



# Master Equation Sheet

Version 1.2

This sheet contains equations typically used in engineering undergraduate classes from freshman through sophomore year.

Subjects include but are not limited to mathematics, physics, dynamics, mechanics of materials, and general engineering. A list of scientific constants and conversions are given at the end as well. Notes on each equation/principle added as necessary. Note that this sheet is not an exhaustive list; it is simply meant as an extra resource.

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For FAQ, mistakes found, comments or suggestions, please read the “README” on the GitHub site: <https://github.com/ntader/MasterEquationSheet/tree/main>

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# Mathematics

## Algebra

### Exponent Laws

$$x^a \cdot x^b = x^{a+b}$$

$$(x^a)^b = x^{ab}$$

$$(xy)^a = (x^a y^a)$$

$$x^{-1} = \frac{1}{x}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

### Quadratic Formula

$$\rightarrow \text{Given } ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Linear Slope Equations

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

### Factoring

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

### Logarithms

$$\ln A^x = x \ln A$$

$$\ln[A \cdot B] = \ln A + \ln B$$

$$\ln\left[\frac{A}{B}\right] = \ln A - \ln B$$

$$\ln\left(\frac{1}{x}\right) = -\ln(x)$$

$$\ln(1) = 0$$

$$\ln(e) = 1$$

$$\ln(e^x) = x$$

$$e^{\ln(x)} = x$$

## Vectors and Matrices

$$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cos(\theta)$$

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$$

$$\hat{u} \cdot \hat{v} = \cos(\theta)$$

$$|\vec{u}| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

$$\tan^{-1}\left(\frac{r_y}{r_x}\right) = \theta$$

$\rightarrow$  Where  $r_x$  and  $r_y$  are vectors in the x-y plane

$$\hat{u} = \frac{\vec{u}}{|\vec{u}|}$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

$$\det(A) = \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

$$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$A^{-1} = \frac{1}{ad - bc} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

$$\hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j} \quad \hat{i} \times \hat{k} = -\hat{j}$$

## Radicals

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

## Geometry

### Circles

$$A = \pi r^2$$

$$C = 2\pi r$$

$$r^2 = (x - a)^2 + (y - b)^2$$

$$s = r\theta$$

$$A_{Hoop} = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$A_{Hoop} = \pi(r_o^2 - r_i^2)$$

$\rightarrow (a, b)$  is the center of the circle.

$\rightarrow \theta$  must be in radians.

### Cylinders

$$A = 2\pi rl + 2\pi r^2$$

$$V = \pi r^2 l$$

### Spheres

$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

$$r^2 = (x - a)^2 + (y - b)^2 + (z - c)^2$$

$\rightarrow (a, b, c)$  is the center of the sphere and  $(x, y, z)$  are coordinates on the surface of the sphere.

### Right Triangles

$$A = \frac{1}{2}bh$$

$$a^2 + b^2 = c^2$$

### Equilateral Triangles

$$A = \frac{\sqrt{3}}{4}a^2$$

$$\theta = 60^\circ$$

## Trigonometry

### Right Angle Ratios

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

→ Using reciprocal identities, the ratios for sec, csc, and cot can be found.

### Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

### Tan/Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Trig Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\sin(2x) = 2 \sin(x) \cdot \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$*\cos^n \theta = [\cos \theta]^n$$

\*Valid for all trigonometric functions (sin, cos, tan, cot, sec, csc).

### Even/Odd Formulas

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\cot(-\theta) = -\cot \theta$$

### Double Angle Formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

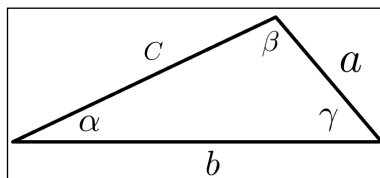
$$= 1 - 2 \sin^2 \theta$$

### Degrees to Radians

→ Where  $D$  is an angle in degrees and  $R$  is an angle in radians.

$$R = D \cdot \frac{\pi}{180} \quad D = R \cdot \frac{180}{\pi}$$

### Law of Sines



$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

### Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

### Small Angle Approx.

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2} \approx 1$$

$$\tan \theta \approx \theta$$

## Calculus

### Derivative Properties

$$\frac{d}{dx}(c) = 0$$

$$(c \cdot f(x))' = c \cdot f'(x)$$

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

### Derivative Power Rule

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

### Derivative Product Rule

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

### Derivative Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

### Derivative Chain Rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

### Standard Derivatives

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x) \cdot \tan(x)$$

$$\frac{d}{dx}(\csc(x)) = -\csc(x) \cdot \cot(x)$$

$$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(n^x) = n^x \cdot \ln(n)$$

$$\frac{d}{dx}(e^{nx}) = n \cdot e^{nx}$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, x \neq 0$$

### Chain Rule Variations

$$\frac{d}{dx}([f(x)]^n) = n \cdot [f(x)]^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x) \cdot e^{f(x)}$$

$$\frac{d}{dx}(\ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\sin[f(x)]) = f'(x) \cdot \cos[f(x)]$$

$$\frac{d}{dx}(\cos[f(x)]) = -f'(x) \cdot \sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)]) = f'(x) \cdot \sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x) \cdot \sec[f(x)] \cdot \tan[f(x)]$$

$$\frac{d}{dx}(\tan^{-1}[f(x)]) = \frac{f'(x)}{1+[f(x)]^2}$$

### Integral Properties

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$\int_a^a dx = 0$$

$$\int_a^b f(x) dx = -\int_b^a f(x) dx$$

$$\int_a^b C \cdot f(x) dx = C \cdot \int_a^b f(x) dx$$

$$\int_a^b C \cdot dx = C \cdot (b-a)$$

### Integral Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

### Standard Integrals

$$\int k dx = k \cdot x + C$$

$$\int e^{nx} dx = \frac{1}{n}e^x + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \sec^2(x) dx = \tan(x) + C$$

$$\int \csc^2(x) dx = -\cot(x) + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + C$$

$$\int \sec(x) \cdot \tan(x) dx = \sec(x) + C$$

$$\int \csc(x) \cdot \cot(x) dx = -\csc(x) + C$$

$$\int \sec(x) dx = \ln |\sec(x) + \tan(x)| + C$$

$$\int \csc(x) dx = -\ln |\csc(x) + \cot(x)| + C$$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln |x| + C$$

$$\int \ln(x) dx = x \cdot \ln(x) - x + C$$

$$\int \tan(x) dx = \ln |\sec(x)| + C$$

$$\int \tan(x) dx = -\ln |\cos(x)| + C$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

### Integration Techniques

Topics include U-Sub, Integration by parts, trigonometric integrals, trig. sub, and PFD.

