y\sqrt{y^2 + \left(\frac{L}{2}\right)^2}}$$

Infinite Line

$$E = \frac{2k\lambda}{y}$$

Infinite Sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

Ring of Charge (radius R)

$$E_z = \frac{kzQ}{(z^2 + R^2)^{3/2}}$$

Disk of Charge

$$E_z = 2\pi k \sigma\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Electric Flux

$$E_n = E \cdot \hat{n} = E\cos\theta$$
$$\phi = E_n A$$
$$\phi_{net} = \int_S E_n dA$$

Gauss's Law

$$\phi_{net} = E_n A = \frac{Q_{in}}{\epsilon_0}$$

Electric Field Near a Conductor

$$\vec{E} = \frac{\sigma}{\epsilon_0}(\hat{r})$$
$$\sigma_{total} = \sigma_{charge} + \sigma_{induced}$$
$$E_{total} = \frac{\sigma_{total}}{\epsilon_0} = E_{external} + E_{charge}$$

Electric Potential

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{x}$$
$$E = -\frac{dV}{dx}$$

Coulomb Potential

$$V = \frac{kq}{r}$$
$$V = \Sigma_i \frac{kq_i}{r_i}$$

$$V = \int \frac{k\,dq}{r}$$

Line charge:

$dq = \lambda dx$

Plane charge:

$dq = \sigma dx dy$

Disk charge:

$dq = \sigma\rho d\theta d\rho = 2\pi\sigma\rho d\rho$

Potential Energy

$$U = q_0 V = \frac{k q_0 q}{r}$$

DC Circuit Equations

Capacitance

$$C = \frac{Q}{V}$$

$$C_{ParallelPlates} = \frac{\epsilon_0 A}{d}$$

$$C_{Cylindrical} = \frac{2\pi\epsilon_0 L}{\ln(r_2/r_1)}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

➔ Where C is capacitance, Q is charge, U is energy in a capacitor, and V is electric potential.

Dielectrics

$$\kappa = \frac{\epsilon}{\epsilon_0}$$

$$C = \kappa C_0$$

$$E = \frac{E_0}{\kappa}$$

➔ Where E is the field with the dielectric, E_0 is the field without the dielectric, and κ is the dielectric constant.

Current

$$I_{Avg} = \frac{\Delta Q}{\Delta t}$$

$$J = \frac{I}{A}$$

$$J = qnv_D$$

➔ Where J is current density, I is current, A is cross-sectional area, q is charge per particle, n is particle density, v_d is drift velocity, and Q is overall charge.

Power

$$P = IV = I^2 R = \frac{V^2}{R}$$

➔ Where V is voltage, R is resistance, and I is current.

Ohm's Law

$$\vec{J} = \sigma\vec{E}$$

$$\sigma = \frac{1}{\rho}$$

$$V = IR$$

$$R = \rho\frac{L}{A}$$

➔ Where $\vec{J}$ is the electric current density (at a point), $\vec{E}$ is the applied electric field, σ is the conductivity of a material, and ρ is the material resistivity.

Batteries

$$V_{terminal} = V_{EMF} - IR_{internal}$$

Resistor Networks

$$R_{Series} = \Sigma_i R_i = R_1 + R_2 + \ldots$$

$$\frac{1}{R_{Parallel}} = \Sigma_i \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

Capacitor Networks

$$\frac{1}{C_{Series}} = \Sigma_i \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

$$C_{Parallel} = \Sigma_i C_i = C_1 + C_2 + \ldots$$

Kirchhoff's Circuit Rules

Loop Rule: $\Sigma V = 0$

Junction Rule: $\Sigma I_{in} = \Sigma I_{out}$

RC Circuits

$\tau = RC$

Discharging:

$$Q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

Charging:

$$Q(t) = CV_{EMF}(1 - e^{-t/RC})$$

$$= Q_{max}(1 - e^{-t/\tau})$$

$$I(t) = \frac{V_0}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

Magnetism Equations

Moving Point Charge

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \text{ x } \hat{r}}{r^2}$$

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl \text{ x } \hat{r}}{4\pi r^2}$$

B for Various Configurations

$$B_{solenoid} = \mu_0 \frac{N}{l} I$$

$$B_{loop} = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

$$B_{loop,center} = \frac{\mu_0 I}{2R}$$

$$B_{\infty wire} = \frac{\mu_0}{2\pi}\frac{I}{R}$$

Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

Point Charge in B

$\vec{F} = q\vec{v} \text{ x } B$

$r = \dfrac{mv}{qB}$

$f_{cyclotron} = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$

$v = \dfrac{E}{B}$ (velocity selector)