### U-Substitution

Take an "X" term to be  $u$ , and then take  $du$  of that  $u$  term. Solve the integral in terms of  $u$ , and then re-substitute into the equation.

If needed, find new limits of integration using the substitution. Example:

$$\int_1^2 5x^2 \cos(x^3) dx$$

so

$$u = x^3 \therefore du = 3x^2 dx$$

or

$$\frac{1}{3} du = x^2 dx$$

resulting in

$$= 5 \int_{**}^* \frac{1}{3} \cos(u) du$$

Notice the substitution chosen allows for all  $x$  terms to be turned into  $u$  terms.

The integral can now easily be solved through standard methods. Once solved, replace  $u$  with the substitution above and replace the limits of integration as well. Solve as normal.

It is possible to complete  $u$ -sub without suppressing the limits of integration, you will just need to plug the given limits into the  $u$  term to find the new limits of integration.

For example, the lower would become  $(1^3) = 1$  and the upper would become  $(2^3) = 8$ . Note that either method works and produces the same solution.

## Integration by Parts

The standard formula for integration by parts is as follows:

$$\int u \, dv = uv - \int v \, du$$

Find  $u$  and  $dv$  in the original equation, then solve for  $du$  and  $v$ . Plug into the formula above and solve.

The  $u$  term can be found according to ILATE: inverse trigonometric, logarithmic, algebraic, trigonometric and exponential.

Example:

$$\int x e^{-x} \, dx$$

so

$$\begin{aligned} u &= x & dv &= e^{-x} \\ du &= dx & v &= -e^{-x} \end{aligned}$$

using the equation above:

$$= -x e^{-x} + \int e^{-x} \, dx$$

resulting in

$$= -x e^{-x} - e^{-x} + C$$

## Trigonometric Integrals

When solving an integral with trigonometric functions (usually involving powers and multiple trig functions multiplied together), a  $u$ -sub may not be able to be applied.

Instead, the integral will need to be separated into multiples of the trig function, apply a trig identity, and then complete the  $u$ -sub.

Example:

$$\int \sin^6 x \cos^3 x \, dx$$

separating  $\cos^3 x$  into  $\cos^2 x \cdot \cos x$  and applying an identity:

$$= \int \sin^6 x (1 - \sin^2 x) \cos x \, dx$$

take  $u = \sin x \therefore du = \cos x \, dx$  and perform the remaining  $u$ -sub:

$$= \int u^6 (1 - u^2) \, du$$

ending with:

$$= \frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C$$

Note that while  $\sin^2 x + \cos^2 x = 1$  is a common substitution, it is also common for other identities such as  $\tan^2 x + 1 = \sec^2 x$  to be used as well.

## Trigonometric Substitution

In certain cases, an integral may contain one of the following roots. In such situation, the following substitutions and formulas will be used to solve the integral.

Case I:

$$\sqrt{a^2 - b^2 x^2} \Rightarrow x = \frac{a}{b} \sin \theta$$

$$\text{uses } \cos^2(\theta) = 1 - \sin^2(\theta)$$

Case II:

$$\sqrt{b^2 x^2 - a^2} \Rightarrow x = \frac{a}{b} \sec(\theta)$$

$$\text{uses } \tan^2(\theta) = \sec^2(\theta) - 1$$

Case III:

$$\sqrt{a^2 + b^2 x^2} \Rightarrow x = \frac{a}{b} \tan(\theta)$$

$$\text{uses } \sec^2(\theta) = \tan^2(\theta) + 1$$

Example:

$$\int \frac{1}{(1 - x^2)^{3/2}} \, dx$$

Because this is a case I problem, use the substitution

$$x = \sin \theta \therefore dx = \cos \theta$$

Apply the substitution(s) back into the original equation:

$$\int \frac{1}{(1 - \sin^2(\theta))^{3/2}} \cdot \cos \theta \, d\theta$$

From here, the integral can be simplified and solved readily:

$$= \int \frac{1}{(\cos^2 \theta)^{3/2}} \cdot \cos \theta \, d\theta$$

$$= \int \frac{1}{\cos^3 \theta} \cdot \cos \theta \, d\theta$$

$$= \int \frac{1}{\cos^2 \theta} \, d\theta$$

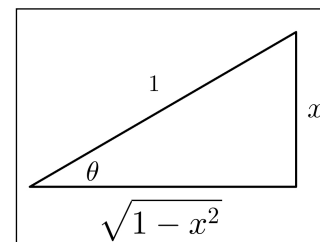
$$= \int \sec^2 \theta \, d\theta$$

$$= \tan \theta + C$$

Although tempting to assume so, the problem is not solved. Because a substitution was applied near the beginning, the final answer must be in terms of  $x$ , not  $\theta$ .

$$\sin \theta = \frac{x}{1} = \frac{\text{opposite}}{\text{hypotenuse}}$$

By creating a right triangle with this definition, the adjacent side  $a$  can be solved:



Recall that  $a^2 + b^2 = c^2$  and as such  $(x)^2 + (a)^2 = (1)^2$ , resulting in

$$a = \sqrt{1-x^2}$$

The final result can finally be expressed in terms of  $x$  as

$$= \frac{x}{\sqrt{1-x^2}} + C$$

## Partial Fractions

Occasionally an integral will involve a fraction which may be difficult to be solved by standard substitution methods.

Using PFD, the integral can be broken up into simpler fractions which can be easier solved.