Force on Current Carrying Wires

$\vec{F} = \vec{I}dl \text{ x } \vec{B}$

$\dfrac{\vec{F}}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi R}$

Torque on a Loop

$\vec{\mu} = NIA\hat{n}$

$\tau = \vec{u} \text{ x } \vec{B}$

$U = -\mu B \cos\theta = -\mu \cdot B$

Magnetic Flux

$\phi_B = \int_{Surface} \vec{B} \cdot \hat{n}\, dA$

$\phi_B = NBA \cos\theta$

Faraday's Law

$EMF = -\dfrac{d\phi_B}{dt}$

Inductance

$L = \dfrac{\phi_B}{I}$

$L_{solenoid} = \dfrac{\mu_0 N^2 A}{l}$

$EMF_i = -L\dfrac{dl}{dt}$

$U = \dfrac{1}{2}LI^2$

LR Circuits (DC)

$\tau = \dfrac{L}{R}$

$I(t) = I_{max}(1 - e^{-t/\tau})$ (closes)

$I(t) = I_{max}e^{-t/\tau}$ ("opens")

LC Circuits (DC)

$\omega = \dfrac{1}{\sqrt{LC}}$

$\omega = 2\pi f$

$I(t) = -\omega Q_{max} \sin(\omega t)$

AC Circuits and EM Waves Equations

Displacement Current

$I_d = \epsilon_0 \dfrac{d\phi_e}{dt}$

Maxwell's Equations

$\oint_S \vec{E} \cdot d\vec{A} = \dfrac{Q}{\epsilon}$

$\oint_S \vec{B} \cdot d\vec{A} = 0$

$\oint_C \vec{E} \cdot d\vec{l} = -\dfrac{d\phi_B}{dt}$

$\oint_C \vec{B} \cdot d\vec{l} = \mu\left(I + \epsilon\dfrac{d\phi_E}{dt}\right)$

Lorentz Force

$\vec{F} = q\vec{E} + q(\vec{v} \text{ x } \vec{B})$

AC Circuits

$\Delta V = NBA\omega \sin(\omega t)$

$\omega = 2\pi f$

$V_{rms} = \dfrac{V_{max}}{\sqrt{2}}$

$X_L = \omega L$

$X_C = \dfrac{1}{\omega C}$

$\omega_0 = \dfrac{1}{\sqrt{LC}}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$EMF = \Delta V_{EMF} = IZ$

$\phi_{\text{phase angle}} = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right)$

$P_{ave} = I_{RMS}^2 R$

$P_{ave} = I_{RMS}V_{RMS} \cos\phi$

EM Waves

$E_x(z, t) = E_0 \sin(kz - \omega t)$

$B_y(z, t) = \dfrac{k}{\omega}E_0 \sin(kz - \omega t)$

$k = \dfrac{2\pi}{\lambda}$

$c = \lambda f = \dfrac{E}{B} = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}}$

Poynting Vector

$\vec{S} \equiv \dfrac{1}{\mu_0}\vec{E} \text{ x } \vec{B}$

$S = \dfrac{EB}{\mu_0} = \dfrac{E^2}{c\mu_0}$

Intensity

$I = \dfrac{\text{Power}}{\text{Area}} = \dfrac{EB}{2\mu_0} = \dfrac{E^2}{2c\mu_0}$

Light Equations

Light

$c = \lambda f = \dfrac{E}{B} = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}}$

$E = hf = \dfrac{hc}{\lambda}$

$\text{Intensity} = \dfrac{E_{max}B_{max}}{2\mu_0}$

$$n_{medium} = \frac{c}{v_{medium}}$$

Polarization

$$I_{transmitted} = I_{incident} \cos^2\theta$$

$$\tan\theta_{brewster} = \frac{n_2}{n_1}$$

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_{crit} = \frac{n_2}{n_1}$$

Mirrors

$$f = \frac{1}{2}R$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{h_{img}}{h_{obj}} = -\frac{s'}{s}$$

Lenses

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

$$m = -\frac{n_1 s'}{n_2 s}$$

$$s' = -\frac{n_2}{n_1}s$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Thin Lens Equation

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

Interference

Constructive:

$$\Delta L = 0, \lambda, 2\lambda \ldots = m\lambda$$

Destructive:

$$\Delta L = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda \ldots = (m + \frac{1}{2})\lambda$$

Phase

$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda}$$

$$= 0, 2\pi, 4\pi \ldots \text{const.}$$

$$= \pi, 3\pi, 5\pi \ldots \text{dest.}$$

Double Slit

Max: $d\sin\theta = m\lambda$

Min: $d\sin\theta = (m + \frac{1}{2})\lambda$

Single Slit

Min: $a\sin\theta = m\lambda$

Grating

Max: $d\sin\theta = m\lambda$

Resolution

$$\alpha_{Rayleigh} = \frac{\lambda}{D} \text{ (rads)}$$

Thin Film

$$\Delta\phi = 2\pi\frac{2t}{\lambda'}$$

$\Delta\phi = \pi$ at $n_2 > n_1$

Modern Physics/ Quantum Mechanics

Heat

$$W = -\int_{V_i}^{V_f} P\,dV$$

$$Q = mc\Delta T$$

$Q = \pm mL_f$ or $Q = \pm mL_v$

$$Q_{gained} = -Q_{lost}$$

$$\Delta E_{int} = nC_V\Delta T = Q_{in} + W$$

➔ Work done on a system is (+), work done by a system is (-).

Gas laws

$$n = \frac{N}{N_A} = \frac{m \text{ (in g)}}{m_{\text{mol}}}$$

$$PV = nRT = Nk_BT$$

$$P_iV_i^\gamma = P_fV_f^\gamma$$

$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$

$$\gamma = \frac{C_p}{C_V}$$

$$K_{avg} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$$

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

Entropy

$$S = k_B \ln\Omega$$

$$\Delta S = \int_i^f \frac{dQ_r}{T}$$

Efficiency and Heat Engines

$$\epsilon = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H} \le 1 - \frac{T_C}{T_H}$$

$$COP = \frac{Q_C}{W_{in}} \le \frac{T_C}{T_H - T_C}$$

Wien's Law

$$\lambda_{peak} = \frac{2.90 \cdot 10^6 \text{ nm} \cdot \text{K}}{T}$$

Mean Free Path

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

Waves

$$v = \lambda f$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$k = \frac{2\pi}{\lambda}$$
$$y(x,t) = A\sin(kx \pm \omega t)$$
$$v_s = \sqrt{\frac{F_T}{\mu}}$$
$$v_s = 331\ \mathrm{m/s}\sqrt{1 + \frac{T_C}{273\ \mathrm{C}}}$$
$$v = \sqrt{\frac{B}{\rho}}$$

Sound Intensity

$$I = \frac{P}{area}$$
$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$
$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

Sound and Standing Waves

$$f' = f\left(\frac{v_{sound} \pm v_{obs}}{v_{sound} \pm v_{source}}\right)$$
$$f_n = n\frac{v}{2L}; n = 1,2,3,\ldots$$
$$f_n = n\frac{v}{4L}; n = 1,3,5,\ldots$$
$$f_{heard} = \frac{1}{2}(f_1 + f_2)$$
$$f_{beat} = f_1 - f_2$$

Photoelectric Effect

$$E = hf = \frac{hc}{\lambda}$$
$$K_{max} = eV_{stop} = hf - \phi$$
$$E_H - E_L = hf$$

Schrödinger Equation

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{(h/2\pi)^2}[E - U(x)]\psi(x)$$

Penetration Distance

$$\eta = \frac{(h/2\pi)}{\sqrt{2m(U_0 - E)}}$$

Uncertainty Equation

$$\Delta x \Delta p_x \geq \frac{h/2\pi}{2}$$

Normalization

$$(\text{Prob. xL to xR}) = \int_{x_L}^{x_R} P(x)dx$$
$$\int_{x_L}^{x_R} P(x)dx = \int_{x_L}^{x_R} \psi(x)^{2}dx = 1$$

Particle in a Box

$$E_n = n^2\frac{h^2}{8mL^2}$$

Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Delta t = \gamma \Delta t_p$$
$$L = \frac{L_p}{\gamma}$$
$$u = \frac{u' \pm v}{1 \pm u' \cdot \frac{v}{c^2}}$$

Relativistic Momentum

$$p = \gamma m_0 v$$

Relativistic Kinetic Energy

$$K = (\gamma - 1)E_0$$

Relativistic Mass

$$E^2 = (pc)^2 = E_0$$

Rest Energy

$$E_0 = m_0 c^2$$

Total Energy

$$E = \gamma m_0 c^2$$

Disintegration Energy

$$B = (Zm_H + Nm_n - m_{atom})^2$$

Half Life

$$t_{1/2} = \tau \ln 2$$
$$N(t) = N_0 e^{-t/\tau}$$

Decay Activity

$$R = R_0 e^{-t/\tau}$$
$$R_0 = \frac{N_0}{\tau}$$