Example:

$$\int \frac{3x+2}{x^2+x} dx$$

This integral is difficult by itself, due to the fact that an easy  $u$ -sub is not available.

To help with this, it can be broken down into simpler integrals. Begin by observing the fraction only and factoring the denominator:

$$\frac{3x+2}{x(x+1)}$$

This fraction can now re-written, with the factors of the denominator for each fraction:

$$\frac{3x+2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

Because the numerators are not known, variables  $A$  and  $B$  are put in place. Note the original factored fraction goes on the left.

From here the denominator of the left (in this case  $x(x+1)$ ) is multiplied through the equation:

$$* 3x+2 = A(x+1) + B(x)$$

Make note that parts of the denominators of terms  $A$  and  $B$

canceled, resulting in a much simpler expression than what was started with.

Multiplying terms:

$$3x+2 = Ax + A + Bx$$

Group terms based on their order (or "power"):

$$3x+2 = (A+B)x + A$$

From here, the coefficient matching game is played. Match the coefficients from the left (with respect to exponents/powers) to the coefficients of the right.

$$3 = A+B$$

$$2 = A$$

Notice it is just the raw coefficients and A/B terms in the new set of equations. From here, it is seen that  $A=2$  and  $B=1$ .

This conclusion could also be reached by revisiting equation \*. Because the equation is true for any value of  $x$ , the equation can be solved by picking "0" as  $x$  and solving from there.

$$3(0)+2 = A(0+1) + B(0)$$

$$2 = A$$

The same could be done for finding  $B$  (notice that  $A$  cancels this time):

$$3(-1)+2 = A(-1+1) + B(-1)$$

$$-1 = -B \therefore B=1$$

Once the numerators are realized, they can be plugged back into the first decomposition:

$$\frac{3x+2}{x(x+1)} = \frac{2}{x} + \frac{1}{x+1}$$

Because of this, the starting integral can now be replaced as well:

$$\int \frac{3x+2}{x(x+1)} dx = \int \frac{2}{x} + \frac{1}{x+1} dx$$

This is now a much easier integral, and can be readily solved using standard methods:

$$\int \frac{2}{x} dx + \int \frac{1}{x+1} dx$$

## The Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t) dt$$

## Laplace Transforms

$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, n=1,2,3,\dots$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}$$

$$\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$$

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$$

$$\mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$$

## Inverse Laplace Transforms

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$$

$$\mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n, n=1,2,3,\dots$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$

$$\mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} = \sin kt$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos kt$$

$$\mathcal{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\} = \sinh kt$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\} = \cosh kt$$

# Physics

## Classical Mechanics

### Kinematic Relationships

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v \frac{dv}{dx}$$

→ These equations will also work for terms  $\theta, \omega, \alpha$  if substituted.

### Uniform Rectilinear Motion

$$x = x_0 + vt$$

$$\theta = \theta_0 + \omega t$$

→ Applies when  $a = 0$

### Uniformly Accelerated Motion

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

→ These equations will also work for terms  $\theta, \omega, \alpha$  if substituted. Also “ $x$ ” is subjective and could be any defined axis ( $y, z, \text{etc.}$ ).

### Circular Motion

$$\theta = \frac{s}{r}$$

$$v = \frac{2\pi r}{T} = r\omega$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$a_{tan} = r\alpha$$

$$\omega = 2\pi f$$

→ Where  $s$  is arc length,  $r$  is the radius of curvature,  $f$  is the frequency, and  $T$  is the period.

### General Force Equations

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_{ab} = -\vec{F}_{ba}$$

$$F_g = mg$$

$$F_{Spring} = -ks$$

$$\vec{F}_{static\ friction} = \mu_s \vec{F}_N$$

$$\vec{F}_{kinetic\ friction} = \mu_k \vec{F}_N$$

### Work and Energy

$$W = \int_{x_0}^{x_f} F(x) dx = \int_{s_0}^{s_f} \vec{F} \cdot d\vec{s}$$

$$W = Fd \cos \theta = \vec{F} \cdot \vec{d}$$

→ Where  $F$  is force,  $d$  is distance traveled, and  $\theta$  is the angle between the two  $\vec{F}$  and  $\vec{d}$  vectors

$$KE_1 + W_{1 \rightarrow 2} = KE_2$$

$$KE_1 + PE_1 = PE_2 + KE_2$$

$$KE = \frac{1}{2}mv^2$$

$$PE_{grav} = mgh$$

$$PE_{Elastic} = \frac{1}{2}kx^2$$

### Universal Gravitation

$$F_G = G \frac{m_1 m_2}{r^2}$$

### Efficiency

$$\eta = \frac{\text{input}}{\text{output}}$$

### Center of Mass

$$x_{CM} = \frac{1}{M} \int x dm$$

$$y_{CM} = \frac{1}{M} \int y dm$$

$$x_{CM} = \frac{\sum_{i=1}^n m_i x_i}{m_T}$$

$$y_{CM} = \frac{\sum_{i=1}^n m_i y_i}{m_T}$$

### Impulse and Momentum

$$p = mv$$

$$m\vec{v}_1 + \int \vec{F} dt = m\vec{v}_2$$

$$I = \int_{t_i}^{t_f} F(t) dt = \Delta p$$

$$I = \vec{F} \Delta t$$

$$\vec{F} \Delta t = \Delta p$$

### Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

$$L = I\omega = mrv \sin \phi$$

$$L = mr^2 \omega \sin \phi$$

$$I_1 \omega_1 = I_2 \omega_2$$

### Collisions

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

### Torque (Moments)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = F \cdot r \sin \theta$$

### Density

$$\rho = \frac{m}{V}$$

$$\rho_{Theor} = \frac{nA}{V_c N_A}$$

### Simple Harmonic Motion

$$T = 2\pi \sqrt{\frac{m}{k}} = \frac{1}{f} = 2\pi \sqrt{\frac{L}{g}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad v = \frac{\lambda}{T}$$

→ Where  $T$  is the period,  $m$  is mass,  $g$  is gravitational acceleration,  $f$  is the frequency,  $L$  is the length of the string, and  $k$  is the spring constant.

### Fluids (Basic Equations)

$$A_1 v_1 = A_2 v_2$$

$$F_2 A_1 = F_1 A_2$$

$$P = \frac{F}{A}$$

$$P_g = \rho g h$$

$$P_T = P_{atm} + \rho g h$$

$$F_{buoyant} = \rho g V_{fluid}$$

$$\frac{V}{t} = A v = Q$$

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

$$\Delta P = \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2$$

→ Where  $\rho$  is density,  $V$  is volume,  $v$  is velocity,  $g$  is gravitational acceleration,  $t$  is time,  $P$  is pressure,  $A$  is cross sectional area,  $Q$  is flow rate,  $h$  is height, and  $F$  is force.

### Temperature

$$T_F = \frac{9}{5} T_C + 32$$

$$T_C = \frac{5}{9} (T_F - 32)$$

$$T_K = T_C + 273$$

### Thermal Expansion

$$L_f = L_i (1 + \alpha \Delta T)$$

$$V_f = V_i (1 + \beta \Delta T)$$

## Electricity and Magnetism

### Electrostatic Equations

#### Coulomb's Law

$$\vec{F}_{1,2} = \frac{k}{r_{1,2}^2} q_1 q_2 \hat{r}$$

#### Electric Field (Discrete Charges)

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E}_i = \frac{k q}{r_{(i,0)}^2} \hat{r}_{(r,0)}$$

$$E(x) = \Sigma E_i(x) = E_1(x) + E_2(x) + \dots$$

#### Electric Field (Continuous Charges)

$$\lambda = \frac{q}{L} \quad \sigma = \frac{q}{A} \quad \rho = \frac{q}{V}$$

$$\vec{E} = \int_{V,A,L} \frac{k dq}{r^2} \hat{r}$$

$$\text{Line: } dq = \lambda dx$$

$$\text{Surface: } dq = \sigma dA$$

$$\text{Volume: } dq = \rho dV$$

#### Finite Line Charge - Parallel

$$E_x = \frac{kQ}{a(a+L)}$$

→ Where  $L$  is length, and  $a$  is the distance from the end

#### Finite Line Charge - Perpendicular

$$E_y = \frac{kQ}{y \sqrt{y^2 + \left(\frac{L}{2}\right)^2}}$$

#### Infinite Line

$$E = \frac{2k\lambda}{y}$$

#### Infinite Sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

#### Ring of Charge (radius R)

$$E_z = \frac{kzQ}{(z^2 + R^2)^{3/2}}$$

#### Disk of Charge

$$E_z = 2\pi k \sigma \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

#### Electric Flux

$$E_n = E \cdot \hat{n} = E \cos \theta$$

$$\phi = E_n A$$

$$\phi_{net} = \int_S E_n dA$$

#### Gauss's Law

$$\phi_{net} = E_n A = \frac{Q_{in}}{\epsilon_0}$$

#### Electric Field Near a Conductor

$$\vec{E} = \frac{\sigma}{\epsilon_0} (\hat{r})$$

$$\sigma_{total} = \sigma_{charge} + \sigma_{induced}$$

$$E_{total} = \frac{\sigma_{total}}{\epsilon_0} = E_{external} + E_{charge}$$

#### Electric Potential

$$\Delta V = V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{x}$$

$$E = - \frac{dV}{dx}$$

#### Coulomb Potential

$$V = \frac{kq}{r}$$

$$V = \Sigma_i \frac{kq_i}{r_i}$$



$$V = \int \frac{k dq}{r}$$

Line charge:

$$dq = \lambda dx$$

Plane charge:

$$dq = \sigma dx dy$$

Disk charge:

$$dq = \sigma \rho d\theta d\rho = 2\pi \sigma \rho d\rho$$

### Potential Energy

$$U = q_0 V = \frac{k q_0 q}{r}$$

## DC Circuit Equations

### Capacitance

$$C = \frac{Q}{V}$$

$$C_{\text{Parallel Plates}} = \frac{\epsilon_0 A}{d}$$

$$C_{\text{Cylindrical}} = \frac{2\pi \epsilon_0 L}{\ln(r_2/r_1)}$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

→ Where C is capacitance, Q is charge, U is energy in a capacitor, and V is electric potential.

### Dielectrics

$$\kappa = \frac{\epsilon}{\epsilon_0}$$

$$C = \kappa C_0$$

$$E = \frac{E_0}{\kappa}$$

→ Where E is the field with the dielectric, E<sub>0</sub> is the field without the dielectric, and κ is the dielectric constant.

### Current

$$I_{\text{Avg}} = \frac{\Delta Q}{\Delta t}$$

$$J = \frac{I}{A}$$

$$J = qn v_D$$

→ Where J is current density, I is current, A is cross-sectional area, q is charge per particle, n is particle density, v<sub>d</sub> is drift velocity, and Q is overall charge.

### Power

$$P = IV = I^2 R = \frac{V^2}{R}$$

→ Where V is voltage, R is resistance, and I is current.

### Ohm's Law

$$\vec{J} = \sigma \vec{E}$$

$$\sigma = \frac{1}{\rho}$$

$$V = IR$$

$$R = \rho \frac{L}{A}$$

→ Where  $\vec{J}$  is the electric current density (at a point),  $\vec{E}$  is the applied electric field, σ is the conductivity of a material, and ρ is the material resistivity.

### Batteries

$$V_{\text{terminal}} = V_{\text{EMF}} - IR_{\text{internal}}$$

### Resistor Networks

$$R_{\text{Series}} = \sum_i R_i = R_1 + R_2 + \dots$$

$$\frac{1}{R_{\text{Parallel}}} = \sum_i \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

### Capacitor Networks

$$\frac{1}{C_{\text{Series}}} = \sum_i \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

$$C_{\text{Parallel}} = \sum_i C_i = C_1 + C_2 + \dots$$

### Kirchhoff's Circuit Rules

Loop Rule:  $\sum V = 0$

Junction Rule:  $\sum I_{\text{in}} = \sum I_{\text{out}}$

### RC Circuits

$$\tau = RC$$

Discharging:

$$Q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

Charging:

$$Q(t) = C V_{\text{EMF}} (1 - e^{-t/RC}) = Q_{\text{max}} (1 - e^{-t/\tau})$$

$$I(t) = \frac{V_0}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

## Magnetism Equations

### Moving Point Charge

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$$

### Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Idl \times \hat{r}}{4\pi r^2}$$

### B for Various Configurations

$$B_{\text{solenoid}} = \mu_0 \frac{N}{l} I$$

$$B_{\text{loop}} = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

$$B_{\text{loop, center}} = \frac{\mu_0 I}{2R}$$

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi R}$$

### Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

### Point Charge in B

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$r = \frac{mv}{qB}$$

$$f_{cyclotron} = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$v = \frac{E}{B} \text{ (velocity selector)}$$

### Force on Current Carrying Wires

$$\vec{F} = I\vec{dl} \times \vec{B}$$

$$\frac{\vec{F}}{L} = \frac{\mu_0 I_1 I_2}{2\pi R}$$

### Torque on a Loop

$$\vec{\mu} = NIA\hat{n}$$

$$\tau = \vec{u} \times \vec{B}$$

$$U = -\mu B \cos \theta = -\mu \cdot \vec{B}$$

### Magnetic Flux

$$\phi_B = \int_{\text{Surface}} \vec{B} \cdot \hat{n} dA$$

$$\phi_B = NBA \cos \theta$$

### Faraday's Law

$$EMF = -\frac{d\phi_B}{dt}$$

### Inductance

$$L = \frac{\phi_B}{I}$$

$$L_{solenoid} = \frac{\mu_0 N^2 A}{l}$$

$$EMF_i = -L \frac{dl}{dt}$$

$$U = \frac{1}{2} LI^2$$

### LR Circuits (DC)

$$\tau = \frac{L}{R}$$

$$I(t) = I_{max}(1 - e^{-t/\tau}) \text{ (closes)}$$

$$I(t) = I_{max}e^{-t/\tau} \text{ ("opens")}$$

### LC Circuits (DC)

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\pi f$$

$$I(t) = -\omega Q_{max} \sin(\omega t)$$

### AC Circuits and EM

#### Waves Equations

#### Displacement Current

$$I_d = \epsilon_0 \frac{d\phi_e}{dt}$$

### Maxwell's Equations

$$\oint_s \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon}$$

$$\oint_s \vec{B} \cdot d\vec{A} = 0$$

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint_c \vec{B} \cdot d\vec{l} = \mu \left( I + \epsilon \frac{d\phi_E}{dt} \right)$$

### Lorentz Force

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

### AC Circuits

$$\Delta V = NBA\omega \sin(\omega t)$$

$$\omega = 2\pi f$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$EMF = \Delta V_{EMF} = IZ$$

$$\phi_{\text{phase angle}} = \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$$

$$P_{ave} = I_{RMS}^2 R$$

$$P_{ave} = I_{RMS} V_{RMS} \cos \phi$$

### EM Waves

$$E_x(z, t) = E_0 \sin(kz - \omega t)$$

$$B_y(z, t) = \frac{k}{\omega} E_0 \sin(kz - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$c = \lambda f = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

### Poynting Vector

$$\vec{S} \equiv \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

### Intensity

$$I = \frac{\text{Power}}{\text{Area}} = \frac{EB}{2\mu_0} = \frac{E^2}{2c\mu_0}$$

### Light Equations

#### Light

$$c = \lambda f = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$E = hf = \frac{hc}{\lambda}$$

$$\text{Intensity} = \frac{E_{max} B_{max}}{2\mu_0}$$

$$n_{medium} = \frac{c}{v_{medium}}$$

### Polarization

$$I_{transmitted} = I_{incident} \cos^2 \theta$$

$$\tan \theta_{brewster} = \frac{n_2}{n_1}$$

### Snell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_{crit} = \frac{n_2}{n_1}$$

### Mirrors

$$f = \frac{1}{2}R$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{h_{img}}{h_{obj}} = -\frac{s'}{s}$$

### Lenses

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

$$m = -\frac{n_1 s'}{n_2 s}$$

$$s' = -\frac{n_2}{n_1} s$$

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

### Thin Lens Equation

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

### Interference

Constructive:

$$\Delta L = 0, \lambda, 2\lambda \dots = m\lambda$$

Destructive:

$$\Delta L = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda \dots = (m + \frac{1}{2})\lambda$$

### Phase

$$\Delta\phi = 2\pi \frac{\Delta L}{\lambda}$$

$$= 0, 2\pi, 4\pi \dots \text{const.}$$

$$= \pi, 3\pi, 5\pi \dots \text{dest.}$$

### Double Slit

$$\text{Max: } d \sin \theta = m\lambda$$

$$\text{Min: } d \sin \theta = (m + \frac{1}{2})\lambda$$

### Single Slit

$$\text{Min: } a \sin \theta = m\lambda$$

### Grating

$$\text{Max: } d \sin \theta = m\lambda$$

### Resolution

$$\alpha_{Rayleigh} = \frac{\lambda}{D} \text{ (rads)}$$

### Thin Film

$$\Delta\phi = 2\pi \frac{2t}{\lambda'}$$

$$\Delta\phi = \pi \text{ at } n_2 > n_1$$

## Modern Physics/ Quantum Mechanics

### Heat

$$W = - \int_{V_i}^{V_f} P dV$$

$$Q = mc\Delta T$$

$$Q = \pm mL_f \text{ or } Q = \pm mL_v$$

$$Q_{gained} = -Q_{lost}$$

$$\Delta E_{int} = nC_V\Delta T = Q_{in} + W$$

### Gas laws

$$n = \frac{N}{N_A} = \frac{m \text{ (in g)}}{m_{mol}}$$

$$PV = nRT = Nk_B T$$

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\gamma = \frac{C_p}{C_v}$$

$$K_{avg} = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_B T$$

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

### Entropy

$$S = k_B \ln \Omega$$

$$\Delta S = \int_i^f \frac{dQ_r}{T}$$

### Efficiency and Heat Engines

$$\epsilon = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H} \leq 1 - \frac{T_C}{T_H}$$

$$COP = \frac{Q_C}{W_{in}} \leq \frac{T_C}{T_H - T_C}$$

### Wien's Law

$$\lambda_{peak} = \frac{2.90 \cdot 10^6 \text{ nm} \cdot \text{K}}{T}$$

### Mean Free Path

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

### Waves

$$v = \lambda f$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$k = \frac{2\pi}{\lambda}$$

$$y(x, t) = A \sin(kx \pm \omega t)$$

$$v_s = \sqrt{\frac{F_T}{\mu}}$$

$$v_s = 331 \text{ m/s} \sqrt{1 + \frac{T_C}{273 \text{ C}}}$$

$$v = \sqrt{\frac{B}{\rho}}$$

#### Sound Intensity

$$I = \frac{P}{\text{area}}$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

#### Sound and Standing Waves

$$f' = f \left( \frac{v_{\text{sound}} \pm v_{\text{obs}}}{v_{\text{sound}} \pm v_{\text{source}}} \right)$$

$$f_n = n \frac{v}{2L}; n = 1, 2, 3, \dots$$

$$f_n = n \frac{v}{4L}; n = 1, 3, 5, \dots$$

$$f_{\text{heard}} = \frac{1}{2}(f_1 + f_2)$$

$$f_{\text{beat}} = f_1 - f_2$$

#### Photoelectric Effect

$$E = hf = \frac{hc}{\lambda}$$

$$K_{\text{max}} = eV_{\text{stop}} = hf - \phi$$

$$E_H - E_L = hf$$

#### Schrödinger Equation

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{(\hbar/2\pi)^2}[E - U(x)]\psi(x)$$

#### Penetration Distance

$$\eta = \frac{(\hbar/2\pi)}{\sqrt{2m(U_0 - E)}}$$

#### Uncertainty Equation

$$\Delta x \Delta p_x \geq \frac{\hbar/2\pi}{2}$$

#### Normalization

$$(\text{Prob. xL to xR}) = \int_{x_L}^{x_R} P(x) dx$$

$$\int_{x_L}^{x_R} P(x) dx = \int_{x_L}^{x_R} \psi(x)^2 dx = 1$$

#### Particle in a Box

$$E_n = n^2 \frac{h^2}{8mL^2}$$

#### Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_p$$

$$L = \frac{L_p}{\gamma}$$

$$u = \frac{u' \pm v}{1 \pm u' \cdot \frac{v}{c^2}}$$

#### Relativistic Momentum

$$p = \gamma m_0 v$$

#### Relativistic Kinetic Energy

$$K = (\gamma - 1)E_0$$

#### Relativistic Mass

$$E^2 = (pc)^2 = E_0^2$$

#### Rest Energy

$$E_0 = m_0 c^2$$

#### Total Energy

$$E = \gamma m_0 c^2$$

#### Disintegration Energy

$$B = (Zm_H + Nm_n - m_{\text{atom}})^2$$

#### Half Life

$$t_{1/2} = \tau \ln 2$$

$$N(t) = N_0 e^{-t/\tau}$$

#### Decay Activity

$$R = R_0 e^{-t/\tau}$$

$$R_0 = \frac{N_0}{\tau}$$

# Dynamics

## Kinematic Relationships

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v \frac{dv}{dx}$$

→ These equations will also work for terms  $\theta, \omega, \alpha$  if substituted.

## Uniform Rectilinear Motion

$$x = x_0 + vt$$

$$\theta = \theta_0 + \omega t$$

→ Applies when  $a = 0$

## Uniformly Accelerated Motion

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

→ These equations will also work for terms  $\theta, \omega, \alpha$  if substituted. Also “x” is subjective and could be any defined axis ( $y, z, etc$ ).

## 3D Rectangular Motion

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

## 2D Motion - Tangential & Normal Components

$$\vec{v} = v\hat{e}_t$$

$$\vec{a} = a_t\hat{e}_t + a_n\hat{e}_n$$

→ Where  $a_t = \frac{dv}{dt}$  and  $a_n = \frac{v^2}{\rho}$

## 2D Motion - Radial & Transverse Components

$$\vec{r} = r\hat{e}_r$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

## Newton's Second Law

$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{M} = I\vec{\alpha}$$

## Work & Energy

$$T_1 + U_{1 \rightarrow 2} = T_2$$

$$\rightarrow \text{Where } U_{1 \rightarrow 2} = \int_{A_1}^{A_2} \vec{F} \cdot d\vec{r}$$

## Mechanical Energy

$$T_1 + V_1 = T_2 + V_2$$

$$V_g = Wy \quad V_e = \frac{1}{2}kx^2$$

$$T_1 + V_1 + U_{NC1 \rightarrow 2} = T_2 + V_2$$

## Power

$$P = \frac{dU}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

## Efficiency

$$\eta = \frac{U_{out}}{U_{in}}$$

## Linear Impulse & Momentum

$$m\vec{v}_1 + \int \vec{F} dt = m\vec{v}_2$$

## Coefficient of Restitution

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

## Rigid Body Kinematics

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

For a 2D slab:

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

## Rotating Frames

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F}$$

$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/F} + \vec{a}_{cor}$$

→ Where  $\vec{a}_{cor} = 2\vec{\Omega} \times \vec{v}_{P/F}$

## Rotational Kinetic Energy

$$T_{rot} = \frac{1}{2}I\omega^2$$

## Moments of Inertia

$$\bar{I}_{disk} = \frac{1}{2}mr^2 \quad \bar{I}_{rod} = \frac{1}{12}ml^2$$

$$\bar{I}_{hoop, symmetry axis} = mr^2$$

$$\bar{I}_{sphere} = \frac{2}{5}mr^2$$

## Angular Momentum

$$\vec{H}_G = I\vec{\omega}$$

# Mechanics

## Average normal/axial stress

$$\sigma_{avg} = \frac{F}{A}$$

## Average Shear Stress

$$\tau_{avg} = \frac{V}{A_s}$$

## Average Normal/Axial Strain

$$\epsilon_{avg} = \frac{\Delta L}{L_0}$$

## Thermal Strain

$$\epsilon_T = \alpha \Delta T$$

## Shear Strain

$$\gamma = \frac{\pi}{2} - \theta$$

$$\gamma = \tan^{-1}\left(\frac{\delta}{L}\right)$$

## Poisson's Ratio

$$\nu = -\frac{\epsilon_L}{\epsilon_{Ax}} = -\frac{\epsilon_{trans}}{\epsilon_{long}}$$

## Shear Modulus

$$G = \frac{E}{2(1 + \nu)}$$

## Factor of Safety

$$F_s = \frac{\sigma_{failure}}{\sigma_{allow}}$$

## Hooke's Law (1D)

$$\sigma_x = E\epsilon_x$$

## Hooke's Law (plane stress)

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) + \alpha\Delta T$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) + \alpha\Delta T$$

$$\epsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

## Elongation in Axial Members

$$e = \int_0^L \frac{F(x) \cdot dx}{A(x) \cdot E(x)}$$

$$e = \frac{FL}{EA}$$

$$e = \frac{FL}{EA} + \alpha\Delta TL$$

## Angle of Twist

$$\phi = \int_0^L \frac{T(x) dx}{I_p(x) G(x)}$$

$$\phi = \frac{TL}{I_p G}$$

## Maximum Shear Stress

$$\tau_{max} = \frac{Tr}{I_p}$$

## Polar Moment of Inertia

$$I_p = \int_A \rho^2 dA \text{ (General)}$$

$$I_p = \frac{\pi d^4}{32} \text{ (Circular Shaft)}$$

$$I_p = \frac{\pi}{32}(d_o^4 - d_i^4) \text{ (Tubular Shaft)}$$

## Equilibrium Beam Relationships

$$\Delta V = P$$

$$\Delta V = V_2 - V_1 = \int_{x_1}^{x_2} p(x) dx$$

$$\frac{dV}{dx} = p(x)$$

$$\Delta M = -M$$

$$\Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V(x) dx$$

$$\frac{dM}{dx} = V(x)$$

## Flexure Stress in Bending Beams

$$\sigma_x = \frac{-My}{I_z}$$

$$S = \frac{I_z}{c}$$

$$S_{design} \geq \frac{M_{max}}{\sigma_{allow}}$$

## Centroids

$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$$

## Moment of Inertia

$$I_z = \int_A y^2 dA$$

Rectangle:

$$I_z = \frac{bh^3}{12}$$

Solid Circular Section:

$$I_z = \frac{\pi d^4}{64}$$

Hollow Circular Section:

$$I_z = \frac{\pi}{64}(d_o^4 - d_i^4)$$

## Parallel Axis Theorem

$$I_{axis, any} = I_c + d^2 A$$

$$I = \sum_i (I_{c,i} + d_i^2 A_i)$$

### Shear Stress in Bending Beams

$$\tau = \frac{VQ}{It}$$

$$Q = \sum_i \bar{y}_i' A_i'$$

Solid Circular Section:

$$Q_{max} = \frac{1}{12} d^3$$

Hollow Circular Section:

$$Q_{max} = \frac{1}{12} [d_o^3 - d_i^3]$$

### Shear Flow

$$q = \frac{VQ}{I}$$

$$n_f \tau_f A_f \geq q \Delta s$$

### Stress Transformations

$$\sigma_n = \left( \frac{\sigma_x + \sigma_y}{2} \right) + \left( \frac{\sigma_x - \sigma_y}{2} \right) \cos(2\theta) + \tau_{xy} \sin(2\theta)$$

$$\sigma_n = \left( \frac{\sigma_x + \sigma_y}{2} \right) - \left( \frac{\sigma_x - \sigma_y}{2} \right) \cos(2\theta) - \tau_{xy} \sin(2\theta)$$

$$\sigma_x + \sigma_y = \sigma_n + \sigma_t$$

$$\tau_{nt} = - \left( \frac{\sigma_x - \sigma_y}{2} \right) \sin(2\theta) + \tau_{xy} \cos(2\theta)$$

### Principal (max/min) Stresses

$$\sigma_{avg} = \left( \frac{\sigma_x + \sigma_y}{2} \right)$$

$$R = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \sigma_{avg} + R$$

$$\sigma_2 = \sigma_{avg} - R$$

$$\tau_{max} = \pm R$$

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\text{If } (\sigma_x - \sigma_y) > 0, \theta_p = \theta_{p,1}$$

$$\text{If } (\sigma_x - \sigma_y) < 0, \theta_p = \theta_{p,2}$$

$$\tan(2\theta_s) = - \frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$$\theta_s = \theta_p \pm 45$$

$$\text{If } \theta_p > 0, \theta_s = \theta_p - 45$$

$$\text{If } \theta_p < 0, \theta_s = \theta_p + 45$$

### Pressure Vessels

$$\sigma_{sphere} = \frac{pr}{2t}$$

$$\sigma_{axial} = \frac{pr}{2t}$$

$$\sigma_{hoop} = \frac{pr}{t}$$

$$\tau_{abs,max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Differential Equations of the

### Deflection Curve

Deflection =  $v(x)$

$$\text{Slope} = \frac{dv}{dx} = \theta(x)$$

$$\text{Moment, } M(x) = EI \frac{d^2v}{dx^2}$$

$$\text{Shear, } V(x) = \frac{dM}{dx} = EI \frac{d^3v}{dx^3}$$

$$\text{Load, } w(x) = \frac{dV}{dx} = EI \frac{d^4v}{dx^4}$$

# Appendix

## Greek Characters

Symbol	Name
$\alpha$	Alpha
$\beta$	Beta
$\chi$	Chi
$\Gamma \gamma$	Gamma
$\Delta \delta$	Delta
$\epsilon$	Epsilon
$\epsilon_0$	Epsilon Nought
$\zeta$	Zeta
$\eta$	Eta
$\Theta \theta$	Theta
$\kappa$	Kappa
$\Lambda \lambda$	Lambda
$\mu$	Mu
$\mu_0$	Mu Nought
$\nu$	Nu
$\Xi \xi$	Xi
$\Pi \pi$	Pi
$\rho$	Rho
$\Sigma \sigma$	Sigma
$\tau$	Tau
$\Phi \phi$	Phi
$\Psi \psi$	Psi
$\Omega \omega$	Omega

## SI Base Units

Name	Symbol	Measure	Dim. Analysis Symbol
Second	s	Time	T
Meter	m	Length	L
Kilogram	kg	Mass	M
Ampere	A	Electric Current	I
Kelvin	K	Temp	$\Theta$
Mole	mol	Amount of substance	N
Candela	cd	Luminous Intensity	J

## SI Prefixes

Prefix	Symbol	Factor	Meaning
Pico	p	$10^{-12}$	Trillionth
Nano	n	$10^{-9}$	Billionth
Micro	$\mu$	$10^{-6}$	Millionth
Milli	m	$10^{-3}$	Thousandth
Centi	c	$10^{-2}$	Hundredth
Deci	d	$10^{-1}$	Tenth
Kilo	K	$10^3$	Thousand
Mega	M	$10^6$	Million
Giga	G	$10^9$	Billion
Tera	T	$10^{12}$	Trillion

## Constants

### Gravitational constant

$$G = 6.67430 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$$

### Earth topics

$$m_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$$

$$r_{\text{Earth}} = 6.38 \times 10^6 \text{ m}$$

### Gravity on earth

$$g = 9.81 \text{ m/s}^2 \text{ or } 32.17 \text{ ft/s}^2$$

### Atmospheric pressure

$$1 \text{ atm} = 101325 \text{ pa} = 760.00 \text{ mmHg}$$

### Avogadro constant

$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

### Gas constant

$$R = 8.31 \text{ J/(mol} \cdot \text{K)}$$

### Boltzmann constant

$$k_b = 1.38 \times 10^{-23} \text{ J/K}$$

### Speed of sound

$$v_s = 343 \text{ m/s}$$

→ When on earth at 20° C or 68° F

### Reference sound intensity

$$I_0 = 10^{-12} \text{ W/m}^2$$

→ Where  $I_0$  is the lowest sound intensity able to be heard by an undamaged human ear (in room conditions)

### Elementary charge

$$e = 1.602 \times 10^{-19} \text{ C}$$

→ This could be the charge of a single proton, or the magnitude of a single electron

### Coulomb constant

$$k_e = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} = \frac{1}{4\pi\epsilon_0}$$

### Vacuum permittivity

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F} \cdot \text{m}^{-1}$$



Permeability of free space

$$4\pi \cdot 10^{-7} \frac{Tm}{A}$$

Mass of a proton

$$m_{proton} = 1.672 \times 10^{-27} \text{ kg} = 938.27 \text{ MeV}/c^2$$

Mass of an electron

$$m_{electron} = 9.11 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV}/c^2$$

Speed of light (vacuum)

$$c = 2.998 \times 10^8 \text{ m/s}$$

Planck constant

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

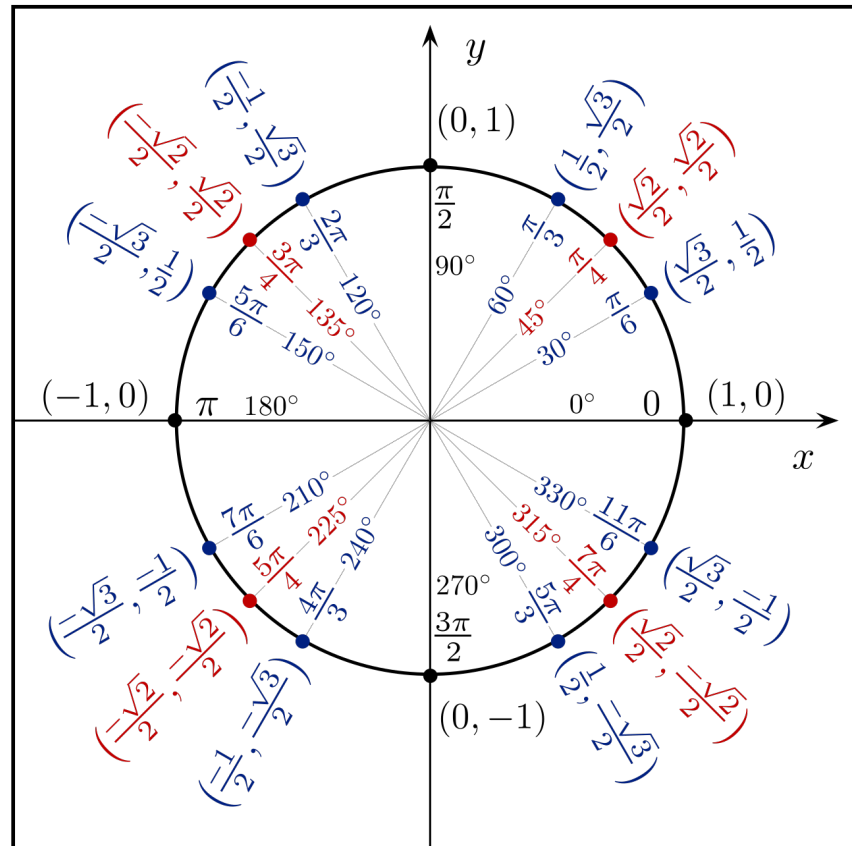
$$h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$\hbar = \frac{h}{2\pi}$$

Bohr Radius

$$a_b = 0.0529 \text{ nm}$$

## Unit Circle



[https://en.wikipedia.org/wiki/Unit\\_circle](https://en.wikipedia.org/wiki/Unit_circle)

